1.8 BLOCK DIAGRAM REDUCTION TECHNIQUES

A system that can change its output in accordance with change in input is known as a closed loop system. This can be implemented by introducing a feedback path in an open-loop system and manipulating the input that is applied to the system. Such as closed-loop system can be represented by using a block diagram shown in figure 1.8.1.

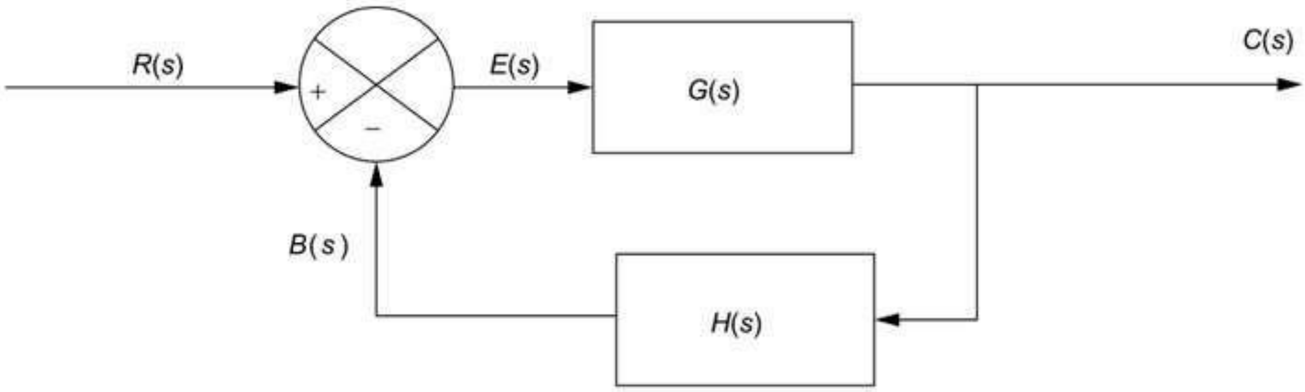
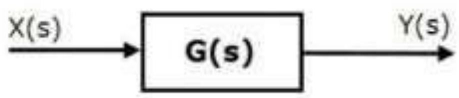


Figure 1.8.1 Simple block diagram representation

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 3.2]

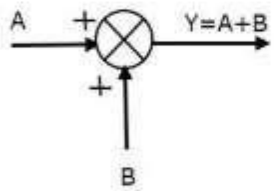
BLOCK

The transfer function of a component is represented by a block. Block has single input and single output.



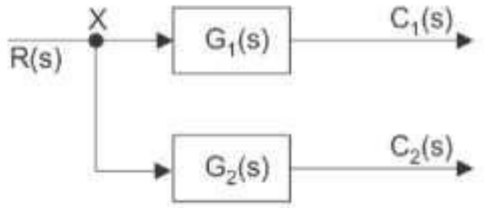
SUMMING POINT

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one. The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as sum of A and B.



NODE

The node is a point from which the same input signal can be passed through more than one branch. That means with the help of node, we can apply the same input to one or more blocks, summing points. In the following figure, the node is used to connect the same input, R(s) to two more blocks.



The advantages of block diagram representation are:

(i) It facilitates easier representation of complex systems.

(ii) Calculation of transfer function by block diagram reduction techniques is easy.

(iii) Performance analysis of a complex system is simplified by determining its transfer function.

(iv) It facilitates easier access of individual elements in a system that is represented by a block diagram.

(v) It facilitates visualization of operation of the whole system by the flow of signals.

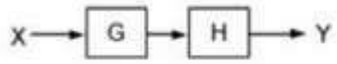
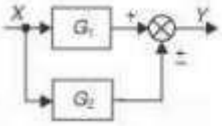
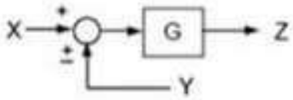
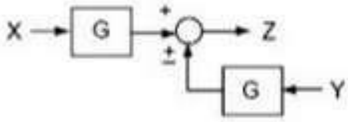
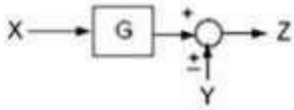
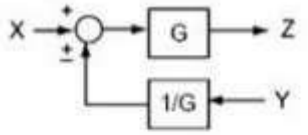
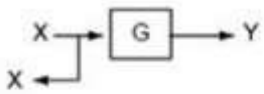
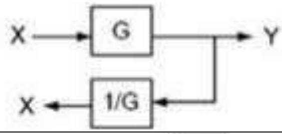
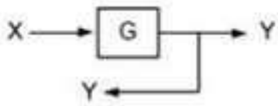
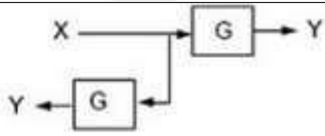
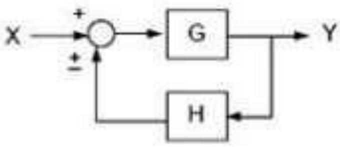
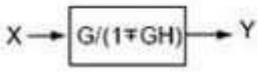
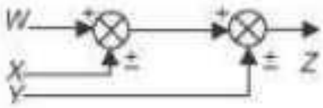
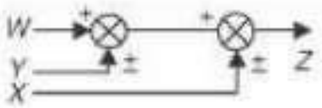
The disadvantages of block diagram representation are:

(i) It is difficult to determine the actual composition of individual elements in a system.

(ii) Representation of a system using block diagram is not unique.

(iii) The main source of signal flow cannot be represented definitely in a block diagram.

RULES FOR BLOCK DIAGRAM REDUCTION

Rule No.	Rule	Block diagram	Equivalent block diagram
1	Blocks in cascade		
2	Blocks in parallel		
3	Moving a summing point behind the block		
4	Moving a summing point ahead of the block		
5	Moving a branch point behind the block		
6	Moving a branch point ahead of the block		
7	Eliminating a feedback loop		
8	Interchanging the summing point		

1.4 ELECTRICAL ANALOGY OF MECHANICAL SYSTEMS

FORCE-VOLTAGE ANALOGY

Consider a simple translational mechanical system as shown in figure 1.4.1.

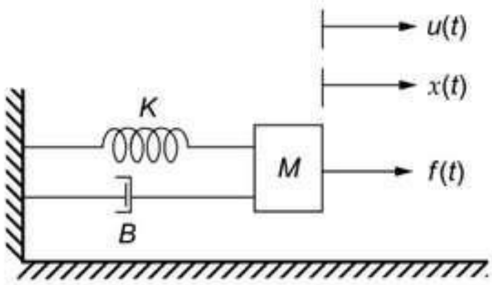


Figure 1.4.1 Translational mechanical system

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.51]

Using D' Alembert's principle, we have,

Sum of the applied forces = Sum of the opposing forces

$$f(t) = M\frac{du(t)}{dt} + Bu(t) + K\int u(t)\,dt$$

Consider a series RLC circuit as shown in figure 1.4.2.

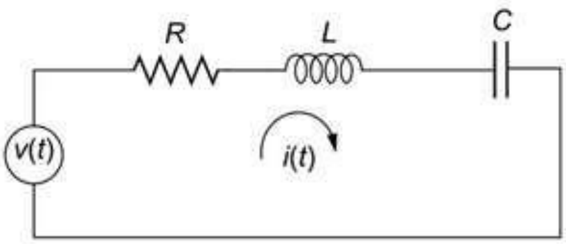


Figure 1.4.2 Series RLC circuit

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.52]

Using KVL, the integro-differential equations can be written as

$$v(t) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)\,dt$$

Translational system	Electrical system
Force (f)	Voltage (v)
Velocity (u)	Current (i)
Displacement (x)	Charge (q)
Mass (M)	Inductance (L)
Damping coefficient (B)	Resistance (R)
Spring constant (K)	1/Capacitance (C)

FORCE-CURRENT ANALOGY

Consider a simple parallel RLC circuit as shown in figure 1.4.3.

