2.1 INTRODUCTION:

SKIN EFFECT:

Consider a conductor made up of a large number of fine strands of wire. A strand at the center is linked by all the internal flux in the conductor, whereas a stand on the surface is not linked by the internal flux. The inductance and reactance of the strand at the center is greater than that of the strand at the surface. The interior strand thus carries less current than the outer so as to produce equal impedance drops along the strands. This phenomenon is known as **skin effect**.

When a line, either open- wire or coaxial, is used at frequencies of a megacycle or more, certain approximations may be employed leading to simplified analysis of line performance.

THE ASSUMPTIONS USUALLY MADE ARE:

1) At very high frequency, **the skin effect is very considerable** so that currents may be assumed as flowing on flowing on conductor surfaces, internal inductance then being zero.

2) Due to skin effect, resistance R increases with $\sqrt{f}$. But the line reactance ωL increases directly with frequency f. Hence $\omega L >> R$.

3) The lines are well enough constructed that G may be considered zero.

ANALYSIS IS MADE IN EITHER OF TWO WAYS:

1) R is merely small with respect to ωL. If **R is small**, the line is considered as one of **small dissipation**, and this concept is useful when the lines are employed as circuit elements or where resonance properties are involved,

2) R is completely negligible as compared to ωL, and the line is considered as one of zero dissipation and this concept is used for transmission of power at high frequency.

LINE CONSTSNTS OF DISSIPATION LESS LINE

In general the line constants for a transmission line are:

$Z_0 = \sqrt{\frac{Z}{Y}}$

Z = R+jωL

Y = G+jωC

$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

γ = α+jβ

$\gamma = \sqrt{ZY}$

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

According to the standard assumption for the line at high frequency

$j\omega L >> R, j\omega C >> G$ R =

0, G = 0

Sub the condition in , γ

$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}}$

$Z_0 = \sqrt{\frac{L}{C}}$

$\gamma = \sqrt{(j\omega L)(j\omega C)}$

$\gamma = \sqrt{(j^2\omega^2 LC)}$ γ

$= \sqrt{(-\omega^2 LC)}$ γ

$= j\omega\sqrt{LC}$ α+jβ =

$j\omega\sqrt{LC}$

Equate the real and image parts,

α = 0

$\beta = \omega\sqrt{LC}$

$v = \frac{\omega}{\beta}$

$v = \frac{1}{\sqrt{LC}}$

$$\lambda = \frac{2\pi}{\beta}$$

$$\lambda = \frac{2\pi}{\omega\sqrt{LC}}$$

There are the line constants for dissipation less line.

2.5 MEASUREMENT OF UNKNOWN LOAD IMPEDANCE:

The unknown value of a load impedance ZR connected to a transmission line may be determined by standing wave measurements on the open wire of slotted line. Bride circuit is used for the measurement of unknown impedance.

At the point of voltage minimum at a distance s' from the load it can be shown that

$$Z_S = R_{min} = \frac{R_O}{S} \qquad \text{......... (1)}$$

S = Standing wave ratio

At any point on the line, the input impedance is given by,

$$Z_S = R_O\left[\frac{Z_R + jR_O\tan\beta s'}{R_O + jZ_R\tan\beta s'}\right] \qquad \text{.......(2)}$$

$$\beta = \frac{2\pi}{\lambda}$$

Sub β value in equ (2),

$$Z_S = R_O\left[\frac{Z_R + jR_O\tan\left(\frac{2\pi s'}{\lambda}\right)}{R_O + jZ_R\tan\left(\frac{2\pi s'}{\lambda}\right)}\right] \qquad \text{.......(3)}$$

Equating equ (1) and (3),

$$R_O\left[\frac{Z_R + jR_O\tan\left(\frac{2\pi s'}{\lambda}\right)}{R_O + jZ_R\tan\left(\frac{2\pi s'}{\lambda}\right)}\right] = \frac{R_O}{S}$$

Solving for Z_R gives,

$$R_O + jZ_R\tan\left(\frac{2\pi s'}{\lambda}\right) = s\left[Z_R + jR_O\tan\left(\frac{2\pi s'}{\lambda}\right)\right]$$

$$R_O + jZ_R\tan\left(\frac{2\pi s'}{\lambda}\right) = sZ_R + SjR_O\tan\left(\frac{2\pi s'}{\lambda}\right)$$

$$-sZ_R + jZ_R\tan\left(\frac{2\pi s'}{\lambda}\right) = -R_O + SjR_O\tan\left(\frac{2\pi s'}{\lambda}\right)$$

$$-Z_R\left[S - j\tan\left(\frac{2\pi s'}{\lambda}\right)\right] = -R_O\left[1 - jS\tan\left(\frac{2\pi s'}{\lambda}\right)\right]$$

$$Z_R = R_O \left[\frac{[1-j\,S\tan(\frac{2\pi s'}{\lambda})]}{[S-j\tan(\frac{2\pi s'}{\lambda})]}\right]$$

Where $\beta=\frac{2\pi}{\lambda}$,

$$Z_R = R_O\left[\frac{1-j\,S\tan\beta S}{-j\tan\beta S}\right]$$

POWER AND IMPEDANCE MEASUREMENT ON LINE:

Expression for voltage and current for the dissipation less line is given by,

$E = I_R \left(\frac{Z_R+Z_O}{2}\right)[1+|K| \angle \emptyset - 2\beta s]$ ……..(1)

$I = I_R \left(\frac{Z_R+Z_O}{2R_O}\right)[1+|K| \angle \emptyset - 2\beta s]$ ………(2)

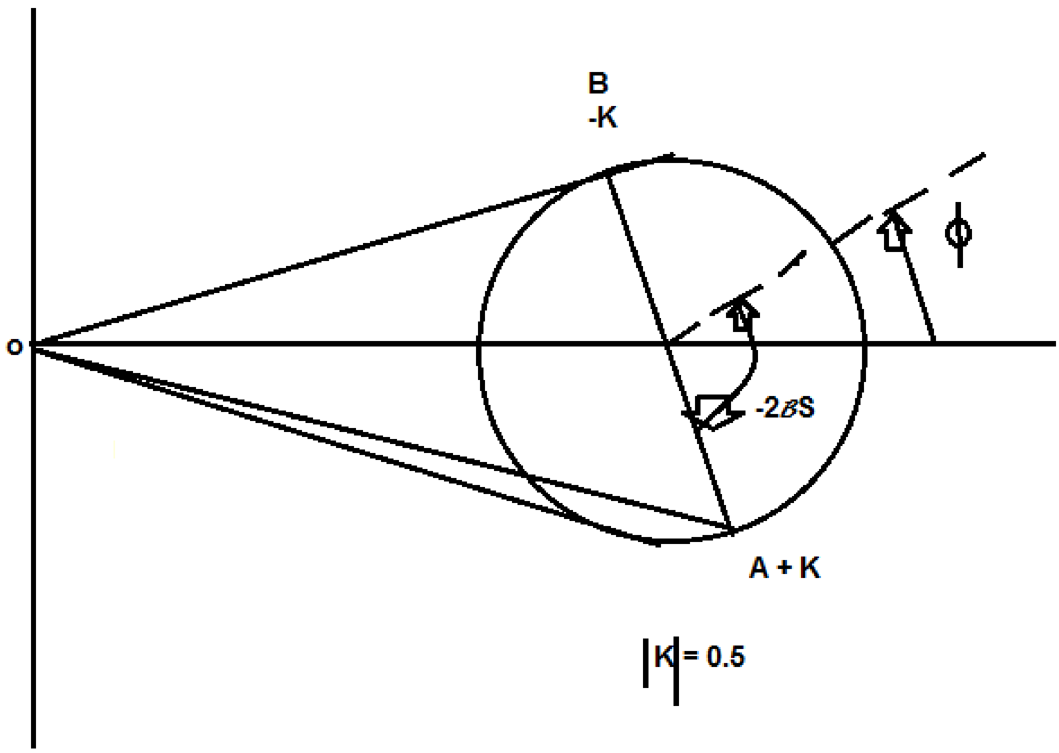


Fig: 2.5.1 Diagram illustrating Equation (1) & (<u>In Fig 2.5.1, phasors A and B being propositional to E and I, respectively, Consider two phases A and B representing voltage and cureent.</u>

When the incident and reflected waves are inphase, we get voltage maximum.

So it can be given by,

$\emptyset - 2\beta s = 0$, sub in (1) and (2),

$$E_{max} = I_R\left(\frac{Z_R+Z_O}{2}\right)[1+|K|] \quad \ldots\ldots (3)$$

$$I_{max} = I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1+|K|] \quad \ldots\ldots (4)$$

$$\frac{E_{max}}{I_{max}} = \frac{I_R\left(\frac{Z_R+Z_O}{2}\right)[1+|K|]}{I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1+|K|]}$$

$$\frac{E_{max}}{I_{max}} = R_O \quad \ldots\ldots(5)$$

From the theory of standing waves it can be obtained that the minimum values of voltage and current occurs at both incident and reflected waves are out-off phase.

$$\emptyset - 2\beta s = \pi$$

$$E_{min} = I_R\left(\frac{Z_R+Z_O}{2}\right)[1-|K|] \quad \ldots\ldots (6)$$

$$I_{min} = I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1-|K|] \quad \ldots\ldots (7)$$

$$\frac{E_{min}}{I_{min}} = \frac{I_R\left(\frac{Z_R+Z_O}{2}\right)[1-|K|]}{I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1-|K|]}$$

$$\frac{E_{min}}{I_{min}} = R_O \quad \ldots\ldots (8)$$

It is clear that the voltage maximum and current minimum at the same point in the transmission line.

