3.2 IMPEDANCE MATCHING BY STUBS:

In microwave and radio-frequency engineering, a stub is a length of transmission line or waveguide that is connected at one end only. The free end of the stub is either left open-circuit or (especially in the case of waveguides) short-circuited. Neglecting transmission line losses, the input impedance of the stub is purely reactive; either capacitive or inductive, depending on the electrical length of the stub, and on whether it is open or short circuit. In Fig 3.2.1 Stubs may thus be considered to be frequency-dependent capacitors and frequency-dependent inductors.

Because stubs take on reactive properties as a function of their electrical length, stubs are most common in UHF or microwave circuits where the line lengths are more manageable. Stubs are commonly used in antenna impedance matching circuits and frequency selective filters.

Smith charts can also be used to determine what length line to use to obtain a desired reactance.

Stub matching

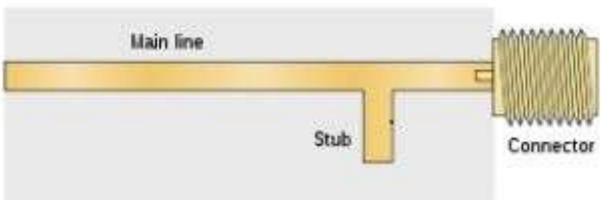


Fig: 3.2.1 Stub matching

Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

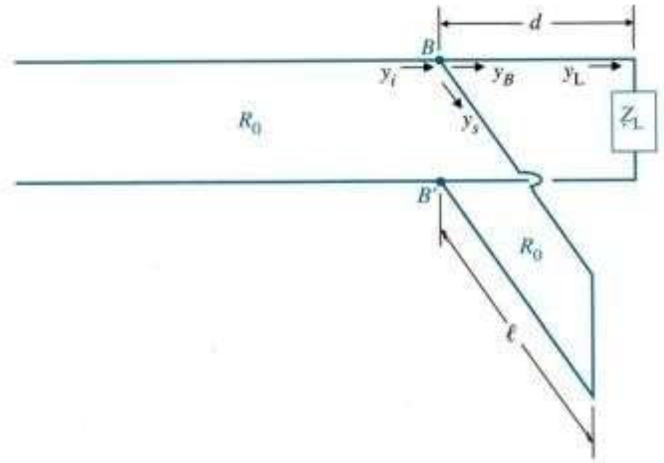
In a strip line circuit, a stub may be placed just before an output connector to compensate for small mismatches due to the device's output load or the connector itself.

Stubs can be used to match a load impedance to the transmission line characteristic impedance. The stub is positioned a distance from the load. This distance is chosen so that at that point the resistive part of the load impedance is made equal to the resistive part of the characteristic impedance by impedance transformer action of the length of the main line. The length of the stub is chosen so that it exactly cancels the reactive part of the presented impedance. That is, the stub is made capacitive or inductive according to whether the main line is presenting an inductive or capacitive impedance respectively. This is not the same as the actual impedance of the load since the reactive part of the load impedance will be subject to impedance transformer action as well as the resistive part.

In the method of impendence matching using stub, an open or closed stub line of suitable length is used as a reactance shunted across the transmission line at a designated distance from the load, to tune the length of the line and the load to resonance with an anti-resonant resistance equal to Ro. Matching stubs can be made adjustable so that matching can be corrected on test.

Single stub will only achieve a perfect match at one specific frequency. For wideband matching several stubs may be used spaced along the main transmission line. The resulting structure is filter-like and filter design techniques are applied. For instance, the matching network may be designed as a Chewy she filter but is optimized for impedance matching instead of pass band transmission. The resulting transmission function of the network has a pass band ripple like the Cheese filter, but the ripples never reach 0dB insertion loss at any point in the pass band, as they would do for the standard filter.

SINGLE STUB MATCHING:



Impedance matching by single-stub method.

Fig: 3.2.2 Location of single stub for impedance matching

Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

The load should be matched to the characteristic impedance of the line so that as much power as possible is transmitted from the generator to the load for radio-frequency power transmission.

The lines should be matched because reflections from mismatched loads and junctions will result in echoes and will distort the information-carrying signal for information transmission.

Short-circuited (instead of open-circuited) stubs are used for impedance-matching on transmission lines is shown in Fig 3.2.2.

Single-stub method for impedance matching: an arbitrary load impedance can be matched to a transmission line by placing a single short-circuited stub in parallel with the line at a suitable location

When a high frequency line terminated in its characteristic impedance it is operated as a smooth line under such conditions the reflections are absent.

Hence we get maximum power delivered to the load efficiency.

But in practice load antennas does not produce resistance equal to R_O.

For example, we use quarter wave line for impedance matching technique.

The input impedance Y is looking towards the load from any point on the transmission line is given by,

$Y_S = G_O + jB$ ……. (A)

G_O - Conductance

Y_S – Susie Trance

These may be the admittance at point A before the stub connected.

The point A located such that at which $G_O = \frac{1}{R_O}$

Then at point A short stub line is connected and this line is selected such that its input susceptance is $\mp$ **jobs**

This stub is connected across the transmission line. The total admittance and I/p can be written as,

$Y_S = G_O \pm jB \mp jB$

$Y_S = G_O = \frac{1}{R_O}$

$Z_S = \frac{1}{Y_S}$

$R_O = \frac{1}{G_O}$

The input impedance is given by,

$$z_{in} = z_S = R_O \left[\frac{1+|K| \angle \emptyset-2\beta s}{1-|K| \angle \emptyset-2\beta s}\right] \quad \ldots\ldots$$

The I/p admittance is given by,

$$Y_S = \frac{1}{z_S} = \frac{1}{R_O}\left[\frac{1-|K| \angle \emptyset-2\beta s}{1+|K| \angle \emptyset-2\beta s}\right] \quad \ldots\ldots(2)$$

$$Y_S = G_O\left[\frac{1-K\cos(\emptyset-2\beta s)-j\,k\sin(\emptyset-2\beta s)}{1+K\cos(\emptyset-2\beta s)+j\,k\sin(\emptyset-2\beta s)}\right]$$

$$Y_S = G_O\left[\frac{[1-K\cos(\emptyset-2\beta s)]-j\,k\sin(\emptyset-2\beta s)}{[1+K\cos(\emptyset-2\beta s)]+j\,k\sin(\emptyset-2\beta s)}\right]$$

$$Y_S = G_O\left[\frac{[1-K\cos(\emptyset-2\beta s)]-j\,k\sin(\emptyset-2\beta s)}{[1+K\cos(\emptyset-2\beta s)]+j\,k\sin(\emptyset-2\beta s)}\right] \times \left[\frac{[1+K\cos(\emptyset-2\beta s)]-j\,k\sin(\emptyset-2\beta s)}{[1+K\cos(\emptyset-2\beta s)]-j\,k\sin(\emptyset-2\beta s)}\right]$$

$$Y_s = G_O \left[\frac{\cdots}{[1+K\cos(\emptyset-2\beta s)]^2+[k\sin(\emptyset-2\beta s)]^2}\right]$$

$$\ldots\ldots = [1+K\cos(\emptyset-2\beta s)-j\,k\sin(\emptyset-2\beta s)-K\cos(\emptyset-2\beta s)-$$
$$K^2cos^2(\emptyset-2\beta s)+jK^2\cos(\emptyset-2\beta s)\sin(\emptyset-2\beta s)-j\,k\sin(\emptyset-2\beta s)-j$$
$$K^2\cos(\emptyset-2\beta s)\sin(\emptyset-2\beta s)-K^2sin^2(\emptyset-2\beta s)]$$

$$Y_s = G_O \left[\frac{1-j2\,k\sin(\emptyset-2\beta s)-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2cos^2(\emptyset-2\beta s)+K^2sin^2(\emptyset-2\beta s)}\right]$$

$$Y_s = G_O \frac{[1-K^2-j2\,k\sin(\emptyset-2\beta s)]}{[1+2K\cos(\emptyset-2\beta s)+K^2]}$$

We know that,

$$\boldsymbol{Y_S = G_S + j\,B_S}$$

$$\boldsymbol{G_S + j\,B_S} = G_O\left[\frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2} - \frac{j2\,k\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2}\right]$$

$$\frac{\boldsymbol{G_S + j\,B}}{G_O} = \left[\frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2} - \frac{j2\,k\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2}\right]$$

$$\frac{\boldsymbol{G_S}}{G_O} + j\frac{S}{G_O} = \left[\frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2} - \frac{j2\,k\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2}\right]$$

$$\frac{\boldsymbol{G_S}}{G_O} = \frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2} \qquad \ldots\ldots (3)$$

$$\frac{\boldsymbol{B}}{G_O} = \frac{-2\,k\sin(\emptyset-2\beta s)}{1+2K\cos(\emptyset-2\beta s)+K^2} \qquad \ldots\ldots (4)$$

From the values of $\frac{\boldsymbol{GS}}{G_O}$ And $\frac{\boldsymbol{BS}}{G_O}$ we can relate this value to the stub length and the

Point at which the stub is to be connected.

From the plot shown in figure it is absorbed that the value of $\frac{\boldsymbol{GS}}{G_O}$ Is maximum if

The value cosine term in the expression in negative i.e.