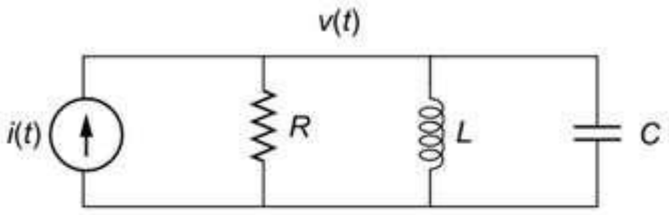


Figure 1.4.3 Parallel RLC circuit

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.52]

Using KCL, the integro-differential equations can be written as follows:

$$i(t) = C\frac{dv(t)}{dt} + Gv(t) + \frac{1}{L}\int v(t)\,dt$$

where, conductance, G=1/R.

On comparing with the mechanical translational system equation, we get,

Translational System	Electrical System
Force (f)	Current (i)
Velocity (u)	Voltage (v)
Displacement (x)	Flux (Φ)
Mass (M)	Capacitance (C)
Damping coefficient (B)	Conductance (G)
Spring constant (K)	1/Inductance (L)

TORQUE-VOLTAGE ANALOGY

Consider a simple rotational mechanical system as shown in figure 1.4.4.

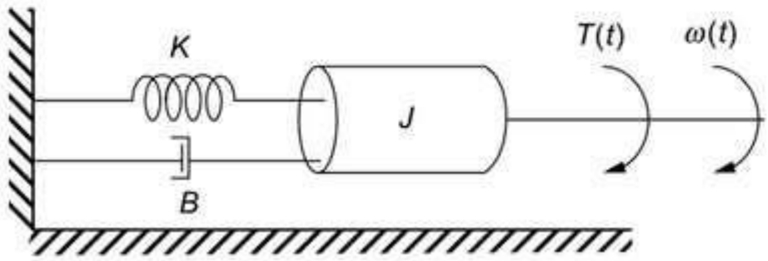


Figure 1.4.4 Rotational mechanical system

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.71]

Using D' Alembert's principle, we have,

Sum of the applied torques = Sum of the opposing torques

$$T(t) = J\frac{d\omega(t)}{dt} + B\omega(t) + K\int \omega(t)dt$$

Consider a series RLC circuit as shown in figure 1.4.5.

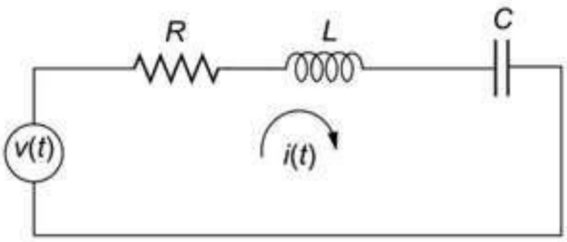


Figure 1.4.5 Series RLC circuit

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.72]

Using KVL, the integro-differential equations can be written as

$$v(t) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt$$

On comparing with the mechanical rotational system equation, we get,

Rotational System	Electrical System
Torque (T)	Voltage (v)
Angular velocity (ω)	Current (i)
Angular displacement (θ)	Charge (q)
Moment of inertia (J)	Inductance (L)
Rotational damping (B)	Resistance (R)
Rotational spring constant (K)	1/Capacitance (C)

TORQUE-CURRENT ANALOGY

Consider a simple parallel RLC circuit as shown in figure 1.4.6.

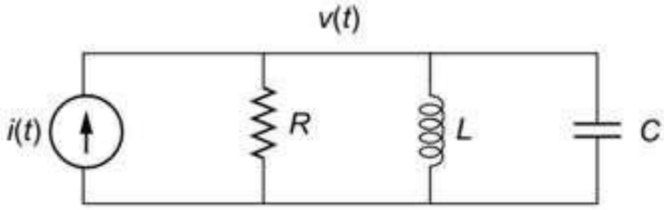


Figure 1.4.6 Parallel RLC circuit

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.73]

Using KCL, the integro-differential equations can be written as follows:

$$i(t) = C\frac{dv(t)}{dt} + Gv(t) + \frac{1}{L}\int v(t)\,dt$$

where, conductance, G=1/R.

On comparing with the mechanical rotational system equation, we get,

Rotational Mechanical System	*T-I* Analogous
Torque (T)	Current (i)
Angular velocity (ω)	Voltage (v)
Angular displacement (θ)	Flux (Φ)
Moment of inertia (J)	Capacitance (C)
Rotational spring constant (K)	1/Inductance (L)
Rotational damping (B)	Conductance (G)

1.5 ELECTRICAL ANALOGY OF THERMAL SYSTEMS

There are two fundamental physical elements that make up thermal networks, thermal resistances and thermal capacitance. There are also three sources of heat, a power source, a temperature source, and fluid flow.

Example:

In practice temperature when we discuss temperature, we will use degree Celsius (ºC), while SI unit for temperature is to use Kelvins (0ºK = - 273.15ºC). Generally reference temperature (T_1) is taken and all temperatures are measured relative to this reference. Reference temperature is assumed to be constant.

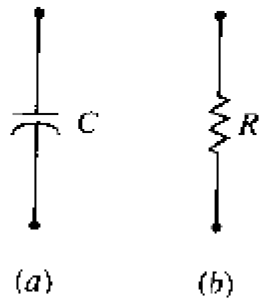


Figure 1.5.1 Network elements of thermal systems

[Source: "Linear Control System Analysis and Design" by John J. D'Azzo, Page: 76]

Thermal resistance

Consider the situation in which there is a wall, one side of which is at a temperature T_1, with the other side at temperature T_2, the wall has a thermal resistance of R_{12}.

Thermal capacitance

In addition to thermal resistance, objects can also have thermal capacitance (also called thermal mass). The thermal capacitance of an object is a measure of how much heat it can store. If an object has thermal capacitance its temperature will rise as heat flows into the object, and the temperature will lower as heat flows out. To understand this, envision a rock in the sun. During the day heat goes in to the rock from the sunlight, and the temperature of the rock increases as energy is stored in the rock as an increased temperature. At night energy is released, and the rock cools down. We represent a thermal capacitance in isolation in diagrams (and equations) as shown in Figure (in the drawing at the left the coil represents a power source and the stippled object is the thermal capacitance). In the thermal analogy, one end of the capacitor is always connected to the constant ambient temperature. The electrical model will always have one side of the capacitance connected to ground, or reference. Also, we could write the equation as = q but since T_1 is constant, it can be removed from the derivative. The thermal capacitance of an object is determined by its mass and specific heat.

$$C = mc_p$$

Where C is the thermal capacitance, m is the mass in kilograms, and c_p is the specific heat in J/(kg-ºK). it is always assumed that the capacitor is at a single uniform temperature, though this is obviously a simplification in many cases.

$$C\frac{dT_2}{dt} = q$$

Power source (or heat source)

A common part of a thermal model is a controlled power source that generates a predetermined amount of power, or heat, in a system. This power can either be constant or a function of time. In the electrical analogy, the power source is represented by a current source. An example of a power source is the quantity *q* in the diagrams for the thermal capacitance, above. In practice a power source is often an electrical heating element comprised of a coil of wire that is heated by a current flowing through it. Therefore, we use a diagram of a coil of wire to represent the power source. An ideal power source generates power that is independent of temperature.

Temperature source

An ideal temperature source maintains a given temperature independent of the amount of power required. Ambient temperature is considered to be reference temperature).

Mass Transfer (Fluid Flow)

If fluid with specific heat c_p J/kg-ºK) flows into a system with a flow rate of G kg/sec and a temperature of T_mºC above reference, and flows out at a temperature of T_{out}ºC below reference then the rate of heat flow into the system is given by

$$q_{in} = G\left\{\frac{kg}{sec}\right\} \cdot c_p\left\{\frac{J}{kg\ K}\right\} \cdot (T_{in} - T_{out})\{°C\} = Gc_p(T_{in} - T_{out})\{W\}$$

We can cancel the K and ºC since a temperature difference $(T_{in} - T_{out})$ is the same in Kelvin r Celsius. If you carefully observe this equation, it makes sense intuitively. Heat into s system goes up with mass flow rate into the system (increased mass flow, yields increased heat flow). Heat into a system also goes up with the specific heat of the mass (Higher specific heat indicates increased capacity to store heat). Finally, heat into system increases with an increased inflow temperature, or a decreased outflow temperature (if the temperature difference between inflow and outflow increases, more heat is being taken from the fluid). Note, the mass flow rate at the input and output must be equal to the mass (and thermal capacitance) of the system would be changing. This is not allowed for the systems being studied (time-invariant systems).

