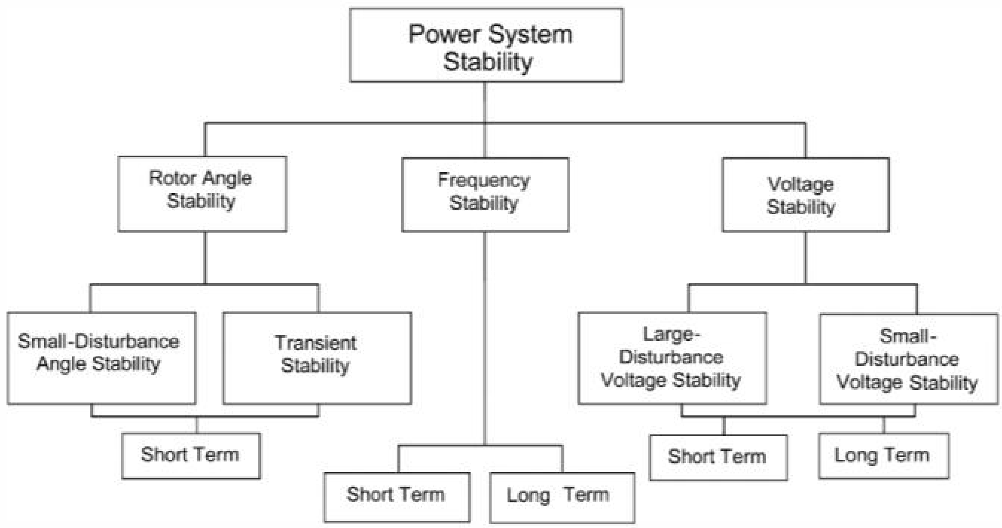
Classification of power system stability Introduction

At present the demand for electricity is rising phenomenally especially in developing country like India. This persistent demand is leading to operation of the power system at its limit.The need for reliable, stable and quality power is on the rise due to electric power sensitiveindustries like information technology, communication, electronics etc. In this scenario, meeting the electric power demand is not the only criteria but also it is the responsibility of the power system engineers to provide a stable and quality power to the consumers. These issues highlight the necessity of understanding the power system stability. In this course we will try to understand how to asses the stability of a power system, how to improve the stability and finally how to prevent system becoming unstable.

Basic Concepts and Definitions of Power System Stability Power system stability

The stability of an interconnected power system means is the ability of the power system is to return or regain to normal or stable operating condition after having been subjected to some form of disturbance

“Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most of the system variables bounded so that practically the entire system remains intact” [1], [2]. The disturbances mentioned in the definition could be faults, load changes, generator outages, line outages, voltage collapse or some combination of these. Power system stability can be broadly classified into rotor angle, voltage and frequency stability. Each of these three stabilities can be further classified into large disturbance or small disturbance, short term or long term. The classification is depicted in Fig.



Rotor angle stability:

“It is the ability of the system to remain in synchronism when subjected to a disturbance”. The rotor angle of a generator depends on the balance between the electromagnetic torque due to the generator electrical power output and mechanical torque due to the input mechanical power through a prime mover.

Remaining in synchronism means that all the generators electromagnetic torque is exactly equal to the mechanical torque in the opposite direction. If in a generator the balance between electromagnetic and mechanical torque is disturbed, due to disturbances in the system, then this will lead to oscillations in the rotor angle. Rotor angle stability is further classified into small disturbance angle stability and large disturbance angle stability.

Small-disturbance or small-signal angle stability:

It is the ability of the system to remain in synchronism when subjected to small disturbances. If a disturbance is small enough so that the nonlinear power system can be approximated by a linear system, then the study of rotor angle stability of that particular system is called as small-disturbance angle stability analysis. Small disturbances can be small load changes like switching on or off of small loads, line tripping, small generators tripping etc. Due to small disturbances there can be two types of instability: non-oscillatory instability and oscillatory instability. In non-oscillatory instability the rotor angle of a generator keeps on increasing due to a small disturbance and in case of oscillatory instability the rotor angle oscillates with increasing magnitude.