$\emptyset-2\beta s = -\pi$

$2\beta s_2 = \emptyset + \pi$

$$s_2 = \frac{\emptyset+\pi}{2\beta}$$

At distance s_2 The maximum value of $\frac{G_S}{G_O}$ Is,

$$\left(\frac{G_S}{G_O}\right)_{max} = \frac{1-K^2}{1+2K\cos(\emptyset-2\beta s)+K^2}$$

$$\left(\frac{G_S}{G_O}\right)_{max} = \frac{1-K^2}{1+2K\cos(-\pi)+K^2}$$

$$\left(\frac{G_S}{G_O}\right)_{max} = \frac{1-K^2}{1-2K+K^2}$$

$$\left(\frac{G_S}{G_O}\right)_{max} = \frac{1-K^2}{(1-K)^2}$$

$$\left(\frac{G_S}{G_O}\right)_{max} = \frac{(1-K)(1+K)}{(1-K)^2}$$

$$\left(\frac{G_S}{G_O}\right)_{max} = \frac{1+K}{1-K}$$

$$\left(\frac{G_S}{G_O}\right)_{max} = S$$

$$G_S = G_O\, S$$

$$\frac{1}{R_s} = \frac{1}{R_O} S$$

$$R_s = \frac{R_O}{S}$$

Thus at the point s_2 the point impedance R_s is resistive and its value is minimum voltage at distance s_2 from the load.

At distance s_1 from the load,

$$G_S = G_O$$

This is the point at which the stub is to be connected where the value of $\frac{G_S}{G_O}$ is Unity.

$$\frac{G_S}{G_O} = 1$$

From equal (3),

$$1 = \frac{1-K^2}{1+2K\cos(\emptyset-2\beta s_1)+K^2}$$

$$1 + 2K\cos(\emptyset - 2\beta s_1) + K^2 = 1 - K^2$$

$$2K\cos(\emptyset - 2\beta s_1) = 1 - K^2 - 1 - K^2$$

$2K\cos(\emptyset - 2\beta s_1) = -2K^2$

$\cos(\emptyset - 2\beta s_1) = \frac{-2K^2}{2K}$

$\cos(\emptyset - 2\beta s_1) = -k$

$\emptyset - 2\beta s_1 = cos^{-1}(-K)$

$\emptyset - 2\beta s_1 = -\pi \pm cos^{-1}(K)$

$\emptyset + \pi \pm cos^{-1}(K) = 2\beta s_1$

$s_1 = \frac{1}{2\beta}(\emptyset + \pi \pm cos^{-1}(K))$

$\beta = \frac{2\pi}{\lambda}$

Sub β value in s_1

$s_1 = \frac{\lambda}{4\pi}(\emptyset + \pi \pm cos^{-1}(K))$

The distance 'd' from the voltage minimum to the point of stub connection is given by,

d = s_2 - s_1

d = $\frac{\emptyset + \pi}{2\beta} - \frac{\emptyset + \pi \pm cos^{-1}(K)}{2\beta}$

d = $\frac{\emptyset + \pi - \emptyset - \pi \pm cos^{-1}(K)}{2\beta}$

d = $\frac{\pm cos^{-1}(K)}{2\beta}$

We know that,

K = $\frac{S-1}{S+1}$, $\beta = \frac{2\pi}{\lambda}$

Sub these values in above equ,

$$d = \frac{\pm cos^{-1}\left(\frac{S-1}{S+1}\right)}{\frac{4\pi}{\lambda}}$$

$$d = \left[\frac{\pm cos^{-1}\left(\frac{S-1}{S+1}\right)}{\pi}\left(\frac{\lambda}{4}\right)\right]$$

hence the stub mat be located at distance 'd' measured in either direction from voltage minimum but for better performance the stub is placed on the load side of the voltage minimum which is nearest to the load.

To calculate the input susceptance of the line at s distance 's_1' ,

From equ (4),

$$\frac{B}{G_0} = \frac{-2k\sin(\emptyset - 2\beta s_1)}{1 + 2K\cos(\emptyset - 2\beta s_1) + K^2}$$

We know that,

$$s_1 = \frac{1}{2\beta}(\emptyset + \pi \pm cos^{-1}(K))$$

$$\cos(\emptyset - 2\beta s_1) = \cos\left(\emptyset - 2\beta\,\frac{\emptyset + \pi \pm cos^{-1}(K)}{2\beta}\right)$$

$$= \cos(\emptyset - \emptyset - \pi \pm cos^{-1}(K))$$

$$= \cos(-\pi \pm cos^{-1}(K))$$

$$= \cos(cos^{-1}(-K))$$

$$\cos(\emptyset - 2\beta s_1) = -K$$

$$\sin(\emptyset - 2\beta s_1) = \sin\left(\emptyset - 2\beta\,\frac{\emptyset + \pi \pm cos^{-1}(K)}{2\beta}\right)$$

$$= \sin(\emptyset - \emptyset - \pi \pm cos^{-1}(K))$$

$$= \sin(-\pi \pm cos^{-1}(K))$$

$$= \sin(cos^{-1}(-K))$$

Let $cos^{-1}(-K) = \theta$

$-K = \cos\theta$

$\sin(\emptyset - 2\beta s_1) = \sin(cos^{-1}(\cos\theta))$ $\sin(\emptyset - 2\beta s_1) = \sin\theta$

[$in^2\theta + cos^2\theta = 1$

$sin^2\theta = 1 - cos^2\theta$

$\sin\theta = \pm\sqrt{1 - cos^2\theta}$]

$$= \pm\sqrt{1 - cos^2\theta}$$

$$\sin(\emptyset - 2\beta s_1) = \pm\sqrt{1 - K^2}$$

$$\frac{B_S}{G_O} = \pm\frac{2k\sqrt{1-K^2}}{1+2(-K)+K^2}$$

$$\frac{B_S}{G_O} = \pm\frac{2k\sqrt{1-K^2}}{1-2K^2+K^2}$$

$$\frac{B_S}{G_O} = \pm\frac{2k\sqrt{1-K^2}}{1-K^2}$$

$$\frac{B}{G_O} = \pm\frac{2k}{\sqrt{1-K^2}}$$

$$B_S = \pm G_O\left(\frac{2k}{\sqrt{1-K^2}}\right)$$

$$B_S = \pm\frac{2k\,G_O}{\sqrt{1-K^2}}$$

The above equation gives susceptance of the line with a distance‘s’ where the stub is to be connected.

So, this is given as,

$$B_{Stub} = \pm\frac{2k\,G_O}{\sqrt{1-K^2}}$$

In general, the input impedance of a short circuited line is given by,

$$z_{sc} = jR_0\tan\beta s$$

$$Y_{sc} = G + jB$$

$$\frac{1}{Z} = Y$$

$$Y_{sc} = \frac{1}{jR_0\tan\beta s}$$

$$G + jB = \frac{1}{jR_0\tan\beta s}$$

$$B = \frac{1}{R_0\tan\beta s}$$

The stub connected to the transmission line is also a short circuited line with a total length L.

$$B_{Stub} = \frac{1}{R_0\tan\beta L} = \pm\frac{2k\,G_O}{\sqrt{1-K^2}}$$

$$2k\,G_O\tan\beta L = \pm\sqrt{1-K^2}$$

$$2k\,G_O\frac{1}{G_O}\tan\beta L = \pm\sqrt{1-K^2}$$

$$\tan\beta L = \pm \frac{\sqrt{1-K^2}}{2k}$$

$$\beta L = \tan^{-1}\left(\pm \frac{\sqrt{1-K^2}}{2k}\right)$$

$$L = \frac{1}{\beta}\tan^{-1}\left(\pm \frac{\sqrt{1-K^2}}{2k}\right)$$

$$\beta = \frac{2\pi}{\lambda}$$

Sub β value in L,

$$L = \frac{\lambda}{2\pi}\tan^{-1}\left(\pm \frac{\sqrt{1-K^2}}{2k}\right)$$

We know that,

$$K = \frac{S-1}{S+1}$$

$$\frac{\sqrt{1-K^2}}{2k} = \frac{\sqrt{1-\left(\frac{S-1}{S+1}\right)^2}}{2\left(\frac{S-1}{S+1}\right)}$$

$$= \frac{\sqrt{\frac{(S+1)^2-(S-1)^2}{(S+1)^2}}}{2\left(\frac{S-1}{S+1}\right)}$$

$$= \frac{\sqrt{\frac{S^2+1+2S-S^2-1+2S}{(S+1)^2}}}{2\left(\frac{S-1}{S+1}\right)}$$

$$= \frac{\frac{\sqrt{4S}}{S+1}}{2\left(\frac{S-1}{S+1}\right)}$$

$$= \frac{\sqrt{4S}}{2(S-1)}$$

$$= \frac{2\sqrt{S}}{2(S-1)}$$

$$\frac{\sqrt{1-K^2}}{2k} = \frac{\sqrt{S}}{(S-1)}$$

$$L = \frac{\lambda}{2\pi}\tan^{-1}\left(\pm\frac{\sqrt{s}}{(s-1)}\right)$$

This is the length of the stub which is short circuited.

