

## **1.2 ERRORS IN MEASUREMENT:**

A measurement cannot be made without errors. These errors can only be minimized but not eliminated completely. It is important to find out the accuracy of measurement and how different errors have entered in to the measurement. Before that it is essential to know the different errors that can possibly enter in to the measurement.

### **Classification of errors**

- Gross Errors
- Systematic Errors
- Random Errors
- Limiting Errors

### **GROSS ERRORS:**

This type of errors mainly covers human mistakes in reading the instruments (misreading the instruments) making adjustments (incorrect adjustments) and application of instruments (improper application). The computational errors are also grouped under this type of error. The human being may grossly misread the scale. For example due to an oversight, he may read the temperature as  $31.5^{\circ}\text{C}$  while the actual reading may be  $21.5^{\circ}$ . He may transpose the reading while recording. For example he may read  $25.8^{\circ}$  and record  $28.5^{\circ}\text{C}$ . When human beings are involved in measurement, gross errors may be committed.

Although complete elimination of gross errors is probably impossible, one should try to anticipate and correct them. One common gross error frequently encountered involves the improper selection of the instrument. When a voltmeter is used to measure the potential difference across two points in a circuit, the input impedance of the voltmeter chosen should be at least 10 times greater than the output impedance of the measuring circuit. As the output impedance of a circuit is normally not known before hand, the selection of the

voltmeter may not be made correctly, leading to a gross error. The error caused by the improper selection of a voltmeter is shown by the following example.

**Example 1.1:**

A voltmeter reads 20 V in its 40 V scale when connected across an unknown resistor as shown in fig (1.3.1). The resistance of the voltmeter coil is 2000ohms/volt. If the ammeter reads 2mA, calculate (a) apparent value of the unknown resistor. (b) Actual value of the unknown resistor (c) Percentage of Error.

(a) Apparent value of resistance

$$R_A = V/I = 20/2 = 10k\Omega$$

(b) Voltmeter resistance

$$R_V = 2000 \times 40 = 80k\Omega$$

Since, the voltmeter is connected in parallel with the unknown resistor,

$$\frac{R}{A} = \frac{R_X R_V}{R_X + R_V}$$

Where  $R_X$  is the unknown resistance value

(b) Percentage of Error

$$\begin{aligned} \% \text{ of error} &= \frac{\text{Apperant value} - \text{Actual value}}{\text{Actual value}} \times 100 \\ &= \frac{10 - 11.43}{11.43} \times 100 \end{aligned}$$

$$= -12.5 \%$$

This error is due to the appreciable current drawn by the voltmeter which is known as loading effect. Gross errors may be avoided by two means. They are

1. Great care should be taken in reading and recording the data.

$$\begin{aligned} R &= \frac{R_A R_V}{R_V - R_A} \\ R &= \frac{10 \times 10^3 \times 80 \times 10^3}{10^3(80 - 10)} \end{aligned}$$

$$= 11.43 \text{ KOhm}$$

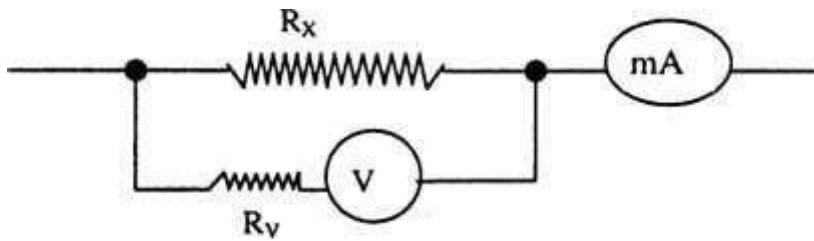


Fig 1.2.1

## **SYSTEMATIC ERRORS**

Systematic errors are due to shortcomings of the instrument and changes in external conditions affecting the measurement. These types of errors are divided into three categories:

- a. Instrumental errors
- b. Environmental errors
- c. Observational errors

### **a. Instrumental errors**

These errors arise due to the following:

- Due to inherent shortcomings of the instrument.
- Due to misuse of the instruments.
- Due to loading effects of instruments.

#### **1. Inherent shortcomings of instruments:**

These errors are inherent in instruments because of their mechanical structure. They may be due to construction, calibration or operation of the instruments or measuring devices. Example: Zero Error.

#### **2. Misuse of instruments:**

Often, the errors caused in measurements are due to the fault of the operator than that of the instrument. A good instrument misused may cause errors. There are some ill practices like using the instrument contrary to manufacturer's instructions and specifications which in addition to producing errors cause permanent damage to the instruments as a result of overloading and overheating.

### **3. Loading effects:**

Errors occur when we use the instrument in an improper manner. For example: a well calibrated voltmeter may give incorrect reading when connected across a high resistance circuit. The same voltmeter, when connected in a low resistance circuit, may give correct reading. This is due to the loading effect of voltmeter.

#### **a. Environmental errors**

Environmental errors are due to changes in the environmental conditions such as temperature, humidity, pressure, electrostatic and magnetic fields. For example: the resistance of a strain gauge changes with variation in temperature.

#### **b. Observational errors**

The observational error may be caused due to parallax. For example: the pointer of a voltmeter rests slightly above the surface of the scale. Thus an error on account of parallax will occur unless the line of vision of the observer is exactly above the pointer. This may be minimized by mirrored scales in the meters.

### **RANDOM (RESIDUAL) ERRORS**

Random errors are unpredictable errors and occur even when all systematic errors are accounted for, although the instrument is used under controlled environment and accurately precalibrated before measurement. Over a period of observation, the readings may vary slightly. The happenings or disturbances about which we are unaware are lumped together and called "Random" or "Residual". Hence the errors caused by these happenings are called Random (or Residual) errors.

Two, three or even more readings should be taken for the quantity under measurement.

### **LIMITING ERRORS (GUARANTEE ERRORS)**

In most instruments the accuracy is guaranteed to be within certain percentage of full scale reading. The manufacturer has to specify the deviations

from the nominal value of a particular quantity. The limits of these deviations from the specified value are defined as limiting errors or Guarantee errors. For example: the nominal magnitude of resistor is  $10\ \Omega$  with a limiting error of 1, i.e. manufacturer guarantees that the value of resistance of the resistor lies between  $9\ \Omega$  and  $11\ \Omega$ .