Divide (3) and (7),

$$\frac{E_{max}}{I_{min}} = \frac{I_R\left(\frac{Z_R+Z_O}{2}\right)[1+|K|]}{I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1-|K|]}$$

$$\frac{E_{max}}{I_{min}} = R_O\left(\frac{1+|K|}{1-|K|}\right)$$

$$\frac{E_{max}}{I_{min}} = R_O\, s$$

$$R_{max} = R_O\, s$$

s = standing wave ratio

$$s = \frac{1+|K|}{1-|K|}$$

Divide (6) and (4)

$$\frac{E_{min}}{I_{max}} = \frac{I_R\left(\frac{Z_R+Z_O}{2}\right)[1-|K|]}{I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1+|K|]}$$

$$\frac{E_{min}}{I_{max}} = R_O\left(\frac{1-|K|}{1+|K|}\right)$$

$$\frac{E_{min}}{I_{max}} = \frac{R_O}{S}$$

$$R_{min} = \frac{R_O}{S}$$

Here, the resistance R_{am} is known as impedance in the voltage loop and R_{min} is represented as impedance in current loop.

The effective power flowing into a resistance R_{am} is the power passing through voltage loop at voltage A_{max}.

$$P = \frac{E^2max}{R_{max}}$$

Similarly, the power can be also calculated as the power passing through current loop at voltage M_{ein}.

$$P = \frac{E^2min}{R_{min}}$$

$$P^2 = \frac{E^2min\, E^2max}{R_{max}R_{min}}$$

$$P^2 = \frac{E^2min\, E^2max}{R_O S \cdot \frac{R_O}{S}}$$

$$P^2 = \frac{E^2min\, E^2max}{R_O^2}$$

$$P = \frac{|E_{max}||E_{min}|}{R_O}$$

$$P = |I_{max}||I_{min}| \cdot R_O$$

REFLECTION LOSSES ON THE UNMATCHED LINE:

If the line is not terminated to its load, the energy delivered by the line to the load is less than if the impedance are properly adjusted. This effect is due to the reflection coefficient at the junction and results in a reflected wave and a standing wave system. The voltage at a maximum voltage point is due to the in-phase sum of the incident and reflected waves is given by:

$$|E_{max}| = |E_i| + |E_r| = \left|\frac{I_R(Z_R+Z_0)}{2}\right| (1+|K|) \quad \text{........ (1)}$$

The minimum voltage is due to the difference of the incident and reflected wave and is given as.

$$|E_{min}| = |E_i| - |E_r| = \left|\frac{I_R(Z_R+Z_0)}{2}\right| (1-|K|) \quad \text{........ (2)}$$

Hence the standing wave ratio is:

$$\frac{E_{max}}{E_{min}} = \frac{|E_i| + |E_r|}{|E_i| - |E_r|} \quad \text{......... (3)}$$

The total power along the line and delivered to the load is given by:

$$P = \frac{|E_{max}||E_{min}|}{R_0} \quad \text{........ (4)}$$

$$P = \frac{(|E_i| + |E_r|)(|E_i| - |E_r|)}{R_0}$$

$$P = \frac{|E_i|^2 - |E_r|^2}{R_0} = P_i - P_r \quad \text{....... (5)}$$

The above equation is the difference of two power flows, one being the power P_i is transmitted in the incident wave, the other being power P_r travelling back in the reflected wave.

The ratio of the power P delivered to the load to the power transmitted by the incident wave is,

$$\frac{P}{P_i} = \frac{P_i - P_r}{P_i} = \frac{|E_i|^2 - |E_r|^2}{|E_i|^2} = 1 - \frac{|E_r|^2}{|E_i|^2} = 1 - |K|^2$$

Where $K = \frac{S-1}{S+1}$

$$\frac{P}{P_i} = 1 - \left(\frac{S-1}{S+1}\right)^2 = \frac{(S+1)^2 - S^2 + 2S - 1}{(S+1)^2}$$

$$\frac{P}{P_i} = \frac{S^2 + 2S + 1 - S^2 + 2S - 1}{(S+1)^2}$$

$$\frac{P}{P_i} = \frac{4S}{(S+1)^2} \quad \text{.......... (6)}$$

The ratio of power absorbed by the load to the transmitter is plotted as a function of S as shown in the following Fig 2.5.2.

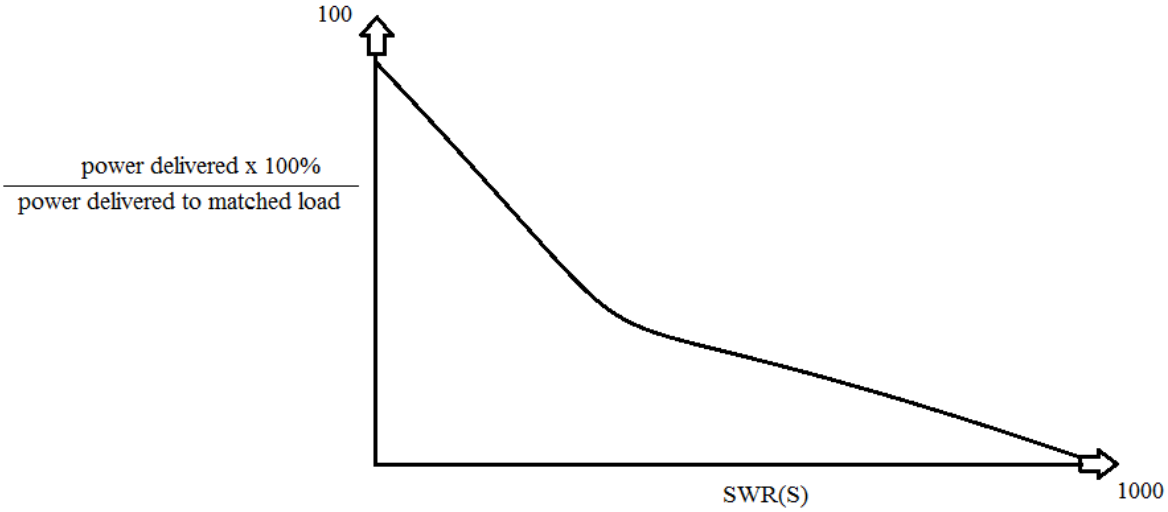


Fig: 2.5.2 Reflection losses as a function of standing wave ratio

2.6 MEASUREMENT OF VSWR AND WAVELENGTH:

(A) Measurement of Voltage Standing Wave Ratio (VAWR)

If V_{max} and V_{min} are known, then SWR can be calculated by using the following equation:

$$SWR = \frac{|V_{max}|}{|V_{min}|}$$

Therefore the determination of SWR in fact, is the determination of V_{max} and

V_{min} .

(i) Open Wire Line:

The value of V_{max} and V_{min} can be readily obtained on open wire line by arranging a simple set up as shown in the following Fig 2.6.1.

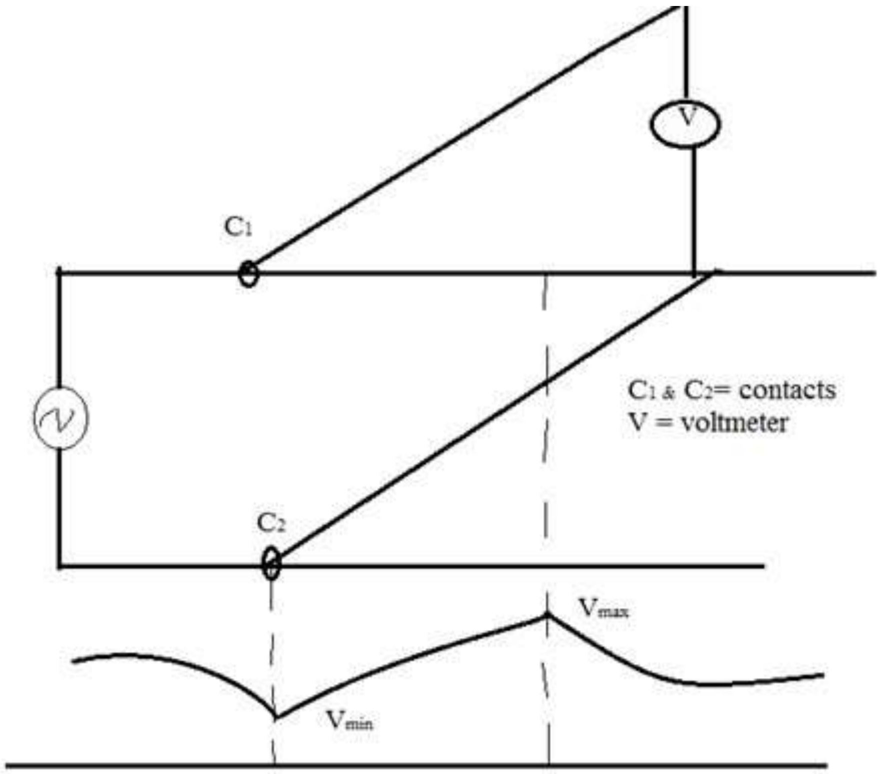


Fig: 2.6.1 Determination of SWR on open wire line

A sliding contact A.C (rams) voltmeter V is used to measure the voltage at different points along the line by sliding contact C_1 and C_2. One thing that becomes obvious is that the ratio of V max to V min becomes larger as the reflection coefficient increases. That is, if the ratio of V max to V min is one, then

There are no standing waves, and the impedance of the line is perfectly matched to the load. If the ratio of V max to V min is infinite, then the magnitude of the reflection coefficient is 1, so that all power is reflected. Hence, this ratio, known as the Voltage Standing Wave Ratio (VSWR) or standing wave ratio is a measure of how well matched a transmission line is to a load.

