I. TRIGONOMETRY

i. Ratios:

$\sin\theta, \cos\theta, \tan\theta$

$cosec\,\theta, \sec\theta, \cot\theta$

$$cosec\,\theta = \frac{1}{sin\theta}$$

$$\sec\theta = \frac{1}{cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

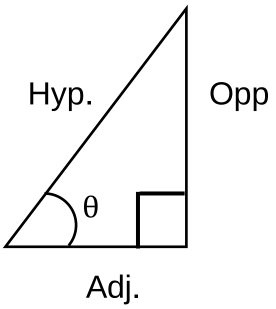
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin\theta = \frac{opp}{Hyp}$$

$$\cos\theta = \frac{Adj}{Hyp}$$

$$\tan\theta = \frac{Opp}{Adj}$$

$$cosec\,\theta = \frac{Hyp}{Opp}$$



Identities:

1. $sin^2\theta + cos^2\theta = 1$

   $\Rightarrow sin^2\theta = 1 - cos^2\theta$

   $\Rightarrow cos^2\theta = 1 - sin^2\theta$

2. $sec^2\theta - tan^2\theta = 1$

   $\Rightarrow sec^2\theta = 1 + tan^2\theta$

   $\Rightarrow tan^2\theta = sec^2\theta - 1$

3. $cosec^2\theta - cot^2\theta = 1$

$\Rightarrow cosec^2\theta = 1 + cot^2\theta$

$\Rightarrow cot^2\theta = cosec^2\theta - 1$

Trigonometric values

θ	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	
	$0°$	$30°$	$45°$	$60°$	$90°$	
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\sqrt{}$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\leftarrow$
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$\div$

Complementary Angles

1. $\sin(90-\theta) = \cos\theta$
2. $\cos(90-\theta) = \sin\theta$
3. $\tan(90-\theta) = cot\theta$

Example:

i. $\sin 60° = \sin(90-30) = \cos 30$
ii. $\cos 45° = \cos(90-45) = \sin 45$
iii. $\sin 10° = \sin(90-10) = \cos 80$
iv. $\sin(360-\theta) = \sin(-\theta) = -sin\theta$
v. $\cos(360-\theta) = \cos(-\theta) = cos\theta$

vi. $\tan(360-\theta)=\tan(-\theta)\ =-tan\theta$

II	I
$silver$	All
$90+\theta$	$90-\theta$
$180-\theta$	$360+\theta$
III	IV
Tea	cup
$180+\theta$	$270+\theta$
$270-\theta$	$360-\theta$

COMPOUND ANGLES:

1. $\sin(A+B)=\sin A\cos B+\cos A\sin B$
2. $\sin(A-B)=\sin A\cos B-\cos A\sin B$
3. $\cos(A+B)=\cos A\cos B-\sin A\sin B$
4. $\cos(A-B)=\cos A\cos B+\sin A\sin B$
5. In '1' put B=A
   - $\sin 2A=\sin A\cos A+\cos A\sin A$

     $=2\sin A\cos A$

     $\sin 2A=2\sin A\cos A$

     Here $A\sim\frac{A}{2}$
   - $\sin A=2sin\frac{A}{2}\ cos\frac{A}{2}$

6. In '3' put B=A

- $\cos(A + A) = cos^2A - sin^2A$

$\cos 2A = cos^2A - sin^2 A$

$= (1 - sin^2A) - sin^2A$

$\cos 2A = 1 - 2sin^2 A$

$2sin^2A = 1 - \cos 2A$

$sin^2A = \frac{1-\cos 2A}{2}$

$A \sim \frac{A}{2}$

$sin^2\frac{A}{2} = \frac{1-cosA}{2}$

$= 1 - \cos A = 2sin^2\frac{A}{2}$

Similarly, $\cos 2A = cos^2 A - (1 - cos^2A)$

$\cos 2A = 2cos^2 A - 1$

$1 + \cos 2A = 2cos^2A$

$cos^2A = \frac{1 + \cos 2A}{2}$

$A \sim \frac{A}{2}$

$cos^2\frac{A}{2} = \frac{1 + \cos A}{2}$

$1 + \cos A = 2cos^2\frac{A}{2}$

7. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan AtanB}$

8. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

9. In 7 put B=A

- $\tan(A+A) = \frac{\tan A+\tan A}{1-\tan A \tan A}$
- $\tan 2A = \frac{2 \tan A}{1-tan^2 A}$

10. $\sin 3A = 3 \sin A - 4sin^3 A$

- $4sin^3 A = 3sinA - sin3A$
- $sin^3 A = \frac{1}{4}(3sinA - sin3A)$

11. $\cos 3A = 4cos^3 A - 3 \cos A$

- $3 \cos A + \cos 3A = 4cos^3 A$
- $cos^3 A = \frac{1}{4}(3 \cos A + \cos 3A)$
- $\tan 3\theta = \frac{3tan\theta - tan^3\theta}{1-3tan^2\theta}$

III Expression of sum or difference into product

1. $\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$
2. $\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$
3. $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$
4. $\cos A - cosB = -2\sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$
5. $sin0 = sin\pi = \sin 2\pi = \cdots = 0; \sin n\pi = 0, n\epsilon z$
6. $sin\frac{\pi}{2} = sin\frac{5\pi}{2} = sin\frac{9\pi}{2} = \cdots = 1$
7. $sin\frac{3\pi}{2} = sin\frac{7\pi}{2} = sin\frac{11\pi}{2} = \cdots - 1$
8. $\cos 0 = \cos 2\pi = cos4\pi = \cdots 1$

9. $\cos\pi = \cos 2\pi = \cos 5\pi = \cdots - 1$

- $\cos n\pi = (-)^{n}, n\epsilon z$
- $\cos(n+1)\pi = (-1)^{n+1}, n\epsilon z$
- $\cos 2n\pi = (-1)^{2n} = ((-1)^{2})^{n} = 1^{n} = 1.., n\epsilon z$

10. $cos\frac{\pi}{2} = cos\frac{3\pi}{2} = \frac{cos5\pi}{2} = \cdots = 0$

$\cos(2n-1)\pi = 0, n\epsilon z$

1. <u>Hyperbolic Functions:</u>

We have already said 'e' whose value is approximately 2.7 is called the exponential constant. Further, if $log_{e}y = x$ then $y = e^{x}$ is called as the exponential function. Hyperbolic functions are defined interm of exponential function as below.

- $sinh\,x = \frac{e^{x}-e^{-x}}{2}$; $cosh\,x = \frac{e^{x}+e^{-x}}{2}$
- $\tanh x = \frac{sinh\,x}{\cosh x}$; $\coth x = \frac{1}{\tanh x} = \frac{\cosh x}{\sinh x}$
- $\operatorname{sech} x = \frac{1}{\cosh x}$; $cosech\,x = \frac{1}{\sinh x}$
- $\frac{d}{dx}(\sinh x) = \cosh x$
- $\frac{d}{dx}(\cosh x) = \sinh x$
- $\cos i\,x = \cosh x$
- $\sin ix = \mathrm{i} \sinh x$

2. Important Hyperbolic Identities:

(i) $cosh^2x - sinh^2x = 1$

(ii) $1 - tanh^2x = sech^2x$

(iii) $coth^2x - 1 = cosech^2x$

(iv) $cosh^2x + sinh^2x = cosh2x$

(v) $2sinhx\ coshx = sinh2x$