DOUBLE STUB MATCHING:

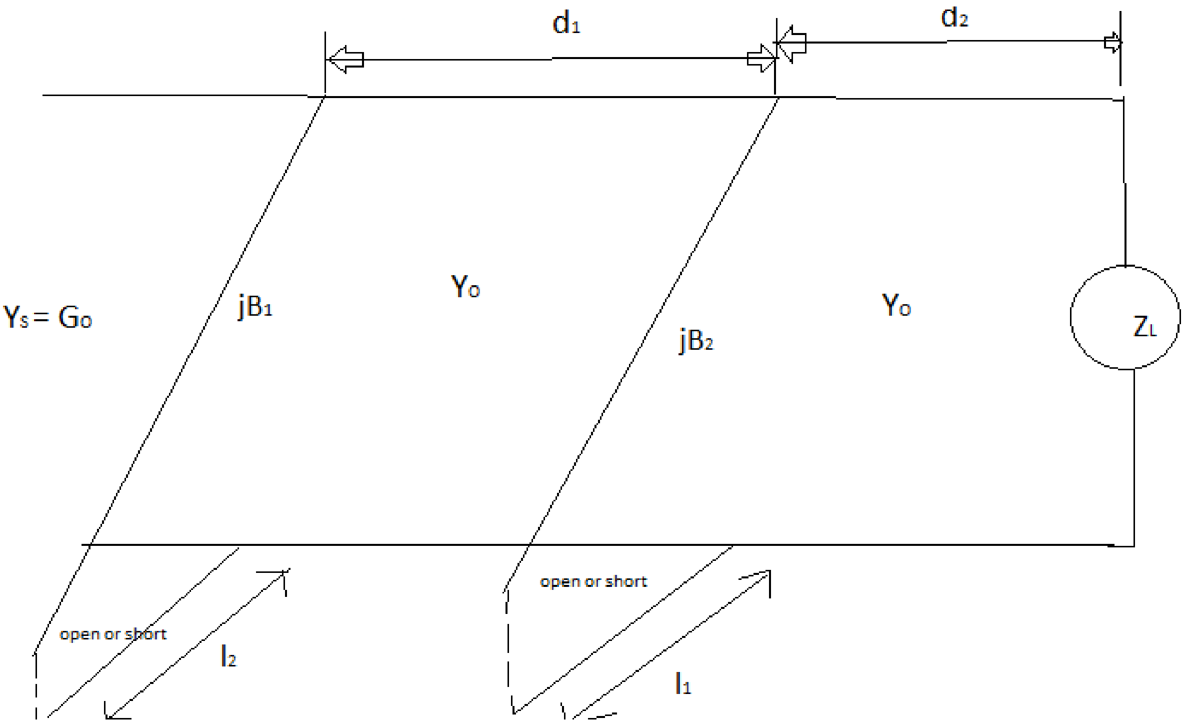


Fig: 3.2.3 illustrating double stub matching

The Fig 3.2.3 shows that the another possible method of impedance matching is to use two stubs in which the locations of the stub are arbitrary, the two stub lengths furnishing the required adjustments. The spacing is frequently made l/4.This is called double stub matching.

Limitations of single stub matching:

It provides matching at single frequency

The stub must be located at a fixed position on the line.

Not easy to tune.

$Y_S = G_O$

$\frac{Y_S}{G_O} = 1 + jb$

$Y_S = 1 + jb - jb$

The input impedance is given by,

$Z_s = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$

$R_O = Z$, $z_R = Z_L$ sub in above equ,

$$z_s = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta s}{Z_0 + jZ_L \tan\beta s} \right]$$

$z_s = \frac{1}{Y_S}$, $Z_0 = \frac{1}{Y_0}$, $Z_L = \frac{1}{Y_L}$ sub in above equ,

$$\frac{1}{Y_S} = \frac{1}{Y_0} \left[\frac{\frac{1}{Y_L} + j\frac{1}{Y_0}\tan\beta s}{\frac{1}{Y_0} + j\frac{1}{Y_L}\tan\beta s} \right]$$

$$\frac{Y_0}{Y_S} = \frac{\frac{\frac{Y_0}{Y_L} + j\tan\beta s}{Y_0}}{\frac{1 + j\frac{Y_0}{Y_L}\tan\beta s}{Y_0}}$$

$$\frac{Y_0}{Y_S} = \frac{\frac{Y_0}{Y_L} \pm j\tan\beta s}{1 + j\frac{Y_0}{Y_L}\tan\beta s}$$

$$\frac{Y_0}{Y_S} = y_S$$

$$\frac{Y_0}{Y_L} = y_L$$

$$y_S = \frac{y_L + j\tan\beta s}{1 + jy_L\tan\beta s}$$

$$y_S = \frac{y_L + j\tan\beta s}{1 + jy_L\tan\beta s} \times \frac{1 - jy_L\tan\beta s}{1 - jy_L\tan\beta s}$$

$$y_S = \frac{y_L + j\tan\beta s - jy_L^2\tan\beta s + y_L\tan^2\beta s}{1 + y_L^2\tan^2\beta s}$$

$$y_S = \frac{y_L + y_L\tan^2\beta s + j\tan\beta s - jy_L^2\tan\beta s}{1 + y_L^2\tan^2\beta s}$$

$$y_S = \frac{y(1 + \tan^2\beta s) + j\tan\beta s(1 - y_L^2)}{1 + y_L^2\tan^2\beta s}$$

$$y_S = \frac{y(1 + \tan^2\beta s)}{1 + y_L^2\tan^2\beta s} + \frac{j\tan\beta s(1 - y_L^2)}{1 + y_L^2\tan^2\beta s}$$

$y = g_1 + jb_1$

$y_S' = g_1 + jb_1'$

$y_S'' = y = 1 \pm jb_2$

3.1 INTRODUCTION:

Transmission lines are used to transmit power from a source to a load. Maximum power is transmitted by the line to the load with minimum losses when no reflected wave is present or when the load impedance is equal to the characteristic impedance of the line, when the load properly matched. Usually this condition is not achieved.

- Transmission lines is used to transmit power from source to a load.
- Maximum power is transmitted by the line to the load with minimum losses when no reflected wave is present or when the load is equal to the characteristic impedance of the line, that is, when the lad is properly matched.
- There are various impedance matching techniques using short length Transmission lines ($\frac{\lambda}{2}, \frac{\lambda}{4}, \frac{\lambda}{8}$) and using stubs (single stub, double stub).

THE EIGHT WAVE LINE: ($\frac{\lambda}{8}$):

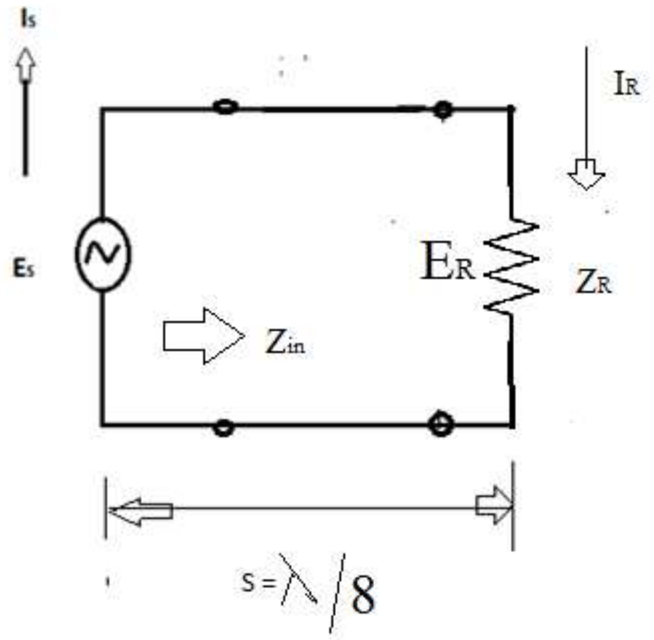


Fig: 3.1.1 The eight wave line

Fig 3.1.1 shows the circuit diagram of a transmission line with finite length $\frac{\lambda}{8}$

The input voltage is E_s and current I_s. The

input impedance of the circuit is Z_{in}

The receiving end voltage and current is E_R and I_R.

The distance between sending and receiving end S = $\frac{\lambda}{8}$

We know, the input impedance of the lossless line is given by,

- $= \frac{E_S}{I_S} = R_O\left[\frac{Z_R + jR_O\tan\beta s}{R_O + jZ_R\tan\beta s}\right]$

- $Z_{in} = \frac{E_S}{I_S} = R_O\left[\frac{Z_R + jR_O\tan\beta s}{R_O + jZ_R\tan\beta s}\right]$

- Sub, S= $\frac{\lambda}{8}$, $\beta = \frac{2\pi}{\lambda}$ in above equ,

$$Z_{in} = R_O\left[\frac{Z_R + jR_O\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right)}{R_O + jZ_R\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right)}\right]$$

$$Z_{in} = R_O\left[\frac{Z_R + jR_O\tan\left(\frac{\pi}{4}\right)}{R_O + jZ_R\tan\left(\frac{\pi}{4}\right)}\right]$$

$$Z_{in} = R_O\left[\frac{Z_R + jR_O}{R_O + jZ_R}\right]$$

$$|Z_{in}| = |R_O|\left[\frac{|Z_R + jR_O|}{|R_O + jZ_R|}\right]$$

$$|Z_{in}| = R_O\left[\frac{\sqrt{R_O^2 + Z_R^2}}{R_O^2 + Z_R^2}\right]$$

$|Z_{in}| = R_O$

Thus the eighth wave line is generally used to transformer any resistance Z_R an impedance z_{in} having its magnitude equal to characteristic resistance R_O of the line.