(ii) Coaxial Cable:

The VSWR measuring setup for a coaxial line is shown in the following Fig 2.6.2. For coaxial lines, it is necessary to use a length of line in which a longitudinal slot, one half wavelength or more long has been cut.

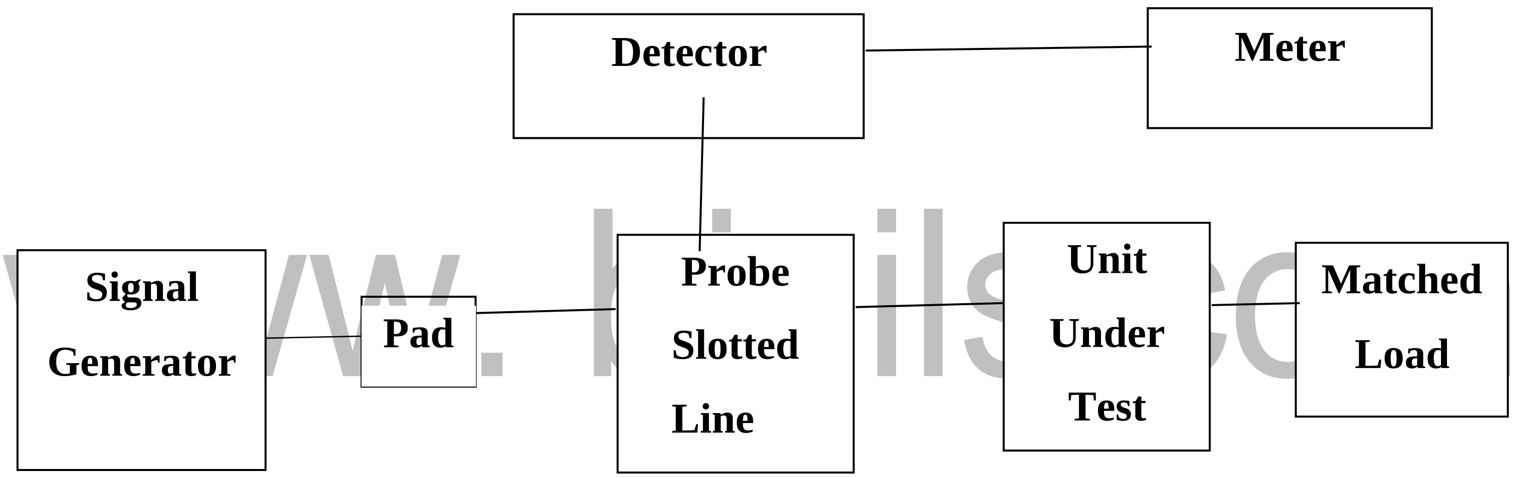


Fig: 2.6.2 Determination of SWR for coaxial cable

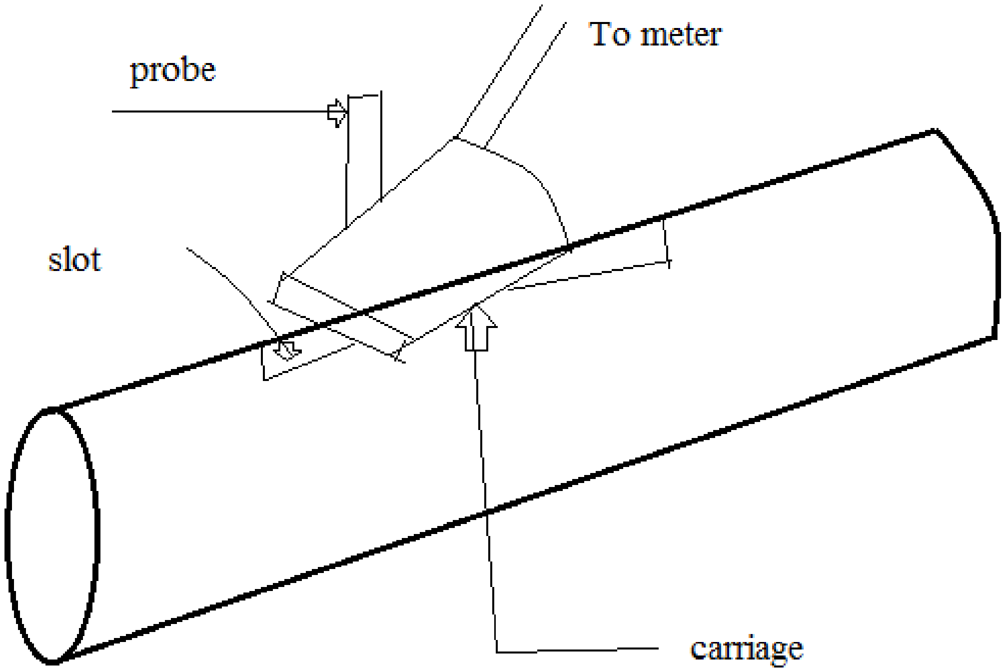


Fig: 2.6.3 Diagram of a slotted line section and a probe voltmeter for SWR measurement

The signal source must present a matched impedance looking back into it otherwise, reflections from the network being tested will reflect again off the signal generator and cause the peaks and nodes to shift in proportion in the standing wave pattern. The Fig 2.6.3 shows that the diagram of a slotted line section and a probe voltmeter for SWR measurement

(iii) Directional Couplers:

For direct indication and measurement of standing waves a device known as directional coupler can be used in the following Fig 2.6.4.

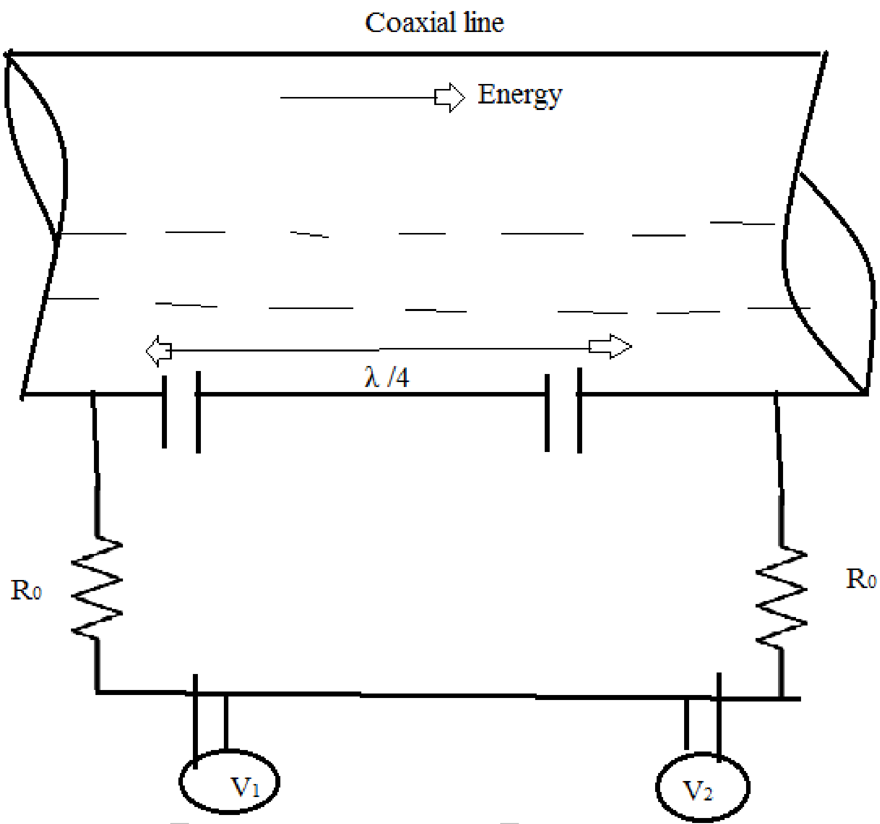


Fig: 2.6.4 A directional coupler on a coaxial line

(iv) The Reflect meters

In this, advantage is taken of the fact that the voltage on transmission line consists of two components travelling in opposite directions. The power goin from the transmitter to the load is represented by incident wave and the power reflected from the load is represented as reflected voltage.

If $R_1 = R_2$, the bridge of Fig 2.6.5 will be balanced when $R_x = R_S$. This is true is the input resistance of a perfectly matched transmission line and R_S is chosen equal to the characteristic impedance Zoo of the line.

The SWR cab be calculated from the following formula:

$$SER = \frac{V_i + V_r}{V_i - V_r}$$

V_i = incident voltage

V_r = reflected voltage

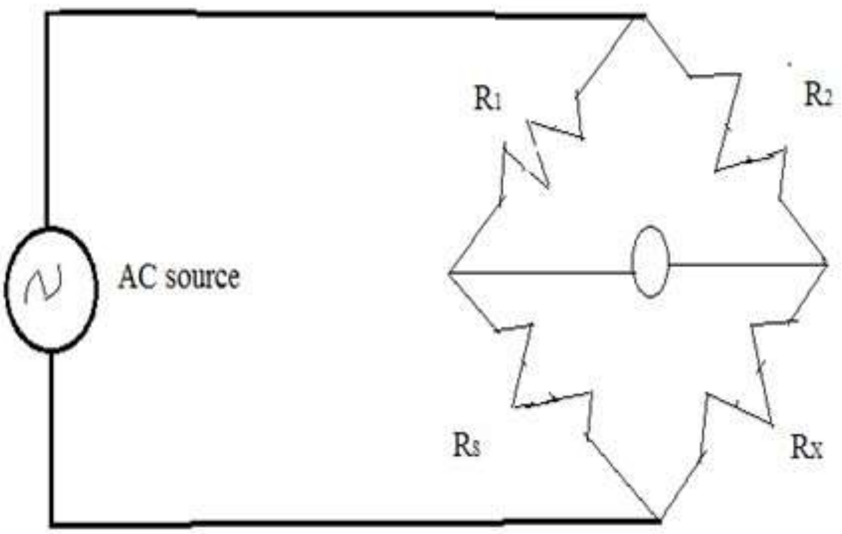


Fig: 2.6.5 SWR measurements by reflect meter

(B) Measurement of Wavelength (λ):

It is based on the fact that the distance between two successive voltage / current Maxima or minima being equal to half wavelength $\frac{\lambda}{2}$. Such measurements when Made on open wire line is called lecher measurements.