Energy balance

To develop a mathematical model of a thermal system we use the concept of an energy balance. The energy balance equation simply states that at any given location, or node, in a system, the heat into that node is equal to the heat out of the node plus any heat that is stored (heat is stored as increased temperature in thermal capacitances). The terms used in the equations is mentioned below:

Symbol	Quantity	U.S. customary units	Metric units
q	Rate of heat flow	Btu/minute	Joules/second
M	Mass	Pounds	Kilograms
S	Specific heat	Btu/(pounds)(°F)	Joules/(kilogram)(°C)
C	Thermal capacitance $C = MS$	Btu/°F	Joules/°C
K	Thermal conductance	Btu/(minute)(°F)	Joules/(second)(°C)
R	Thermal resistance	Degrees/(Btu/minute)	Degrees/(joule/second)
θ	Temperature	°F	°C
h	Heat energy	Btu	Joules

Additional heat stored in a body whose temperature is raised from θ_1 to θ_2 is given by

$$h = \frac{q}{D} = C(\theta_2 - \theta_1)$$

$$q = CD(\theta_2 - \theta_1)$$

Rate of heat flow through a body in terms of the two boundary temperatures θ_3 to θ_4

$$q = \frac{\theta_3 - \theta_4}{R}$$

The thermal resistance determines the rate of heat flow through the body. This is analogous to the resistance of a resistor in an electric circuit, which determines the current flow.

SIMPLE MERCURY THERMOMETER

Consider a thin glass-walled thermometer filled with mercury that has stabilized at a temperature θ_1. It is plunged into a bath of temperature θ_0 at t=0. In its simplest form, the thermometer can be considered to have a capacitance C that stores heat and a resistance R that limits the heat flow. The temperature at the center of the mercury is θ_m.The flow of heat into the thermometer is

$$q = \frac{\theta_0 - \theta_m}{R}$$

The heat entering the thermometer is stored in the thermal capacitance and is given by

$$h = \frac{q}{D} = C(\theta_m - \theta_1)$$

These equations can be combined to form

$$\frac{\theta_0 - \theta_m}{RD} = C(\theta_m - \theta_1)$$

Differentiating the above equation and rearranging the terms gives,

$$RC\, D\theta_m + \theta_m = \theta_0$$

The thermal network is drawn as in figure 1.5.2. Thus, the state equation is

$$\dot{x}_1 = -\frac{1}{RC}x_1 + \frac{1}{RC}u$$

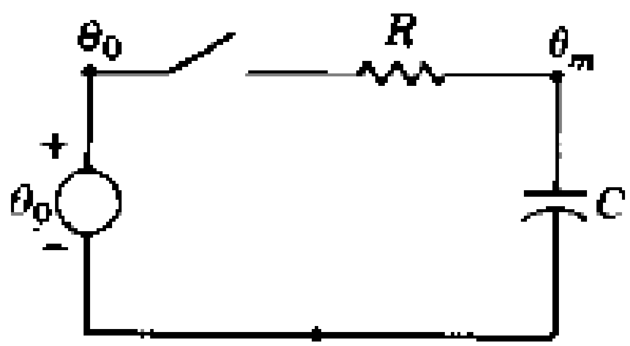


Figure 1.5.2 Network representation of a thermometer

[Source: "Linear Control System Analysis and Design" by John J. D'Azzo, Page: 77]

In general, *Heat in = Heat out + Heat stored*

$$q = \frac{T_r - T_a}{R_{ra}} + C\frac{dT_r}{dt}$$

1.1 INTRODUCTION

A system is a combination of components connected to perform a required action. The control component of a system plays a major role in altering or maintaining the system output based on our desired characteristics. There are two types of control systems: manual control and automatic control. For example, in manual control, a man can switch ON or OFF the bore well motor to control the level of water in a tank. On the other hand, in automatic control, level switches and transducers are used to control the level of water in a tank. Control systems have naturally evolved in our ecosystem. In almost all living things, automatic control regulates the conditions necessary for life by tackling the disturbance through sensing and controlling functionalities. They operate complex systems and processes and achieve control with desired precision. The application of control systems facilitates automated manufacturing processes, accurate positioning and effective control of machine tools. They guide and control space vehicles, aircrafts, ships and high-speed ground transportation systems. modern automation of a plant involves components such as sensors, instruments, computers and application of techniques that involve data processing and control. It is essential to understand a system and its characteristics with the help of a model, before creating a control for it. The process of developing a model is known as modeling. Physical systems are modeled by applying notable laws that govern their behavior. For example, mechanical systems are described by Newton's laws and electrical systems are described by Ohm's law, Kirchhoff's laws, Faraday's laws and Lenz's law. These laws form the basis for the constitutive properties of the elements in a system.

BASIC ELEMENTS IN CONTROL SYSTEMS

In recent years, control systems have gained an increasingly importance in the development and advancement of the modern civilization and technology. Disregard the complexity of the system; it consists of an input (objective), the control system and its output (result). Practically our day-to-day activities are affected by some type of control systems. There are four basic elements of a typical motion control system. These are

- Controller
- Amplifier

- Actuator
- Feedback
- Error detector

The complexity of each of these elements will vary depending on the types of applications for which they are designed and built. A dynamical system manipulates entities such as energy, material, information, capital investment etc. It is characterized by relationships among certain variables that are chosen in its description. Usually inputs (causes) and outputs (effects) are important variables, which are connected by relations. Although a relationship is a function of time, the properties embedded in it may be time-invariant. A system may have only one input and one output. Such a system is termed a single-input-single-output (SISO) system. Some may be multiple-input-multiple-output (MIMO) systems. Large systems are characterized by several levels of organization, in a hierarchy. Figure 1 shows the schematic diagrams of systems indicating such features. The fields of systems, control and information processing are closely related to the science of cybernetics. Cybernetics attempts to understand the behavior of the system in nature. This understanding leads to the knowledge enabling us to improve the performance of natural or man-made processes.

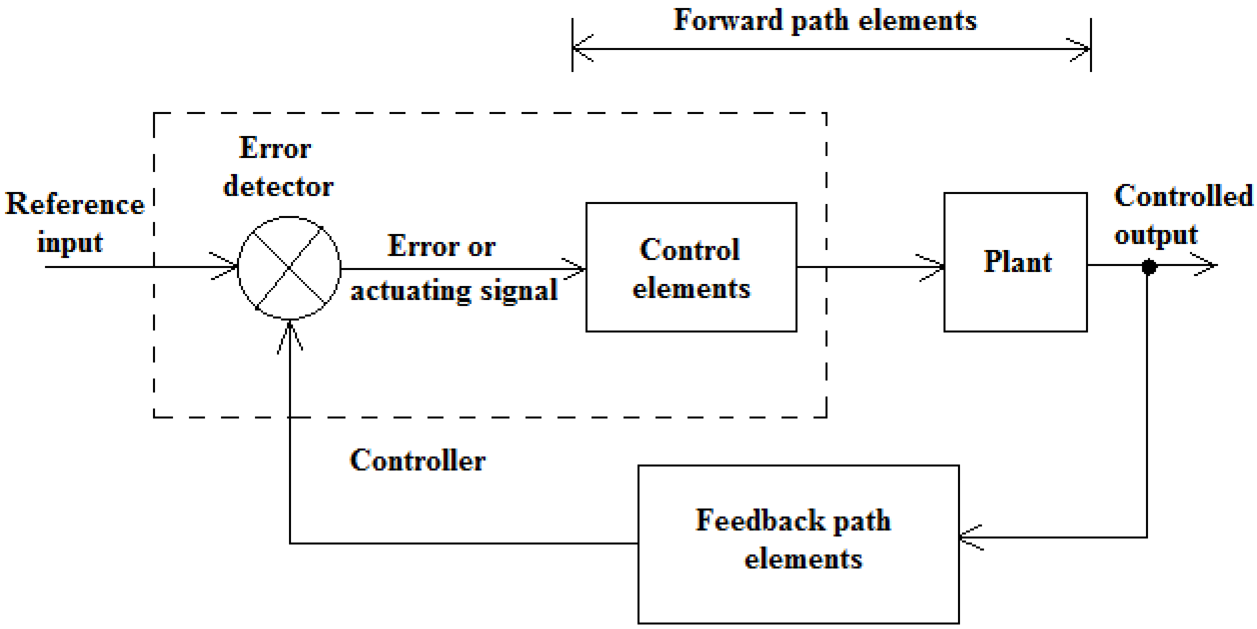


Figure 1.1.1 Basic Elements in Control Systems

[Source: "Control Systems Engineering" by I.J.Nagrath, M.Gopal, Page: 5]

1.3 MECHANICAL TRANSLATIONAL AND ROTATIONAL SYSTEMS

The general classification of mechanical system is of two types namely translational and rotational systems.