THE QUARTER WAVE LINE: ($\frac{\lambda}{4}$) :

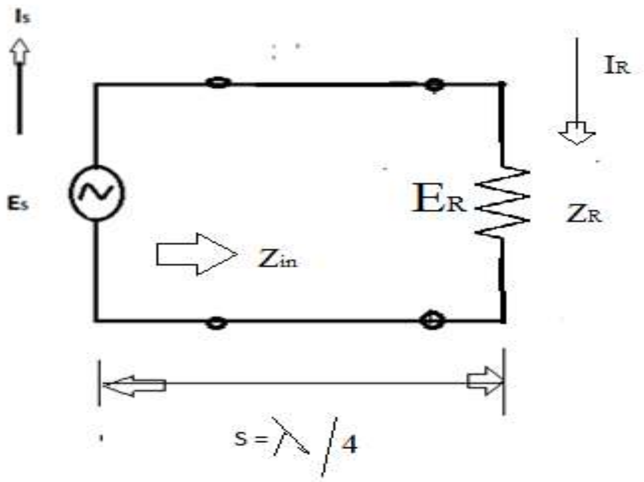


Fig: 3.1.2 Quarter wave section as a impedance inverter

Fig 3.1.2 shows the circuit diagram of a transmission line with finite length $\frac{\lambda}{4}$

The input voltage is E and current I_S. The

input impedance of the circuit is Z_{in}

The receiving end voltage and current is E and I_R.

The distance between sending and receiving end S = $\frac{\lambda}{4}$

We know, the input impedance of the lossless line is given by,

$$Z_{in} = \frac{E_S}{I_S} = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$$

Sub, S= $\frac{\lambda}{4}$, $\beta = \frac{2\pi}{\lambda}$ in above equ,

$$Z_{in} = R_O \left[\frac{Z_R + jR_O \tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)}{R_O + jZ_R \tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)}\right]$$

$$Z_{in} = R_O \left[\frac{Z_R + jR_O \tan\left(\frac{\pi}{2}\right)}{R_O + jZ_R \tan\left(\frac{\pi}{2}\right)}\right]$$

$$Z_{in} = R_O \frac{\tan\left(\frac{\pi}{2}\right)}{\tan\left(\frac{\pi}{2}\right)} \left[\frac{\frac{Z_R}{\tan\left(\frac{\pi}{2}\right)} + jR_O}{\frac{R_O}{\tan\left(\frac{\pi}{2}\right)} + jZ_R}\right]$$

$$Z_{in} = R_O \left[\frac{\frac{1}{\infty} + jR_O}{\frac{1}{\infty} + jZ_R}\right]$$

$$Z_{in} = \frac{{R_O}^2}{Z_R}$$

The input impedance equation is similar to the equation of transformer.

Thus the quarter wave line can be used as a transformer for impedance matching to the load Z_R with input impedance z_{in}.

A quarter wave transformer can transfer low impedance into high impedance And vice versa.

So, it can be considered as impedance inverter. The short circuit quarter wave line behaves like a open circuit in the other end.

While, the open circuited quarter wave line will behave like short circuit in the other end.

APPLICATION:

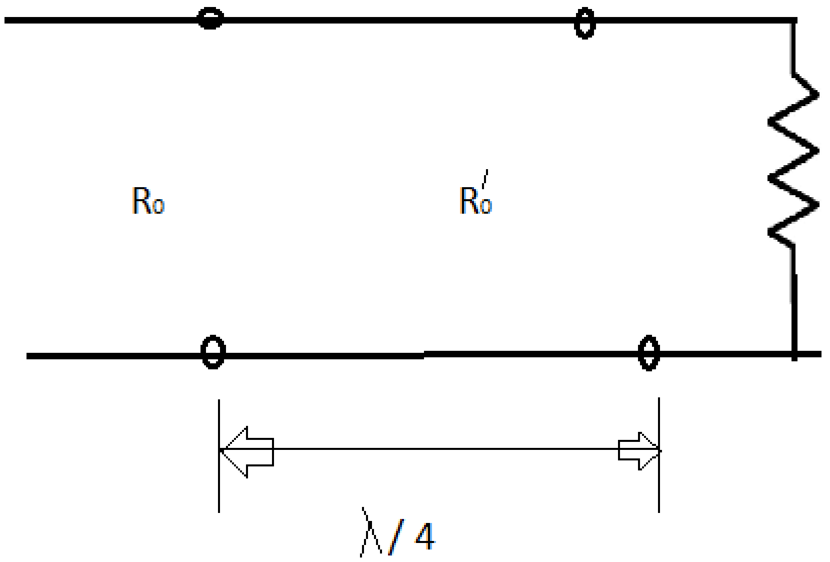


Fig: 3.1.3 Quarter wave line as-insulator

In Fig 3.1.3 the major application of $\frac{\lambda}{4}$ line is impedance transformer is coupling A transmission line to a resistive load such as an antenna. From the input Impedance equation of $\frac{\lambda}{4}$ line.

$$Z_{in} = \frac{R_0^2}{Z_R}$$

If antenna is a load having a resistance of R_a the quarter wave section is designed such that its characteristic impedance R'_O transforms antenna resistance R_a to the characteristic impedance of the line R_O.

$$R_O = \frac{R'^2_O}{R_a}$$

$$R'^2_O = R_O R_a$$

$$R'_O = \sqrt{R_O R_a}$$

THE HALF WAVE LINE: $(\frac{\lambda}{2})$:

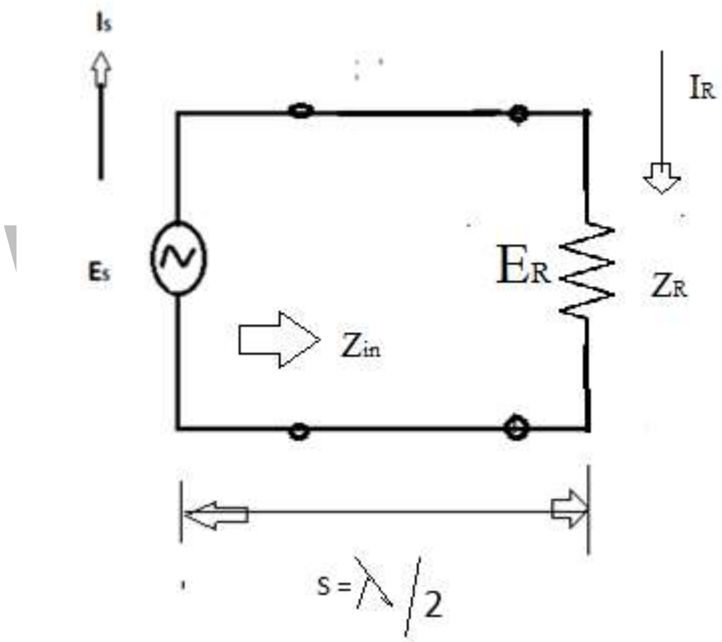


Fig: 3.1.4 the half wave line

Fig 3.1.4 shows the circuit diagram of a transmission line with finite length $\frac{\lambda}{2}$

The input voltage is E_s and current I_s. The

input impedance of the circuit is Z_{in}

The receiving end voltage and current is E_R and I_R.

The distance between sending and receiving end S = $\frac{\lambda}{2}$

We know, the input impedance of the lossless line is given by,

$$z_{in} = \frac{E_S}{I_S} = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$$

Sub, $S = \frac{\lambda}{2}$, $\beta = \frac{2\pi}{\lambda}$ in above equ,

$$z_{in} = R_O \left[\frac{Z_R + jR_O \tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)}{R_O + jZ_R \tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)}\right]$$

$$z_{in} = R_O \left[\frac{Z_R + jR_O \tan(\pi)}{R_O + jZ_R \tan(\pi)}\right]$$

$$z_{in} = R_O \left[\frac{Z_R}{R_O}\right]$$

$$z_{in} = Z_R$$

The half wave line repeats terminating impedance. So the half wave line can be considered as 1:1 transformer.

3.5 SINGLE STUB MATCHING USING SMITH CHART:

PROBLEM 1:

A 30m long lossless transmission line with Z_0 = 50 ohm operating at 2 MHz is terminated with a load Z_L=60+j40 ohm. If v=0.6 c on the line, find Refection coefficient, the standing wave ratio and the input impedance.

STEP 1:

- To find The normalized load impedance is
- $Z\ ' = \frac{Z_L}{Z_0}$
- $Z\ ' = \frac{60+j40}{50}$
- $Z_L{}' = 1.2 + j0.8$

STEP 2:

- Fig 3.5.1, draw the normalized load impedance in smith chart

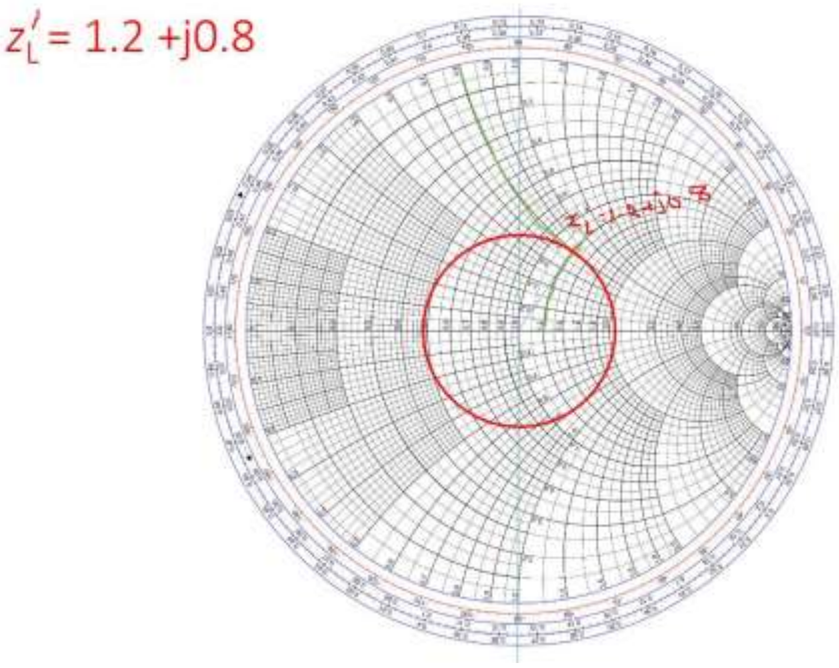


Fig: 3.5.1 Normalized load impedance

STEP 3:

- Fig 3.5.2, mark the value of Standing Wave Ratio in smith chart

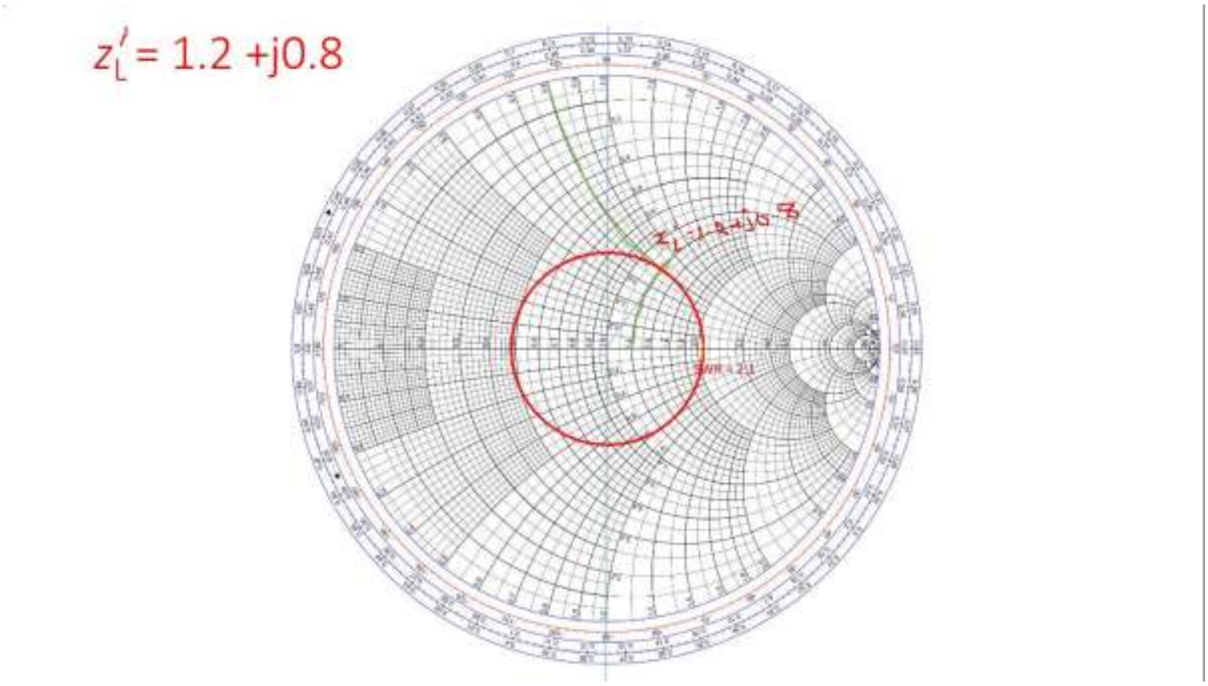


Fig 3.5.2 Standing Wave Ratio

STEP 4:

- Fig 3.5.3, mark the wavelength in smith chart

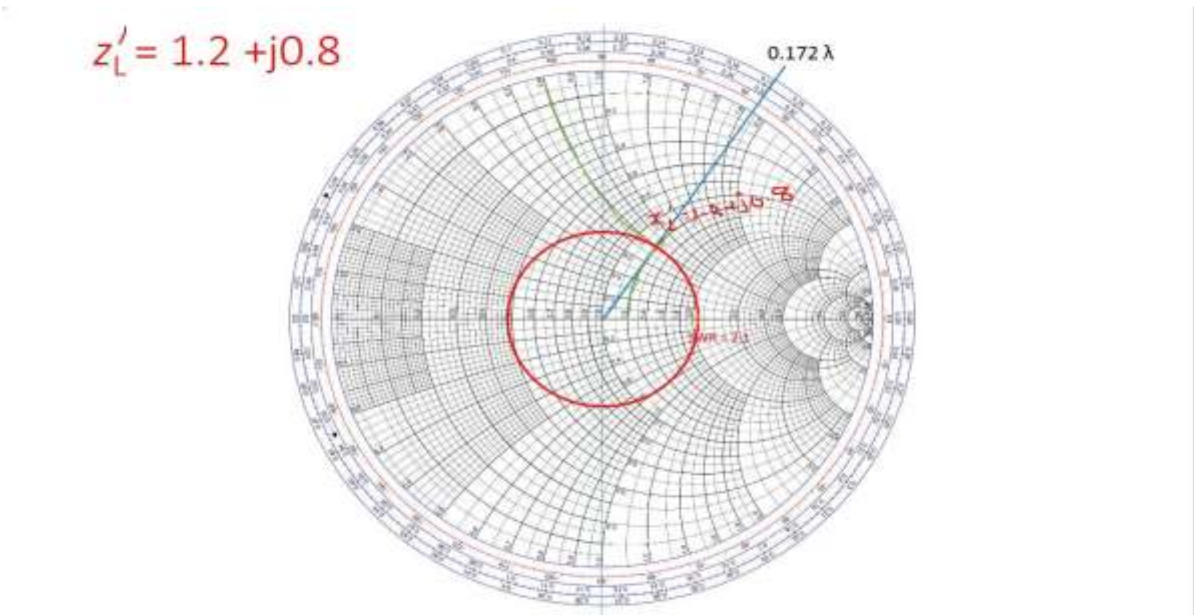


Fig: 3.5.3 wavelength

STEP 5:

- Calculate the velocity and wavelength using 30m long transmission line

v = 0.6c

= 0.6 X 3X10^8

= 1.8X10^8

$\lambda = \frac{v}{f}$

$\lambda = \frac{1.8X10^8}{2\ X\ 10^6}$

λ = 90m

L = 30m

1 m = 30 X $\frac{\lambda}{90}$

L = $\frac{\lambda}{3}$

L = 0.333 λ

STEP 6:

- Fig 3.5.4, draw the new wavelength in smith chart

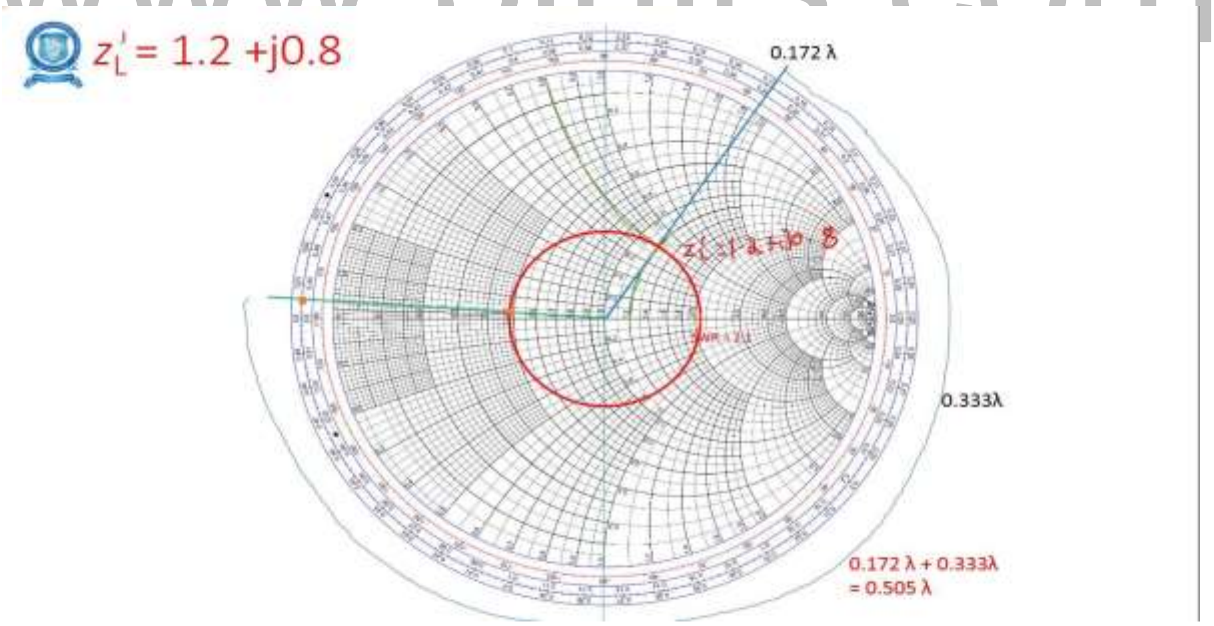


Fig: 3.5.4 new wavelength

STEP 7:

Fig 3.5.5, calculate the normalized input impedance and mark the Z_{in} in smith chart

