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Question Paper Code : X 10655

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020/
APRIL/MAY 2021
Second Semester

MA8251 : ENGINEERING MATHEMATICS – II
[Common to all (Except Marine Engineering)]
(Regulations 2017)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Given that α, β are the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, form the matrix whose eigenvalues are α^2, β^2 .
2. If the canonical form in the three variables u, v, w is given by $3v^2 + 15w^2$ corresponding to a quadratic form, then state the nature, index, signature and rank of the quadratic form.
3. Check whether the vector
$$\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$$
is solenoidal or not.
4. State Green's theorem in a plane.
5. State the polar form of the Cauchy Riemann equations.
6. Find the invariant points of the mapping $w = \frac{z-i}{1-iz}$.
7. State the Taylor series representation of an analytic function $f(z)$ about $z = a$.
8. State the nature of the singularity of $f(z) = z \cos\left(\frac{1}{z}\right)$.
9. Using Laplace transform of derivatives, find the Laplace transform of $\cos^2 t$.
10. Given $L\{f(t)\} = \frac{1}{s(s+1)(s+2)}$, find $\lim_{t \rightarrow 0} f(t)$.

PART – B

(5×16=80 Marks)

11. a) i) The eigenvectors of a real symmetric matrix A corresponding to the eigenvalues 2, 3, 6 are respectively $(1, 0, -1)^T$, $(1, 1, 1)^T$ and $(-1, 2, -1)^T$. Find the matrix A. (8)

ii) Show that A satisfies its own characteristic equation and hence find A^8 if $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$. (8)

(OR)

b) i) Using Cayley-Hamilton theorem, find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$. (8)

ii) Reduce the quadratic form $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$ into a canonical form using an orthogonal transformation. (8)

12. a) i) Find the angle between the normals to the surface $xy = z^2$ at the points $(-2, -2, 2)$ and $(1, 9, -3)$. (6)

ii) Verify Stokes' theorem for $\vec{F} = xy\hat{i} - 2yz\hat{j} - zx\hat{k}$ where S is the open surface of the rectangular parallelepiped formed by the planes $x = 0$, $x = 1$, $y = 0$, $y = 2$ and $z = 3$ above the xoy-plane. (10)

(OR)

b) i) Find the values of a, b, c so that $\vec{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. For these values of a, b, c, find also the scalar potential of \vec{F} . (8)

ii) Using Gauss' divergence theorem, evaluate $\iiint_S \vec{F} \cdot \hat{n} \, dS$ where $\vec{F} = y\hat{i} + x\hat{j} + z^2\hat{k}$ and S is the surface of the cylindrical region bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = b$. (8)

13. a) i) Show that $u = e^x \cos y$ is harmonic. Find the analytic function $w = u + iv = f(z)$ using Milne-Thompson method and hence find the conjugate harmonic function v. (10)

ii) Given $w = u + iv = z^3$, verify that the families of curves $u = C_1$ and $v = C_2$ cut orthogonally. (6)

(OR)

b) i) If $f(z)$ is an analytic function, then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(10)**

ii) Find the image of the triangular region in the z -plane bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ under the transformation $w = e^{i\pi/4}z$ **(6)**

14. a) i) If $f(z) = \oint_C \frac{3z^2 + 7z + 1}{z - a} dz$, C is the circle $|z| = 2$, then find the values of $f(3)$, $f'(1 + i)$ and $f''(1 - i)$. **(8)**

ii) Using Laurent's series expansion, find the residue of $f(z) = \frac{z^2}{(z - 1)(z + 2)^2}$ at its simple pole. **(8)**

(OR)

b) Using contour integral, evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)^2 (x^2 + 2x + 2)}$. **(16)**

15. a) i) Using Laplace transform, evaluate $\int_0^{\infty} \left(\frac{\cos at - \cos bt}{t}\right) dt$. **(8)**

ii) Using convolution theorem, find $L^{-1}\left(\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right)$. **(8)**

(OR)

b) i) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} t & 0 \leq t \leq a \\ 2a - t, & a \leq t \leq 2a \end{cases} \quad \text{with period } 2a. \quad \mathbf{(8)}$$

ii) Using Laplace transform, solve $(D^2 + 4D + 13)y = e^{-t} \sin t$ given $y = Dy = 0$ at $t = 0$, $D \equiv \frac{d}{dt}$. **(8)**
