2.2 DYNAMIC CHARACTERISTICS OF A TRANSDUCER:

DYNAMIC SIGNALS:

Dynamic means that something is changing (with time).

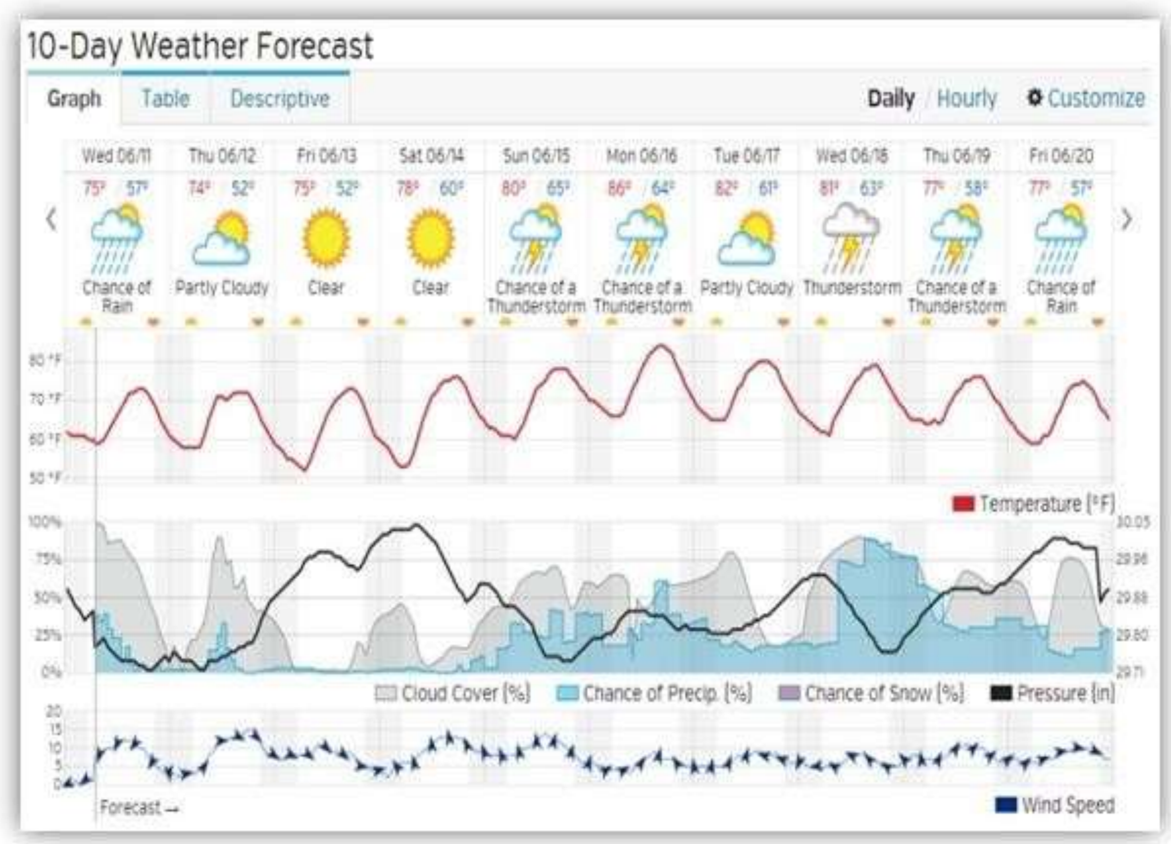


Figure 2.2.1 Example of Dynamic Signals
[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 353]

DYNAMIC CHARACTERISTICS:

Dynamic characteristics relate the response of the device to variations of the measurand with respect to time.

A certain amount of time is required before the measuring instrument can indicate any output based on the input received by the measuring instrument.

The number of parameters required to define the dynamic behaviour of a transducer is decided by the group to which the transducer belongs in the above Fig 2.2.1.

DYNAMIC CHARACTERISTICS OF A MEASUREMENT SYSTEM

Measurement Systems especially in industrial, aerospace and Biological applications are subject to inputs which are not static but dynamic in nature i.e. the inputs vary with time and also the output vary with time.

The Dynamic characteristics of any measurement system described by

- Speed of Response
- Response Time
- Measurement Lag
- Fidelity

SPEED OF RESPONSE:

It is defined as the rapidity with which an instrument or measurement system responds to changes in measured quantity.

RESPONSE TIME:

Time elapsed between an input is applied and the time in which the system gives an output corresponding to some specified percentage.

Example : 95%, of its final value

MEASURING LAG:

An instrument does not immediately react a change in output.

Measuring Lag is defined as the delay in the response of an instrument to a change in a Measuring quantity.

Two types of Measuring Lag:

Retardation type: In this case the response of the instrument begins immediately after a change in the measured has

Time Delay type: In this case the response of the system begins after a “Dead Time” that means after the application of the input.

FIDELITY:

Fidelity of a measurement system is defined as the ability of the system to reproduce the output in the same variation of the input.

In Fidelity measurement system , there is no time lag or Phase shift between the input and output.

TERMINOLOGIES USED IN DYNAMIC CHARACTERISTICS

DYNAMIC ERROR:

Dynamic Error is the difference between the measured value of the instrument changing with time and the value indicated by the instrument if no static error is assumed is shown in Fig 2.2.2.

$\delta(\omega) = M(\omega) - 1$, a measure of the inability of a system to adequately reconstruct the amplitude of the input for a particular frequency.

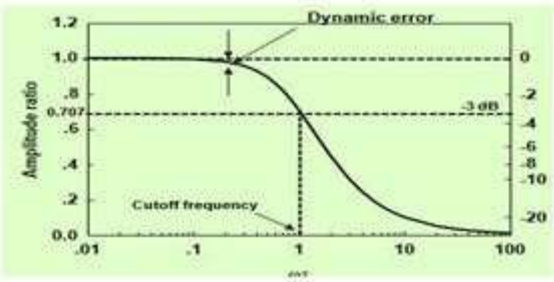


Figure 2.2.2 Dynamic Error

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 353]

TIME CONSTANT:

(1st order system) The time for the output to change by 63.2% of its maximum possible change is shown in Fig 2.2.3.

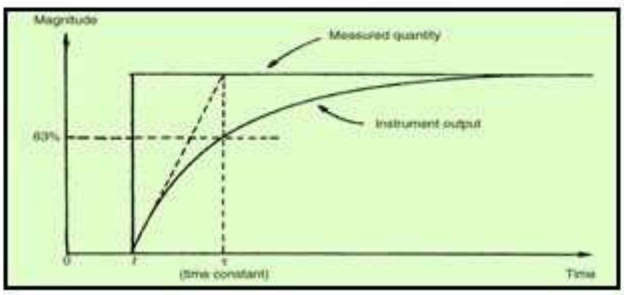


Figure 2.2.3 Time Constant

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 353]

RISE TIME:

Time taken for the output to rise to some specified percentage of the steady-state output. Often the rise time refers to the time taken for the output to rise from 10% of the steady-state value to 90 of the steady-state value.

SETTLING TIME:

This is the time taken for the output to settle to within some percentage.

Example 2%, of the steady-state value.

OVERSHOOT:

- The overshoot is evaluated as the maximum amount by which moving system moves beyond the steady state position.
- An excessive overshoot is undesirable.
- A typical overshoot response graph can be shown as the response time stated in terms of rise time, peak percentage overshoot and settling time is shown in Fig 2.2.4.

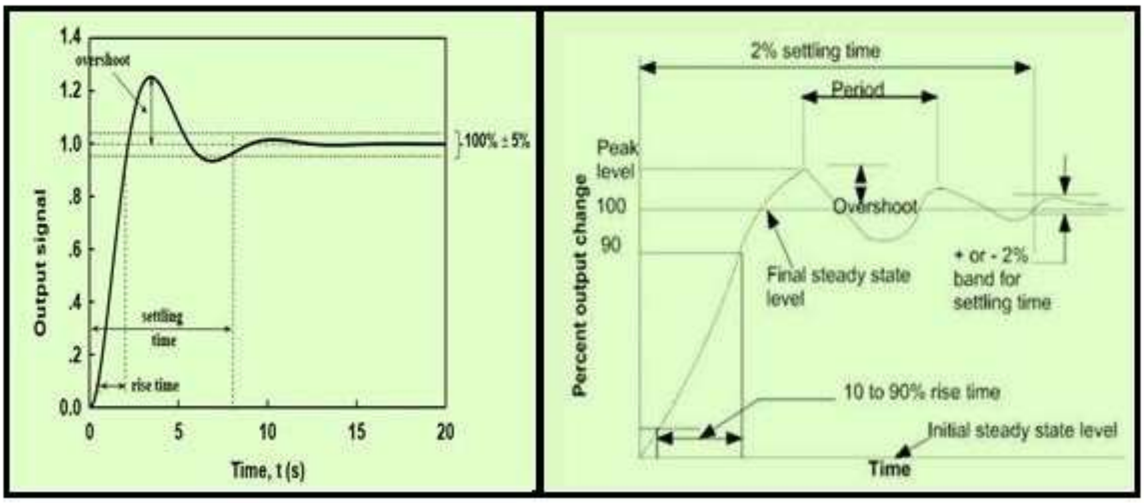


Figure 2.2.4 Typical Overshoot response
[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 353]

TRANSFER FUNCTION:

A simple, concise and complete way of describing the system performance

$$H(s) = \frac{Y(s)}{X(s)}$$

where Y(s) and X(s) are the Laplace Transforms of the input and output respectively.

FREQUENCY RESPONSE:

It describes how the ratio of output and input changes with the input frequency. (sinusoidal input)

BANDWIDTH:

The frequency band over which M(.). 0.707 (-3 dB in decibel unit)

CUTOFF FREQUENCY:

The frequency at which the system response has fallen to 0.707 (-3 dB) of the stable low frequency

$$t_r = \frac{0.35}{f_c}$$

TEST INPUTS:

The transducers are normally subjected to inputs of random nature.