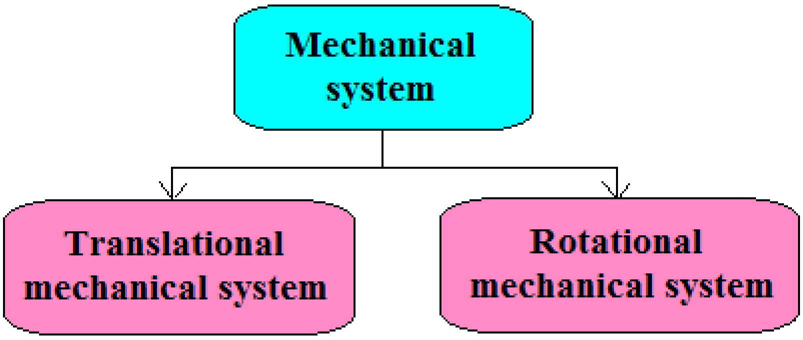


Figure 1.3.1 Classification of mechanical system

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.21]

MECHANICAL TRANSLATIONAL SYSTEMS

The model of mechanical translational systems can obtain by using three basic elements mass, spring and dashpot. When a force is applied to a translational mechanical system, it is opposed by opposing forces due to mass, friction and elasticity of the system. The force acting on a mechanical body is governed by Newton's second law of motion. For translational systems it states that the sum of forces acting on a body is zero.

Force balance equations of idealized elements:

Inertia force, $f_m(t)$

Consider an ideal mass element shown in figure, which has negligible friction and elasticity. Let a force be applied on it. The mass will offer an opposing force which is proportional to acceleration of a body.

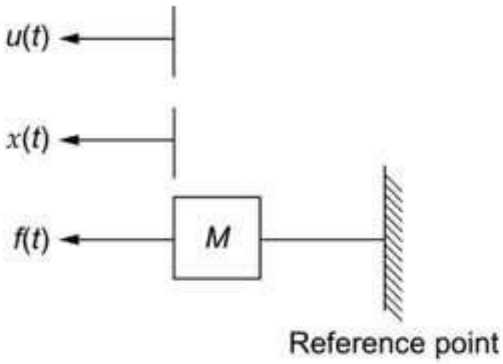


Figure 1.3.2 Mechanical translational element: Mass

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.21]

Let f(t) - applied force, f_m - opposing force due to mass,

$$f_m \propto \frac{d^2x}{dt^2}$$

By Newton's second law,

$$f = f_m = M\frac{d^2x}{dt^2}$$

Damper force, $f_b(t)$

Consider an ideal frictional element dash-pot shown in fig. which has negligible mass and elasticity. The dashpot's opposing force which is proportional to velocity of the body.

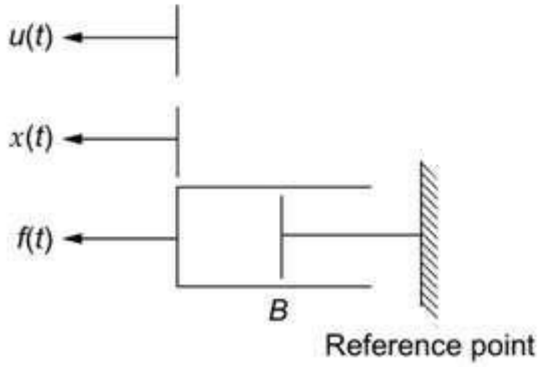


Figure 1.3.3 Mechanical translational element: Dashpot

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.23]

Let f = applied force, f_b = opposing force due to friction

$$f_b \propto \frac{dx}{dt}$$

By Newton's second law,

$$f = f_b = B\frac{dx}{dt}$$

Spring force, $f_k(t)$

Consider an ideal elastic element spring is shown in fig. This has negligible mass and friction.

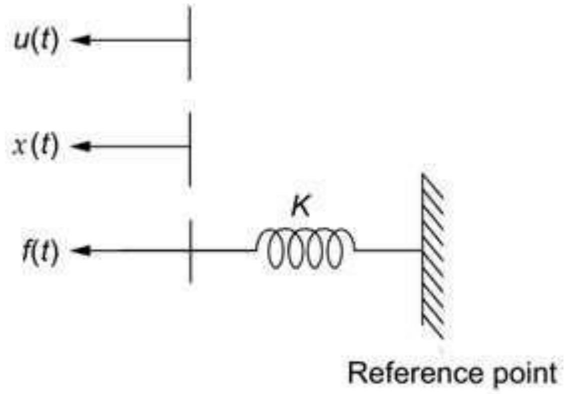


Figure 1.3.4 Mechanical translational element: Spring

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.24]

Let f = applied force, f_k = opposing force due to elasticity

$$f_k \propto x$$

By Newtons second law,

$$f = f_k = kx$$

According to D'Alembert's principle, "The algebraic sum of the externally applied forces to any body is equal to the algebraic sum of the opposing forces restraining motion produced by the elements present in the body." A simple translational mechanical system and its free body diagram are shown in figures 1.3.5 (a) and (b) respectively.

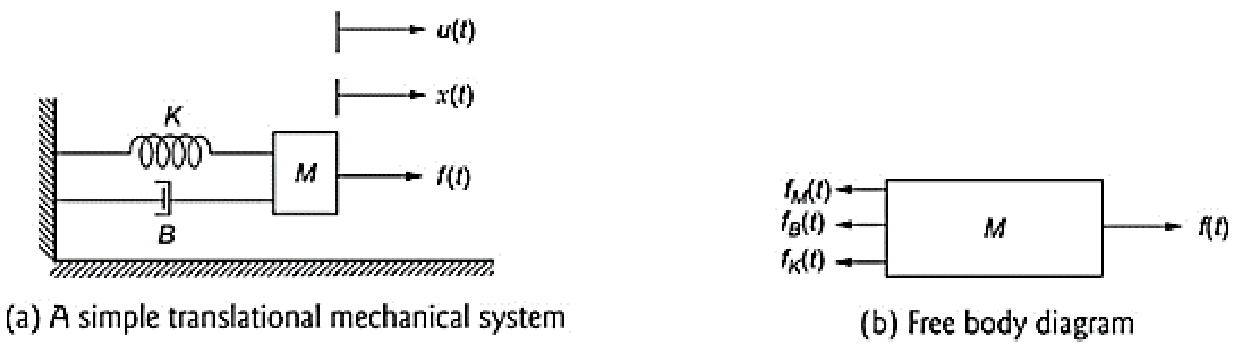


Figure 1.3.5 Mechanical translational system and its free body diagram

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.25]

$$f_m = M\frac{d^2x}{dt^2}$$

$$f_b = B\frac{dx}{dt}$$

$$f_k = kx$$

$$f(t) = f_m + f_b + f_k = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

MECHANICAL ROTATIONAL SYSTEM

The modeling of a linear passive rotational mechanical system can be obtained by using three basic elements: inertia, rotational spring and rotational damper. The modeling of a rotational mechanical system is similar to that of a translational mechanical system except that the elements undergo a rotational instead of a translational movement. The opposing torques due to inertia, rotational spring and rotational damper act on a system when the system is subjected to a torque. Using D'Alembert's principle, for a linear passive rotational mechanical system, the sum of all the torques acting on a body is zero (i.e., the sum of applied torques is equal to the sum of the opposing torques on a body). Angular displacement, angular velocity and angular acceleration are the variables used to describe a linear passive rotational mechanical system. In rotational mechanical systems, the energy storage elements are inertia and rotational spring and the energy dissipating element is the rotational viscous damper. The analogous of the energy storage elements in ana electrical circuit are the inductors and the capacitors and the analogous of energy dissipating element in an electrical circuit is the resistor.

Torque balance equations of idealized elements:

Inertia Torque, $T_j(t)$

When a torque T(t) is applied to an inertia element J, it experiences an angular acceleration and it is shown in figure 1.3.6.