Large-disturbance or transient angle stability

"It is the ability of the system to remain in synchronism when subjected to large disturbances". Large disturbances can be faults, switching on or off of large loads, large generators tripping etc. When a power system is subjected to large disturbance, it will lead to large excursions of generator rotor angles. Since there are large rotor angle changes the power system cannot be approximated by a linear representation like in the case of small-disturbance stability. The time domain of interest in case of large-disturbance as well as small-disturbance angle stability is any where between 0.1- 10 s. Due to this reason small and large- disturbance angle stability are considered to be short term phenomenon. It has to be noted here that though in some literature "dynamic stability" is used in place of transient stability

Voltage stability

"It is the ability of the system to maintain steady state voltages at all the system buses when subjected to a disturbance. If the disturbance is large then it is called as large-disturbance voltage stability and if the disturbance is small it is called as small-disturbance voltage stability". Unlike angle stability, voltage stability can also be a long term phenomenon. In case voltage fluctuations occur due to fast acting devices like induction motors, power electronic drive, HVDC etc then the time frame for understanding the stability is in the range of 10-20 s and hence can be treated as short term phenomenon. On the other hand if voltage variations are due to slow change in load, over loading of lines, generators hitting reactive power limits, tap changing transformers etc then time frame for voltage stability can stretch from 1 minute to several minutes. The main difference between voltage stability and angle stability is that voltage stability depends on the balance of reactive power demand and generation in the system where as the angle stability mainly depends on the balance between real power generation and demand.

Frequency stability:

It refers to the ability of a power system to maintain steady frequency following a severe disturbance between generation and load. It depends on the ability to restore equilibrium between system generation and load, with minimum loss of load. Frequency instability may lead to sustained frequency swings leading to tripping of generating units or loads. During frequency excursions, the characteristic times of the processes and devices that are activated will range from fraction of seconds like under frequency control to several minutes, corresponding to the response of devices such as prime mover and hence frequency stability may be a short-term phenomenon or a long-term phenomenon. Though, stability is classified into rotor angle, voltage and frequency stability they need not be independent isolated events. A voltage collapse at a bus can lead to large excursions in rotor angle and frequency. Similarly, large frequency deviations can lead to large changes in voltage magnitude

Classification of power system

stability Rotor angle stability

Rotor angle stability is the ability of interconnected synchronous machines of a power system to remain in synchronism.

Steady state stability

Steady state stability is defined as the ability of the power system to bring it to a stable condition or remain in synchronism after a small disturbance.

Steady state stability limit

The steady sate stability limit is the maximum power that can be transferred by a machine to receiving system without loss of synchronism

Transient stability

Transient stability is defined as the ability of the power system to bring it to a stable condition or remain in synchronism after a large disturbance.

Transient stability limit

The transient stability limit is the maximum power that can be transferred by a machine to a fault or a receiving system during a transient state without loss of synchronism. Transient stability limit is always less than steady state stability limit **Dynamic stability**

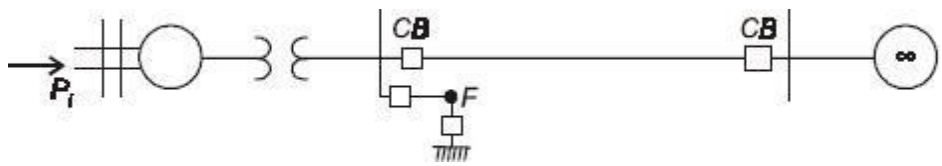
It is the ability of a power system to remain in synchronism after the initial swing (transient stability period) until the system has settled down to the new steady state equilibrium condition

Voltage stability

It is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance.