## **PRECISION AND ACCURACY**

### **Precision**

It is a description of **random errors**, a measure of statistical variability.

Precision is the degree to which several measurements provide answers very close to each other.

It is an indicator of the scatter in the data. The lesser the scatter, higher the precision.

### **Accuracy**

It is a description of **systematic errors**, a measure of statistical bias.

Accuracy describes the nearness of a measurement to the standard or true value, i.e., a highly accurate measuring device will provide measurements very close to the standard, true or known values.

## **1.4 TRANSDUCERS:**

Instrument Society of America defines a sensor or transducer as a device which provides a usable output in response to a specified measurand. Here the measurand is a physical quantity to be measured and the output may be an electrical, mechanical or optical quantity. The words 'sensor' and 'transducer' are often used interchangeably although they are different devices with different characteristics.

- Transducer contains two parts.
- The sensing element
- Transduction element

### **The sensing element**

Sensor senses the difference or change in the environment they are exposed to and gives an output in the same format. Sensor is defined as a device which measures a physical quality (light, sound, space etc.) and converts them into an easily readable format. If calibrated correctly, sensors are highly accurate devices. Not all transducers are sensors but most sensors are transducers.

For example, a thermistor is a type of sensor; it will respond to the change in temperature but does not convert the energy into a different format to what it was originally sensed in.

### **Transduction element**

Transducer converts a specified measurand into usable output using transduction principle.

For example, a properly cut piezoelectric crystal can be called a sensor where as it becomes a to it. So, the sensor is the primary element of a transducer. transducer with appropriate electrodes and input/output mechanisms attached

## **1.9 Classification of transducers**

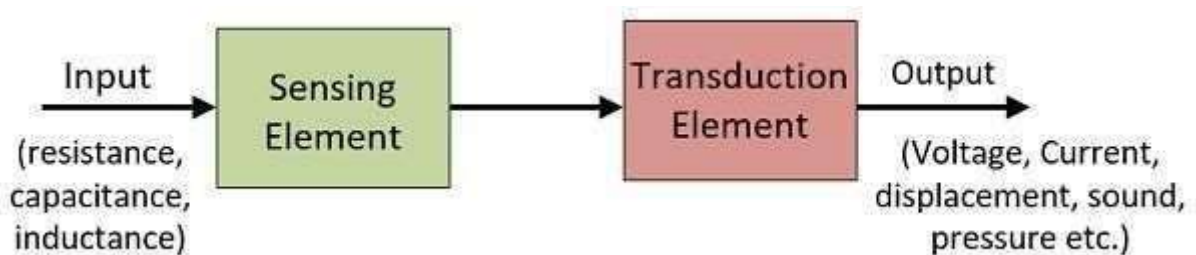
The transducers may be classified based on:

- The physical effect employed .
- The physical quantity measured.
- The source of energy.
- The method of energy conversion.
- The nature of output.
- Transducer and Inverse Transducer.

### Classification based on physical effect (Transduction Principle)

The physical quantity applied as measurand (quantity to be measured) to the transducer causes some physical changes in its element. By this physical effect the transducer converts the physical quantity into electrical quantity.

For example, a change in temperature to be measured causes variation of resistance (physical change) in a copper wire and this effect could, be used to convert temperature into an electrical output. The physical effects commonly employed are:



**Fig 1.4.1 Transducer**

- Variation of resistance
- Variation of inductance
- Variation of capacitance
- Piezo electric effect
- Magnetostrictive effect

- Elastic effect
- Hall effect
- Electromagnetic effect
- Optical effect

### **Variation of resistance**

The transducer whose resistance varies because of the environmental effects is known as the resistive transducer

The resistance of a length of metallic wire is given by

- The resistance of the transducers can vary because of the change in environmental conditions as well as the physical properties of the conductor. Some of the transducers based on this principle are potentiometer, strain gauge, resistance thermometer, carbon microphone, and photoconductive cell.
- The resistance thermometer is based upon thermo resistive effect which is the change in electrical resistivity of a metal or semiconductor due to change in temperature co-efficient of resistivity.
- Carbon microphone works on the principle of change in contact resistance due to applied pressure.
- Photoconductive cell is based on photoconductive effect which is the change in electrical conductivity due to incident light.
- Potentiometer works on the principle of change in resistance due to linear or rotational motion.
- Strain gauge works on the principle of change in resistance due to applied pressure.

### **Variation of inductance**

$$R = \frac{\rho L}{A}$$



Where, R- Resistance in ohm.

P – Resistivity (or specific resistance) of the material in ohmmeter

L – Length of the wire in meter

A – Area of cross section of the wire in m<sup>2</sup>

The inductive transducers work on the principle of the electromagnetic induction. The inductance of the magnetic material depends on a number of variables like the number of turns of the coil on the material, the size of the magnetic material, and the permeability of the flux path. In the inductive transducers the magnetic materials are used in the flux path and there are one or more air gaps. The change in the air gap also results in change in the inductance of the circuit and in most of the inductive transducers it is used for the working of the instrument.

The inductance of a coil is given by

$$L = N \frac{d\phi}{dt}$$

$$L = \frac{N^2 \mu_0 \mu_r A}{l}$$

Where,

L - Inductance in henrys

N -No. of turns

$\mu_0$  = Absolute permeability

$\mu_r$  = relative permeability

A – Area of cross section of the core

l - Length of the magnetic path

As L is a function, of N,, A and I, when anyone of these quantities changes, the inductance changes. This leads to the design of a variable inductance transducer. Some of the transducers based on variation of inductance are induction potentiometer, linear variable differential transformer (LVDT) and synchros.

## Variation of capacitance

The capacitance between two conductor plates is given by

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

Where,

$\epsilon_0$  = Absolute Permittivity

$\epsilon_r$  = Relative permittivity of the separating medium

A - Area of cross-section of the plates

C - Capacitance in farad

As 'C' is a function of A and d, when anyone of these quantities changes, the capacitance varies. This leads to the design of a variable capacitance transducer. The capacitive transducer is used in the measurement of the linear and angular displacement, force, pressure and humidity in gases.

## Piezoelectric effect

When a piezoelectric crystal like quartz or Rochelle salt is subjected to mechanical stress, the mechanical energy is converted into charge or voltage. This is known as piezoelectric effect. The transducer based on this effect is piezoelectric transducer.