(i) Resonant air insulated lines are normally used to measure wavelength. The line is arranged with a movable short circuit at one end is loosely coupled to the generator at the other, when the line is adjusted to a resonant length, the energy coupled from the generator is able to built-up an oscillation of large amplitude of the line.

A parallel-wire line P used for this purpose is called Lecher-wire system and shown in Fig 2.6.6. Resonance is determined by the reading of a current indicator connected in the short circuiting slider S.

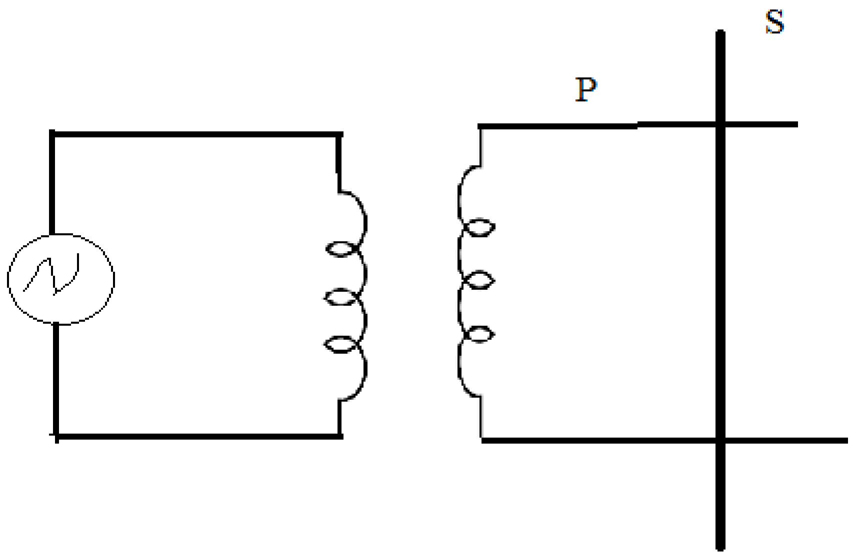


Fig: 2.6.6 Measurements of λ with Lecher-wire system

(ii) At short wavelengths, the preceding method is used with a coaxial cable and an instrument built especially for this purpose is called coaxial wave meter. It consists of an air insulated coaxial line which is closed at one end and has a movable short at the other. When the coaxial meter has been adjusted to one of these lengths, the current in the coupling loop is able to build up an oscillation of large amplitude within the wave meter.

2.2 PARAMETERS OF THE OPEN WIRE LINW AT RADIO FREQUENCY

a) LOOP INDUCTANCE:

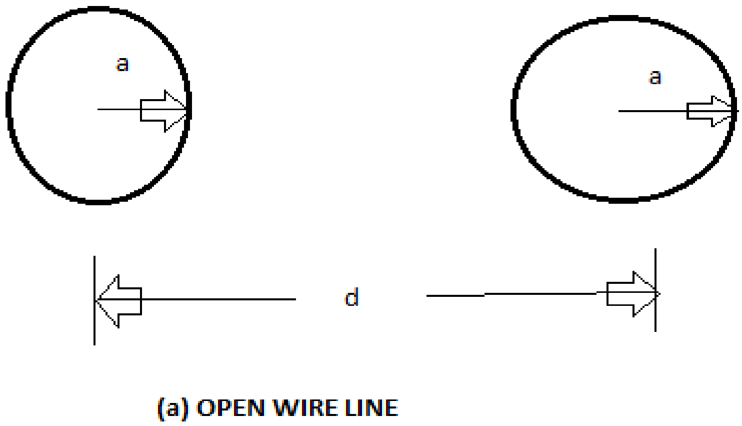


Fig: 2.2.1 Loop inductance of Open wire line

In Fig 2.2.1 shows that it consists of two spaced parallel wire supported by insulators; at proper distance to give a desired value of inductance.

a = Radius of the each line

d = Spacing between two parallel lines

Inductance of an open wire line is given by,

$$L = \frac{\mu_0}{2\pi} \ln \frac{d}{a}$$

$$L = \frac{4\pi X 10^{-7}}{2\pi} \ln \frac{d}{a} \text{ H/m}$$

The self-inductance of an open wire lines together is given by,

$$L = 0.1\,\mu_r + 0.921\,log_{10} \frac{d}{a} \text{ H/m}$$

Where,

a = radius of the conductor

d = distance b/w the conductor

μ_r = Relative permeability of the conductor

b) LOOP INDUCTANCE IN COAXIAL CABLE:

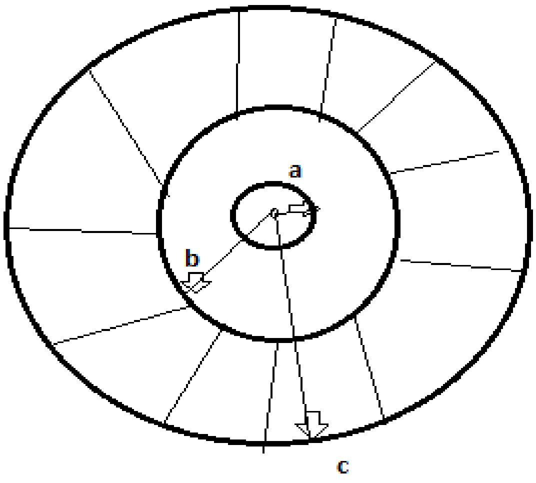


Fig : 2.2.2 Loop inductance of Coaxial cable

In Fig 2.2.2,

$$L = \frac{\mu_d}{2\pi} log_e\left(\frac{d}{a}\right) + \frac{\mu_c}{2\pi}\left[\frac{4c}{c^2 - b^2} log\left(\frac{c}{b}\right) - \frac{2c^2}{c^2 - b^2}\right]$$

g

a = radius of the inner conductor

b = inner radius of the outer conductor

c = outer radius of the outer conductor

μ_c = permeability of the conductor

μ_d = permeability of the dielectric.

SHUNT CAPACITANCE:

a) FOR OPEN WIRE LINE:

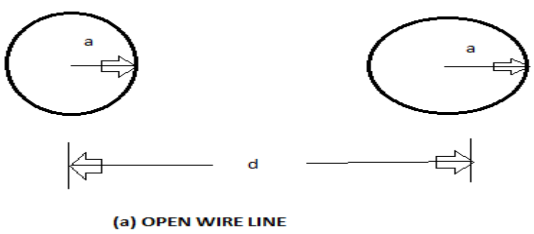


Fig: 2.2.3 Shunt capacitance of Open wire line

In Fig 2.2.3 shows that it consists of two spaced parallel wire supported by insulators; at proper distance to give a desired value of capacitance.

$$C = \frac{\pi\varepsilon_d}{log_e\left(\frac{d}{a}\right)} \text{ F/m}$$

ε_d = Permeability of the dielectric

= radius of the conductor

d = distance b/w the conductor

b) FOR COAXIAL CABLE:

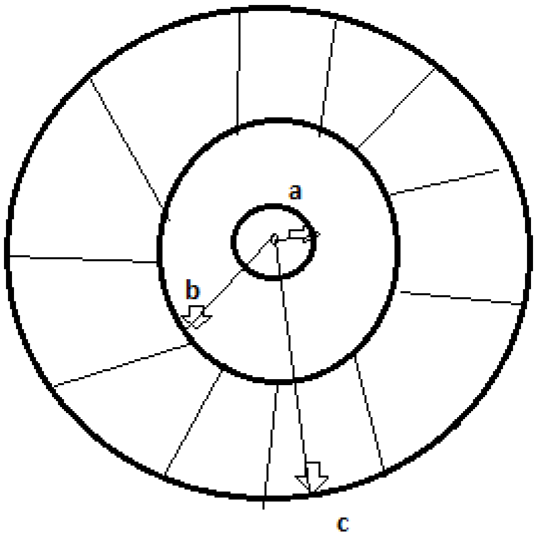


Fig: 2.2.4 Shunt capacitance of Coaxial cable

In Fig 2.2.4,

$$C = \frac{\pi\varepsilon_d}{log_e\left(\frac{d}{a}\right)} \text{ F/m}$$

a = radius of the inner conductor

b = inner radius of the outer conductor

LOOP RESISTSNCE:

a) FOR OPEN WIRE LINE:

In Fig 2.2.5 shows that it consists of two spaced parallel wire supported by insulators; at proper distance to give a desired value of resistance.