$Z_{in} = Z_{in}{'} Z_0$

$Z_{in}{'} = 0.48 - 0.03$

$Z_{in} = (0.48 - j0.03)\ 50$

$Z_{in} = 24 + j1.5$ ohm

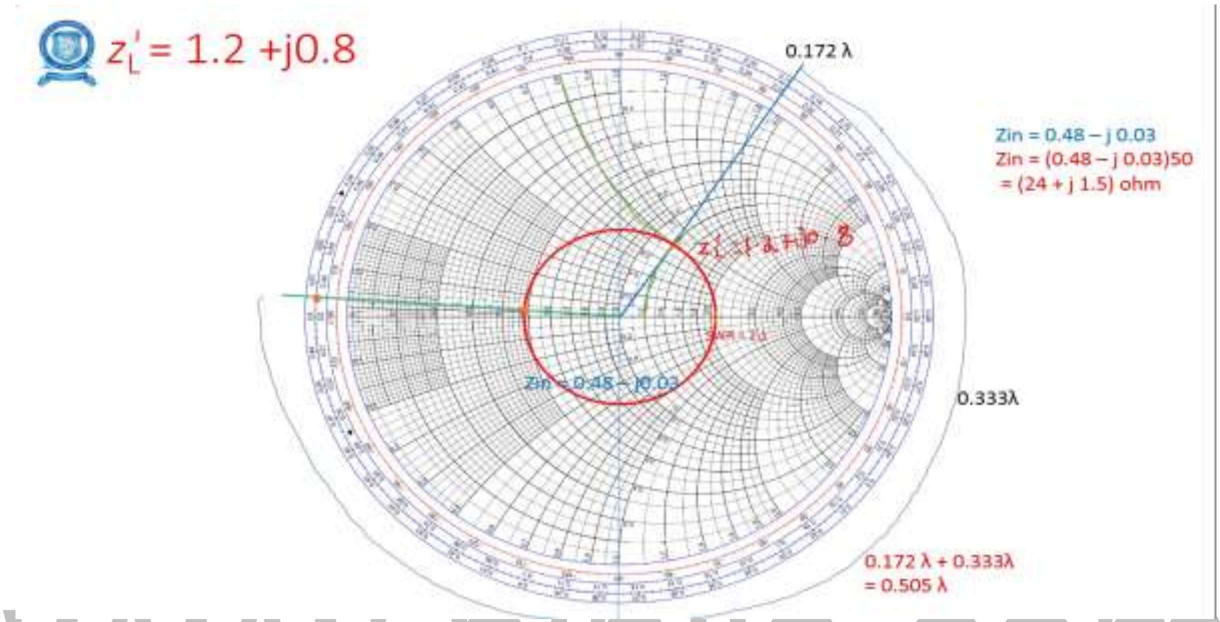


Fig: 3.5.5 Normalized input impedance

STEP 8:

- Fig 3.5.6, mark the reflection coefficient and phase angle in smith chart

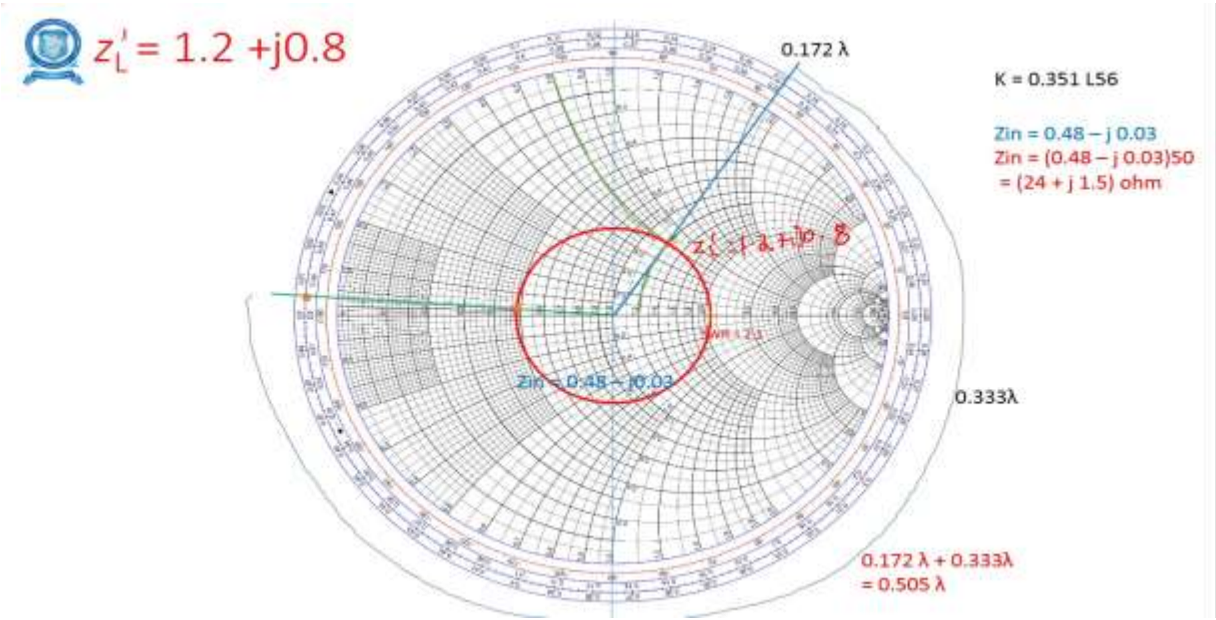


Fig: 3.5.6 Reflection coefficient and phase angle

\[Source: John D Ryder, —Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

3.3 SMITH CHART, SOLUTIONS OF PROBLEMS USING SMITH CHART:

Smith Chart:

The Smith Chart is a fantastic tool for visualizing the impedance of a transmission line and antenna system as a function of frequency. Smith Charts can be used to increase understanding of transmission lines and how they behave from an impedance viewpoint. Smith Charts are also extremely helpful for impedance matching, as we will see. The Smith Chart is used to display a real antenna's impedance when measured on a Vector Network Analyzer (VNA).

Smith Charts were originally developed around 1940 by Phillip Smith as a useful tool for making the equations involved in transmission lines easier to manipulate. See, for instance, the input impedance equation for a load attached to a transmission line of length L and characteristic impedance Z0 is shown in Fig 3.3.1. With modern computers, the Smith Chart is no longer used to the simplify the calculation of transmission line equations; however, their value in visualizing the impedance of an antenna or a transmission line has not decreased.

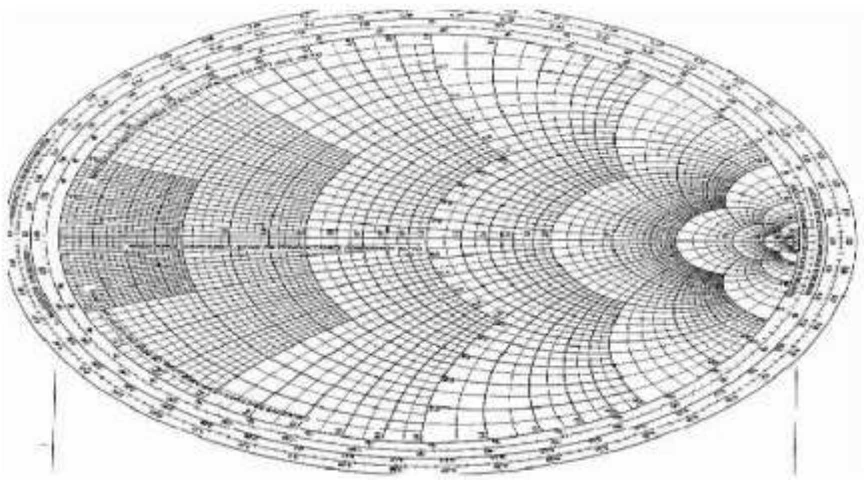


Figure should look a little intimidating, as it appears to be lines going everywhere. There is nothing to fear though. We will build up the Smith Chart from scratch, so that you can understand exactly what all of the lines mean. In fact, we are going to learn an even more complicated version of the Smith Chart known as the imminence Smith Chart, which is twice as complicated, but also twice as useful. But for now, just admire the Smith Chart and its curvy elegance. This section of the antenna theory site will present an intro to the Smith Chart basics.

<u>THE CIRCLE DIAGRAM FOR DISSIPATION LESS LINE:</u>

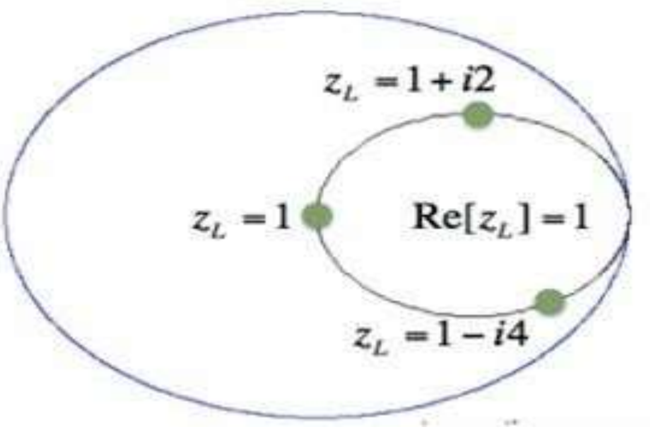
Constant Resistance Circles

For a given normalized load impedance zL, we can determine and plot it on the Smith Chart. Now, suppose we have the normalized load impedance given by:

$z_{1} = 1 + \text{ivy}$ .....(1)

Source: John D Ryder, —Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

In Fig 3.3.2, the outer blue ring represents the boundary of the smith chart. The black curve is a constant resistance circle: this is where all values of z1 = 1 +



