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QUESTION PAPER CODE: X10657

B.E. / B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Third Semester
Computer Science and Engineering
MA8351 –DISCRETE MATHEMATICS
(Common to Information Technology)
(Regulations 2017)
Answer ALL Questions

Time: 3 Hours **Maximum Marks:100**

PART-A

(10×2=20 Marks)

1. Show that $\{\neg, \wedge\}$ is a functionally complete set of connectives.

2. Write the negation of the statement $\forall(x^2 > x) \wedge \exists x(x^2 = 4)$.

3. Using the principle of mathematical induction, show that $1 + 3 + 5 + \ldots + (2n - 1) = n^2$, $\forall \quad n \geq 1$.

4. In how many ways a foot ball team of eleven players can be chosen out of 17 players, when
   (i) five particular players are to be always included.
   (ii) two particular players are to be always exclued.

5. Obtain the adjacency matrix of the complement of the graph $K_{1,4}$.

6. Check whether the complete bipartite graph $K_{3,3}$ is Hamiltonian or Eulerian.

7. In a group $(G, *)$, show that $(a * b)^{-1} = b^{-1} * a^{-1}$, $\forall\, a, b \in G$.

8. Show that if every element of group is self-inverse then it must be abelian.

9. Show that in a partially ordered set $(A, \leq)$, if greatest lower bound of a subset $S \subseteq A$ exists, then it must be unique.

10. In a lattice $(L, \leq)$, show that $a \leq b$, if and only if $a * b = a$.

PART-B

(5×16=80 Marks)

11. (a) (i) Use the indirect method to show that

$$R \to \neg Q, \quad R \cup S, \quad S \to \neg Q, \quad P \to Q \implies \neg P$$

(8)

(ii) Show that the premises "A student in the class has not read the book" and "Every one in this class passed the semester exam" imply the conclusion "Some one who passed the semester exam "has not read the book". (8)

(OR)

(b) (i) Using indirect method, prove the following statements.

(A) If n is an integer and $3n+2$ is odd, then n is odd. (4

(B) If $n = ab$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$. (4

(ii) Construct an argument to show that the following premises to show that the following premises imply the conclusion "It rained". "If it does not rain or if there is no traffic dislocation, then the soports day will be held and the cultural programme will go on"; "If the sports day is held, the trophy will be awarded" and "The trophy was not awarded". (8)

12. (a) (i) Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$, where $n \geq 2$ and $a_0 = 1$, $a_1 = 2$. (10)

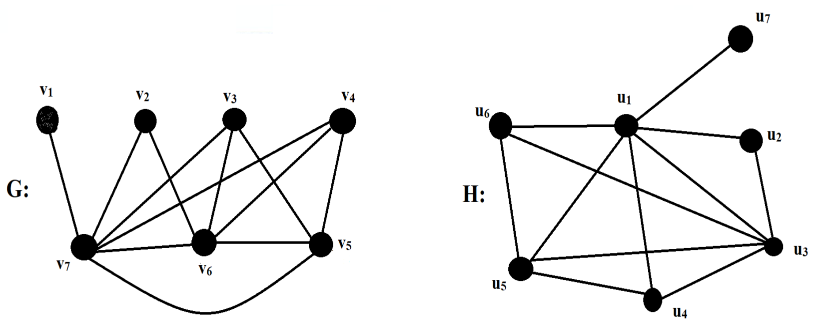
(ii) Prove that every positive integer $n \geq 2$ is either a prime or it is a product of primes. (6)

(OR)

(b) (i) Determine the number of positive integers n, $1 \leq n \leq 2000$ that are not divisible by 2, 3, or 5 but are divisible by 7. (10)

(ii) An odd positive integer n such that m denotes $2^n - 1$. (6)

13. (a) (i) State the necessary condition for two graphs to be isomorphic. Show that the following two graphs are isomorphic. (10)



(ii) State and prove Hand-Shake lemma for graphs. (6)

(OR)

(b) (i) When do we say a graph is self-complementary. If a graph G is self-complementary then prove that $|V(G)| \equiv 0 \;\; 1 \pmod{4}$ (6)

(ii) Let G be a graph with $S(G) \geq \dfrac{|V(G)|}{2}$ and $|V(G)| \geq 3$. Then prove that G is Hamiltonian. (6)

14. (a) (i) IF $(G, *)$ is a finite group, then prove that order of any subgroup divides the order of the group. (10)

(ii) Prove that group homomorphism preserves identity and inverse. (6)

(OR)

(b) (i) Obtain the composition table of $(S_3, \Diamond)$ and show that $(S_3, \Diamond)$ is a group/ Check whether $(S_3, \Diamond)$ is abelian. Justify your answer. (10) (8)

(ii) Show that in a cycle group every subgroup is a normal subgroup. (6)

15. (a) (i) Lat (A, R) be a partially ordered set. Then show that $A, R^{-1})$ is also partially set, where R_{-1} is defined as $R^{1-} = \{(a, b) \in A \times A/(b, a) \in R\}$. (6)

(ii) Show that in a lattice "isotone property" and "distributive inequalities" are true. (10)

(OR)

(b) (i) Show that in a distributive lattice cancellation law is true. Hence, show that in a distributive lattice if compliment of an element exists then it must be unique. (6)

(ii) Show that the complemented and distributive lattice, the following are true.

$$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$$

(10)

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