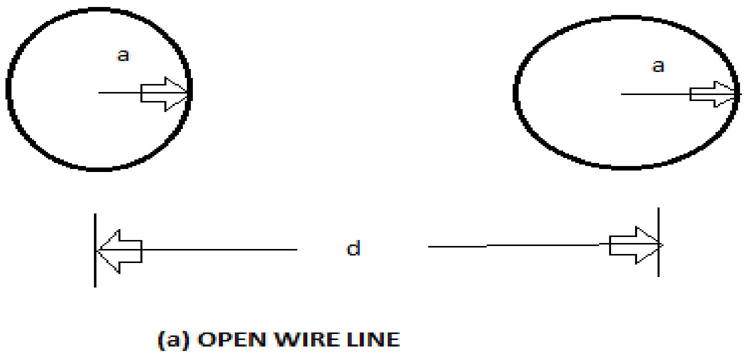


Fig: 2.2.5 Loop resistance of Open wire line

$$R_{dc} = \frac{2}{\pi \sigma a^2}$$

$$R_{ac} = \frac{R_{dc}}{2} a \sqrt{\pi \sigma \mu_c f}$$

σ = conductivity

μ_c = permeability of a conductor

f = frequency

a = radius of the conductor

b) <u>FOR COAXIAL LINE:</u>

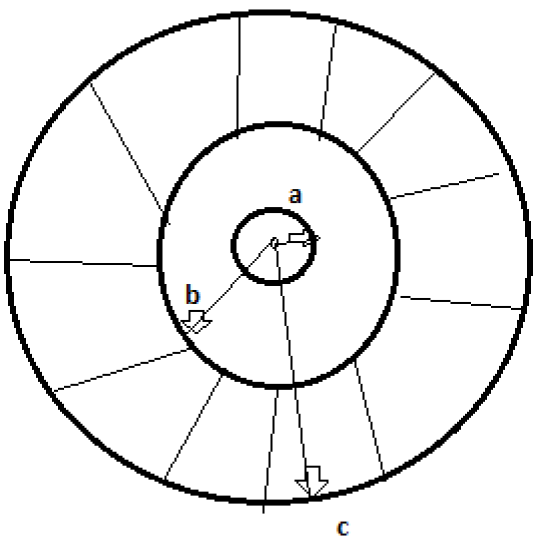


Fig: 2.2.6 Loop resistance of Coaxial cable

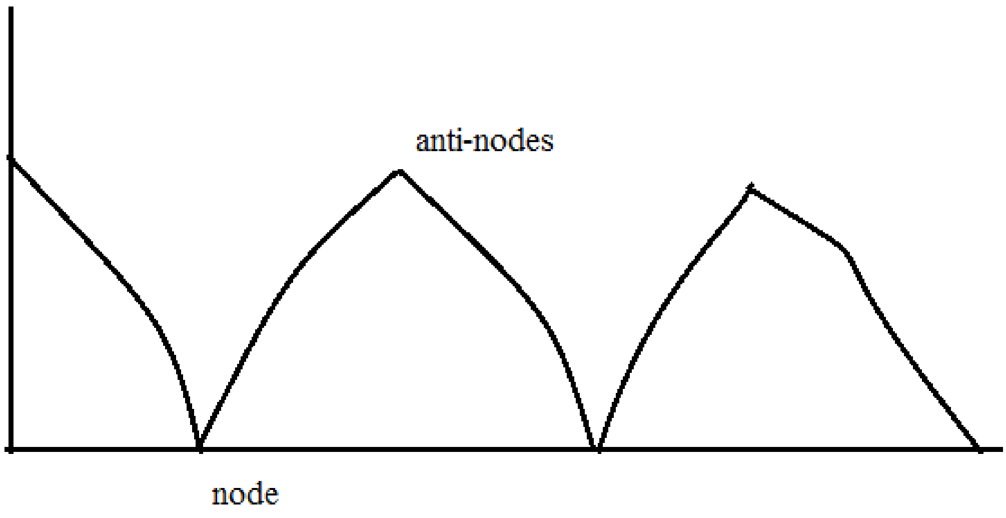
In Fig 2.2.6,

$$R_{dc} = \frac{1}{\pi\sigma}\left[\frac{1}{a^2} + \frac{1}{c^2 - b^2}\right]$$

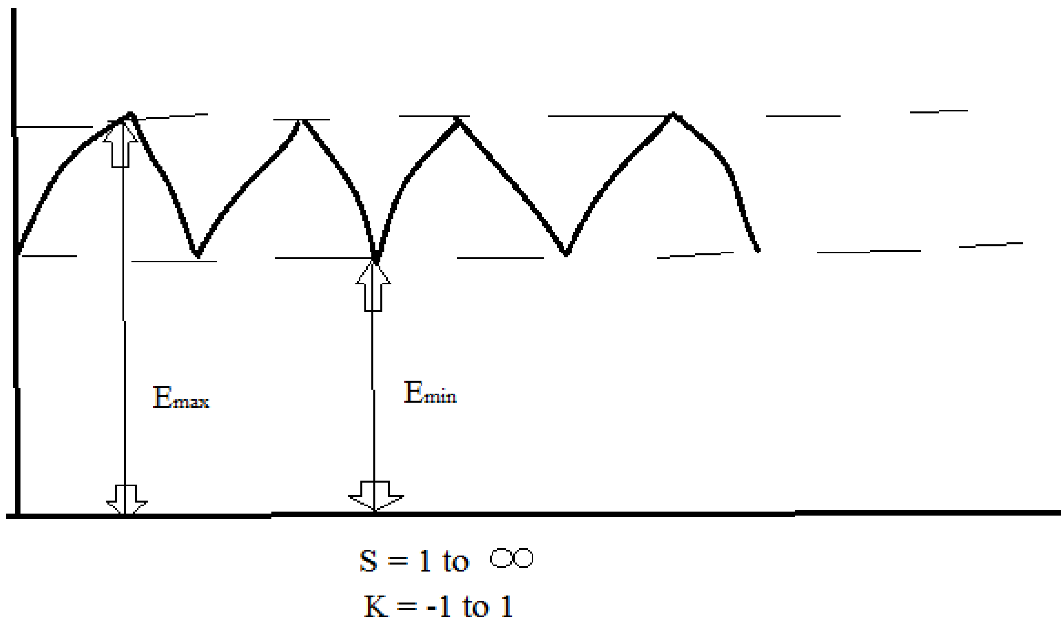
$$R_{ac} = \sqrt{\frac{\mu_c f}{4\pi\sigma}}\left(\frac{1}{a} + \frac{1}{b}\right)$$

All the parameters of R, L, G, and C will change with respect to weather condition.

2.3 STANDING WAVES:



(a)



(b)

Fig : 2.3.1 (a) Standing waves on a dissipation less line terminated in a load not equal to R_O; (b) Standing waves on a line having open-or short-circuit termination.

If voltage magnitudes are measured long the length of a line terminated in a load other than R_O, the plotted values will appear as in Fig 2.3.1 (a). Fig 2.7 (b) is drawn for a resistive load of value not equal to R_O, and Fig 2.3.1 (b) is the case

For either open or short circuit.

When the load is terminated properly with characteristic impedance the distribution of voltage and current along the line consists of maximum and minimum values of voltage and current. These values are called as standing waves.

NODES:

The points along the line were magnitude of current and voltage is zero are called nodes.

ANTINODES:

The points along the line where the magnitude of voltage and current are maximum then it is called anti-modes or Zoops.

STANDING WAVE RATIO:

The ratio of maximum to minimum magnitude of voltage or current on a line having standing waves is called standing wave ratio.

SWR = S = $\frac{|E_{max}|}{|E_{min}|} = \frac{|I_{max}|}{|I_{min}|}$

VSWR = voltage standing wave ratio

ISWR = current standing wave ratio

When the line is not terminated properly standing wave are produced. Then the total power absorption is not possible.

The ratio of E_{max} to E_{min} is referred to as voltage standing wave ratio (VSWR).

The ratio of I_{max} to I_{min} is referred to as current standing wave ratio (ISWR).

But is practice ISWR calculation is not used. Hence, practically VSWR calculation will be done. VSWR is nothing but SWR. Theoretically the value of's' lies b/w 1 to ∞

RELATION SHIP BETWEEN STANDING WAVE RATIO AND REFLECTION COEFFICIENT:

In the high frequency transmission line at high frequencies reflections takes place.

The incident wave amplitude is E^+ and reflected wave magnitude is E^-

If both of them in phase then it will be added and becomes E_{max}

If it is out of phase then it will be subtracted and becomes E_{min}

$E_{max} = E^+ + E^-$

$E_{min} = E^+ - E^-$

SWR $= \frac{|E_{max}|}{|E_{min}|}$

SWR $= \frac{|E^+ + E^-|}{|E^+ - E^-|}$

SWR $= \frac{E^+|1 + \frac{E^-}{E^+}|}{E^+|1 - \frac{E^-}{E^+}|}$

Wit,

$\frac{E^-}{E^+} = K$

S $= \frac{1+|k|}{1-|k|}$

(1 – K) S= 1 + K

S – KS = 1+K

S = 1+ K + KS

S = 1 + K (1+S)

S – 1 = K (1 + S)

K $= \frac{S-1}{S+1}$

RELATION SHIP BETWEEN STANDING WAVE RATIO AND MAGNITUDE OF REFLECTION COEFFICIENT:

The voltage at a point’s’ away from the receiving end is given by,

E = E_{R} cash (jβs) + $I_R Z_O$ sin (jβs) ....(1)

Cash$\theta = \frac{e^{\theta} + e^{-\theta}}{2}$

$$\text{Sinha}\theta = \frac{e^{\theta} - e^{-\theta}}{2}$$

$$E = E_R \left(\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right) + I_R Z_O \left(\frac{e^{j\beta s} - e^{-j\beta s}}{2}\right) \quad \text{......(2)}$$

$$E = \frac{E_R}{2} e^{j\beta s} + \frac{E_R}{2} e^{-j\beta s} + \frac{I_R Z_O}{2} e^{j\beta s} - \frac{I_R Z_O}{2} e^{-j\beta s}$$

$$E = \frac{e^{j\beta s}}{2}[E_R + I_R Z_O] + \frac{e^{-j\beta s}}{2}[E_R - I_R Z_O]$$

$$E_R = I_R Z_R$$

sub E_R value in above equ....

