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**Question Paper Code : X 10655**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020/  
APRIL/MAY 2021  
Second Semester

MA8251 : ENGINEERING MATHEMATICS – II  
[Common to all (Except Marine Engineering)]  
(Regulations 2017)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Given that  $\alpha, \beta$  are the eigenvalues of the matrix  $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ , form the matrix whose eigenvalues are  $\alpha^2, \beta^2$ .
2. If the canonical form in the three variables  $u, v, w$  is given by  $3v^2 + 15w^2$  corresponding to a quadratic form, then state the nature, index, signature and rank of the quadratic form.
3. Check whether the vector  $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$  is solenoidal or not.
4. State Green's theorem in a plane.
5. State the polar form of the Cauchy Riemann equations.
6. Find the invariant points of the mapping  $w = \frac{z - i}{1 - iz}$ .
7. State the Taylor series representation of an analytic function  $f(z)$  about  $z = a$ .
8. State the nature of the singularity of  $f(z) = z \cos\left(\frac{1}{z}\right)$ .
9. Using Laplace transform of derivatives, find the Laplace transform of  $\cos^2 t$ .
10. Given  $L\{f(t)\} = \frac{1}{s(s+1)(s+2)}$ , find  $\lim_{t \rightarrow 0} f(t)$ .

PART – B

**(5×16=80 Marks)**

11. a) i) The eigenvectors of a real symmetric matrix A corresponding to the eigenvalues 2, 3, 6 are respectively  $(1, 0, -1)^T$ ,  $(1, 1, 1)^T$  and  $(-1, 2, -1)^T$ . Find the matrix A. (8)

ii) Show that A satisfies its own characteristic equation and hence find  $A^8$  if  $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ . (8)

(OR)

b) i) Using Cayley-Hamilton theorem, find the inverse of the matrix  $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ . (8)

ii) Reduce the quadratic form  $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$  into a canonical form using an orthogonal transformation. (8)

12. a) i) Find the angle between the normals to the surface  $xy = z^2$  at the points  $(-2, -2, 2)$  and  $(1, 9, -3)$ . (6)

ii) Verify Stokes' theorem for  $\vec{F} = xy\hat{i} - 2yz\hat{j} - zx\hat{k}$  where S is the open surface of the rectangular parallelepiped formed by the planes  $x = 0$ ,  $x = 1$ ,  $y = 0$ ,  $y = 2$  and  $z = 3$  above the xoy-plane. (10)

(OR)

b) i) Find the values of a, b, c so that  $\vec{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$  is irrotational. For these values of a, b, c, find also the scalar potential of  $\vec{F}$ . (8)

ii) Using Gauss' divergence theorem, evaluate  $\iiint_S \vec{F} \cdot \hat{n} \, dS$  where  $\vec{F} = y\hat{i} + x\hat{j} + z^2\hat{k}$  and S is the surface of the cylindrical region bounded by  $x^2 + y^2 = a^2$ ,  $z = 0$  and  $z = b$ . (8)

13. a) i) Show that  $u = e^x \cos y$  is harmonic. Find the analytic function  $w = u + iv = f(z)$  using Milne-Thompson method and hence find the conjugate harmonic function v. (10)

ii) Given  $w = u + iv = z^3$ , verify that the families of curves  $u = C_1$  and  $v = C_2$  cut orthogonally. (6)

(OR)

b) i) If  $f(z)$  is an analytic function, then prove that  $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ . **(10)**

ii) Find the image of the triangular region in the  $z$ -plane bounded by the lines  $x = 0$ ,  $y = 0$  and  $x + y = 1$  under the transformation  $w = e^{i\pi/4}z$  **(6)**

14. a) i) If  $f(z) = \oint_C \frac{3z^2 + 7z + 1}{z - a} dz$ ,  $C$  is the circle  $|z| = 2$ , then find the values of  $f(3)$ ,  $f'(1 + i)$  and  $f''(1 - i)$ . **(8)**

ii) Using Laurent's series expansion, find the residue of  $f(z) = \frac{z^2}{(z - 1)(z + 2)^2}$  at its simple pole. **(8)**

(OR)

b) Using contour integral, evaluate  $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)^2 (x^2 + 2x + 2)}$ . **(16)**

15. a) i) Using Laplace transform, evaluate  $\int_0^{\infty} \left(\frac{\cos at - \cos bt}{t}\right) dt$ . **(8)**

ii) Using convolution theorem, find  $L^{-1}\left(\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right)$ . **(8)**

(OR)

b) i) Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} t & 0 \leq t \leq a \\ 2a - t, & a \leq t \leq 2a \end{cases} \quad \text{with period } 2a. \quad \mathbf{(8)}$$

ii) Using Laplace transform, solve  $(D^2 + 4D + 13)y = e^{-t} \sin t$  given  $y = Dy = 0$  at  $t = 0$ ,  $D \equiv \frac{d}{dt}$ . **(8)**

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