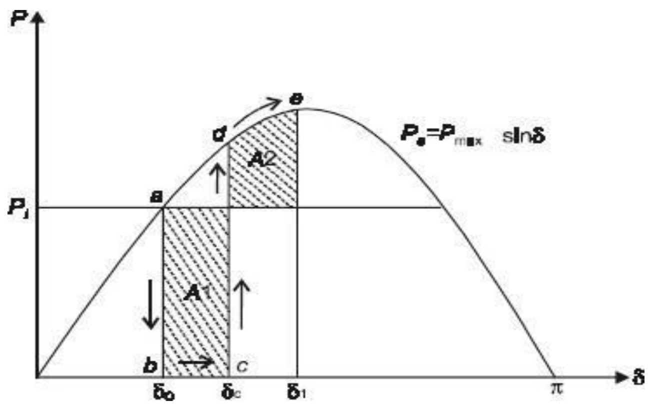
Critical Clearing Angle and Critical Clearing Time:-

If a fault occurs in a system, δ begins to increase under the influence of positive accelerating power, and the system will become unstable if δ becomes very large. There is a critical angle within which the fault must be cleared if the system is to remain stable and the equal area criterion is to be satisfied. This angle is known as the **critical clearing angle**.



(Fig. 8 Single machine infinite bus system)

Consider a system as shown in Fig. 8 operating with mechanical input P_i at steady angle $_0$. $P_i = P_e$ as shown by point 'a' on the power angle diagram as shown in Fig. 9. Now if three phase short circuit occur at point F of the outgoing radial line , the terminal voltage goes to zero and hence electrical power output of the generator instantly reduces to zero i.e., $P_e = 0$ and the state point drops to 'b'. The acceleration area A_1 starts to increase while the state point moves along b-c. At time t_c corresponding clearing angle δ_C, the fault is cleared by the opening of the line circuit breaker. t_c is called clearing time and δ_C is called clearing angle. After the fault is cleared, the system again becomes healthy and transmits power $P_e = P_{max} \sin\delta$, i.e., the state point shifts to'd' on the power angle curve. The rotor now decelerates and the decelerating area A_2 begins to increase while the state point moves along d-e. For stability, the clearing a ngle, δ_C, must be such that area A_1 = area A_2.



(Fig. 9 $P_e \sim \delta$ characteristics)

Expressing area A1 =Area A2 mathematically we have,

$$P_i(\delta_c - \delta_0) = \int_{c}^{1} (P_e - P_i) d\delta$$

$$\therefore \quad P_i(\delta_c - \delta_0) = \int_{c}^{1} P_{max} \sin \delta \, . \, d\delta - P_i(\delta_1 - \delta_c)$$

$$\therefore \quad P_i\delta_c - P_i\delta_0 = P_{max}(-\cos\delta_1 + \cos\delta_c) - P_i\delta_1 + P_i\delta_c$$

$$\therefore \quad P_{max}(\cos\delta_c - \cos\delta_1) = P_i(\delta_1 - \delta_0) \quad \text{.................................(60)}$$

Also $= \sin \quad _0$ ....(61)

Using equation (60) and (61) we get,

$$P_{max}(\cos\delta_c - \cos\delta_1) = P_{max}(\delta_1 - \delta_0)\sin \quad _0$$

$$\therefore \quad \cos\delta_c = \cos\delta_1 + (\delta_1 - \delta_0)\sin \quad _0 \quad \text{...(62)}$$

Where δ_c= clearing angle, δ_0 = initial power angle, and δ_1 = power angle to which the rotor advances (or overshoots) beyond δ_c.