$$E = \frac{e^{j\beta s}}{2}[I_R Z_R + I_R Z_O] + \frac{e^{-j\beta s}}{2}[I_R Z_R - I_R Z_O]$$

$$E = \frac{I_R e^{j\beta s}}{2}[Z_R + Z_O] + \frac{I_R e^{-j\beta s}}{2}[Z_R - Z_O]$$

$$E = \frac{I_R e^{j\beta s}}{2}[(Z_R + Z_O) + \frac{e^{-j\beta s}}{e^{j\beta s}}(Z_R - Z_O)]$$

$$E = \frac{I_R e^{j\beta s}}{2}[(Z_R + Z_O) + e^{-j2\beta s}(Z_R - Z_O)]$$

$$E = \frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 + e^{-j2\beta s}\left(\frac{Z_R - Z_O}{Z_R + Z_O}\right)]$$

$$E = \frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 + e^{-j2\beta s} . k]$$

$$\left(k = \frac{Z_R - Z_O}{Z_R + Z}\right)$$

$$E = \frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 \angle 0° + 1 \angle -2\beta s \, |k| \angle \emptyset]$$

$$E = \frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 \angle 0° + |k| \angle \emptyset - 2\beta s]$$

i) <u>At E_{max} :</u>

$$\emptyset - 2\beta s = 0$$

$$E_{max} = \frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 \angle 0° + |k| \angle 0°]$$

$$E_{max} = \frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 + |k|]$$

ii) <u>At E_{min} :</u>

$$\emptyset - 2\beta s = \pi = 180°$$

$$E_{min} = \frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 \angle 0^\circ + |k| \angle \pi]$$

$$E_{min} = \frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 - |k|]$$

We know that,

$$S = \frac{E_{max}}{E_{min}}$$

$$S = \frac{\frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 + |k|]}{\frac{I_R e^{j\beta s}}{2}(Z_R + Z_O)[1 - |k|]}$$

$$S = \frac{1 + |k|}{1 - |k|}$$

S (1-k) = 1+k

S - KS = 1 +

k S - 1 = k +

KS

S - 1 = k (1 + S)

$$k = \frac{S-1}{S+1}$$

INPUT IMPEDANCE FOR THE DISSIPATION-LESS LINE:

In Fig 2.3.2 the voltage and current of a transmission line at a distance 's' from the receiving end is given by,

$E_S = E_R \text{coos}\ \beta s + j\ I_R R_O \sin\beta s$

$I_S = I_R \text{Cos}\ \beta s + j \frac{E_R}{} \sin\beta$

ion less line

The input impedance of the transmission line is given by,

$$z_{in} = z_s = \frac{E_s}{I_s} \qquad \text{...... (1)}$$

$$Z_{in} = Z_S = \frac{E_R cos\beta s + j\, I_R R_O sin\beta s}{I_R cos\beta s + j\, \frac{E_R}{R_o} sin\beta s}$$

$$Z_s = R_o \left[\frac{E_R cos\beta s + jI_R R_o sin\beta s}{I_R R_O cos\beta s + jE_R sin\beta s}\right]$$

We know,

$E_R = I_R Z_R$

Sub E_R value in above equal,

$$z_s = R_O \left[\frac{I_R Z_R \cos\beta s + j\, I_R R_O \sin\beta s}{I_R R_O \cos\beta s + j\, I_R Z_R \sin\beta s}\right]$$

$$z_s = \frac{R_O I_R \cos\beta s}{I_R \cos\beta s}\left[\frac{Z_R + \frac{jR_O \sin\beta s}{\cos\beta s}}{R_O + \frac{jZ_R \sin\beta s}{\cos\beta s}}\right]$$

$$z_s = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right] \qquad \text{......... (3)}$$

The another method to represent input impedance of the transmission line is given by,

$$z_s = R_O \left[\frac{I_R Z_R \cos\beta s + j\, I_R R_O \sin\beta s}{I_R R_O \cos\beta s + j\, I_R Z_R \sin\beta s}\right]$$

$$z_s = \frac{I_R R_O}{I_R}\left[\frac{Z_R \cos\beta s + jR_O \sin\beta s}{R_O \cos\beta s + j\, Z_R \sin\beta s}\right]$$

$$Cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$Sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$z_s = R_O \left[\frac{Z_R\left(\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right) + jR_O\left(\frac{e^{j\beta s} - e^{-j\beta s}}{2j}\right)}{R_O\left(\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right) + jZ_R\left(\frac{e^{j\beta s} - e^{-j\beta s}}{2j}\right)}\right]$$

$$Z_s = \frac{2R_O}{2}\left[\frac{Z_R e^{j\beta s} + Z_R e^{-j\beta s} + R_O e^{j\beta s} - R_O e^{-j\beta s}}{R_O e^{j\beta s} + R_O e^{-j\beta s} + Z_R e^{j\beta s} - Z_R e^{-j\beta s}}\right]$$

$$Z_s = R_O\left[\frac{e^{j\beta s}(Z_R + R_O) + e^{-j\beta s}(Z_R - R_O)}{e^{j\beta s}(Z_R + R_O) - e^{-j\beta s}(Z_R - R_O)}\right]$$

$$Z_s = R_O \frac{e^{j\beta s}(Z_R + R_O)}{e^{j\beta s}(Z_R + R_O)}\left[\frac{1 + \frac{e^{-j\beta s}(Z_R - R_O)}{e^{j\beta s}(Z_R + R_O)}}{1 - \frac{e^{-j\beta s}(Z_R - R_O)}{e^{j\beta s}(Z_R + R_O)}}\right]$$

$$Z_s = R_O\left[\frac{1 + K\ e^{-j2\beta s}}{1 - K\ e^{-j2\beta s}}\right]$$

$$Z_s = R_O\left[\frac{1 + |K|\ 1\angle\emptyset . 1\ \angle -2\beta s}{1 - |K|\ 1\angle\emptyset . 1\ \angle -2\beta s}\right]$$

$$Z_s = R_O\left[\frac{1 + |K|\ \angle\emptyset - 2\beta s}{1 - |K|\ \angle\emptyset - 2\beta s}\right]$$

CONDITION FOR z_{max}:

The input impedance will be maximum when both incident and reflected waves are in phase.

$\emptyset - 2\beta s = 0$

$\emptyset = 2\beta s$

$$S = \frac{\emptyset}{2\beta}$$

$Z_{max} = R_o[1 + |K|\lfloor 0^0]$

$$Z_{max} = R_0\left[\frac{1 + |K|}{1 - |K|}\right]$$

CONDITION FOR z_{min}:

The input impedance will be maximum when both incident and reflected waves are in phase.

$\emptyset - 2\beta s = -\pi$

$\emptyset = -\pi + 2\beta s$

$$\emptyset + \pi = 2\beta S$$

$$S = \frac{\emptyset + \pi}{2\beta}$$

$$s = \frac{\emptyset}{2\beta} + \frac{\pi}{2\beta}$$

$$s = \frac{\emptyset}{2\beta} + \frac{\lambda}{4} \qquad (\lambda = \frac{2\pi}{\beta})$$

$$z_{min} = R_O \left[\frac{1+|\ K|\ \angle -\pi}{1-|\ K|\ \angle -\pi}\right]$$

$$z_{min} = R_O \left[\frac{1-|\ K|}{1+|\ K|}\right]$$

$$z_{min} = \frac{R_O}{s}$$

s = standing wave ratio

$$s = \frac{1+|\ K|}{1-|\ K|}$$

2.4 VOLTAGE AND CURRENT FOR DISSIPATION LESS LINE:

The voltage at any point at distance‘s’ from the receiving end of a transmission line is given by,

$$E = \frac{E_R}{2}\left[\frac{Z_R + Z_o}{Z_R}\right]\left[e^{\sqrt{ZY}S} + \left[\frac{Z_R - Z_o}{Z_R + Z_o}\right] e^{-\sqrt{ZY}S}\right]$$

For dissipation less line,

$\gamma = \sqrt{ZY}$

$\gamma = j\omega \sqrt{LC}\beta$

$= \omega \sqrt{LC}\ \gamma$

$= j\ \beta$

$$E = \frac{ER}{2Z_R}[(Z_R + Z_o) e^{j\beta S} + (Z_R - Z_o) e^{-j\beta S}]$$

$$E = \frac{ER}{2Z_R}[Z_R\ e^{j\beta S} + Z_o\ e^{j\beta S} + Z_R\ e^{-j\beta S} - Z_o\ e^{-j\beta S}]$$

$$E = \frac{ER}{2Z_R}[Z_R\ (e^{j\beta S} + e^{-j\beta S}) + Z_o\ (e^{j\beta S} - e^{-j\beta S})]$$

- $\cos\ \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
- $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$$E = \frac{ER}{Z_R}\left[Z_R \frac{(e^{j\beta S} + e^{-j\beta S})}{2} + Z_o \frac{(e^{j\beta S} - e^{-j\beta S})}{2}\right]$$

$$E = \frac{ER}{Z_R}[Z_R \cos\beta S + jZ_o \sin\beta S]$$

$$E = E_R \cos\beta S + \frac{jZ_oER}{Z_R} S,in\ \beta$$

$E = E_R \text{Cos}\, \beta S + jZ_0 I_R \sin \beta S$

This is the voltage equation for the

less line.