## Magneto strictive effect

Magneto striction can be explained as the change in length of a magnetic material produced as a result of magnetization. The material should be Magneto strictive in nature. This phenomenon is known as Magneto strictive Effect. The same effect can be reversed in the sense that, if an external force is applied on a magnetostrictive material, there will be a proportional change in the magnetic state of the material. This property was first discovered by James Prescott Joule by noticing the change in length of the material according to the change in magnetization. He called the phenomenon as

Joule effect. The reverse process is called Villari Effect or Magneto strictive effect. This effect explains the change in magnetization of a material due to the force applied.

Joule effect is commonly applied in magneto strictive actuators and Villari effect is applied in magneto strictive sensors.

### **Elastic effect**

When an elastic member is subjected to mechanical stress it is deformed. The transducer based on this effect is called elastic transducer. Most pressure measuring devices use elastic members for sensing pressure at the primary stage. These elastic members convert the pressure into mechanical displacement which is later converted into an electrical form using a secondary transducer.

### **Hall effect**

The Hall-effect element is a type of transducer used for measuring the magnetic field by converting it into an E.M.F. The direct measurement of the magnetic field is not possible. When a magnetic field is applied to a current carrying conductor at right angles to the direction of current, a transverse electric potential gradient is developed in the conductor. This effect is called as Hall Effect and the transducer based on this effect is called as Hall effect transducer. The Hall effect element is mainly used for magnetic measurement and for sensing the current.

### **Electromagnetic effect**

In this type, the measurand is converted to the voltage induced in a conductor by a change in the magnetic flux in the absence of excitation.

These transducers are active transducers and change in flux is done by the motion between a piece of magnet and electromagnet.

### **Optical effect**

The optical transducer converts light into electrical quantity. They are also called as photoelectric transducers.

The optical transducer can be classified as photo emissive, photoconductive and photovoltaic transducers.

- The photo emissive devices operate on the principle that radiation falling on a cathode causes electrons to be emitted from the cathode surface.
- The photoconductive devices operate on the principle that whenever a material is illuminated, its resistance changes.
- The photovoltaic cells generate an output voltage that is proportional to the radiation intensity. The radiation that is incident may be x-rays, gamma rays, ultraviolet, infrared or visible light.

### **Classification based on physical quantity measured**

The transducers may be classified based on the quantity they measure as follows:

- Temperature transducers
- Pressure transducers
- Flow transducers
- Liquid level transducers
- Force/Torque transducers
- Velocity/Speed transducers
- Humidity transducers
- Acceleration/vibration transducers
- Displacement transducers
- Ultraviolet, infrared or visible light.

### **Classification based on source of energy**

Transducers may be, classified based on source of energy into two types.

- Passive transducer
- Active transducer

### **Passive transducer**

Passive transducer converts a form of energy into another (electrical) by making use of an external source of power. Passive transducer is a device which converts the given non-electrical energy into electrical energy by external force. The capacitive, resistive and inductive transducers are the example of the passive transducer.

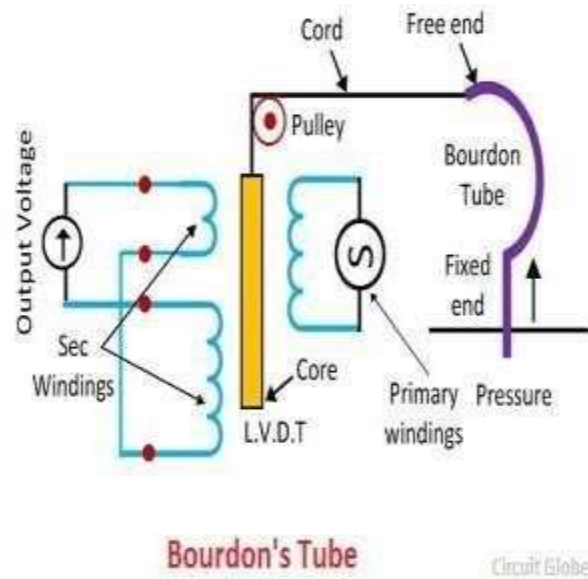
### **Active transducer**

The transducer which does not require the external power source is known as the active transducer. Such type of transducer develops their own voltage or current, hence known as a self-generating transducer. The output signal is obtained from the physical input quantity.

The physical quantity like velocity, temperature, force and the intensity of light is induced with the help of the transducer. The piezoelectric crystal, tachogenerator, thermocouples, photovoltaic cell and electromagnetic transducer are the examples of the active transducers.

### **Classification based on method of energy conversion**

- Primary Transducers
- Secondary Transducers



**Fig 1.4.2 Bourdon's Tube**

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 289]

### Primary Transducers

Some transducers contain the mechanical as well as electrical device. The mechanical device converts the physical quantity to be measured into a mechanical signal. Such mechanical device are called as the primary transducers, because they deal with the physical quantity to be measured

### Secondary Transducers

The secondary transducer converts the mechanical signal into an electrical signal. The magnitude of the output signal depends on the input mechanical signal.

Bourdon's Tube shown in the figure below. The tube act as a primary transducer. It detects the pressure and converts it into a displacement from its free end. The displacement of the free ends moves the core of the linear variable displacement transformer. The movement of the core induces the output voltage which is directly proportional to the displacement of the tube free end.

## **Nature of Output**

- Analog Transducer
- Digital Transducer

The transducer can also be classified by their output signals. The output signal of the transducer may be continuous or discrete.

## **Analog Transducer**

The Analog transducer changes the input quantity into a continuous function. The strain gauge, L.V.D.T, thermocouple and thermistor are the examples of the analogue transducer.

## **Digital Transducer**

These transducers convert an input quantity into a digital signal or in the form of the pulse. The digital signals work on high or low power. Example: Shaft Encoders.

## **Transducer and Inverse Transducer**

**Transducer:** The device which converts the non-electrical quantity into an electric quantity is known as the transducer.

**Inverse Transducer:** The transducer which converts the electric quantity into a physical quantity, such type of transducers is known as the inverse transducer. The transducer has high electrical input and low nonelectrical output. The piezoelectric ultrasonic transducer is an example of the inverse transducer.