For a three phase fault with P_e =0,

$$\frac{d^2}{dt^2} = \frac{fP_i}{H} \quad \text{.. (63)}$$

Integrating equation (63) twice and utilizing the fact that $\frac{d}{dt} = 0$ and t = 0 yields

$$\delta = \frac{fP_i}{2H} t^2 + \delta_0 \quad \text{.............................. (64)}$$

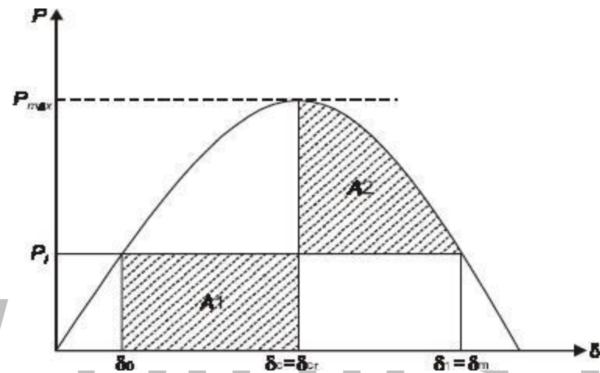
If t_c is the clearing time corresponding to a clearing angle δ_c, then we obtain from equation (64),

$$\delta_c = \frac{\pi f P_i}{2H} t_c^2 + \delta_0$$

So

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_i}} \quad \text{.........(65)}$$

Note that δ_c can be obtained from equation (62). As the clearing of faulty line is delayed, A1 increases and so does δ_1 to find A2=A1 till $\delta_1 = \delta_m$ as shown in Fig. 10.



(Fig. 10 Critical clearing angle)

For a clearing angle (clearing time) larger than this value, the system would be unstable. The maximum allowable value of the clearing angle and clearing time for the system to remain stable are known as critical clearing angle and critical clearing time respectively.

From Fig. 10, $\delta_m = \pi - \delta_0$. Substituting this in equation (62), we have,

$$\cos\delta_{cr} = \cos\delta_m + (\delta_m - \delta_0)\sin\delta_0$$

$$\cos\delta_{cr} = \cos\delta_m + (\pi - \delta_0 - \delta_0)\sin\delta_0$$

$$\cos\delta_{cr} = \cos(\pi - \delta_0) + (\pi - 2\delta_0)\sin\delta_0$$

$$\cos\delta_{cr} = (\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0$$

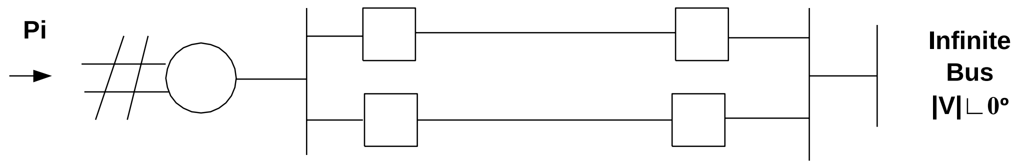
$$\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right] \quad \text{.........(66)}$$

Using equation (65) critical clearing angle can be obtained as

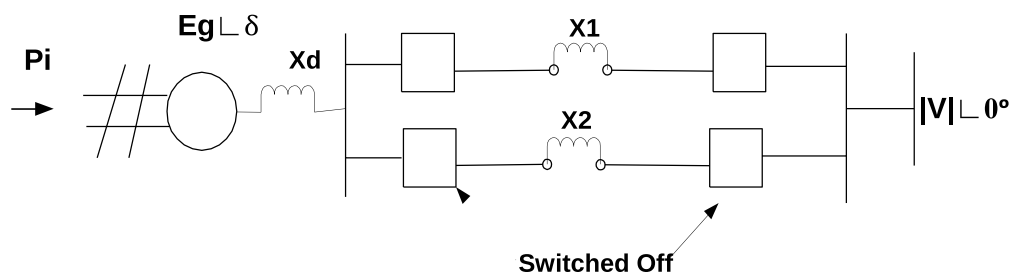
$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_i}} \quad \text{.........(67)}$$

Application of the Equal Area Criterion:-

(1) Sudden Loss of One of parallel Lines:-



(a)



(b)

(Fig. 11 Single machine tied to infinite bus through two parallel lines)

Consider a single machine tied to infinite bus through parallel lines as shown in Fig. 11(a). The circuit model of the system is given in Fig. 11(b).