The following test inputs are applied to the transducer to determine its dynamic behaviour,

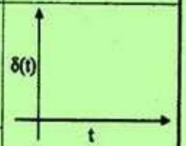
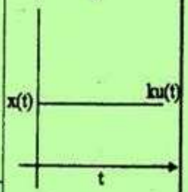
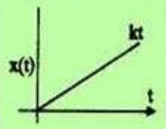
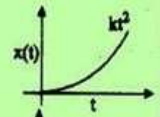
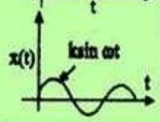
Sl.No.	Name of the input	Time function	Laplace function	Pictorial representation
1.	Impulse input	$x(t) = \delta(t)$ $= 1$ for $t = 0$ $= 0$ for $t \neq 0$	1	
2.	Step input	$x(t) = K$ for $t > 0$ $= 0$ for $t < 0$ If $K = 1$ $x(t) = u(t)$ = unit step	$\frac{K}{S}$	
3.	Ramp input	$x(t) = Kt$ for $t \geq 0$ $= 0$ for $t \leq 0$	$\frac{K}{S^2}$	
4.	Parabolic input	$x(t) = Kt^2$ for $t \geq 0$ $= 0$ for $t \leq 0$	$\frac{2K}{S^3}$	
5.	Sinusoidal input	$x(t) = K \sin wt$ for $t > 0$ $= 0$ for $t \leq 0$	$\frac{Kw}{s^2 + \omega^2}$	

Table 2.2.1 Test Inputs to determine the dynamic behaviour of a Transducer

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 353]

MATHEMATICAL MODEL TO STUDY DYNAMIC RESPONSE:

Ordinary linear differential equation with constant coefficients is the most widely used mathematical model to study dynamic response in the form.

Usually linear time-invariant (LTI) systems are considered.

In an LTI system, input and output, for time t > 0, are related as:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{d^o q_o}{dt^o} = b_m \frac{d^m q_o}{dt^m} + b_{m-1} \frac{d^{m-1} q_o}{dt^{m-1}} + \dots + b_1 \frac{dq_i}{dt} + b_0 q_i \quad \dots\dots(1)$$

where q_i is the measured (input) quantity, q_0 is the output reading and a_o. . . an, b_o. . . b_m. are constants

Using the method of Laplace-transform, equation (1) becomes

$(a_n s^n + a_{n-1} s^{n-1} + \ldots..+ a_1 S + a_o) Q_o (S) = (b_m s^m + b_{m-1} s^{m-1} + \ldots..+ b_1 S + b_o) Q_i (S)$

(or)

$$\frac{Q_o (S)}{Q_i (S)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots..+ b_1 S + b_o}{a_n s^n + a_{n-1} s^{n-1} + \cdots..+ a_1 S + a_o} \qquad \ldots\ldots(2)$$

If we limit consideration to step changes in the measured quantity only, then Equation (1) reduces to:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \ldots.+ a_o \frac{d^o q_o}{dt^o} = b_o q_i \qquad \ldots\ldots(3)$$

So that, $H(s) = \frac{b_o}{a_n s^n + a_{n-1} s^{n-1} + \cdots..+ a_1 S + a_o}$

2.4 SECOND-ORDER TRANSDUCER:

If all coefficients $a_3 \ldots a_n$ other than a_o, a_1 and a_2 in equation (3) are assumed zero, then we get:

$$a_2 \frac{d^2 q_0}{dt} + a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i \qquad \ldots\ldots(1)$$

Applying Laplace transform and rearranging:

$$H(S) = \frac{bo}{a_2 S^2 + a_1 S + a_0}$$

Defining K (static sensitivity), ω_n (un-damped natural frequency) and ξ (damping ratio) as

$$K = \frac{bo}{a_0}$$

$$\omega_n = \sqrt{\frac{a_0}{a_2}}$$

$$\xi = \frac{a_1}{2(\sqrt{a_0 a_2})}$$

Equation (1), in terms of K, ω and ξ, becomes

$$H(S) = \frac{K}{\frac{S^2}{\omega_n^2} + \frac{2\xi S}{\omega_n} + 1} = \frac{K\omega_n^2}{S^2 + 2\xi S\omega_n + \omega_n^2} \qquad \ldots\ldots(2)$$

If equation (2) is solved analytically, the shape of the step response depends on the value of ξ

ξ= 0 is no damping case and constant amplitude oscillations

ξ= 0.2, we is still oscillatory response, but the oscillations gradually die down When ξ is increased further, oscillations reduces and overshoot (see curves (C) and (D))

Overdamped response as shown by curve in Fig 2.4.1. (E)

Output reading creeps up slowly towards the correct reading

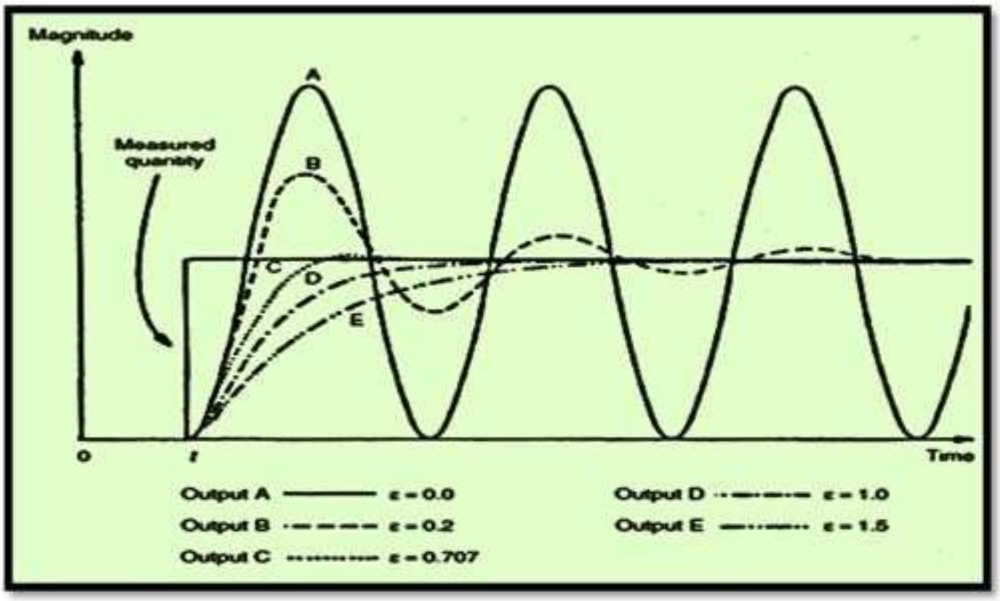


Figure 2.4.1 Response of a Second Order Transducers

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 349]

$\xi = 1$: critically damped

$\xi < 1$: under damped $\xi > 1$:

Over damped

RESPONSE OF A SECOND ORDER TRANSDCUERS FOR STEP INPUT:

When a second-order transducer is subjected to an unit step input,

$X(s) = \frac{1}{s}$

The Laplace transform of the output is given by

$$Y(s) = \frac{K\omega_n^2}{s^2 + 2\xi s\omega_n + \omega_n^2} \cdot \frac{1}{s}$$

$$Y(s) = \frac{K}{s\left[\frac{s^2}{\omega_n^2} + \frac{2\xi s}{\omega n} + 1\right]} \qquad \ldots..(3)$$

Y (t) for different damping conditions is given by

$$\frac{y(t)}{k} = \left[1 - \frac{e^{-\omega_n \xi t}}{\sqrt{1-\xi^2}} \sin\{\omega_n \sqrt{(1-\xi^2)}\, t + \sin^{-1}\sqrt{(1-\xi^2)}\}\right] \text{ for } \xi < 1$$

$$\frac{y(t)}{k} = [1 - (1 + \omega_n t)\, e^{-\omega_n t}] \text{ for } \xi = 1$$

$$\frac{y(t)}{k} = \left[1 - \frac{e^{-\omega_n \xi t}}{\sqrt{1-\xi^2}} \sinh\{\omega_n \sqrt{(\xi^2-1)}\, t + \sinh^{-1}\sqrt{(\xi^2-1)}\}\right] \text{ for } \xi > 1$$

The step responses of second-order transducer for various values of damping ratios are shown in Fig 2.4.2.

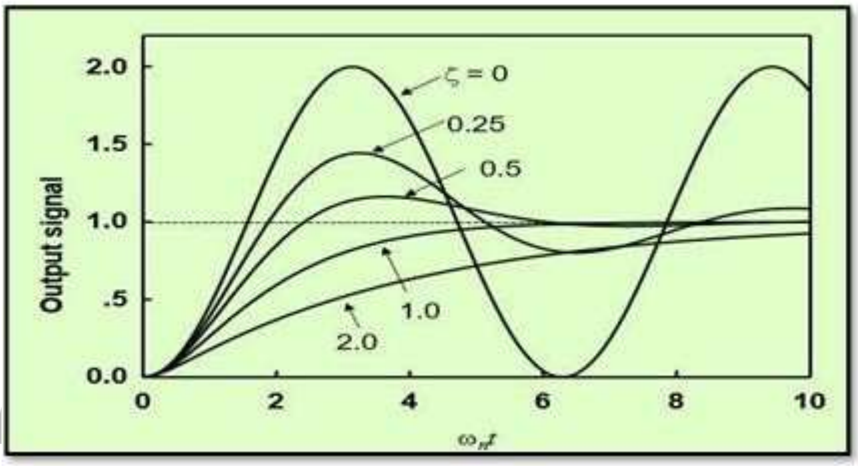


Figure 2.4.2 Response for Step Input of a Second Order Transducers

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 355]

- \Whenever a second-order transducer is suddenly connected to an input, it is equivalent to the application of step input.
- To have a quick indication of the measured values, the time taken for the transducer-response to reach the steady state value should be minimum.
- As the second-order system subjected to step-input-takes infinite time to reach

- the steady-state value, it is customary to define settling time for such systems.
- The settling time is the time taken for the output to reach, and stay within a specified percentage of steady-state value.
- For example, 1Q% settling time means, the time taken for the system output to reach and stay within 90% to 110% of the steady-state value.