## **Selection criteria for transducers**

Transducers are the instruments which converts non-electric signals into an electric signal.

So while selecting any type of transducers for any special purpose, we should think about its specifications or characteristics. Any transducer is based on a simple concept that physical property of a sensor must be altered by an external stimulus to cause that property either to produce an electric signal or to

modulate an external electric signal. Selection criteria of a transducer are based on different factors, such as availability, cost, power consumption, environmental conditions, etc. After considering all these factors we can select a best one for our use.

Selection of the transducer among the many available mainly depends upon:

- Input characteristics.
- Transfer characteristics.
- Output characteristics.
- Life span: It determines how long the selected transducer will work.
- Availability: While selecting a transducer we should think about its availability.
- Cost.
- Stability and reliability.
- Purpose: indication, recording or control.

### **Input characteristics**

This is one of the most important characteristic, while selecting a transducer. By considering input characteristics we can determine, what type of input is needed for that transducer? What is the operating range for that transducer? What is the loading effect on that transducer?

- Type of input
- Operating range
- Loading effect

### **Transfer Characteristics**



Transfer characteristics also plays very important role in selection of transducer. Transfer characteristics means, the effects on the signal when it is being processed. Errors and hysteresis also occurs when the signal is being processed. Following are some major transfer characteristics which we should keep in mind while selecting a transducer for any special purpose:

- Transfer function ( input
- Accuracy and precision output relation)
- Error and hysteresis
- Response of transducer

### **Output Characteristics**

As we all know, while we are doing some work, we always set some goal or aimed for output.

Similarly for our use we should first think about what type of output we required? So here output characteristics play a vital role while selecting a special type of transducer. Some of the output characteristics are summarized below:

- Type of output
- Output impedance
- Useful Range

### **1.3 ERROR ANALYSIS:**

The analysis of the measurement data is necessary to obtain the probable true value of the measured quantity. Any measurement is associated with a certain amount of uncertainty. The best method of analysis is the statistical method. For the statistical analysis, a large number of measurements are required. Also the systematic errors should be small compared with random errors. When temperature of liquid in a tank is to be measured, 10 readings are taken over a period of time by means of a thermocouple. Each of these 10 readings may be different from the others. We cannot find which reading is correct. Here the statistical methods will give the most probable true value of temperature. For statistical methods the terms like arithmetic mean, deviation, mode & median are to be determined.

#### **a. Arithmetic mean**

The most probable value of measured variable is the arithmetic mean of the number of readings taken. The best approximation is made when the numbers of readings of the same quantity are very large. Theoretically, an infinite number of readings would give the best result. But practically, only a finite number of measurements can be made. The arithmetic mean is given by:

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \dots + X_n}{N}$$

$\bar{X}$  = Arithmetic Mean

$X_1, X_2, X_3, \dots, X_n$  = Readings or Variants or

Samples  $N$  = Number of Readings

#### **b. Deviation**

Deviation is departure of the observed reading from the arithmetic mean of the group of readings. Let the deviation of reading  $X_1$  be  $d_1$  and that of reading  $X_2$  is  $d_2$ , etc.

$$d_1 = X_1 - \bar{X}$$

$$d_2 = X_2 - \bar{X}$$

$$d_n = X_n - \bar{X}$$

Average deviation is defined as the average of the modulus of the Individual deviations and is given by

### c. Standard deviation

Another term in the statistical analysis of Random errors is the standard deviation or the root mean square deviation. The standard deviation of an infinite number of data is defined as the square root of the sum of individual deviations squared, divided by the number of readings.

Standard deviation,

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \dots + d^2}{N}}$$

### d. Variance

Variance is another term which is sometimes used in statistical analysis.

This is the square of the standard deviation and is given by

$$V = \sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \dots + d^2}{N}, \quad n > 20 \quad \text{for population}$$

$$V = \sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \dots + d^2}{N}, \quad n < 20 \quad \text{for sample}$$

### e. Median

Median is also used to indicate the most probable value of the measured quantity when a set of readings are taken. When the readings are arranged in the ascending or descending order of magnitude, the middle value of the set is taken as the median in Fig 1.3.1.

For example, the temperature of a bath is noted by ten observers as follows:

75.5°C, 73.7°C, 77.5°C, 75.7°C, 74.8°C, 77.0°C, 75.9°C, 75.3°C,  
73.9°C, 77.5°C.

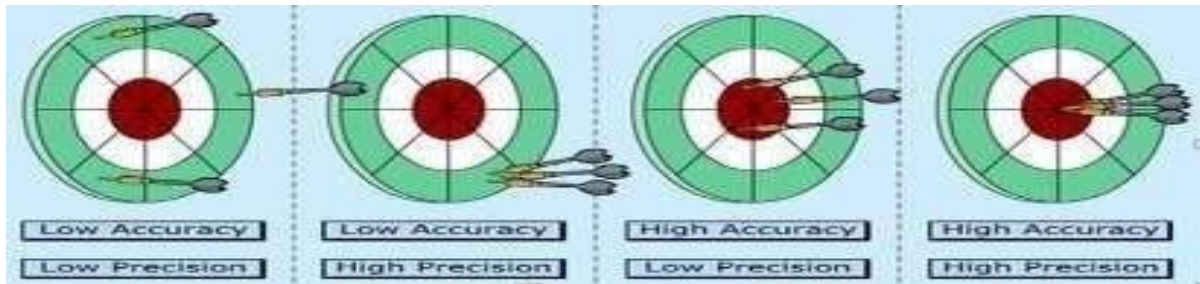
It is rearranged in ascending order as follows:

73.7°C, 73.9°C, 74.8°C, 75.3°C,

75.5°C, 75.7°C, 75.9°C, 77.0°C,  
77.5°C

Now the median is 75.5°C

$$D = \frac{|d_1| + |d_2| + |d_3| + \dots + |d_n|}{n}$$



**Fig 1.3.1 Example of accuracy and Precision**

[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 266]

#### f. Mode

Mode is the value which occurs most frequently in a set of observations and around which other items of the set cluster.

For example, the frequency distribution of a set of 100 observations is given below:

The value of temperature reading 33 has occurred 25 times (Maximum). Hence the **mode** is 33 °C

### **STATISTICAL METHODS OF ERROR ANALYSIS**

#### **a. Probability of errors**

By the nature of the random errors, the uncertainty associated with any measurement cannot be predetermined. Only the probable error can be specified using statistical error analysis. The following are some of the statistical methods of analyzing the errors.