$I = \frac{I_R}{2} [\frac{Z_0 + Z_R}{Z_0}] [e^{\sqrt{ZY}S} - \frac{I_R}{2} [\frac{Z_R - Z_0}{Z_R + Z_0}] e^{-\sqrt{ZY}S}]$

$\gamma = \sqrt{ZY} = j\beta$

$I = \frac{I_R}{2Z_0} [(Z_R + Z_o) e^{j\beta S} - (Z_R - Z_o) e^{-j\beta S}]$

$I = \frac{I_R}{2Z_0} [Z_R e^{j\beta S} + Z_o e^{j\beta S} - Z_R e^{-j\beta} + Z_o e^{-j\beta S}]$

$I = \frac{I_R}{2Z_0} [Z_R (e^{j\beta} - e^{-j\beta S}) + Z_o (e^{j\beta S} + {}^{-j\beta S})]$

$I = \frac{I_R}{Z_0} [Z_R \frac{(e^{j\beta S} - e^{-j\beta S})}{2} + Z_o \frac{(e^{j\beta} + e^{-j\beta S})}{2}]$

$I = \frac{I_R}{Z_0} [Z_0 \text{Cos}\beta S + jZ_R \text{Sin}\, \beta S]$

$I = I_R \cos \beta S + \frac{jZ_R I_R}{Z_0} \text{Sin}\, \beta S$

$E_R = Z_R I_R$

$I = I_R \text{Cos}\, \beta S + \frac{jE_R}{Z_0} \text{Sin}\, \beta S$

This is the current equation for the dissipation less line. We know that

$$Z_o = \sqrt{\frac{L}{C}} = R_o$$

$E = I_R \text{ Cos } \beta S + \frac{jE_R}{R_0} \text{ Sin } \beta S$

These are the voltage and current equation of the dissipation less line.

The above equation represents the voltage in terms of receiving end voltage and current as well as current in terms of receiving end voltage and current.

The voltage and current distribution is the sum of cosine and sine distribution.

Wit,

$\beta = \frac{2\pi}{\Lambda}$

Sub in above equal,

$E = E_R \text{Cos} (\frac{2\pi}{\lambda}) S + jR_0 I_R \text{ Sin} (\frac{2\pi}{\lambda}) S$

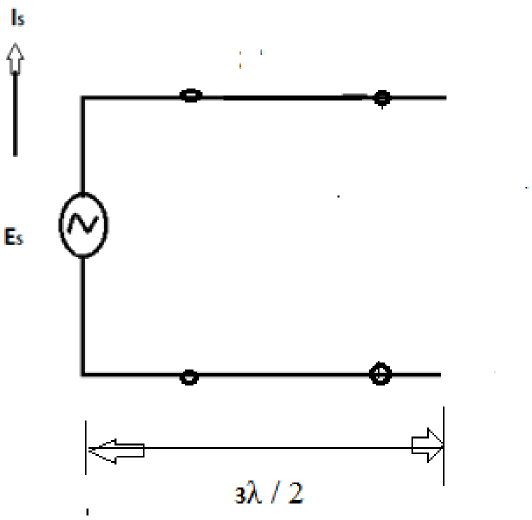
$I = I_R \text{Cos} (\frac{2\pi}{\lambda}) S + \frac{jE_R}{R_0} \text{ Sin} (\frac{2\pi}{\lambda}) S$

CASE (i):

When the line is open circuited at the receiving end. $Z_R = \infty$, $I_R = 0$

$$E_{OC} = E_R \cos\left(\frac{2\pi}{\lambda}\right) S$$

$$I_{OC} = \sin\left(\frac{2\pi}{\lambda}\right) S$$



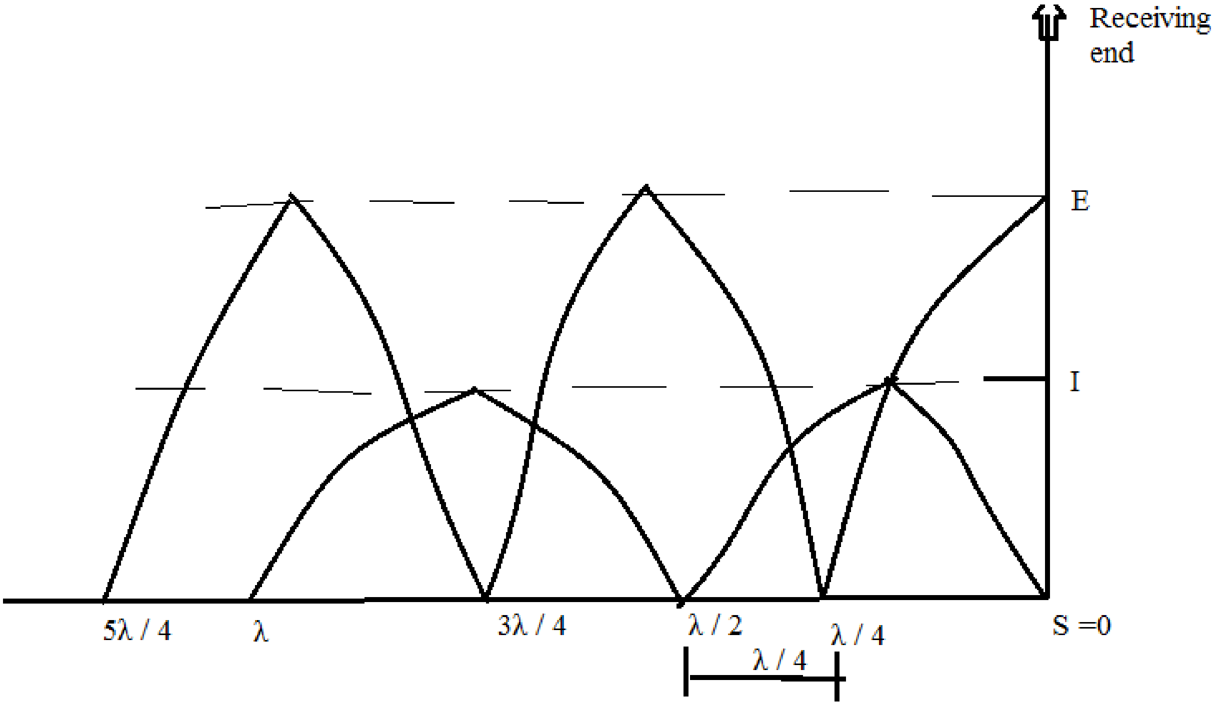


Fig: 2.4.1 Open circuit

The magnitude of voltage and current distribution for a open circuit line for 3 λ / 2 distance is draw in Fig 2.4.1. For every λ / 4 distance the voltage changes from maximum to minimum and vice versa.

CASE (ii):

If the line is short circuited $Z_R = 0$, $E_R = 0$

$$E_{SC} = jR_o I_R \sin\left(\frac{2\pi}{\lambda}\right) S$$

$$I_{SC} = I_R \cos\left(\frac{2\pi}{\lambda}\right) S$$

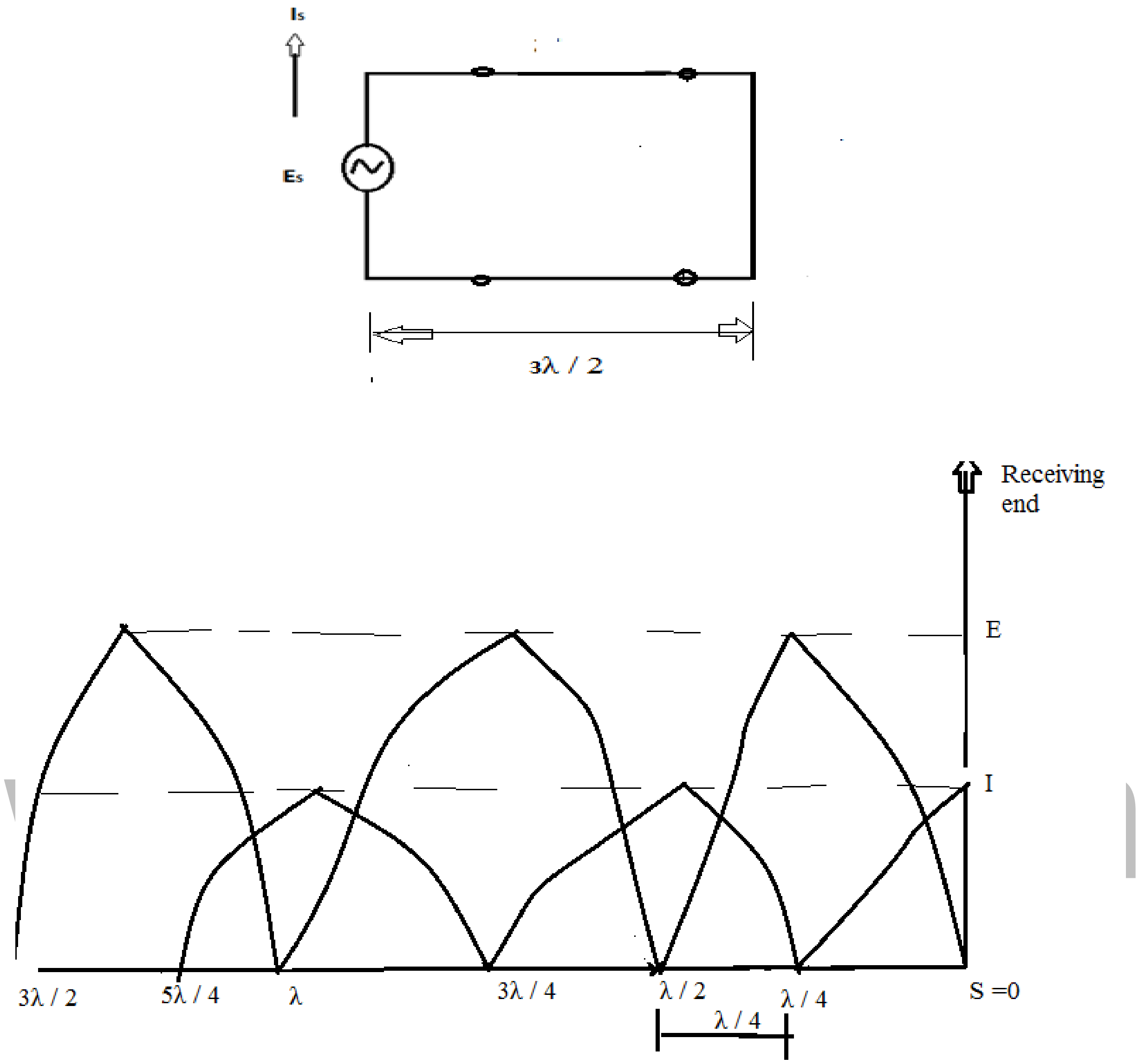


Fig: 2.4.2 Short circuit

The similarity of performance of short-circuited lines to that lines to that of series-resonant or entire sonant circuits may be readily noted by comparison of the curves of Fig 2.4.2.