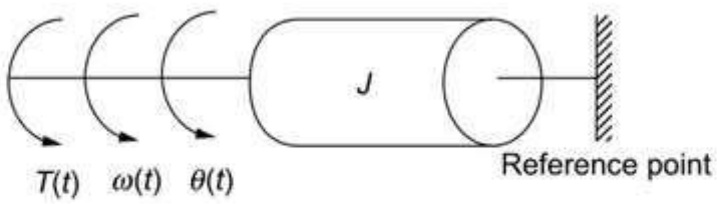


Figure 1.3.6 Mechanical rotational element: Inertia

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.38]

According to Newton's second law, the inertia torque is proportional to the angular acceleration.

$$T_j(t) \propto \frac{d^2\theta}{dt^2}$$

$$T_j(t) = j\,\frac{d^2\theta}{dt^2}$$

where J is the moment of inertia (kg-m^2/rad), θ(t) is the angular displacement (rad) and $T_j(t)$ is measured in Newton-meter (N-m).

Damping Torque, $T_b(t)$

When a torque, T(t) is applied to a damping element, B, it experiences an angular velocity and it is shown in figure 1.3.7.

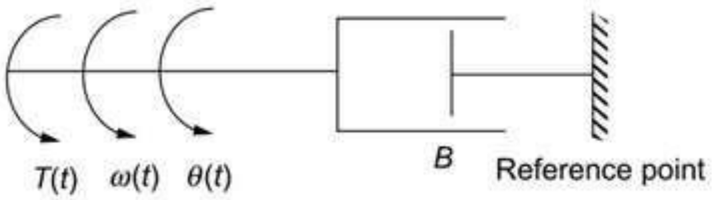


Figure 1.3.7 Mechanical rotational element: Dashpot

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.38]

The damping torque is proportional to the angular velocity. Therefore,

$$T_b(t) \propto \frac{d\theta}{dt}$$

$$T_b(t) = B\frac{d\theta}{dt}$$

where, B is the viscous friction coefficient (N-s/m), θ(t) is the angular displacement (rad). Damper element with two angular displacements and a single applied torque is shown in figure 1.3.8.

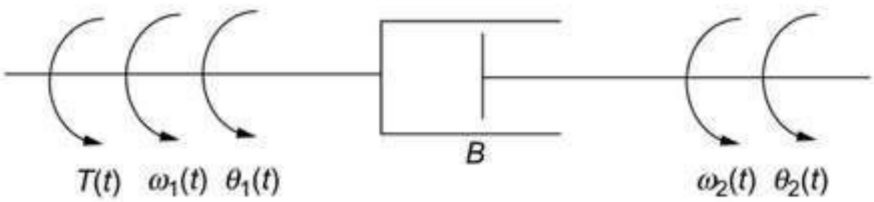


Figure 1.3.8 Mechanical rotational element: Dashpot

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.39]

$$T_b(t) = B\left(\frac{d_{\theta1}}{dt} - \frac{d_{\theta2}}{dt}\right)$$

Here, $T_b(t)$ is measured in Newton-meter.

Torsional/Rotational Spring Torque, $T_k(t)$

When a torque T(t) is applied to a spring element, K, it experiences ana angular displacement and it is shown in figure 1.3.9.

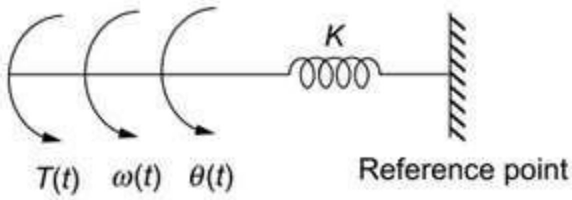


Figure 1.3.9 Mechanical rotational element: Dashpot

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.39]

According to Hooke's law, spring torque is proportional to the angular displacement.

$$T_k(t) \propto \theta$$

$$T_k(t) = K\theta$$

where, K is the spring constant (N-m/rad).

A spring element with two angular displacements is given in figure 1.3.10.

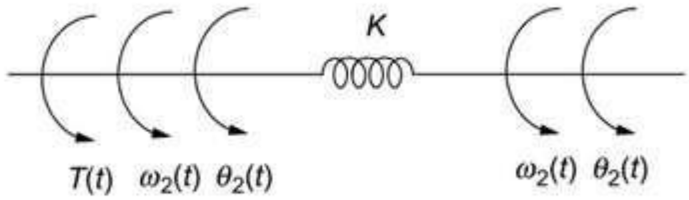


Figure 1.3.10 Mechanical rotational element: Dashpot

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.40]

Here, $T_k(t)$ is measured in Newton-meter.

According to D'Alembert's principle, "The algebraic sum of the externally applied torques to any body is equal to the algebraic sum of the opposing torques restraining motion produced by the elements present in the body." A simple rotational mechanical system and its free body diagram are shown in figures 1.3.11 (a) and (b) respectively.

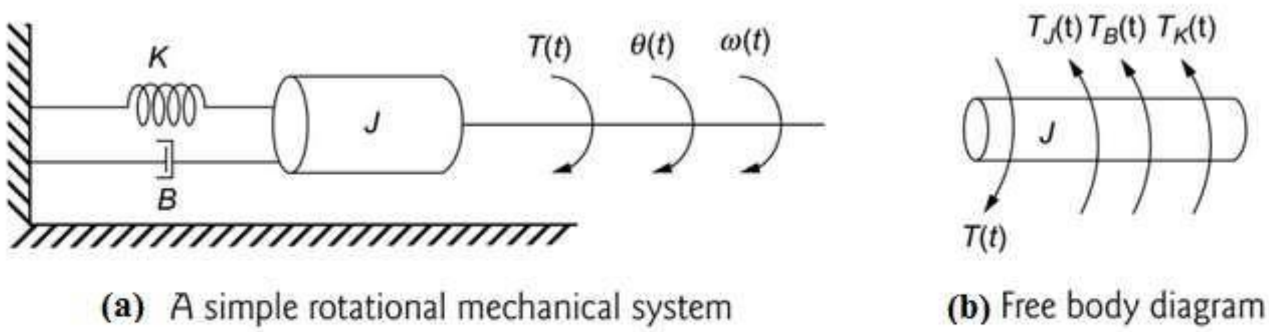


Figure 1.3.11 Mechanical rotational system and its free body diagram

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.40]

$$T_J = J\frac{d^2\theta}{dt^2}$$

$$T_b = B\frac{d\theta}{dt}$$

$$T_K = k\theta$$

$$T(t) = T + T_b + T_k = j\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$

Translational mechanical system	Rotational mechanical system
Force (F)	Torque (T)
Velocity (v)	Angular velocity (ω)
Displacement (x)	Angular displacement (θ)
Mass (M)	Moment of inertia (J)
Damping coefficient (B)	Rotational damping (B)
Spring constant (K)	Rotational spring constant (K)

1.2 OPEN AND CLOSED LOOP SYSTEMS

In recent years, control systems have gained an increasingly importance in the development and advancement of the modern civilization and technology. Figure shows the basic components of a control system. Disregard the complexity of the system; it consists of an input (objective), the control system and its output (result). Practically our day-to-day activities are affected by some type of control systems. There are two main branches of control systems:

1) Open-loop systems and 2) Closed-loop systems

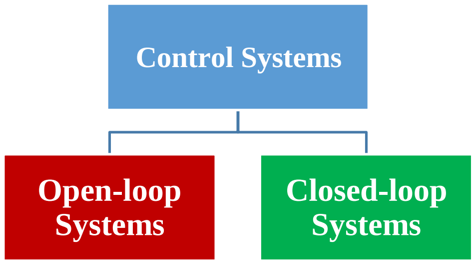


Figure 1.2.1 Classification of Control Systems

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.2]

OPEN LOOP SYSTEMS

A control system that cannot adjust itself to the changes is called open-loop control system. In general, manual control systems are open-loop systems. The block diagram of open-loop control system is shown in figure.