RESPONSE OF A SECOND-ORDER TRANSDUCER TO IMPULSE INPUT:

When a second order transducer is subjected to a unit impulse input, the Laplace transform of the output is given by,

$$Y(s) = \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\xi s}{\omega_n} + 1} R(s)$$

$R(s) = 1$

$$Y(s) = \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\xi s}{\omega_n} + 1}$$

$y(t) = L^{-1}\, Y(s)$

$$y(t) = L^{-1} \frac{K\omega_n^2}{s^2 + 2\xi s\omega_n + \omega_n^2}$$

$$y(t) = L^{-1} \frac{K\omega_n^2}{(s + \xi\omega_n)^2 + \omega_n^2(1+\xi^2)}$$

Y(t) can be obtained from the Laplace Transform as follows,

For under damped conditions ($\xi < 1$)

$$y(t) = \frac{k\omega n}{\sqrt{1-\xi^2}} e^{-\omega_n \xi t} \sin \omega_n \sqrt{(1-\xi^2)}\, t$$

For critically damped conditions ($\xi = 1$)

$y(t) = K\omega_n{}^2 t e^{-\omega_n t}$

For over damped conditions (ξ > 1)

$y(t) = \frac{k\omega n}{2\sqrt{\xi^2-1}} e^{-\xi+\sqrt{\xi^2-1}\,\omega_n t} - e^{-\xi-\sqrt{\xi^2-1}\,\omega_n t}$ ——

The response of the second order Transducer for unit impulse input is shown in below Fig 2.4.3.

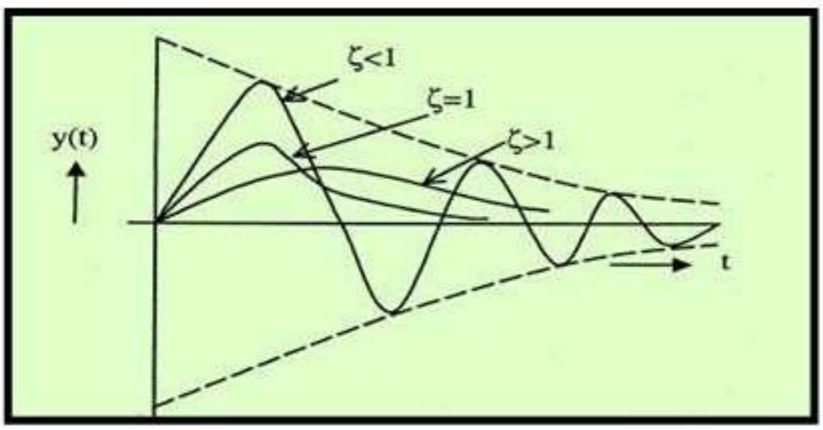


Fig 2.4.3 Response of Impulse input of a second order transducers [Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 358]

RESPONSE OF A SECOND ORDER TRANSDCUERS FOR RAMP INPUT:

Let us consider a second-order transducer subjected to ramp input given by

r(t) = At

$R(s) = \frac{A}{s^2}$

The response of a second-order system for ramp input is given by

$Y(s) = \frac{K\omega n^2}{S^2+2\xi S\omega_n+\omega_n{}^2} \frac{A}{s^2}$

$Y(s) = \frac{KA}{s^2(\frac{S^2}{\omega_n{}^2}+\frac{2\xi S}{\omega n}+1)}$

By partial- fraction,

$$Y(s) = \frac{B_1}{s} + \frac{B_2}{s^2} + \frac{B_3 s + B_4}{s^2 + 2\xi s\omega_n + \omega_n^2}$$

By comparing the coefficients of S, B1,B2, B3, B4 are determined

The output y(t) for [$\xi < 1$] is

$$y(t) = KAt - \frac{KA2\xi}{\omega_n}\left[1 - \frac{e^{-\omega_n \xi t}}{2\xi\sqrt{1-\xi^2}}\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \varnothing\right)\right]$$

Where,

$$\varnothing = \tan^{-1}\frac{2\xi\sqrt{1-\xi^2}}{2\xi^2-1}$$

The ramp response of a II order system is shown in the below Fig 2.4.4.

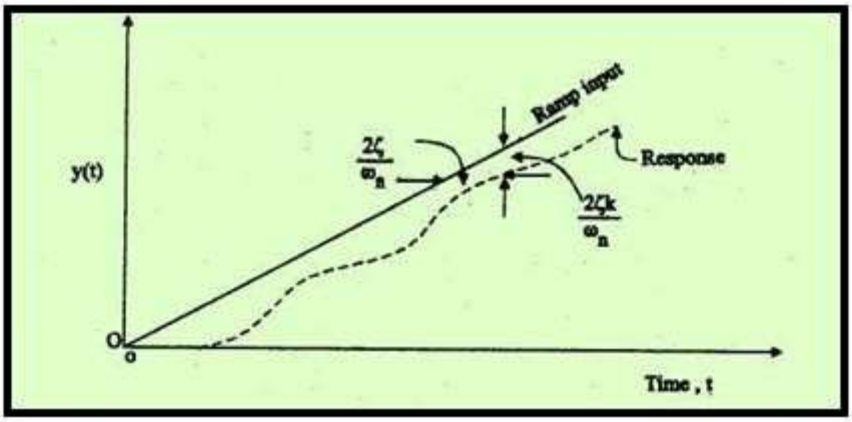


Figure 2.4.4 Response for Ramp Input of a Second Order Transducers

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 368]

- It is seen that there is a steady state error of 2 ξ(K/ ωn).
- The steady state error decreases as ωn increases and is proportional to ξ.
- Under steady state conditions, there is a time lag of 2 ξ/ ωn; in the indication of the true value.

- For a given ωn if ξ . is reduced, oscillations persist for a longer time, but the steady state time lag and steady error becomes less.
- The output y(t) for [ξ = 1] is

$$y(t) = KAt - \frac{KA2}{\omega_n}\left[1 - e^{-\omega_n t}\left(1 + \frac{\omega_n t}{2}\right)\right]$$

The output y(t) for [ξ > 1] is

$$y(t) = KAt - \frac{KA2\xi}{\omega_n}\left[1 + \frac{2\xi^2 - 1 - 2\xi\sqrt{\xi^2-1}}{4\xi\sqrt{\xi^2-1}}\, e^{(-\xi + \sqrt{\xi^2-1})\omega_n t} + \frac{-2\xi^2 + 1 - 2\xi\sqrt{\xi^2-1}}{4\xi\sqrt{\xi^2-1}}\, e^{(-\xi + \sqrt{\xi^2-1})\omega_n t}\right]$$

FREQUENCY RESPONSE OF A SECOND-ORDER TRANSDUCER:

The frequency response of the II-order system is obtained from its transfer function and is given by

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{\left[-\left(\frac{\omega}{\omega_n}\right)^2 + \frac{2j\omega\xi}{\omega_n} + 1\right]}$$

Writing ω/ ωn, the ratio of the frequency of the forcing function to its natural frequency, the response is expressed as,

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{[(1-\eta^2)^2 + 4\xi^2\eta^2]^{\frac{1}{2}}} \angle \emptyset$$

Where,

$$\emptyset = \tan^{-1} \frac{2\xi\eta}{\sqrt{1-\eta^2}}$$

$$= |M| \emptyset$$

The frequency response of a second order transducer is shown in the below Fig 2.4.5,

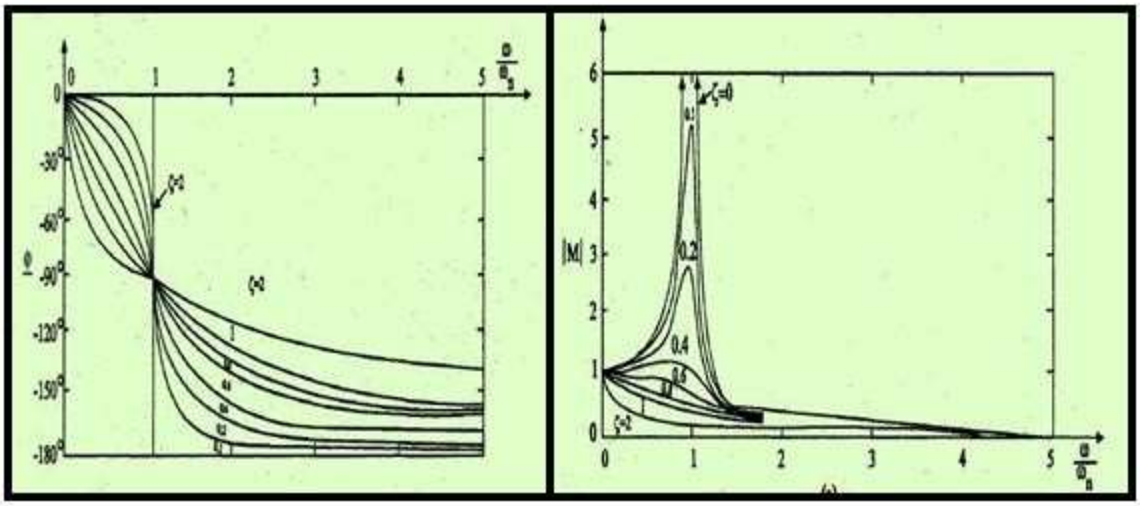


Figure 2.4.5 Frequency Response of a Second Order Transducer

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 371]

For overdamped $\zeta > 1$ or critical damped $\zeta = 1$, there is neither overshoot nor steady- state dynamic error in the response.

In an under damped system $\zeta < 1$ the steady-state dynamic error is zero, but the speed and overshoot in the transient are related is shown in Fig 2.4.6.