#### **b. Normal distribution of errors**

The general laws governing normal distributions are stated as follows:

Positive and negative errors occur with equal probability and equal frequency.

Small errors are more common than large errors.

Large errors seldom occur, and there is a limit to the size of the greatest random error that will occur in any set of observations.

**Examples:**

- The body temperature for healthy humans.
- The heights and weights of adults.
- The thickness and dimensions of a product.
- IQ and standardized test scores.
- Quality control test results.
- Errors in measurements

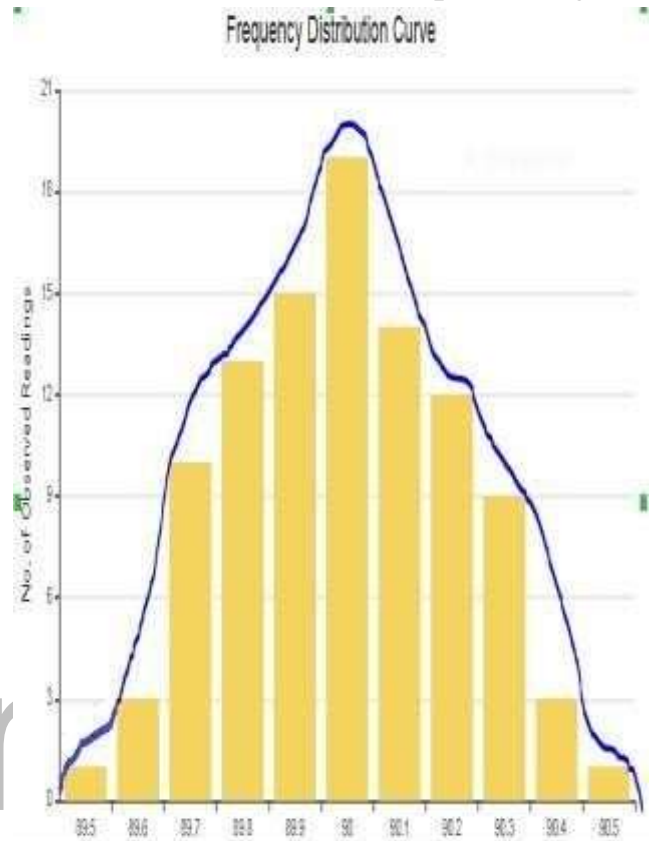
**Table 1.3.1 Frequency Distribution**

Temperature in °C	30	31	32	33	34	35	36	37
No. of Readings	9	4	15	25	15	22	7	3

### c. Histogram

When a number of multi sample observations are taken experimentally there is a scatter of the data about some central value. One method of presenting test results is in the form of a

"Histogram". It is a tool to visually explore the data. It is an efficient graphical method for describing the distribution of data. It is always a good practice to plot your data in a histogram after collecting the data. This will give you an insight about the shape of the distribution. If the data is symmetrically distributed and most results are located in the middle, we can assume that the data is normally distributed.



**Fig: 1.3.2 Histogram**

**[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 285]**

The table below shows a set of 100 readings of a length measurement. The most probable or central value of length is. 90mm.

Fig (1.3.2) shows the histogram which represents these data where the ordinate indicates the number of observed readings (frequency of occurrence) of a particular value. The histogram is also called a "frequency distribution curve".

**Table 1.3.2 Readings of a length measurement**

<b>Length (cm)</b>	<b>89.</b>	<b>89.6</b>	<b>89.</b>	<b>89.</b>	<b>89.</b>	<b>90</b>	<b>90.1</b>	<b>90.2</b>	<b>90.3</b>	<b>90.4</b>	<b>90.</b>
	<b>5</b>		<b>7</b>	<b>8</b>	<b>9</b>						<b>5</b>
<b>No of Readings</b>	<b>1</b>	<b>3</b>	<b>10</b>	<b>13</b>	<b>15</b>	<b>19</b>	<b>14</b>	<b>12</b>	<b>9</b>	<b>3</b>	<b>1</b>

**d. Normal or Gaussian curve of Errors**

The normal or Gaussian curve of errors is the basis for the major part of study of random errors. The law of probability states that the normal occurrence of deviations from average value of an infinite number of measurements or observations can be expressed by:

$$y = \frac{h}{\sigma\sqrt{\pi}} \exp(-h^2 x^2)$$

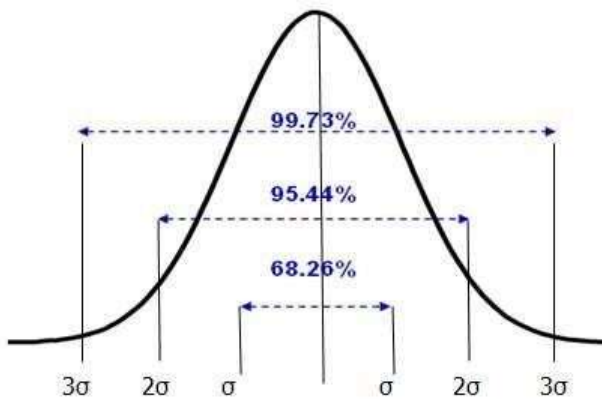
Where, x - Magnitude of deviation from mean

y - Number of readings at any deviation x (the probability of occurrence of deviation x)

h = a constant precision index

Normal or Gaussian curve is the graphical representation of the normal distribution. It is determined by the mean and the standard deviation. It is a symmetric bellshaped curve. Its tails extends infinitely in both directions. The wider the curve, the larger the standard deviation and the more the variation exists in the process. The spread of the curve is equivalent to six times the standard deviation of the process.

The probabilities are represented by the area under the normal curve. The total area under the curve is equal to **100%** (or **1.00**). Since the normal curve is symmetrical, **50 percent** of the data lie on each side of the curve.



**Fig 1.3.3 Normal probability Curve**

**[Source: Neubert H.K.P., Instrument Transducers – An Introduction to their Performance and Design, Page: 286]**

Fig. 1.6 shows the Normal probability Curve

For any normally distributed data: •**68%** of the data fall within **1** standard deviation of the mean. •**95%** of the data fall within **2** standard deviations of the mean. •**99.7%** of the data fall within **3** standard deviations of the mean.