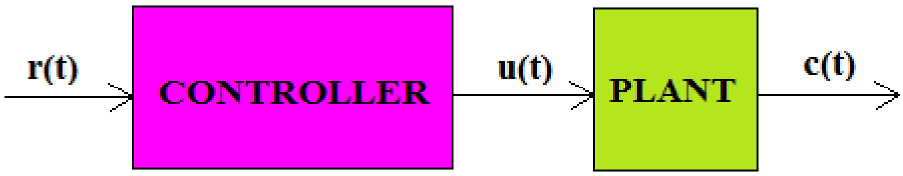


Figure 1.2.2 Block diagram of open loop system

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.2]

Here, r(t) is the input signal, u(t) is the control signal/actuating signal and c(t) is the output signal. In this system, the output remains unaltered for a constant input. In case of any discrepancy, the input should be manually changed by an operator. An open loop control system is suited when there is tolerance for fluctuation in the system and when the system parameter variation can be handled irrespective of the environmental conditions.

PRACTICAL EXAMPLES OF OPEN LOOP CONTROL SYSTEM

1. Electric Hand Drier-Hot air (output) comes out as long as you keep your hand under the machine, irrespective of how much your hand is dried.

2. Automatic Washing Machine-This machine runs according to the pre-set time irrespective of washing is completed or not.
3. Bread Toaster-This machine runs as per adjusted time irrespective of toasting is completed or not.
4. Automatic Tea/Coffee Maker-These machines also function for pre adjusted time only.
5. Timer Based Clothes Drier-This machine dries wet clothes for pre-adjusted time, it does not matter how much the clothes are dried.
6. Light Switch-Lamps glow whenever light switch is on irrespective of light is required or not.
7. Volume on Stereo System-Volume is adjusted manually irrespective of output volume level.

Advantages of Open Loop Control System

a) Simple in construction and design
b) Economical
c) Easy to maintain
d) Generally stable
e) Convenient to use as output is difficult to measure.

Disadvantages of Open Loop Control System

a) They are inaccurate
b) They are unreliable
c) Any change in output cannot be corrected automatically.

CLOSED LOOP SYSTEMS

Any system that can respond to the changes and make corrections by itself is known as closed loop control system. The only difference when compared to open loop system is the presence of feedback action. The block diagram of a closed loop system is shown in the figure. Here, r(t) is the input signal, e(t) is the error signal/actuating signal, u(t) or m(t) is the control signal/manipulated signal, b(t) is the feedback signal and c(t) is the controlled output. Here, the output of the machine is fed back to a comparator (error detector). The output signal is compared with the reference input, r(t) and the error signal, e(t) is sent to the controller. Based on the error, the controller adjusts the air conditioners input [control signal u(t)]. This process is continued till the error gets nullified.

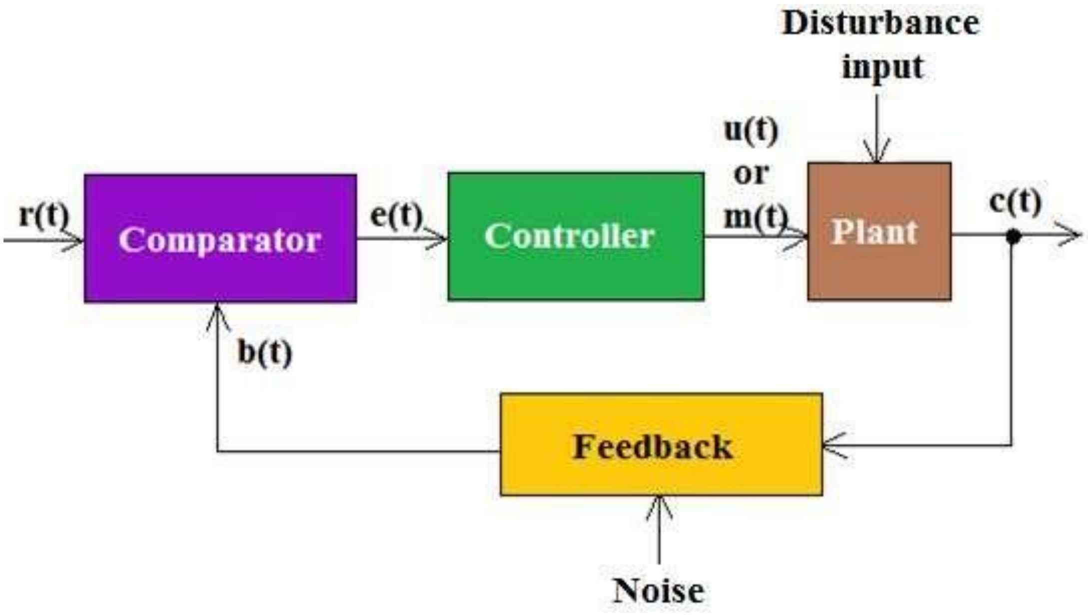


Figure 1.2.3 Block diagram of closed loop system

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.3]

Both the manual and automatic controls can be implemented in a closed loop system.

PRACTICAL EXAMPLES OF CLOSED LOOP CONTROL SYSTEM

1) Automatic Electric Iron-Heating elements are controlled by output temperature of the iron.
2) Servo Voltage Stabilizer-Voltage controller operates depending upon output voltage of the system.
3) Water Level Controller-Input water is controlled by water level of the reservoir.
4) Missile Launched and Auto Tracked by Radar-The direction of missile is controlled by comparing the target and position of the missile.
5) An Air Conditioner-An air conditioner functions depending upon the temperature of the room.
6) Cooling System in Car-It operates depending upon the temperature which it controls.

Advantages of Closed Loop Control System

a) Closed loop control systems are more accurate even in the presence of non-linearity.
b) Highly accurate as any error arising is corrected due to presence of feedback signal.

c) Bandwidth range is large.
d) Facilitates automation.
e) The sensitivity of system may be made small to make system more stable.
f) This system is less affected by noise.

Disadvantages of Closed Loop Control System

a) They are costlier.
b) They are complicated to design.
c) Required more maintenance.
d) Feedback leads to oscillatory response.
e) Overall gain is reduced due to presence of feedback.
f) Stability is the major problem and more care is needed to design a stable closed loop system.

S. No.	Open loop control system	Closed loop control system
1	Inaccurate	Accurate
2	Unreliable	Reliable
3	Stable	Unstable. It can be stabilized using the feedback or by reducing sensitivity
4	Bandwidth is small	Bandwidth is large
5	System is affected by noise	System is less affected by noise
6	Cheap	Costly
7	Simple in construction	Complex construction since a greater number of components are present
8	Requires less maintenance	Requires more maintenance
9	Overall gain is high	Overall high is reduced due to feedback

1.7 SERVOMOTOR

Servo Motor also called control motors, are used in feedback control systems as output actuators and does not use for continuous energy conversion. The principle of the Servomotor is similar to that of the other electromagnetic motor, but the construction and the operation are different. Their power rating varies from a fraction of a watt to a few hundred watts. Rotor inertia of the motors is low and have a high speed of response. The rotor of the Motor has the long length and smaller diameter. They operate at very low speed and sometimes even at the zero speed. Servo motor is widely used in radar and computers, robot, machine tool, tracking and guidance systems, processing controlling.

AC SERVOMOTORS

Servo motors are generally an assembly of four things: a DC motor, a gearing set, a control circuit and a position-sensor (usually a potentiometer). The position of servo motors can be controlled more precisely than those of standard DC motors, and they usually have three wires (power, ground & control). AC Servo Motors are divided into two types 2 and 3 Phase AC servomotor. Most of the AC servomotor are of the two-phase squirrel cage induction motor type. They are used for low power applications. The three phase squirrel cage induction motor is now utilized for the applications where high-power system is required. An AC servo motor is essentially a two-phase induction motor with modified constructional features to suit servo applications as shown in figure 1.7.1.