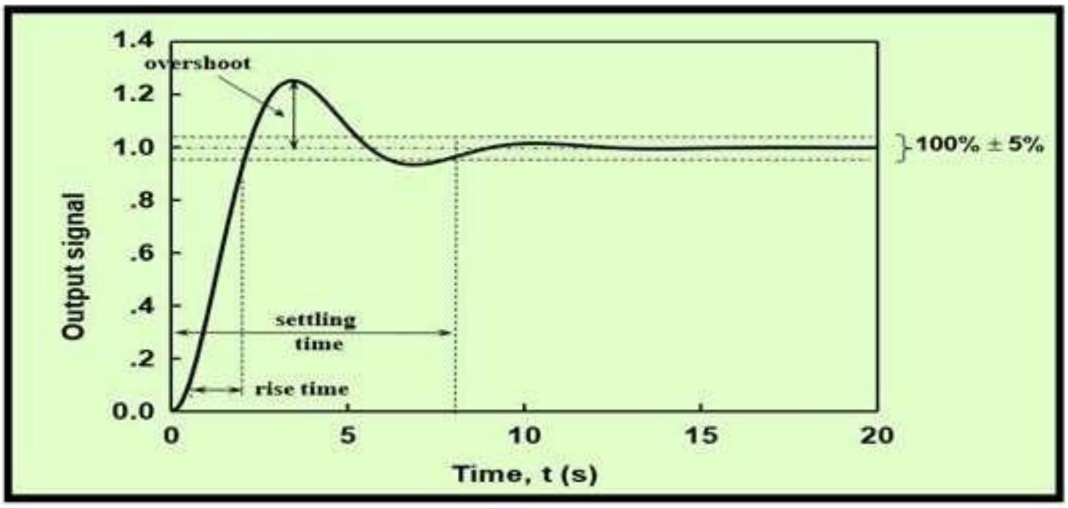


Figure 2.4.6 Typical Response of a Second Order Transducer

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to

their Performance and Design, Page: 375]

Rise time:

$$t_r = \frac{\arctan(\frac{-\omega_d}{\delta})}{\omega_d}$$

Peak time:

$$t_p = \frac{\pi}{\omega_d}$$

Maximum Overshoot:

$$M_p = \exp(-\pi\boldsymbol{\xi}\sqrt{\mathbf{1}-\boldsymbol{\xi}^2})$$

HIGHER ORDER TRANSDUCERS:

- The system which can be described by higher order differential equations is higher order system.
- Many transducers have higher order dynamics which can be described by higher order differential equations.
- For analysis, they can be represented by either first-order or second-order differential equations with some assumptions.
- However, for accurate analysis, the higher order equations can be taken as it is and solved.
- The response of the higher order transducers would be similar to that of second-
- order transducers with a sluggish rise in the initial period.

FREQUENCY RESPONSE:

- The response of a system to a frequency input is called frequency response of a system.
- The, response of a transducer to a frequency input (frequency response of

transducer) is an Important characteristic, since most of the signals can be considered to be a combination of signals of different frequencies.

- The sensitivity of a transducer should be same for all frequencies and phase shift should be either zero or it should increase linearly with frequency.
- That means, the amplitude plot of the frequency response should be flat for all frequencies.
- In general, this plot drops at higher frequencies.
- The term bandwidth is used to quantify the flat useful region of the amplitude plot of the frequency response.
- The bandwidth is defined as the frequency range in which the amplitude ratio is more than 0.707 of the final value which is shown in the below Fig 2.4.7.

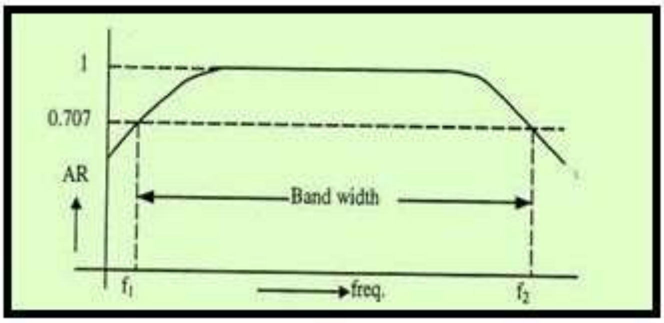


Figure 2.4.7 Bandwidth of a Transducer-Frequency Response

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 379]

- If all the frequency components of the input. lie within the bandwidth of the transducer, then the transducer will-faithfully reproduce the input.
- If' the frequency components of the input signal are outside the bandwidth

of the transducer, then-the output will be distorted.

- If important information is in the frequencies outside the band width, then this information may be missed.

2.1 INTRODUCTION:

- To select the most suitable transducer while designing an instrumentation system.
- To indicate how well the instrument measures the desired input and rejects the spurious (or undesired) inputs.

TYPES OF PERFORMANCE CHARACTERISTICS

The performance characteristics of a transducer are of two types,

- Static Characteristics
- Dynamic Characteristics

STATIC CHARACTERISTICS

In Fig 2.1.1, Static characteristics are the set of performance characteristics that describes the quality of measurement while the measurand are either constant or vary slowly.

Simply, Both input and output are time invariant.

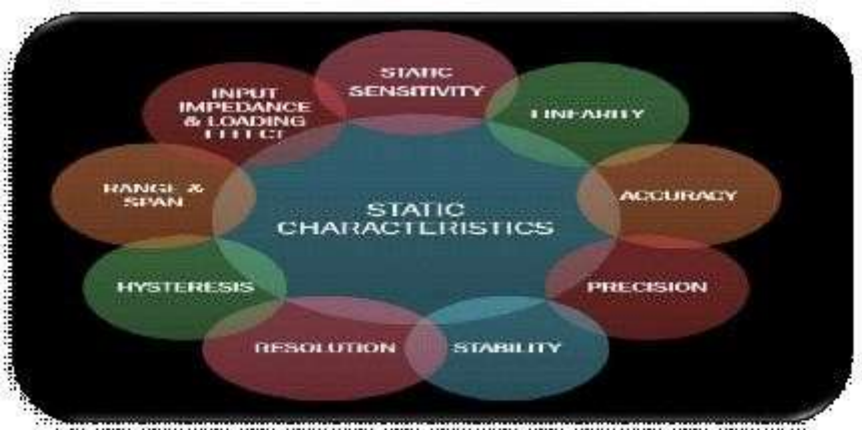


Figure 2.1.1 Static Characteristics Of A Transducer

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 311]

HOW TO OBTAIN THE STATIC CHARACTERISTICS?

All the static characteristics are obtained by one form or another of the process of Static Calibration.

STATIC CALIBRATION:

Static calibration refers to the input-output relations obtained when only one input of the instrument is varied at a time, all other inputs being kept constant.

DETAILED VIEW OF STATIC CHARACTERISTICS

STATIC SENSITIVITY

Static Sensitivity is defined as magnitude of unit change in output per unit change in measurand.

Static Sensitivity of a transducer can be defined as the slope of the static calibration curve is shown in Fig 2.1.2.

CLASSIFICATION OF STATIC SENSITIVITY:

i) Based on Input – Output Relations

- High Sensitivity
- Low Sensitivity

ii) Based on slope of Calibration Curve

- Constant Sensitivity
- Variable Sensitivity

HIGH AND LOW SENSITIVITY:

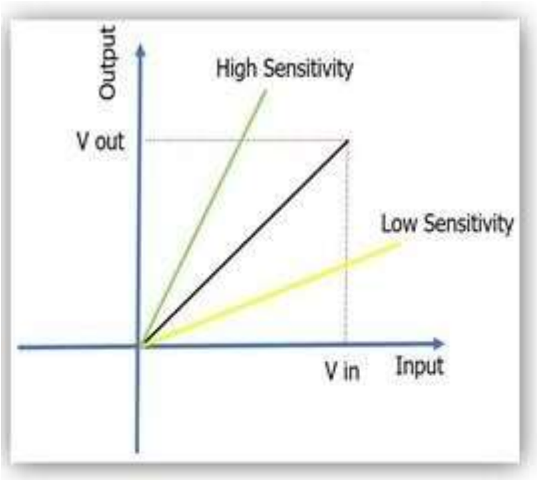


Fig: 2.1.2 Graph of High and Low Sensitivity

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 321]

- For example, a blood pressure transducer with 1 mV per mmHg sensitivity shows an output of 80 mV to 80 mmHg pressure.
- The transducer has **high sensitivity** if a small variation in the blood pressure is indicated in its output as a large voltage variation.
- The transducer has **high sensitivity** if a small variation in the blood pressure is indicated in its output as a large voltage variation.

CONSTANT & VARIABLE SENSITIVITY:

In Fig 2.1.3, Based on the slope of a calibration curve, sensitivity can be defined as follows

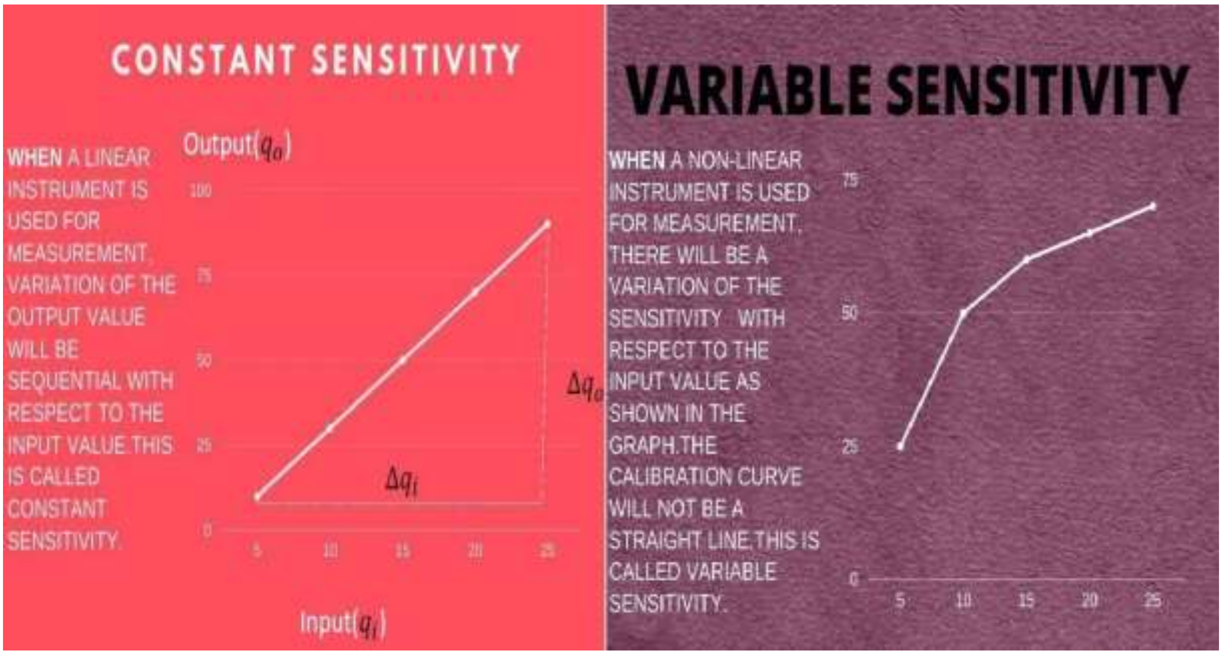


Fig: 2.1.3 Graph of Constant and Variable Sensitivity

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 325]

SENSITIVITY DRIFT:

- Also known as scale factor drift/span drift.
- It defines the amount of drift for a unit change in each environmental parameter to which the instruments are sensitive.
- It is quantified by sensitivity drift coefficients.