I*Y will lie on. Several points are plotted along this curve, z1 = 1, z1 = 1 + I*2, and all = 1 - I*4. Suppose we want to know what the curve z2 = 0.3 + I*Y looks like on the Smith Chart. The result is shown in Fig 3.3.3:

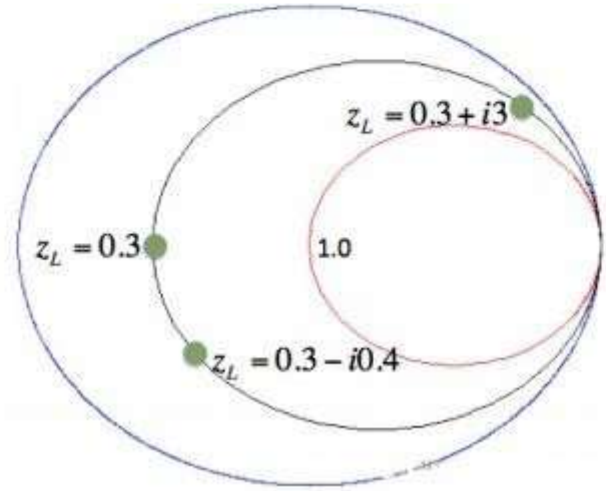


Fig: 3.3.3 Constant resistance circle all = 0.3 on smith chart

Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

In Fig 3.3.3, the black ring represents the set of all impedances where the real part of z2 equals 0.3. A few points along the circle are plotted. We've left the resistance circle of 1.0 in red on the Smith Chart. These circles are called constant resistance curves. The real part of the load impedance is constant along each of these curves. We'll now add several values for the constant resistance, as shown in Fig 3.3.4:

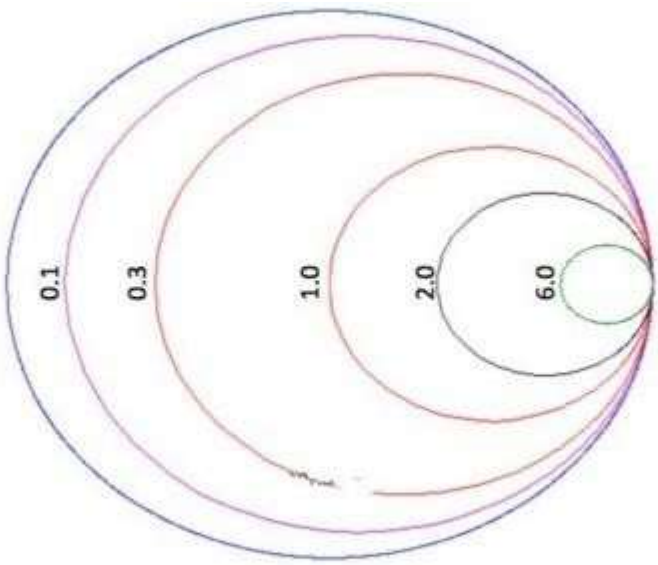


Fig: 3.3.4 Constant resistance circle on smith chart

Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

In Fig 3.3.4, the all=0.1 resistance circle has been added in purple. The all=6 resistance circle has been added in green, and all=2 resistance circle is in black. Look at the set of curves defined by all = R + ivy, where Y is held constant and R varies from 0 to infinity. Since R cannot be negative for antennas or passive devices, we will restrict R to be greater than or equal to zero. As a first example, let all = R + I. The curve defined by this set of impedances is shown in Fig 3.3.4:

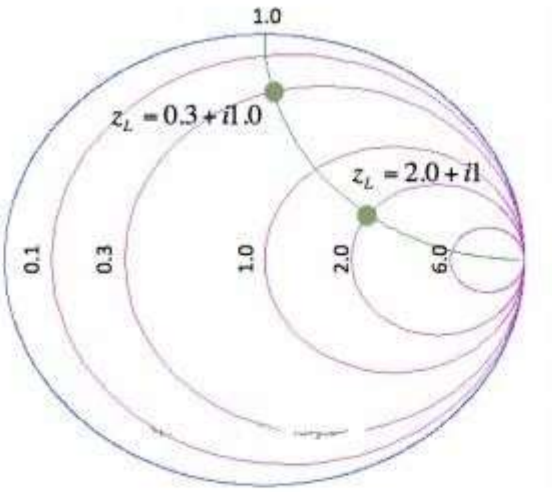


Fig: 3.3.4 Constant resistance Curve all = R + I*1
Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

The resulting curve all = R + I is plotted in green in Fig 3.3.4. A few points along the curve are illustrated as well. Observe that all = 0.3 + I is at the intersection of the Re [all] = 0.3 circle and the I'm [all] =1 curve. Similarly, observe that the all = 2 + i point is at the intersection of the Re[all]=2 circle and the I'm[all]=1 curve. (For a quick reminder of real and imaginary parts of complex numbers, see complex math primer.) The constant reactance curve, defined by I'm[all]=-1 is shown in Fig 3.3.5:

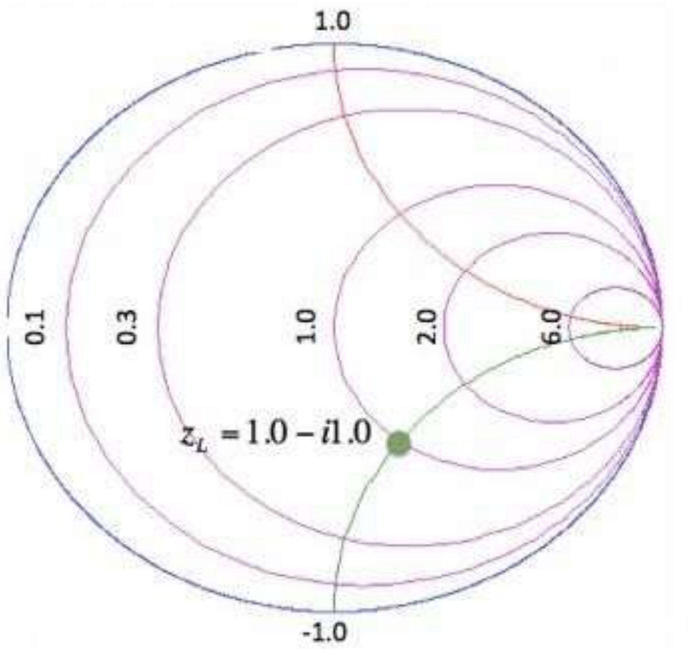


Fig: 3.3.5 Constant reactance Curve zL = R - i

Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

The resulting curve for Im[zL]=-1 is plotted in green in Figure 3.3.5. The point zL=1-i is placed on the Smith Chart, which is at the intersection of the Re[zL]=1 circle and the Im[zL]=-1 curve.

An important curve is given by Im[zL]=0. That is, the set of all impedances given by zL = R, where the imaginary part is zero and the real part (the resistance) is greater than or equal to zero. The result is shown in Fig 3.3.6:

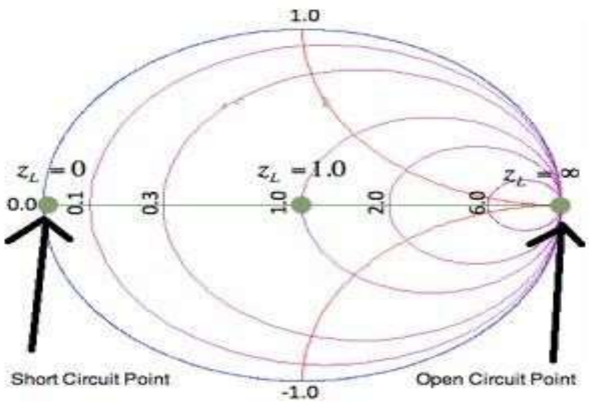


Fig 3.3.6 Constant Reactance Curve for zL=R

Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

Fig 3.3.6. Constant Reactance Curve for all=R. The reactance curve given by I'm[all]=0 is a straight line across the Smith Chart. There are 3 special points along this curve. On the far left, where sly = 0 + i0, this is the point where the load is a short circuit, and thus the magnitude of is 1, so all power is reflected. In the center of the Smith Chart, we have the point given by all = 1. At this location, is 0, so the load is exactly matched to the transmission line? No power is reflected at this point.

The point on the far right in Fig 3.3.6 is given by zL = infinity. This is the open circuit location. Again, the magnitude of is 1, so all power is reflected at this point, as expected. Finally, we'll add a bunch of constant reactance curves on the Smith Chart, as shown in Fig 3.3.7.

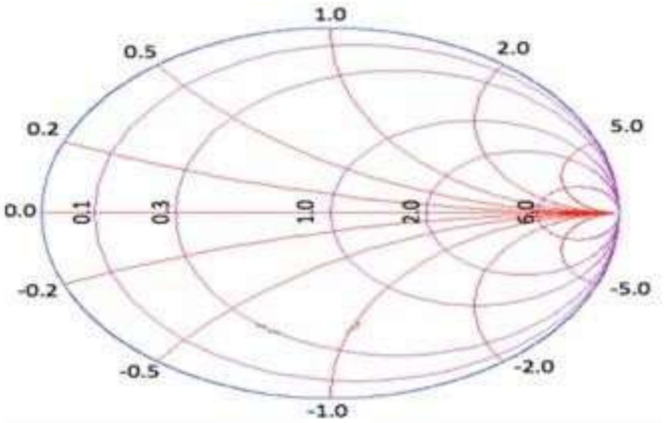


Fig 3.3.7 Smith chart with reactance curves and Resistance circles
Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015

In Fig 3.3.7, we added constant reactance curves for I'm[all]=2, I'm[all]=5, I'm[all]=0.2, I'm[all]=0.5, I'm[all]=-2, I'm[all]=-5, I'm[all]=-0.2, and I'm[all] = -0.5.
Figure 4 shows the fundamental curves of the Smith Chart.