#### **f. Probable error**

The most probable or best value of a Gaussian distribution is obtained by taking arithmetic mean of the various values of the variate. The confidence in the best value (most probable value) is connected with the sharpness of the distribution curve.

### **ODDS AND UNCERTAINTY**

#### **Specifying Odds**

The probability of occurrence can be stated in terms of Odds. Odd is the number of chances that a particular reading will occur when the error limit is specified. Odds is probability that the event will occur divided by the probability that the event will not occur. For example, if the error limits are specified as  $\pm 0.6745\sigma$ , the chances are that 50% of the observations will lie between the above limits or in other words we can say that odds are 1 to 1. The odds can be calculated by the



following formula,

$$\text{Odds of event} = Y / (1-Y)$$

where,

Y is probability of an event and Probability of occurrence is

$$y = \frac{\text{odds}}{\text{odds}+1}$$

### Uncertainty

Uncertainty is expressed as the range of variation of the indicated value from the true value.

**Uncertainty** is a reported value that characterizes the range of values within which the true value is asserted to lie. An uncertainty estimate should address error from all possible effects (both systematic and random) and, therefore, usually is the most appropriate means of expressing the **accuracy** of results. However, in many measurement situations the systematic error is not addressed and only random error is included in the uncertainty measurement. When only random error is included in the uncertainty estimate, it is a reflection of the **precision** of the measurement.

**Table 1.3.3 shows the corresponding values of Deviation and probability.**

Probability(%)	Deviation	Odds
50	$\pm 0.6745 \sigma$	1 to 1
68.3	$\pm \sigma$	2.15 to 1
95.4	$\pm 2\sigma$	21 to 1
99.7	$\pm 3\sigma$	256 to 1

### Example:

The measured the width of a standard piece of notebook paper is stated as  $8.53 \pm 0.08$  inches.

By stating the uncertainty to be 0.08 inches you are claiming with confidence that every reasonable measurement of this piece of paper by other experimenters will produce a value not less than 8.45 inches and not greater than 8.61 inches

### Uncertainty Analysis

Many times the data available is a single sample data and therefore the statistical methods discussed earlier cannot be applied directly. Hence, Kline and McClintock have proposed a method based upon probability and statistics which analyses the data employing uncertainty distribution rather than frequency distribution.

Kline and McClintock suggest that a single sample result may be expressed in terms of a mean value and an uncertainty interval based upon stated odds. The result may be written as follows:

$$X = \bar{X} \pm W \quad (b \text{ to } 1)$$

$\bar{X}$  = The value if only one reading is available on the arithmetic mean of several readings  
 $W$  = uncertainty interval  
 $b$  = odds or the chance that the true value lies within the stated range, based upon the opinion of the experimenter

For example, the results of a temperature measurement may be expressed as  $T = 90^{\circ}\text{C} \pm 0.1$ .

This means that there is an uncertainty of  $\pm 1^{\circ}\text{C}$  in the result. Kline and McClintock proposed that the experimenter specify certain odds for the uncertainty.

So,  $T = 90^{\circ}\text{C} \pm 0.1$  (20 to 1). The experimenter is willing to bet 20 to 1 odds that the temperature measurement which he has made is within  $\pm 1^{\circ}\text{C}$  of  $90^{\circ}\text{C}$ . It means, out of 21 measurements 20 measurements are within the specified range.

## Propagation of Uncertainties

The uncertainty analysis in measurements when many variates are involved is done on the same basis as is done for error analysis when the results are expressed as standard deviations or probable errors. Suppose X is a function of several variables,

$$X = f(x_1, x_2, x_3, \dots, x_n)$$

Where  $x_1, x_2, x_3, \dots, x_n$  are independent variables with the same degree of odds.

The uncertainty in the result is

$$W_X = \sqrt{\left(\frac{dX}{dx_1}\right)^2 W_{x_1}^2 + \left(\frac{dX}{dx_2}\right)^2 W_{x_2}^2 + \dots + \left(\frac{dX}{dx_n}\right)^2 W_{x_n}^2}$$

Where,

$W_X$  = resultant uncertainty.  $W_{x_1}, W_{x_2}, \dots$  are uncertainties in the independent variables  $x_1, x_2, x_3, \dots, x_n$

## **1.1 INTRODUCTION:**

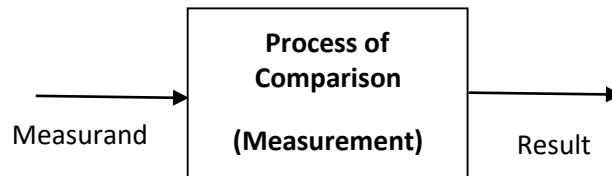
The study of any subject matter in engineering should be motivated by an appreciation of the uses to which the material might be put in the everyday practice of the profession. Measurement systems are used for many detailed purposes in a wide variety of application areas. The easiest way to assess the amount of use of science and technology is to I examine the number of measurements that are being made and how they are being used. All the successful achievements in science and technology are entirely due to the ability to measure the state, condition or characteristics of the physical systems, in quantitative terms with sufficient accuracy. Lord-Kelvin stressed the importance of measurement in this context, by saying: "When you can measure what you are speaking about, and express it in numbers, you know something about it".

## **MEASUREMENT:**

The measurement is usually undertaken to ascertain and present the state, condition or characteristic of a system in quantitative terms. To reveal the performance of a physical or chemical system~ the' first operation carried out on it is measurement. The process or the act of measurement consists of obtaining a quantitative comparison between a pre defined standard and a measurand. The word measurand is used to designate the particular physical parameter being observed and quantified that is, the input quantity to the measuring process. Measurements are generally made:

- To understand an event or an operation.
- To monitor an event or an operation.
- To control an event or an operation.
- To collect data for future analysis
- To validate an engineering design.

Fig 1.1.1 shows the fundamental measuring process



**Fig 1.1.1 Fundamental measuring process**

### **FUNDAMENTAL METHODS OF MEASUREMENT:**

There are two basic methods of measurement

- Direct comparison with either a primary or a secondary standard.
- Indirect comparison through the use of a calibrated system.