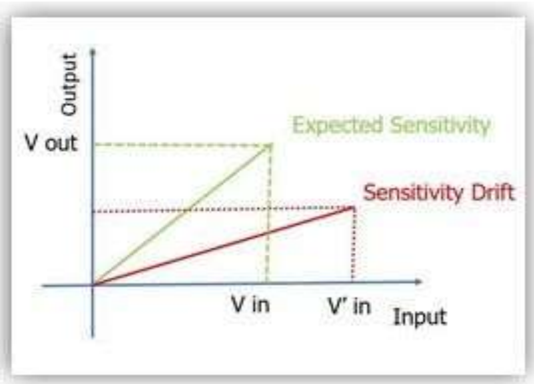


Figure 2.1.3 Graph of sensitivity drift

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 329]

- For **Example**, consider temperature as an input to the pressure gauge.
- Temperature can alter the modulus of elasticity of the pressure-gauge spring, thereby affecting the pressure sensitivity. Here, it is a **modifying input**. This effect is a sensitivity drift or scale-factor drift.

ZERO DRIFT

Zero drift is sometimes known by the alternative term, bias.Zero drift or bias describes the effect where the zero reading of an instrument is modified by a change in ambient conditions. In Fig 2.1.4, This causes a constant error that exists over the full range of measurement of the instrument.

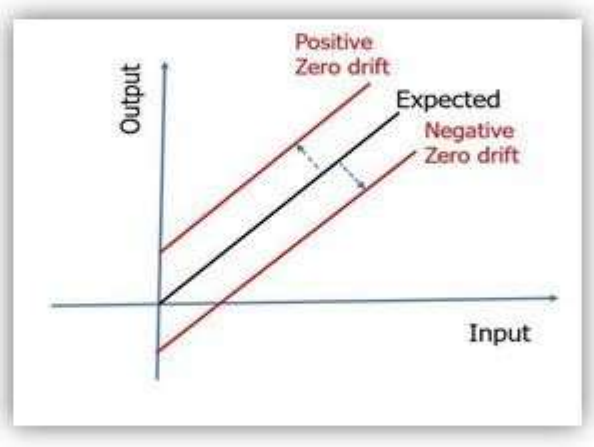


Fig: 2.1.4 Graph of Zero drift

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 333]

- **For Example**, consider temperature as an input to the pressure gauge.
- Temperature can cause a relative expansion and contraction that will

result in a change in output reading even though the pressure has not changed. Here, the temperature is an interfering input. This effect is called a zero drift.

LINEARITY

- Linearity is the transducer characteristic of providing proportional outputs to distinct inputs.
- If to an input Vin1, the output is Vout1, and for Vin2, is Vout2, then to an input (a1Vin1+a2Vin2) the output will be a1Vout1+a2Vout2; a1 and a2 are constants.

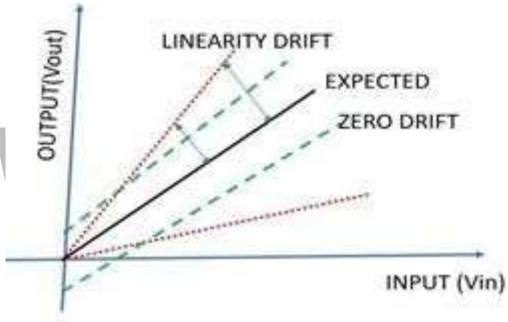


Figure 2.1.5 Graph of Linearity drift

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 337]

- Linearity error is a measure of the maximum deviation of the plotted transducer response from a specified straight line.
- The allowed maximum error of linearity for a transducer is defined by the valuesin the Vout versus Vin region delimited(bounded) by linearity drift and zero drift

ACCURACY

- Accuracy is the closeness/extent to which the result of a reading of a transducer approaches the true value of the quantity being measured.

- In Fig, 2.1.6, Thus, it is a comparison between true and measured values.
- The closer to the real value, the smaller the error and more accurate is the measurement.

Figure 2.1.6 Accuracy of a measurement

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 337]

Accuracy can also be indicated as an error percentage value: Error % = $\frac{yt - ym}{y_t}$

Where,

y_t ⟶ is the true value of the variable,

y_m ⟶ is the transducer output or the measured value of the variable. Accuracy can be expressed in the following ways,

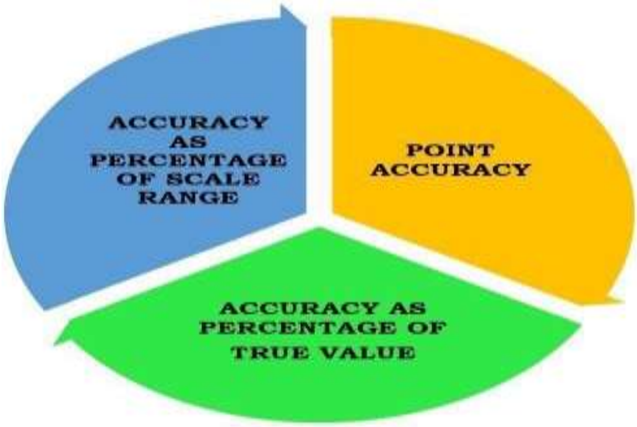


Figure 2.1.7 Ways to Express Accuracy

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 337]

POINT ACCURACY:

- This is the accuracy of the instrument only at one point on its scale.
- The specification of this accuracy does not give any information about the accuracy at other points on the scale.
- In other words, this accuracy does not give any information about the general accuracy of the instrument.

ACCURACY AS 'PERCENTAGE OF SCALE RANGE':

- When an instrument has uniform scale, its accuracy may be expressed in terms of scale range.
- For example, the accuracy of a thermometer having a range of 500 °C may be expressed as ±0.5 percent of scale range.
- This, means that the accuracy of the thermometer when the reading is 500°C is
- ±O.5 percent.

ACCURACY AS 'PERCENTAGE OF TRUE VALUE':

- This type of accuracy of the instruments is determined by identifying the measured value regarding their true value.
- The accuracy of the instruments is neglected up to ±0.5 percent from the true value.

PRECISION:

- The term precision means two or more values of the measurements are closed to each other.
- The value of precision differs because of the observational error.
- The precision is used for finding the consistency or reproducibility of the measurement.
- The conformity and the number of significant figures are the characteristics of the precision.
- The high precision means the result of the measurements are consistent or the repeated values of the reading are obtained.
- The low precision means the value of the measurement varies.

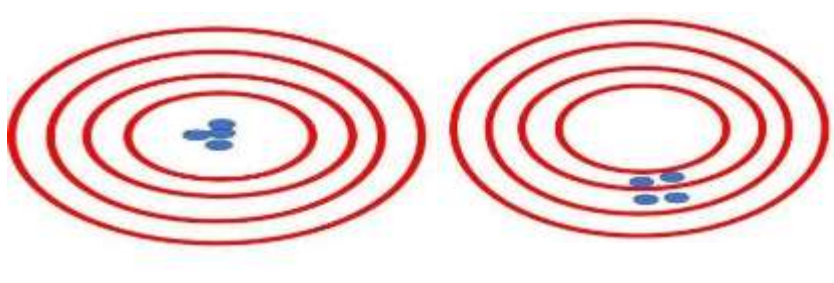


Figure 2.1.8 Ways to Express Accuracy

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 340]

- The high precision means the result of the measurements are consistent or the repeated values of the reading are obtained.
- The low precision means the value of the measurement varies.
- Example – Consider the 100V, 101V, 102V, 103V and 105V are the different readings of the voltages taken by the voltmeter. The readings are nearly close to each other. They are not exactly same because of the error. But as the readings are close to each other then we say that the readings are precise.

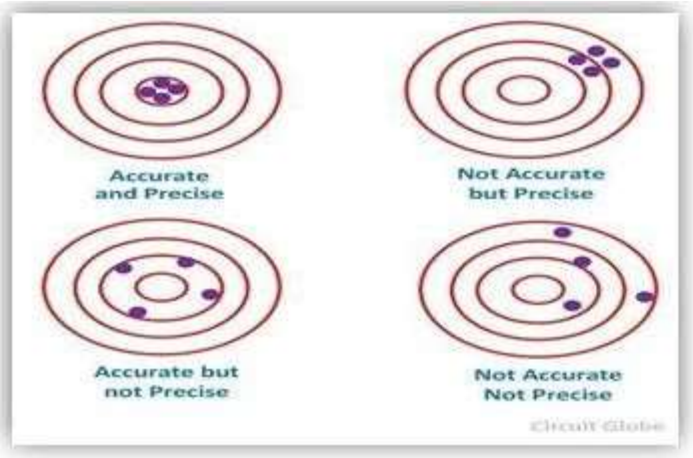


Figure 2.1.9 Accuracy and Precision

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 340]

SIGNIFICANT FIGURE:

- A Significant Figure is a number or digit, which tells how accurate and precise our measurement is. (or)
- The term "significant figures" refers to the number of important single digits (0 to 9 inclusive) in the coefficient of expression in the scientific notation.
- The more the significant figures, the greater the precision of measurement.