CASE (iii):

In Fig 2.4.3, when the line is terminated with an impedance $Z_R = Z_O$

Wit,

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

At high frequencies $Z_O = R_O$

$K = \frac{R_O - R_O}{R_O + R_O}$

$K = 0$

This means that, the reflected wave is absent.

The voltage and current in the line is given by,

$E = E_R \, e^{j\,\beta S}$

$I = I_R \, e^{j\,\beta S}$

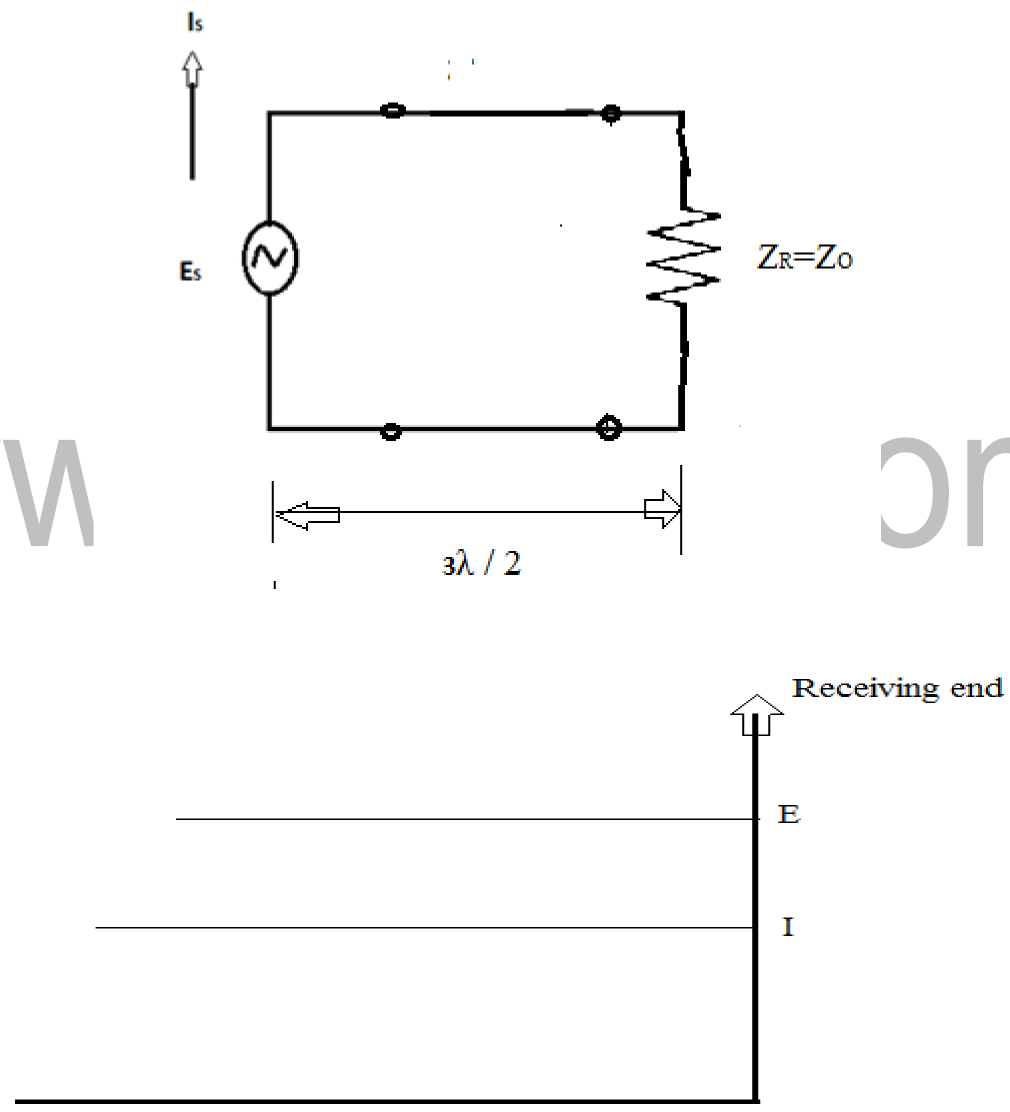


Fig: 2.4.3 Transmission line is terminated with an impedance $Z_R = Z_O$

OPEN AND SHORT CIRCUITED IMPEDANCE:

To find short circuit impedance:

In Fig 2.4.4 shows that the wave is progressing from the receiving end toward the load, the initial value equal to the reflected voltage at the load for open circuit. This is incident wave.

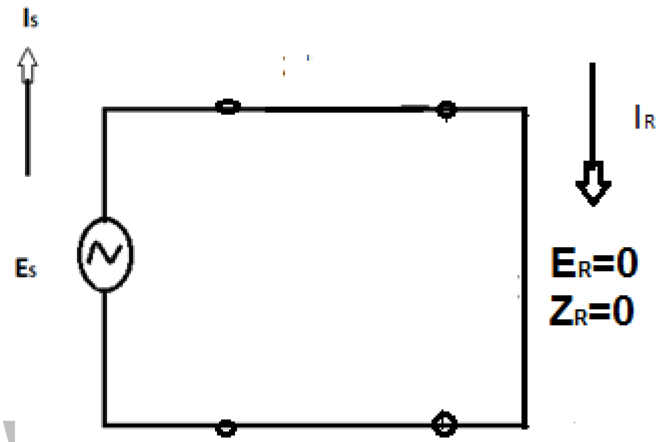


Fig: 2.4.4 Short circuit

At short circuit condition Z_R=0, we know,

At short circuit condition Z_R=0, we know,

$$z_{in} = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$$

Sub $Z_R = 0$,

$$z_{in} = R_O \left[\frac{jR_O \tan\beta s}{R_O}\right]$$

$$z_{in} = jR_O \tan\beta s$$

We know that,

$$\beta = \frac{2\pi}{\lambda}$$

Sub β value in Z_{in},

$$z_{sc} = jR_O \tan\left(\frac{2\pi}{\lambda}\right) s$$

$$R_s + jX_s = jR_O \tan\left(\frac{2\pi}{\lambda}\right) s$$

Equating real and image parts,

$R_s = 0$

$$X_s = R_O \tan\left(\frac{2\pi}{\lambda}\right) s$$

$$\frac{X_s}{R_O} = \tan\left(\frac{2\pi}{\lambda}\right) s$$

Open Circuit impedance:

In Fig 2.4.5 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

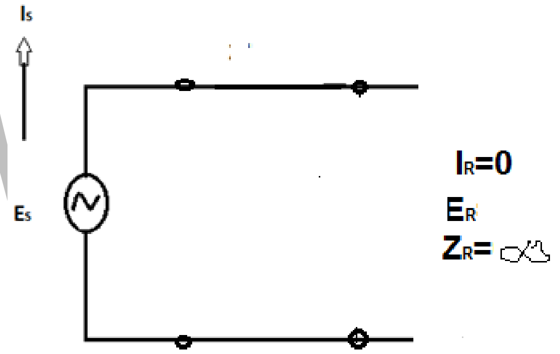


Fig: 2.4.5 Open circuit

$$Z_{in} = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$$

$$Z_{in} = R_O \frac{Z_R}{Z_R}\left[\frac{1 + j\frac{R_O}{Z_R}\tan\beta s}{\frac{R_O}{Z_R} + j\tan\beta s}\right]$$

$Z_R = \infty$ sub in above equ,

$$Z_{oc} = R_O \left[\frac{1}{j\tan\beta s}\right]$$

$$Z_{oc} = \frac{-jR_O}{\tan\beta s}$$

$$Z_{oc} = -jR_O \cot\beta s$$

$z_{oc} = -jR_O \cot\beta s$

$\beta = \frac{2\pi}{\lambda}$

Sub β value in Z_{oc},

$z_{oc} = -jR_O \cot\left(\frac{2\pi}{\lambda}\right) s$

$R_s + jX_s = -jR_O \cot\left(\frac{2\pi}{\lambda}\right) s$

Equating real and imag parts,

$R_s = 0$

$X_s = -R_O \cot\left(\frac{2\pi}{\lambda}\right) s$

$\frac{X_s}{R_O} = -\cot\left(\frac{2\pi}{\lambda}\right) s$