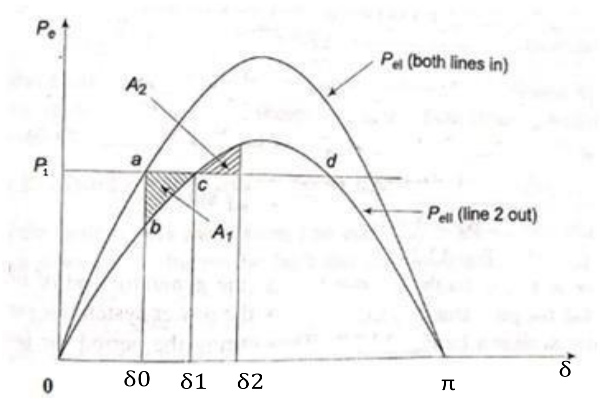
Let us study the transient stability of the system when one of the lines is suddenly switched off with the system operating at a steady load. Before switching off, power angle curve is given by

$$P_{eI} = \frac{|E_g|\,|V|}{X_d + X_1 \,||\, X_2} \sin\delta = P_{maxI} \sin\delta$$

Immediately on switching of line 2, power angle curve is given by

$$P_{eII} = \frac{|E_g|\,|V|}{X_d + X_1} \sin\delta = P_{maxII} \sin\delta$$

In Fig. 12, wherein $P_{maxII} < P_{maxI}$ as $X_d + X_1 > X_d + X_1 \,||\, X_2$. The system is operating initially with a steady state power transfer $P_e = P_i$ at a torque angle δ_0 on curve I.



(Fig. 12 Equal area criterion applied to the opening of one of the two lines in parallel)

On switching off line2, the electrical operating point shifts to curve II (point b). Accelerating energy corresponding to area $A_1$is put into rotor followed by decelerating energy for $\delta > \delta_1$. Assuming that an area A_2 corresponding to decelerating energy (energy out of rotor) can be found such that $A_1 = A_2$, the system will be stable and will finally operate at c corresponding to a new rotor angle is needed to transfer the same steady power.

If the steady load is increased (line P_i is shifted upwards) a limit is finally reached beyond which decelerating area equal to A_1 cannot be found and therefore, the system behaves as an unstable one. For the limiting case, δ_1 has a maximum value given by

$$\delta_1 = \delta_{max} = \pi - \delta_0$$

Transient Stability-Equal Area Criterion:-

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. This may be sudden application of load, loss of generation, loss of large load, or a fault on the system.

A method known as the equal area criterion can be used for a quick prediction of stability. This method is based on the graphical interpretation of the energy stored in the rotating mass as an aid to determine if the machine maintains its stability after a disturbance. This method is only applicable to a one-machine system connected to an infinite bus or a two-machine system. Because it provides physical insight to the dynamic behavior of the machine.

Now consider the swing equation (18),

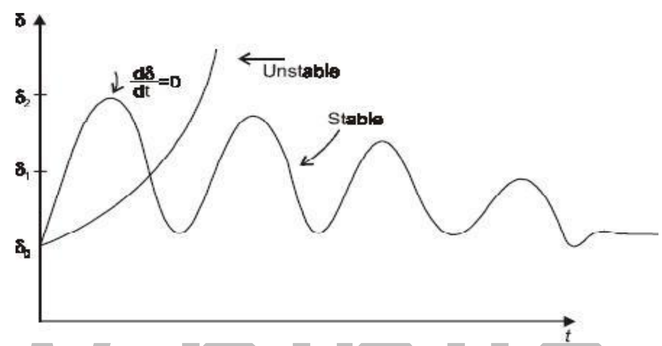
$$M_{(pu)} \frac{d^2\delta}{dt^2} = (P_i - P_e)$$

or

$$M_{(pu)} \frac{d^2\delta}{dt^2} = P_a$$

or $$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \quad \text{.........................} (52)$$

As shown in Fig.5, in an unstable system, δ increases indefinitely with time and machine looses synchronism. In a stable system, δ undergoes oscillations, which eventually die out due to damping. From Fig.4, it is clear that, for a system to be stable, it must be that $\frac{d\delta}{dt} = 0$ at some instant. This criterion ($\frac{d\delta}{dt} = 0$) can simply be obtained from equation (52).