EXAMPLES OF SIGNIFICANT FIGURES:

- For example, if a voltage is specified as 230 V its value should be taken as closer to 230 V than to either 231 V or 229 V.
- If the value of voltage is specified as 230.0 V, it means that the voltage is closer to 230.0 V than it is to 230.1 V or 229.9 V.
- In 230 there are three significant figures while in 230.0 V there are four.

BASIC LAWS OF SIGNIFICANT FIGURES:

1) All non zero digits are significant.

For example, in 2345 cm there are four significant figures and in 0.234 there are three significant figures.

If any zero precedes the non-zero digit then it is not significant. The preceding zero indicates the location of the decimal point, in 0.005 there is only one and the number 0.00232 has 3 figures.

2) Zeroes between non zero digits are significant.

For Example, 4.5006 have five significant figures.

3) A trailing zero or final zero in the decimal portion only are significant.

For Example, 0.500 has three significant figures & 86.00 has four significant figures.

INFINITE FIGURES:

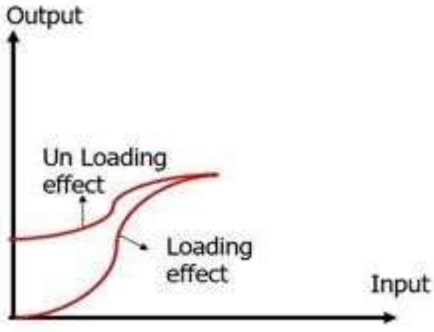
Counting the number of objects for example 5 Cars 10 bikes have infinite figures as these are inexact numbers.

STABILITY

- Stability deals with the degree to which transducer characteristics remain constant over time.
- Zero stability defines the ability of an instrument restore to zero reading after the input quantity has been brought to zero, while other conditions remain the same.

HYSTERESIS

- Transducer output has hysteresis if the relation between the output value and the input variable depends on their previous state.
- In Fig 2.1.10, Hysteresis is a phenomenon which depicts different output effects when loading and unloading ·whether it is a mechanical system or an electrical system.
- Hysteresis is non-coincidence of loading and unloading curves.
- In other words, if the input is monotonically incremented and then decremented, the output values do not coincide to the same input value in the upward and downward curves is shown in Fig 2.1.11.
- For example, the electric energy applied as AC current to the primary coil of an inductive displacement transducer is stored as magnetic energy and dissipated as heat.



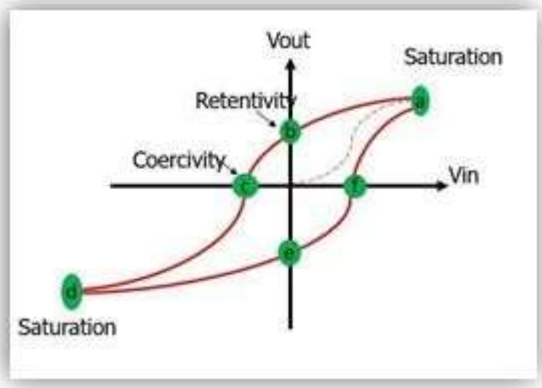
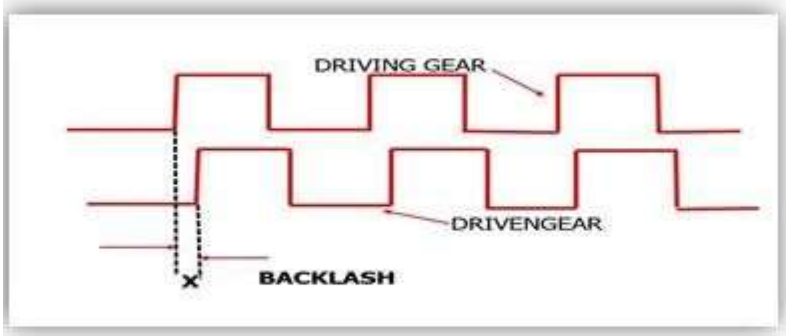


Figure 2.1.10 Hysteresis **Figure 2.1.11 Hysteresis Effects**

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 340]

THRESHOLD

- The threshold is the minimum value of the input below which no output is detected.
- It defines the smallest measurable input, starting from rest. This phenomenon is due to input hysteresis in Fig 2.1.12.
- For Example, In mechanical instruments, the first noticeable measurable change may not occur on account of backlash.



2.1.12 Threshold of an instrument

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 340]

- In the figure which shows a gear train, the driven gear will not move i.e. there will be no noticeable change in the movement of the driven gear unless the driving gear moves through a distance x which is the backlash between the gears.

DEAD TIME:

- Dead time is defined as the time required by a measurement system to begin to respond to a change in the measurand.
- It is basically the time before the instrument begins to respond after the measurand has been changed in Fig 2.1.13.

Figure 2.1.13 Graph of Dead Time

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 340]

DEAD ZONE:

- Dead zone is defined as the largest change of input quantity for which there is no output of the instrument.
- It is basically range of input value for which output is zero.
- Dead zone is also known as Dead band or dead space or neutral zone in

Fig 2.1.14.

- For example, If the input applied to the instrument is insufficient to overcome the friction, it will not move at all. It will only move when the input is such that it produces a driving force which can overcome friction forces.

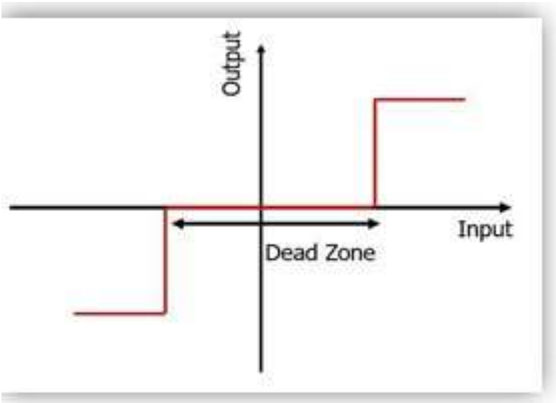


Figure 2.1.14 Graph of Dead Zone

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 345]

RESOLUTION

- When the input to a transducer is slowly increased from some arbitrary (non-zero) value, the change in output is not detected at all until a certain input increment is exceeded.
- The increment is called resolution or discrimination of the instrument.
- Thus the smallest increment in input (the quantity being measured) which can be detected with certainty by an instrument is its resolution or discrimination.
- The resolution of digital instruments is decided by the number of digit used for display.
- For example, the resolution of a four-digit voltmeter with a range of 999.9 volts is 0.1 volt. Whereas for a five-digit voltmeter of the same range, the

resolution would be 0.01 volt.

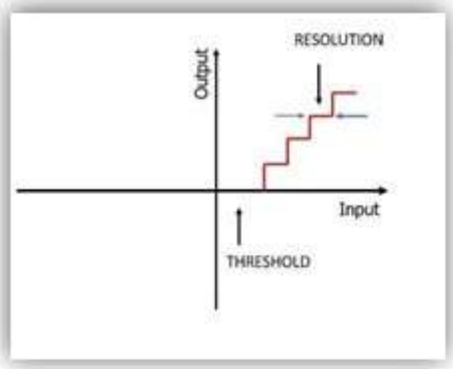


Figure 2.1.15 Threshold & Resolution of an Instrument

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 345]

- From the Fig 2.1.15, It is clear that Resolution defines the smallest measurable input change.
- The threshold defines the smallest measurable input.

RANGE & SPAN

- The range of the transducer is specified as from the low value of input to the high value of input. It is defined as the largest reading that an instrument can read.
- The span of the transducer is specified as the difference between the high and the low limits of the input values.
- Let us consider an instrument that can take reading to the largest value of Y units. It can take smallest of X units

Range of Instrument: Y units or (X-Y range)

Span of Instrument: (Y – X) units.

For Example,

- When an ammeter is specified to be used between 0 and 100 mA,
- Its Range is 0 to 100 mA &
- Its span is 100 mA (i.e. 100 mA - 0 mA=100 mA).
- If a temperature transducer is recommended to be used between 1000 °C and 500°C,
- Its range is specified as 1000 °C to 500°C &
- Its span is 400°C (i.e. 500°C - 100°C = 400°C).

INPUT IMPEDANCE AND LOADING EFFECT

LOADING EFFECT :

- A transducer used for any measurement normally extracts some energy from the measuring medium and thereby disturbs the value of the measured quantity.
- This property is known as the loading effect of the transducer.
- The loading effect is usually expressed in terms of input impedance and stiffness.

IDEAL TRANSDUCER:

- An ideal transducer is one which does not absorb any energy and hence does not disturb the prevailing state of the measured quantity.

INPUT IMPEDANCE:

- The magnitude of the impedance of element connected across the signal source is called "Input Impedance“.
- For Example :The input impedance of a voltmeter is nothing but the input impedance of the deflection coil plus any resistance connected in series.

When the input impedance is high, it draws less current from the circuit.

- The definition of input impedance and stiffness is comprised of two terms called The flow variable (or) across variable
- The effort Variable (or) through variable

Input impedance = Flow variable/ Effort Variable.

- Consider a Voltmeter connected across two nodes in an electrical circuit is shown in 2.1.16.

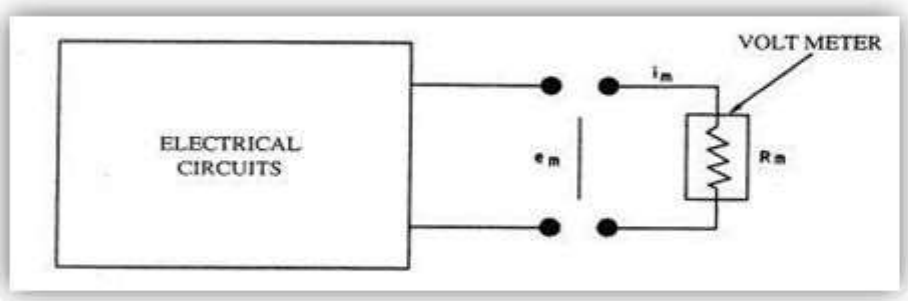


Figure 2.1.16 Input Impedance of a Voltmeter

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 345]

Voltmeter The current drawn by the voltmeter i_m is the flow variable & The voltage across the meter e_m **is the** **e**ffort variable.

