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## Question Paper Code : X10605

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2021 Sixth Semester Instrumentation and Control Engineering IC 8651 – ADVANCED CONTROL SYSTEM (Common to Electrical and Electronics Engineering) (Regulations 2017)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions

PART – A

(10×2=20 Marks)

- 1. A system is characterized by differential equation  $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 7y u = 0$ . Determine its state space model.
- 2. The state-space model of a system is given by X(t) = AX(t) + bu(t) y(t) = CX(t)

where A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 10 \end{bmatrix}$ , b =  $\begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$ , c =  $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ 

Find the transfer function.

- 3. Write the state space equations in the controller canonical forms for the following systems described by the transfer functions  $G_1(s) = \frac{s+1}{s^2+5s+5}$ ,  $G_2 = \frac{s+1}{4s^2+4s+1}$ .
- 4. Write the state space equations in the observer canonical forms for the following systems described by the transfer functions  $G_1(s) = \frac{s+1}{s^2+5s+5}$ ,  $G_2 = \frac{s+1}{4s^2+4s+1}$ .
- 5. Calculate the z-transform for the unit impulse sequence.
- 6. Determine the inverse z-transform  $F(z) = \frac{1}{1 1.5z^{-1} + 0.5z^{-2}}$ .

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7. Draw the phase trajectory of a 1<sup>st</sup> order nonlinear system represented by a differential equation  $\dot{y} + 2y - 2y^3 = 0$ .

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8. For a nonlinear autonomous system the state equations are

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 &= -\mathbf{x}_1 - \mathbf{x}_1^2 - \mathbf{x}_2 \end{aligned}$$

Determine its equilibrium states.

- 9. What are the performance indices for minimum-time optimal control, minimum energy optimal control problem, state regulator optimal control problem and output regulator problem ?
- 10. What is meant by quadratic optimal control problem ? Give its performance index.

(5×13=65 Marks)

11. a) Find the states  $x_1(t)$ ,  $x_2(t)$  and  $x_3(t)$  of the system described by

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \\ \dot{\mathbf{x}}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix}$$

where the initial conditions are :

$$\begin{bmatrix} \mathbf{x}_1(0) \\ \mathbf{x}_2(0) \\ \mathbf{x}_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Compute the state transition matrix using any two approaches. Compute the eigenvalues of the system.

(OR)

b) Obtain the response y(t) of the following system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 0.5 \\ 1 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} \mathbf{u}(t) \ \mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

 $y(t) = [1 \ 0] x(t)$  where u(t) is the unit-step input. Write the MATLAB command to plot the unit-step response.

12. a) A regulator system has the plant  $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}; \mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}.$ 

Design a state-feedback controller which will place the closed-loop poles at  $-2\pm j3.464,-5$  using

- i) Direct comparison of coefficients
- ii) Ackermann's formulae. Give a block diagram of the control configuration.

b) Consider the second-order system  $\dot{X}(t) = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$  and  $Y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} X(t)$ .

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The desired characteristic equation of the observer is given by  $\phi(s) = s^2 + 2\xi\omega_n s + \omega_n^2$ where  $\xi = 0.8$  and  $\omega_n = 10$ . Design a full-state observer such that the desired performance specifications for the observer are satisfied using

- i) Comparison of coefficients methods
- ii) Ackermann's formula
- 13. a) Consider a discrete time system  $y(k+1) = \frac{1}{4}y(k+1) \frac{1}{8}y(k) = 3r(k+1) r(k)$ with input  $r(k) = (-1)^k u(k)$  and initial conditions y(-1) = 5, y(-2) = 6. Find the output, y(k), for  $k \ge 0$ .

(OR)

- b) A plant is described by the transfer function  $\frac{Y(s)}{U(s)} = \frac{5}{s(s+5)}$  and the systems input and output are sampled with a sampling interval T = 0.1 second.
  - 1) Obtain the z transfer function between the input and the output.
  - 2) Obtain the difference equation relating y(k) and u(k).
- 14. a) Derive the describing function for a nonlinear element with dead-zone in its input-output characteristics.

(OR)

b) Obtain the describing function of the following ON-OFF nonlinearity.



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15. a) Consider the system described by the state model:

$$\begin{split} \dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 20 \end{bmatrix} \mathbf{u}; \ \mathbf{y} = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}.\\ \text{Find the optimal control law that minimizes } \mathbf{J} &= \frac{1}{2} \int_{0}^{\infty} \begin{bmatrix} (\mathbf{y}(t) - 1)^2 + \mathbf{u}^2 \end{bmatrix} dt.\\ \text{(OR)} \end{split}$$

b) Consider a first order dynamic system  $\dot{x} = x + u$ . The control input is designed as u = -kx such that the system is stable. Evaluate the performance index  $J = \int_0^\infty x^2 dt$  with x(0) = 2 and hence obtain an optimal value of k such that J is minimum.

- 16. a) Given a continuous time plant as  $G(s) = \frac{1}{s(s+1)}$ 
  - i) Use a Zero Order Hold device prior to this plant and obtain a discretized plant transfer function considering sampling time 1 sec. (3)
  - ii) Consider a step input (discrete-time) obtain step response of the closed-loop system.
    (4)
  - iii) Apply final-value theorem to obtain the steady-state error. (4)
  - iv) Obtain the velocity error constants of the continuous-time and discrete-time plants. (4)

(OR)

b) Investigate the stability of a system with ON-OFF controller shown in below figure using describing function analysis.



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