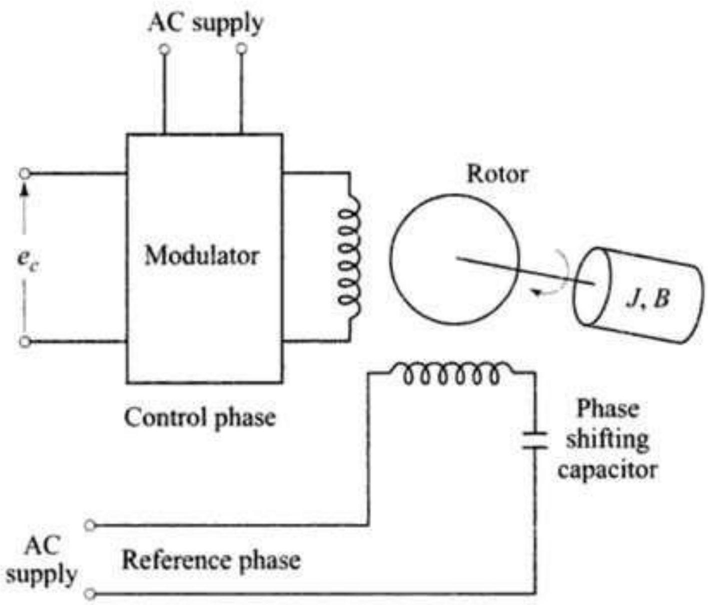


Figure 1.7.1 Symbolic representation of AC Servomotor

[Source: "Control Systems: Principles and Design" by M. Gopal, Page: 132]

It has two windings displaced by 90° on the stator. One winding, called as reference winding, is supplied with a constant sinusoidal voltage. The second winding, called control winding, is supplied with a variable control voltage which is displaced by -- 90° out of phase from the reference voltage.

The major differences between the normal induction motor and an AC servo motor are

1. The rotor winding of an ac servo motor has high resistance (R) compared to its inductive reactance (X) so that its X / *R* ratio is very low.
2. For a normal induction motor, X / *R* ratio is high so that the maximum torque is obtained in normal operating region which is around 5% of slip.

The torque speed characteristics of a normal induction motor and an ac servo motor are shown in figures 1.7.2 and 1.7.3.

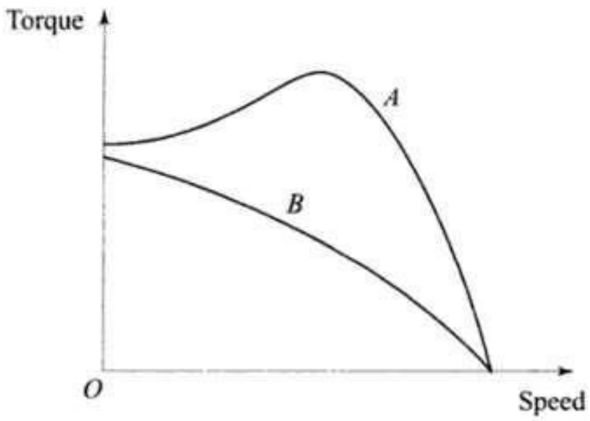


Figure 1.7.2 Torque speed characteristics of AC motors

[Source: "Control Systems: Principles and Design" by M. Gopal, Page: 131]

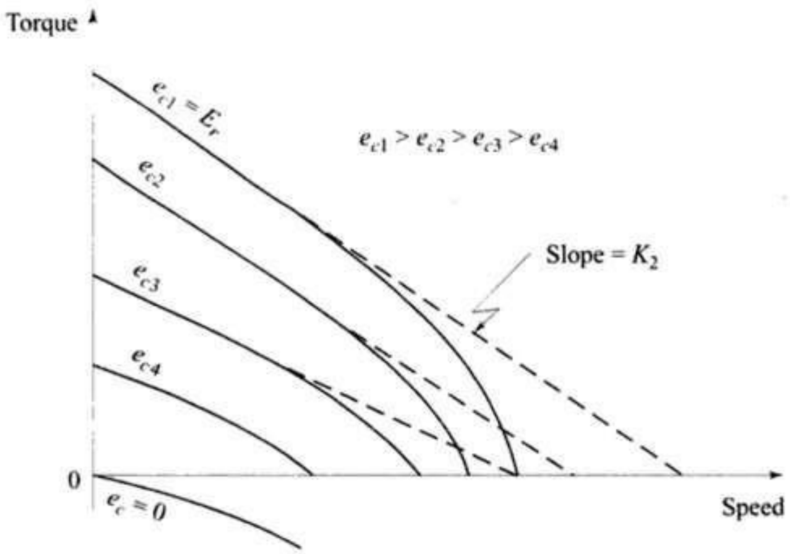


Figure 1.7.3 Torque speed characteristics of AC servomotor

[Source: "Control Systems: Principles and Design" by M. Gopal, Page: 133]

The torque-speed characteristic of a normal induction motor is highly nonlinear and has a positive slope for some portion of the curve. This is not desirable for control applications. as the positive slope makes the systems unstable. The torque speed characteristic of an ac servo motor is fairly linear and has negative slope throughout. The rotor construction is usually squirrel cage or drag cup type for an ac servo motor. The diameter is small compared to the length of the rotor which reduces inertia of the moving parts. Thus, it has good accelerating characteristic and good dynamic response. The supplies to the two windings of ac servo motor are not balanced as in the case of a normal induction motor. The control voltage varies both in magnitude and phase with respect to the constant reference vulture applied to the reference winding. The direction of rotation of the motor depends on the phase (± 90°) of the control voltage with respect to the reference voltage. For different rms values of control voltage the torque speed characteristics are shown in Figure. The torque varies approximately linearly with respect to speed and also controls voltage. The torque speed characteristics can be linearized at the operating point and the transfer function of the motor can be obtained.

DC SERVOMOTOR

A DC servo motor is used as an actuator to drive a load. It is usually a DC motor of low power rating. DC servo motors have a high ratio of starting torque to inertia and therefore they have a faster dynamic response. DC motors are constructed using rare earth permanent magnets which have high residual flux density and high coercively. As no field winding is used, the field copper losses am zero and hence, the overall efficiency of the motor is high. The speed torque characteristic of this motor is flat over a wide range, as the armature reaction is negligible. Moreover, speed in directly proportional to the armature voltage for a given torque. Armature of a DC servo motor is specially designed to have low inertia. DC Servo Motors are separately excited DC motor or permanent magnet DC motors. The figure (a) shows the connection of Separately Excited DC Servo motor and the figure (b) shows the armature MMF and the excitation field MMF in quadrature in a DC machine. This provides a fast torque response because torque and flux are decoupled. Therefore, a small change in the armature voltage or current brings a significant shift in the position or speed of the rotor. Most of the high-power servo motors are mainly DC.

(a) Armature controlled DC servo motor

The physical model of an armature controlled DC servo motor is given in

The armature winding has a resistance Ra and inductance La.

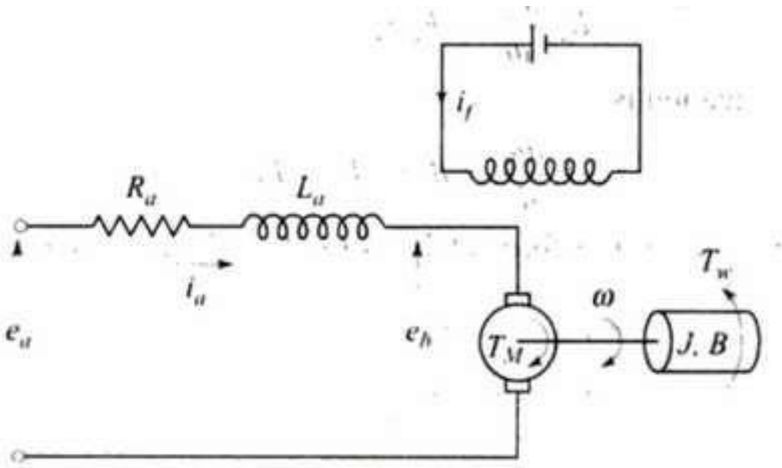


Figure 1.7.4 Armature controlled DC motor with load

[Source: "Control Systems: Principles and Design" by M. Gopal, Page: 117]

The field is produced either by a permanent magnet or the field winding is separately excited and supplied with constant voltage so that the field current, i_f is a constant. When the armature is supplied with a DC voltage of e_a volts, the armature rotates and produces a back emf, e_b. The armature current i_a depends on the difference of e_b and e_n. The armature has a permanent of inertia J, frictional coefficient B_0. The angular displacement of the motor is θ. The torque produced by the motor is given by

$$T_M = K_T i_a$$

$$e_b = K_b \omega$$

$$L_a \frac{di_a}{dt} + R_a$$

Taking Laplace transform,

$$T_M(s) = K_T I_a(s)$$

$$E_b(s) = k_b \omega(s)$$

$$L_a s I_a(s) + R_a I_a(s) + E_b(s) = E_a(s)$$

$$Js\omega(s) + B\omega(s) + T\omega(s) = T_M(s)$$

where K_T is the motor torque constant. The back emf is proportional to the speed of the motor and hence

On solving,

$$\frac{\omega(s)}{E_a(s)} = \frac{K_T/R_a}{js + B + K_T K_b/R_a}$$

$$\frac{\omega(s)}{E_a(s)} = \frac{k_m}{\tau_m s + 1}$$

where,

$$K_m = \frac{KT}{\quad}$$

K_m – motor gain constant, τ_m – motor time constant

(b) Field controlled DC servo motor

The schematic diagram of a field controlled DC servo motor is shown in figure 1.7.5.