### **DIRECT COMPARISON:**

To measure the length of a bar, we compare the length of the bar with a standard, and find that the bar is how many inches long. So that inch-units on the standard has the same length as the bar. Thus we have determined the length by direct comparison. The standard that we have used is called a secondary standard. Measurement by direct comparison is less common than the measurement by indirect comparison.

### **INDIRECT COMPARISON:**

Indirect comparison makes use of some form of transducing device. This device converts the basic form of input into analogous form, which it then processes and presents at the output as a known function of the input.

### **FUNCTIONAL ELEMENTS OF A MEASUREMENT SYSTEM:**

Fig 1.1.2 shows the functional elements of an instrument or a measurement system.

### **PRIMARY SENSING ELEMENT**

The primary sensing element is the one which first receives energy from the measured medium and produces an output depending in some way on the measured quantity (measurand).

### **VARIABLE CONVERSION ELEMENT:**

The output signal of the primary sensing element is some physical variable, such as displacement or voltage. For the instrument to perform the desired function, it may be necessary to convert this variable to another more suitable variable while preserving the information content of the original signal. An element that performs such a function is called a variable conversion element.

### **VARIABLE MANIPULATION ELEMENT:**

The element that performs "manipulation" by which the numerical value of the variable is changed according to some definite rule but the physical nature of the variable is preserved is called a variable manipulation element.

### **DATA TRANSMISSION ELEMENT:**

When the functional elements of an instrument are actually physically separated, it becomes necessary to transmit the data from one to another. An element performing this function is called a data transmission element.

### **DATA PRESENTATION ELEMENT:**

If the information about the measured quantity is to be communicated to a human being for monitoring, control, or analysis purposes, it must be put in to a form recognizable by one of the human senses. An element that performs this "translation" function is called data presentation element. This function includes the simple indication of a pointer-moving over a scale and the recording of a pen moving over a chart.

### **DATA STORAGE / PLAYBACK ELEMENT:**

Although data storage in the form of pen/ink recording is often employed, some applications require a distinct data storage/play back function which can easily recreate the stored data upon command. The magnetic tape recorder/reproducer is the example.

### **UNITS AND STANDARDS OF MEASUREMENT:**

The term "dimension" denotes the defining characteristics of an entity. The "unit" is a basis for quantification of the entity. For example, length is a dimension where as centimeter is a unit of length, time is a dimension and the second is a unit of time.

### **UNITS:**

We measure a physical quantity by the measurement system. The result of the measurement of the physical quantity must be defined both in kind and magnitude. The standard measure of each kind of physical quantity is called a "Unit", In general, we can write:

$$\text{Magnitude of a physical quantity} = (\text{Numerical ratio}) \times (\text{unit})$$

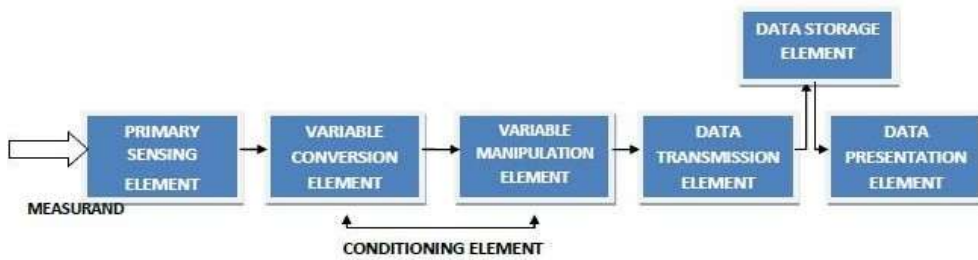
The Numerical Ratio is the number of times the unit occurs in any given amount of the same quantity and therefore, is called the number of measures. This may be otherwise called a numerical multiplier.

For e.g., if we measure distance of

10metre, its magnitude may be,

$$= \text{Distance } (10) \times (\text{m})$$

Here, metre (m) is the unit of length and 10 is the number of units in the length. The physical quantity, distance, in this case is defined by the unit, metre. Without unit, the numerical ratio has no physical meaning.



**Fig 1.1.2 Functional elements of a measurement system**

### **TYPES OF UNITS:**

Major types of units are:

- Fundamental units
- Derived units

Units which are fundamental to most other physical quantities are called fundamental units.

Fundamental units are measures of length, mass and time. Since length, mass and time are fundamental to most of the other physical quantities; they are called the "Primary Fundamental Units". Measures of certain physical quantities in the thermal, electrical, illumination fields are also represented by fundamental units. These units are used only where these particular disciplines are involved and therefore they are called Fundamental Units. All other units which can be expressed in terms of fundamental units with the help of physical equations are called Derived Units. Every derived unit originates from some physical law or equation which defines that unit. For e.g., the area(A) of a room is equal to the product of its length l, and breadth b.

Therefore,

$$A = l \times b$$

If metre is chosen as the unit of length, then the area of a room 6m x 4m is 24 square metres. Note that the number of measures ( $6 \times 4 = 24$ ) as well as the units ( $m \times m = m^2$ ) are multiplied. The derived unit of area is  $m^2$ .



## **SYSTEM OF UNITS:**

A number of systems of units are in use since 16th century. The important systems of units are

- FPS system (foot, pound, second)
- CGS system (centimetre, gram, second)
- MKS system (meter, kilogram, second)
- Rationalized MKSA system (meter, kilogram, second, ampere)
- SI system (seven fundamental units, two supplementary units and twenty seven derived units)

## **CGS SYSTEM OF UNITS:**

The most commonly used units in electrical work were CGS units. These units involve the use of unit of a fourth quantity in addition to units of mass, length and time. Two systems of CGS units are

- Electromagnetic Units (EMU)
- Electrostatic Units (ESU)

### **a) Electromagnetic Units**

Units based on electromagnetic effects are known as electromagnetic units and the system is known as electromagnetic system of units. This system involves the units of four quantities: permeability ( $\mu$ ) of the medium and the units of length, mass and time. The value of permeability of free space (vacuum) is taken as unity in this system.