Input Impedance, $Z_m = e_m / i_m$

Power extracted from the device, $P_{n,m}$ $= i^m \, e \frac{e^2}{Z_m}$

From the equation, It is clear that a low input impedance device connected across the voltage signal source draws more current and drains more power from signal source than a high input impedance device.

INPUT ADMITTANCE:

When the signal is of the form of current then series input devices, are used. Consider an ammeter connected between source and load is shown in Fig 2.1.17.

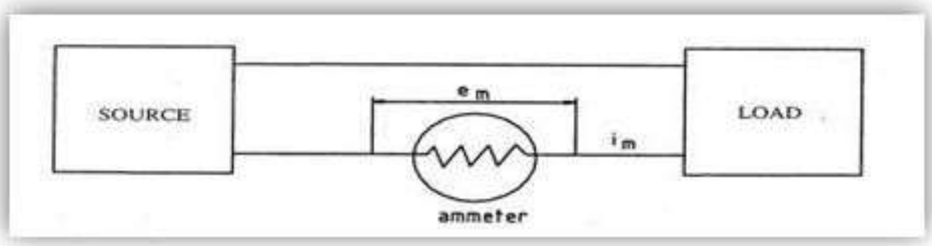


Figure 2.1.17 Input Admittance of a Ammeter

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 345]

Where Y_m is the input admittance.

$$Y_m = \frac{i_m}{e_m} = \frac{1}{Z_m}$$

$$P = i_m e_m = \frac{i^2_m}{Y_m} = Z_m i^2_m$$

From the above equations, it is clear that if the input admittance of the device is high, then the power drawn from the current signal source is small in case of series elements (i.e.) input impedance is low.

Therefore, the loading effects are small when their input admittance is large.

STATIC STIFFNESS:

In case of Mechanical System, Input Impedance and Admittance are not used. Hence the concept of stiffness is introduced is shown in Fig 2.1.18.

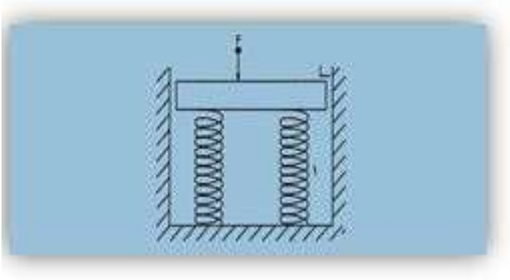


Figure 2.1.18 Stiffness Of A Spring Scale

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 345]

Here a force measuring instrument called spring scale is considered. Force is effort variable and velocity is the flow variable.

S= effort variable/(flow variable)dt

S= force/velocity * dt

S= force/displacement

Here if we calculate mechanical impedance of the instrument, it becomes infinity as velocity of motion becomes zero at steady state. So, we calculate Static stiffness.

2.3 CLASSIFICATION OF TRANSDUECRS BASED ON THE ORDER OF EQUATION:

Based on the order of Equation (3), transducers are classified as are classified as

i. Zero Order Transducers
ii. First Order Transducers
iii. Second Order Transducers

ZERO ORDER TRANSDCUERS:

When all the coefficients $a_1 \ldots a_n$ other than a_o and b_o are assumed to be zero, Equation (3) then degenerates into

$$a_0 q_0 = b_0 q_i \qquad \ldots\ldots(1)$$

Transducers that closely obeys Equation (1) is defined to be a zero-order transducer.

The two constants can be combined to give

$$q_0 = \frac{b_o}{a_0} q_i = K q_i \qquad \ldots..(2)$$

where K= $\frac{b_o}{a_0}$ is static sensitivity

From Equation (2), no matter how qi might vary with time, the output follows it perfectly with no distortion or time lag.

Zero-order instrument represents ideal or perfect dynamic performance is shown in Fig 2.3.1.

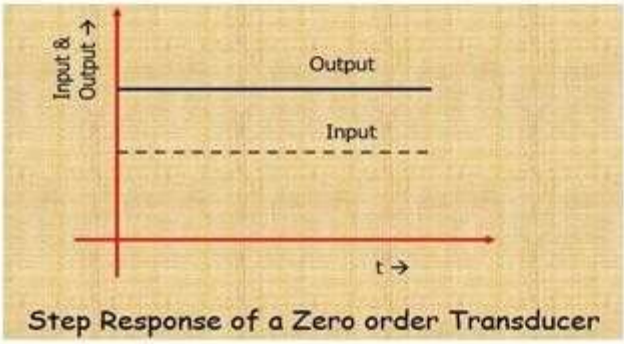


Figure 2.3.1 Step Response of a Zero order Transducer

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 311]

Example of Zero order Transducer : Displacement measuring potentiometer

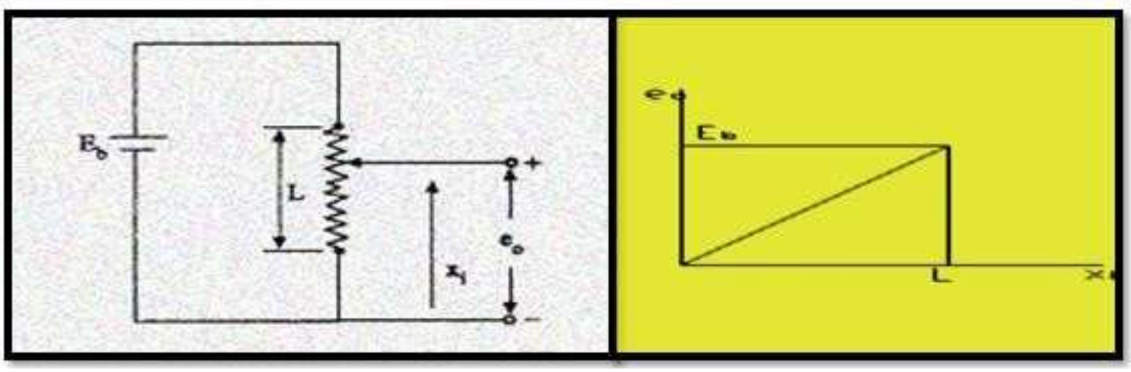


Figure 2.3.2 Displacement Measuring Potentiometer

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 315]

In the above Fig 2.3.2, The potentiometer behaves as a zero-order instrument when it is a pure resistance.

Given linear distribution of resistance along length L, the output voltage e_0 can be written as

$e_0 = \frac{x_i}{L} E_b = K q_i$ …..(3)

The static sensitivity of the potentiometer is K = $\frac{E_b}{L}$

Measurement error $e_m = Kq_i - e_0 = 0$ (ideal)

FIRST ORDER TRANSDUCERS:

When all the coefficients $a_o \ldots a_n$ other than a_o, a_1 and b_o are assumed to be zero, Equation (3) then degenerates into

$$a_1 \frac{dq_o}{dt} + a_0 q_0 = b_0 q_i \qquad \ldots\ldots(4)$$

Using Laplace Transform and Re arranging, We get

$$Q_o = \frac{\frac{b_o}{a_o}}{1+(\frac{a_1}{a_0})s} Q_i \qquad \ldots\ldots(8)$$

Defining $K = \frac{b_o}{a_0}$ as the static sensitivity, $\tau = \frac{a_1}{a_0}$ as the time constant of the system, equation (8) becomes

$$Q_o = \frac{K}{1+\tau s} Q_i \qquad \ldots\ldots(5)$$

Example for First Order Transducer:

Thermocouple used for temperature measurements is an example for first-order transducer.

Let us consider a thermocouple immersed in fluid in a bath is shown in Fig 2.3.3.

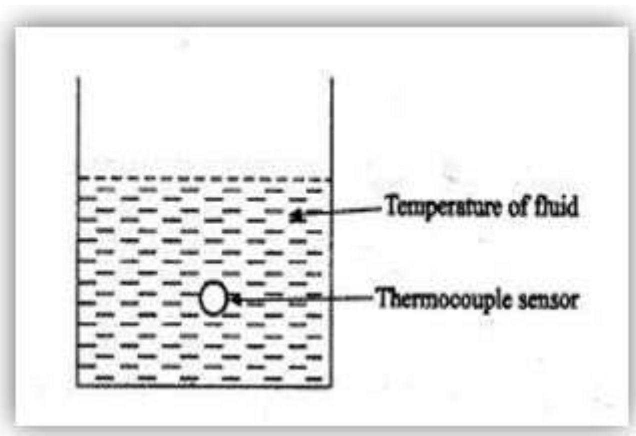


Figure 2.3.3 Thermo couple immersed in a fluid bath

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 319]

The heat balance equation is

$$QA(T_2 - T_1) = MS\frac{dT_1}{dt} \qquad .....(6)$$

Q = Overall heat-transfer coefficient A

T1 = Temperature indicated by the thermocouple

T2 = Temperature of the fluid

M = Mass of the sensing portion of the thermocouple

S = Specific heat of the sensing bead

The transfer function is given by

Heat transferarea

$$\frac{T_1(S)}{T_0(S)} = \frac{K}{1+\tau_S} \quad ; \tau = \frac{MS}{QA} \qquad ....(7)$$

The voltage output of a thermocouple is proportional to the difference in temperature of hot junction and cold junction.

$$V \propto T_1 - T_2 \qquad ....(8)$$

As the cold junction is kept constant at 0°C, the voltage output is proportional to the temperature of the bead at the hot junction. (i.e.)

$$V \propto T_1 ; V = KT_1 \qquad ......(9)$$

where,

V = Thermocouple output in volts

K = probability Constant

The overall transfer function of the thermocouple is given by,

$$\frac{V(S)}{T_2(S)} = \frac{V(S)}{T_1(S)} * \frac{T_1(S)}{T_2(S)} = \frac{K}{1+\tau_S} \qquad ....(10)$$

The equation (14) shows that the thermocouple is a first order transducer.