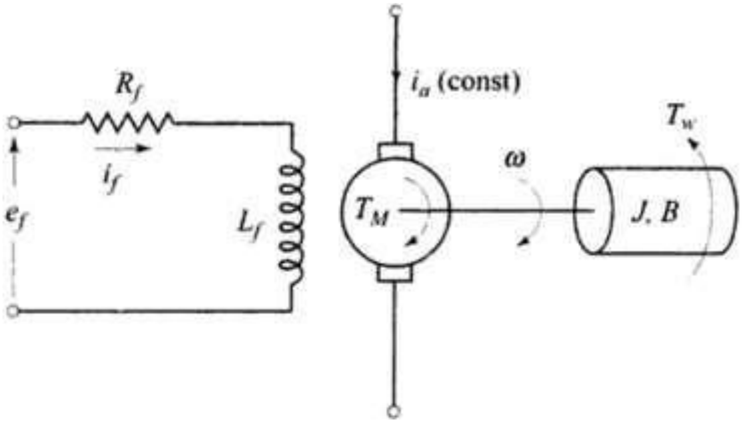


Figure 1.7.5 Field controlled DC servomotor

[Source: "Control Systems: Principles and Design" by M. Gopal, Page: 120]

$$T = K_{Tf} i_f$$

$$L_f \frac{di_f}{dt} + R_f i_f = e_f$$

Taking Laplace transform,

$$T(s) = K_{Tf} I_f(s)$$
$$L_f s I_f(s) + R_f I_f(s) = E_f(s)$$
$$Js^2\theta(s) + BS\theta(s) + T_w(s) = T_M(s)$$

$$\frac{\theta(s)}{E_f(s)} = \frac{K_{Tf}}{s(js + B)(R_f + sL_f)} = \frac{K_{Tf}/R_f\, B}{s\left(\frac{j}{B}s + 1\right)\left(1 + s\frac{L_f}{R_f}\right)} = \frac{K_m}{s(\tau_{m_s} + 1)(1 + s\tau_f)}$$

where, motor gain constant, $K_m = K_{Tf}/R_f B$

motor time constant, $\tau_m = \frac{j}{B}$

field time constant, $\tau_f = \frac{L_f}{R_f}$

1.9 SIGNAL FLOW GRAPH

The diagrammatic or pictorial representation of a set of simultaneous linear algebraic equations of a more complicated system is known as signal flow graph (SFG). It shows the flow of signals in the system. It is important to note that the flow of signals in SFG is only in one direction. To represent the set of algebraic equations using SFG, it is necessary that those algebraic equations are to be represented in the s-domain. The transfer function of the system which is represented by SFG can be obtained by using Mason's gain formula. The dependent and independent variables in the set of algebraic equations are represented by the nodes in the SFG. The branches are used to connect different nodes present in SFG. The connection between the different nodes is based on the relationship given in the algebraic equation. The arrow and the multiplication factor indicated on the branch of SFG represent the signal direction. The SFG and the block diagram representation of a system yield the same transfer function; but when a system is represented by SFG, the transfer function is obtained easily and quickly without using the SFG reduction techniques. The terminologies used in SFG rae explained with the help of SFG of a system as shown in figure 1.9.1.

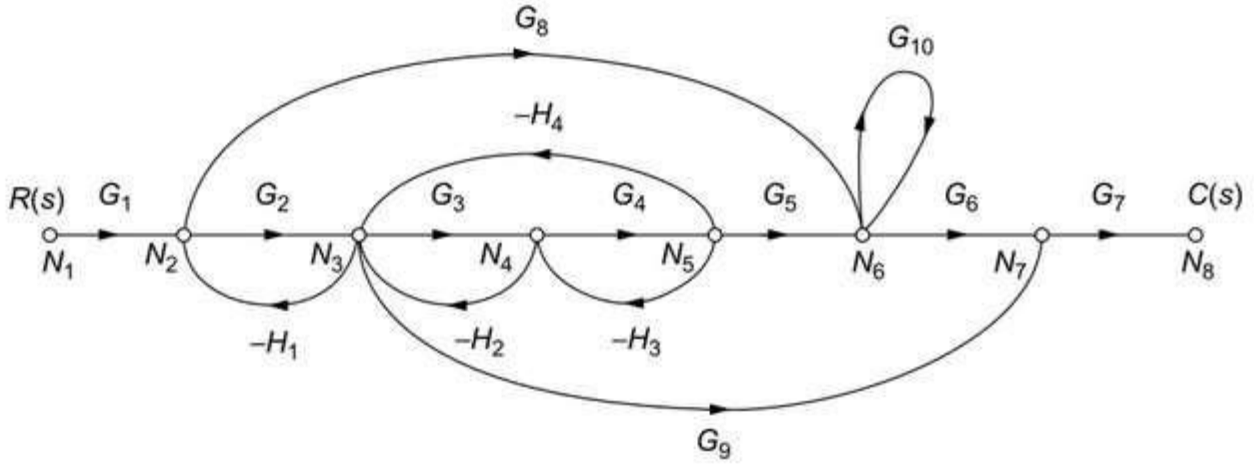


Figure 1.9.1 Signal flow graph of a system

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 4.1]

Node: The variables present in the set of algebraic equations are represented by a point called node.

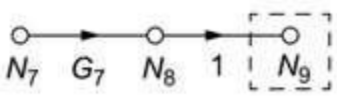


Figure 1.9.2 Node in signal flow graph

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 4.2]

Branch

The line segment joining the two nodes with a specific direction is known as a branch. The specific direction is indicated by an arrow in the branch.

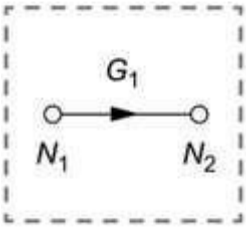


Figure 1.9.3 Branch in signal flow graph

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 4.2]

MASON'S GAIN FORMULA

A technique to reduce a signal flow graph to a single transfer function requires the application of one formula. The transfer function of a system represented by a signal flow graph is

$$T = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

where, k – number of forward path

P_i – ith forward path gain

Δ – 1- (sum of individual loop gains)+(sum of product of two non-touching loop gains)-(sum of product of three non-touching loop gains)+……

Δ_i – 1- (Δ of the loop non-touching the ith forward path)

Steps to determine the transfer function of a system using SFG Method

Step 1: Identify the number of forward paths.

Step 2: Identify the individual loops and find their respective loop gains.

Step 3: Identify the two non-touching loops and find the product of their gains.

Step 4: Identify the three non-touching loops and find the gain product and so on…

Step 5: Calculate the Δ value.

Step 6: Calculate the Δ_i value.

Step 7: Use Mason's gain formula to calculate the transfer function value, T.

Characteristics	Block Diagram	Signal flow graph
Time Consumption	More since the diagrams have to be redrawn repeatedly	Less since there is no necessary to redraw the diagrams
Technique applied	Block Diagram reduction technique	Mason's gain formula
Representation of elements	Blocks are used to represent the element.	Nodes are need to represent the elements
Representation of transfer function of each element	Represented inside the block of each element	Represented along the branches above the arrow ahead
Feedback paths	Present and hence the formula, (G/(1±GH)) is used to reduce the paths	Present, but there is no need for any formulae to reduce the paths
Self-loops	Absence of self-loops	Presence of self-loops
Summing points and takeoff points	Present in block diagram	Absence in SFG

1.6 TRANSFER FUNCTION

The *transfer function* of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Open loop transfer function: G(s)

Loop transfer function: G(s)H(s)

Closed loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(S)}$$