SINGLE AND DOUBLE STUB MATCHING USING SMITH CHART:

Applications of smith Chart:

- Plotting an impedance

- Measurement of VSWR
- Measurement of reflection coefficient (magnitude and phase)
- Measurement of input impedance of the line
- It is used to find the input impendence and input admittance of the line.
- The smith chart may also be used for loss lines and the locus of points on a line then follows a spiral path towards the chart center, due to attenuation.
- The difficulties of the smith chart are

Single stub impedance matching requires the stub to be located at a definite point on the line. This requirement frequently calls for placement of the stub at an undesirable place from a mechanical view point.

For a coaxial line, it is not possible to determine the location of a voltage minimum without a slotted line section, so that placement of a stub at the exact required point is difficult

In the case of the single stub it was mentioned that two adjustments were required, these being location and length of the stub.

3.4 SMITH CHART:

- Developed in 1939 by P. W. Smith as a graphical tool to analyze and design transmission-line circuits
- Today, it is used to characterize the performance of microwave circuits
- Impedances, voltages, currents, etc. all repeat every half wavelength
- The magnitude of the reflection coefficient, the standing wave ratio (SWR) do not change, so they characterize the voltage & current patterns on the line
- If the load impedance is normalized by the characteristic impedance of the line, the voltages, currents, impedances, etc. all still have the same properties, but the results can be generalized to any line with the same normalized impedances
- The Smith Chart is a clever tool for analyzing transmission lines
- The outside of the chart shows location on the line in wavelengths
- The combination of intersecting circles inside the chart allow us to locate the normalized impedance and then to find the impedance anywhere on the line

STEP 1:

Fig 3.4.1, Draw the positive XL circles

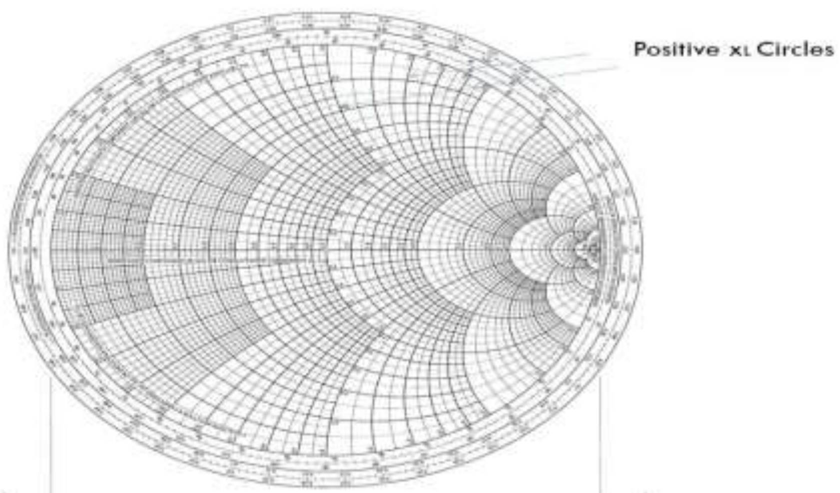


Fig: 3.4.1 Positive xL circles

STEP 2:

Fig 3.4.2, Draw the rL circles

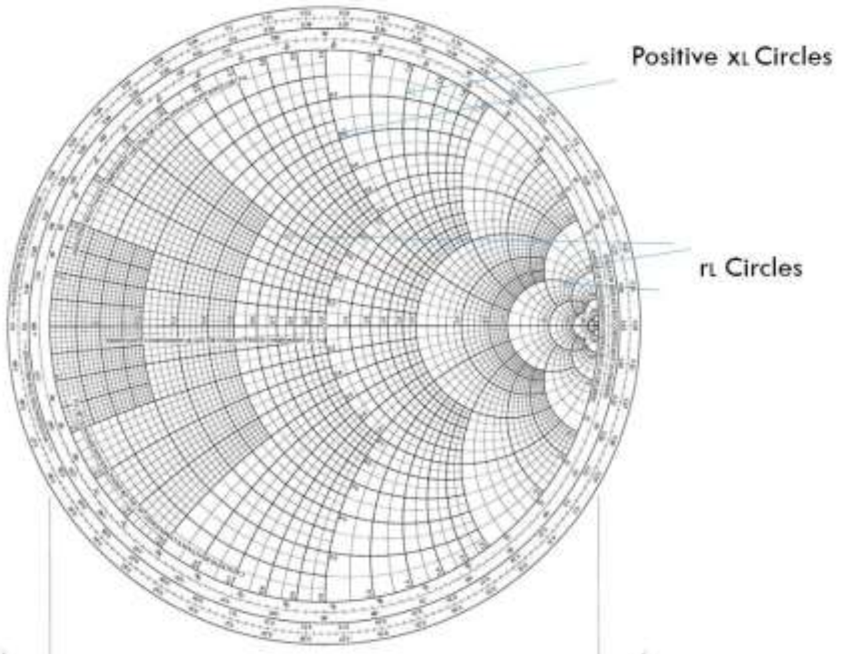


Fig: 3.4.2 Positive rL circles

STEP 3:

Fig 3.4.3, Draw the negative XL circles

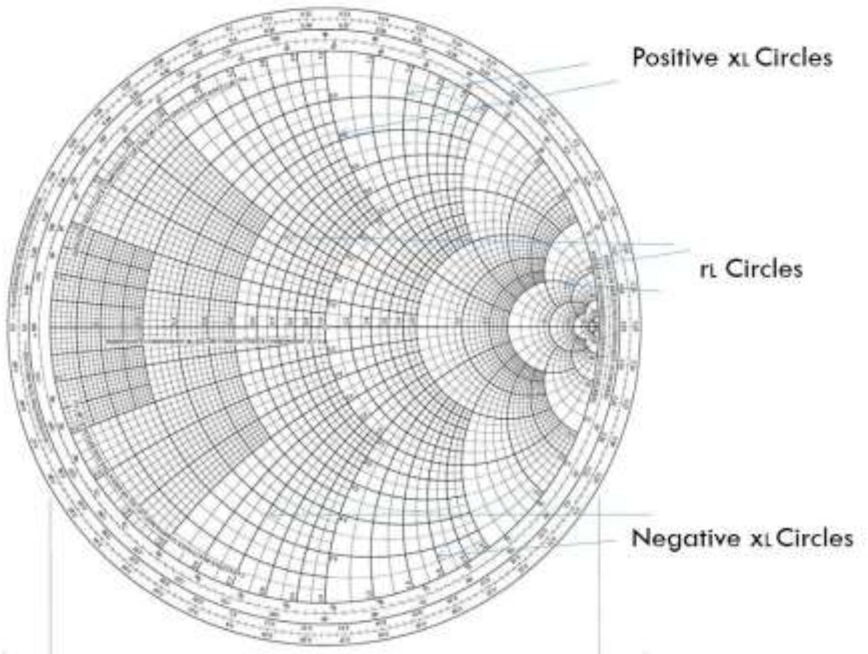


Fig: 3.4.3 Negative xL circles

STEP 4:

Fig 3.4.4, Mark the impedance (3 + j3) in the smith chart

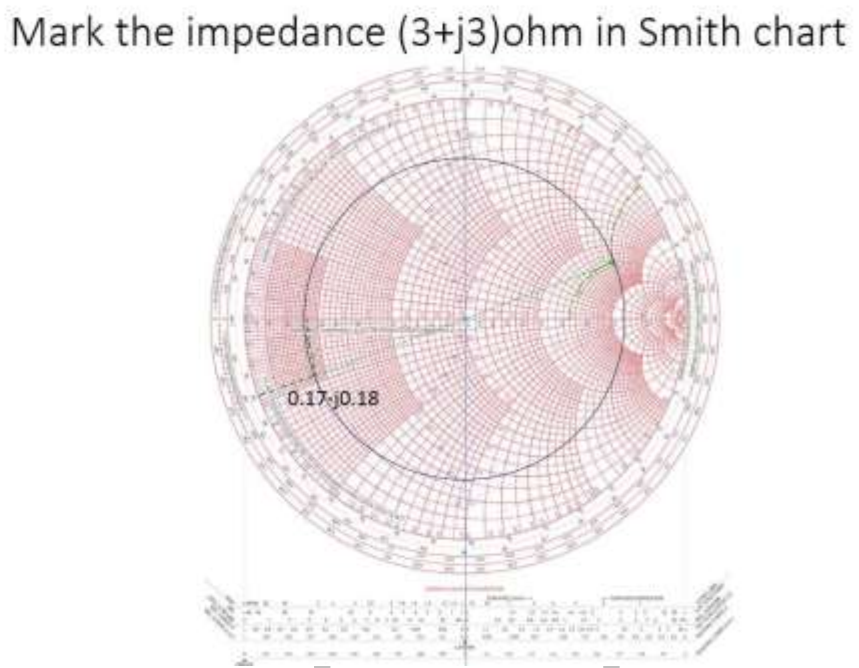


Fig: 3.4.4 Mark the impedance (3 + j3) in the smith chart

[Source: John D Ryder, ―Networks, lines and fields‖, 2nd Edition, Prentice Hall India, 2015]