### **b) Electrostatic Units**

Units based on electrostatic effects are known as electrostatic units and the system is electrostatic system. This system involves the units of four quantities:

Table 1.1.1 Practical Units

No.	Quantity	Practical Unit	Symbol
1	Charge	coulomb	Q
2	Current	ampere	I
3	Potential Difference	volt	E
4	Resistance	ohm	R
5	Inductance	henry	L
6	Capacitance	farad	C
7	Power	watt	P
8	Energy	joule	W

permittivity ( $\epsilon$ ) of the medium and the units of length, mass and time. The value of permittivity of free space is taken as unity in this system.

**c) Absolute units:**

An absolute system of units is defined as a system in which the various units are all expressed in terms of a small number of fundamental units. Absolute measurements do not compare the measured quantity with arbitrary units of the same type but are made in terms of Fundamental Units.

**d) Practical units:**

Practical units are derived either from the absolute units or by reference to arbitrary standards; Table (1.1) shows the symbols and magnitudes of practical units.

**M.K.S SYSTEM (GIORGI SYSTEM)**

The CGS system suffers from the following disadvantages:

- There are two, systems of units (EMU and ESU) for fundamental

theoretical work and third (practical units) for practical engineering work.

- There are two sets of dimensional equations for the same quantity.

In, MKS system, metre, kilogram and second are the three fundamental mechanical units. In order to connect the electrical and mechanical quantities, a fourth fundamental quantity has to be used. This fourth quantity is usually permeability. The permeability of free space is taken as  $\mu_0 = 4\pi \times 10^{-7}$  H/m. The permeability  $\mu$  of any other medium is given by  $\mu = \mu_0 \mu_r$ , where  $\mu_r$  is the relative permeability. The permeability of free space in C.G.S system is unity.

MKS unit of permeability =  $4\pi \times 10^{-7}$  x C.G.S. unit of permeability.

### **ADVANTAGES OF M.K.S SYSTEM**

- This system connects the practical units directly, with the fundamental laws of electricity and magnetism.
- This system gives specified formulae for expressions of electromagnetism involving only practical units.

### **SI UNITS:**

In the Eleventh General conference of Weights and Measures in October, 1960 recommended a unified systematically constituted coherent system of fundamental supplementary and derived units for international use. This system is called the International system of Units and designated by the abbreviation SI, It has been accepted internationally.

The SI unit consists of seven fundamental units and two supplementary units.

They are given in the table 1.1.2.

Table 1.1.2 SI Units

<b>Fundamental Quantity</b>	<b>SI Unit</b>	<b>Symbol</b>
Length	metre	m
Mass	kilogram	Kg
Time	Second	s

Temperature	kelvin	K
Electric Current	ampere	A
Luminous Intensity	candela	cd
Amount of Substance	Mole	mol

Two supplementary units are Radian: It is used to measure plane angle.

Steradian: It is used to measure solid angle.

### **STANDARDS:**

Standards of mass, length and such other physical quantities are physical devices and systems representing the fundamental unit of the particular quantity.

Standards have been developed for all the fundamental units as well as some of the derived- mechanical and electrical units. They are classified as follows:

- International standards
- Primary standards
- Secondary standards
- Tertiary Standards
- Working Standards

### **INTERNATIONAL STANDARDS:**

These standards are those defined and agreed upon internationally, They are maintained at the International Bureau of Weights and Measures and are not accessible outside for calibration of instruments.

### **PRIMARY STANDARDS:**

These standards are those maintained by national standards laboratories in different parts of the world and they are also not accessible

outside for calibration. The primary standards established for the fundamental and some derived units are independently calibrated by absolute measurements at each of the national standards laboratories and an average value for the primary standard is obtained with the highest accuracy possible. These are used for verification and calibration of the secondary standards.

### **SECONDARY STANDARDS:**

These standards are usually fixed standards for use in industrial laboratories, where as working standards are for day-to-day use in measurement laboratories.

### **TERTIARY STANDARDS:**

Tertiary standards are reference standards employed by NPL and are used as the first standards for reference in laboratories and workshops.

These standards are replicas of secondary standards and are usually used as references for working standards.

### **WORKING STANDARDS:**

Working standards may be lower in accuracy in comparison to secondary standards. The accuracy of secondary standards is maintained by periodic comparison with the primary standards, where as working standards may be checked against secondary standards.

### **CALIBRATION:**

Calibration is an essential process to be undertaken for each instrument and measuring system frequently. A reference standard at least ten times more accurate than the instrument under test is normally used. Calibration is the process where the test instrument (the instrument to be calibrated) is compared with the standard instrument. It consists of reading the standard and test instruments simultaneously when the input quantity is held constant at several values over the

range of the test instrument. The calibration is better carried out under the stipulated environmental conditions.

All industrial grade instruments can be checked for accuracy in the laboratory by using the working standards. Generally, certification of an instrument manufactured by, an industry is undertaken by the National Physical Laboratory and other authorized laboratories where the secondary standards and the working standards are kept.

### **GENERALIZED PERFORMANCE CHARACTERISTICS OF INSTRUMENTS:**

The instrument performance characteristics are generally broken down in to two areas

- Static characteristics
- Dynamic characteristics

### **STATIC CHARACTERISTICS:**

Some applications involve the measurement of quantities that are constant or vary only slowly. Under these conditions, it is, possible to define a set of performance criteria that give a meaningful description of the quality of measurement. So "Static characteristics are a set of performance criteria that give a meaningful description of the quality of measurement while the measured quantities are either constant or vary slowly".

### **DYNAMIC CHARACTERISTICS:**

Dynamic characteristics describe the quality of measurement when the measured quantities are rapidly varying quantities.

### **STATIC CALIBRATION:**

The static performance characteristics are obtained by one form or another of the process of static calibration. In general, static calibration refers to a situation in which all inputs except one are kept at some constant values..

Then the one input under study is varied over some range of constant values, which causes the outputs to vary over some range of constant values. The input-output relations developed in this way comprise a static calibration valid under, the stated constant conditions of all the other inputs. This procedure may be repeated, by varying in turn each input considered to be of interest and thus developing a family of static input-output relations

### **PROCEDURE FOR CALIBRATION:**

The procedure for Calibration is as follows:

- Examine the construction of the instrument, and identify and list all the possible inputs.
- Decide which of the inputs will be significant in the application for which the instrument is to be calibrated.
- Select the apparatus that will allow you to vary all the significant inputs over the ranges considered necessary. Select standards to measure each input.
- By holding some inputs constant, varying others and recording the outputs develop the desired static input-output relations.