RESPONSE OF A FIRST ORDER TRANSDCUERS FOR STEP

INPUT:

In Fig 2.3.4, If the First order transducer is excited by, a unit step input function

X(s) = 1/s,

then Y(s) is given by,

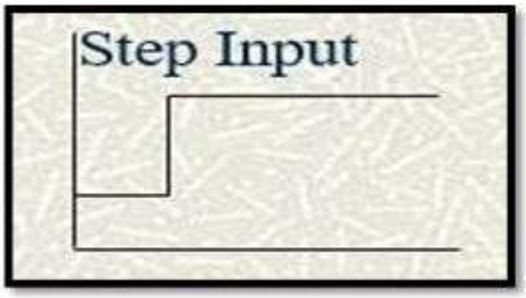


Figure 2.3.4 Response of step Input

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 324]

$$Y(s) = \frac{1}{s} * \frac{K}{1+\tau s}$$

$$y(t) = k\,(1 - e^{-\frac{t}{\tau s}}) \qquad \ldots..(11)$$

Equation (11) reveals the fact that y (t) assumes a final value of k slowly with time.

The speed of response is dependent on the value of τ,

The smaller the value of τ the higher the speed of response is shown in Fig 2.3.5.

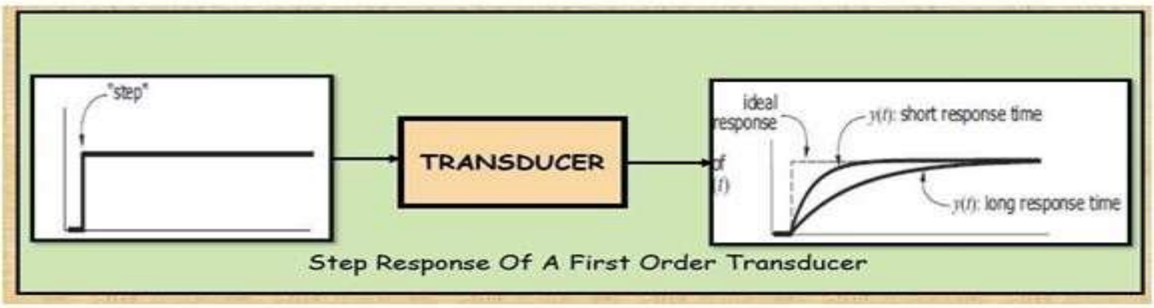


Figure 2.3.5 Step response of a First order Transducer

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 329]

RESPONSE OF A FIRST ORDER TRANSDCUERS FOR RAMP INPUT:

If the Input function is of the unit-ramp type, then the input-output relationship of a I-order transducer is given by

$Y(s) = \frac{1}{s^2} * \frac{K}{1+\tau s}$

$y(t) = k(1 - e^{-\frac{t}{\tau}})$ ……(12)

When Y(t) is solved, it is given by

$y(t) = k(e^{-\frac{t}{\tau}} + t - \tau)$ ……(13)

The response of the ramp input is given by the below graph

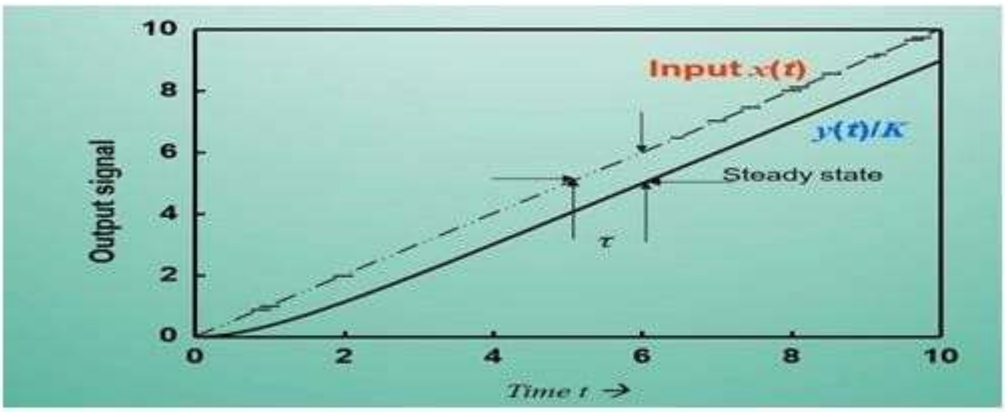


Figure 2.3.6 Response of First Order Transducer for Ramp Input

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 334]

If the transducer is ideal, it should result in an output signal y (t) = Kt, but there is a deviation from this value due to its time constant.

Hence the dynamic error is given by

$y(t) = K(\tau e^{-\frac{t}{\tau}}) - k\tau$ …….(14)

The first term of the net dynamic error dies with time and hence it constitutes transient-error, whereas the second term Kτ becomes the steady state error.

Under steady state conditions, the amplitude of output attains the true value after τ seconds only.

RESPONSE OF A FIRST ORDER TRANSDUCERS FOR IMPULSE INPUT:

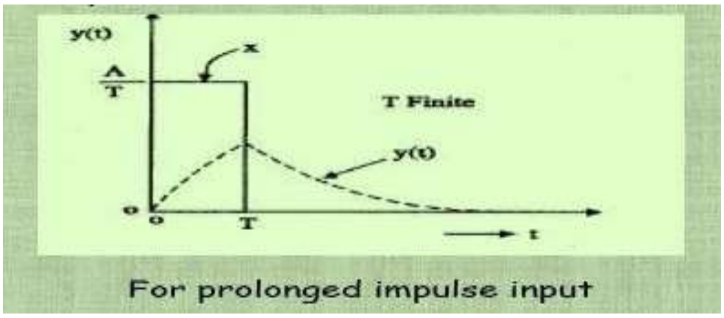
The response for a unit - impulse function is represented by

$$Y(s) = \frac{K}{1+\tau s}$$

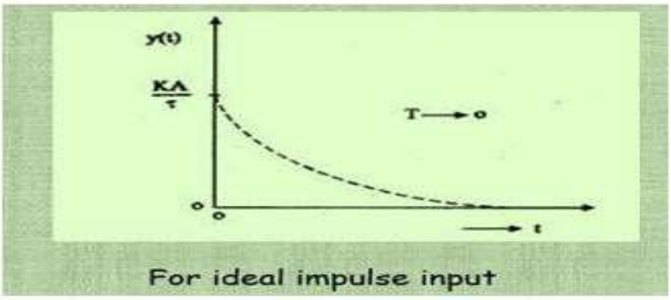
$$y(t) = \frac{k}{\tau}\left(e^{-\frac{t}{\tau}}\right) \qquad \text{.....(15)}$$

If the strength of the impulse is A· units, the response becomes A times the one given by equation. (19).

Response for Impulse input of a First Order Transducers shown in the below graph is shown in Fig 2.3.6.



For prolonged impulse input



For ideal impulse input

Figure 2.3.6 Response for Impulse input of a First Order Transducers [Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 339]

FREQUENCY RESPONSE OF FIRST ORDER TRANSDUCER:

For sinusoidal input functions, the frequency response is determined from the relation

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{1+j\omega\tau}$$

$$= \left|\frac{K}{1+\omega^2\tau^2}\right| \angle \tan^{-1} - \omega\tau$$

$$= |M| \angle \varnothing$$

At zero frequency, i.e., under de excitation, the value of $|M|$ becomes equal to K with $\varnothing = 0$.

Treating the natural frequency of the system, ω_n as given by $\frac{1}{\tau}$ the frequency response curve relating $|M|$ and $\varnothing$ relating $\frac{\omega}{\omega_n} = \omega_\tau$ are shown in the below Fig 2.3.7.

FREQUANCY AND PHASE RESSPONSE OF A FIRST ORDER TRANSDUCER:

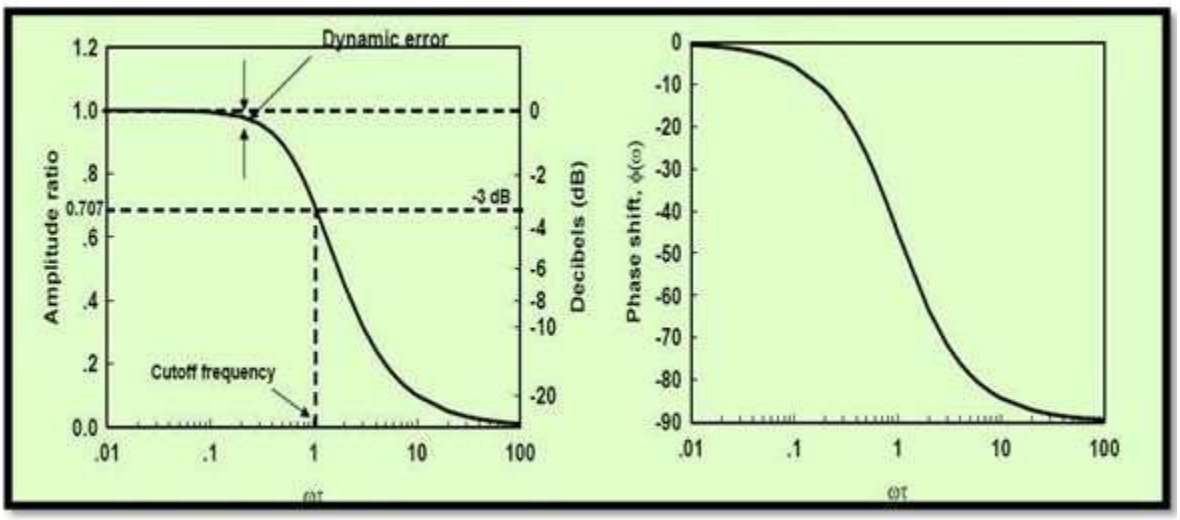


Figure 2.3.7 Frequency Response of a First Order Transducers
[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 345]

Amplitude ratio:

$$M(\omega) = \frac{1}{\sqrt{(\omega\tau)^2+1}}$$

Phase angle:

$$\emptyset(\omega) = -\tan^{-1}(\omega\tau)$$