(Fig. 5 A plot of δ (t))

Multiplying equation (52) by $\frac{2d\delta}{dt}$, we have

$$\frac{2d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{2P_a}{M} \cdot \frac{d\delta}{dt} \quad \text{.........................} (53)$$

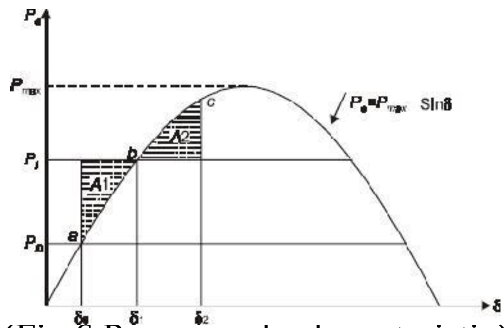
This upon integration with respect to time gives

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta \quad \text{.........................} (54)$$

Where $P_a = P_i - P_e$ = accelerating power and δ_0 is the initial power angle before the rotor begins to swing because of a disturbance. The stability ($\frac{d\delta}{dt} = 0$) criterion implies that

$$\int_{\delta_0}^{\delta} P_a d\delta = 0 \quad \text{.........................} (55)$$

For stability, the area under the graph of accelerating power P_a versus δ must be zero for some value of δ; i.e., the positive (accelerating) area under the graph must be equal to the negative (decelerating) area. This criterion is therefore know as the equal area criterion for stability and is shown in Fig. 6.



(Fig.6 Power angle characteristic)

<u>Application to sudden change in power input</u>:-

In Fig. 6 point ‘a’ corresponding to the δ_0 is the initial steady-state operating point. At this point, the input power to the machine,$P_{i0} = P_{e0}$, where P_{e0} is the developed power. When a sudden increase in shaft input power occurs toP_i, the accelerating powerP_a, becomes positive and the rotor moves toward point ‘b’

We have assumed that the machine is connected to a large power system so that $|V_t|$ does not change and also x_d does not change and that a constant field current maintains $|E_g|$. Consequently, the rotor accelerates and power angle begins to increase. At point $P_i = P_e$ and $\delta = \delta_1$. But $\frac{d}{dt}$ is still positive and δ overshoots ‘b’, the final steady-state operating point. Now P_a is negative and δ ultimately reaches a maximum value δ_2 or point ‘c’ and swing back towards point ‘b’. Therefore the rotor settles back to point ‘b’, which is ultimate steady-state operating point.

In accordance with equation (55) for stability, equal area criterion requires

$$\text{Area } A_1 = \text{Area } A_2$$

or $$\int_{0}^{1}(P_i - P_{max} \sin\delta)d\delta = \int_{1}^{2}(P_{max} \sin\delta - P_i)d\delta \text{......................} (56)$$

or $$_i(\ _1 - \ _0) + \ (\cos_1 - \cos_0) =$$

$$_i(\ _1 - \ _2) + \ (\cos_1 - \cos_2) \text{...} (57)$$

But $$_i = \ \sin$$

Which when substituted in equation (57), we get

$$(\ _1 - \ _0)\sin \ + \ (\cos_1 - \cos_0) = (\ _1 - \ _2)\sin \ + \ (\cos_1 - \cos_2) \text{...} (58)$$

On simplification equation (58) becomes

$$(\delta_2 - \delta_0)\sin\delta_1 + \cos\delta_2 - \cos\delta_0 = 0 \quad \text{.....................................} (59)$$

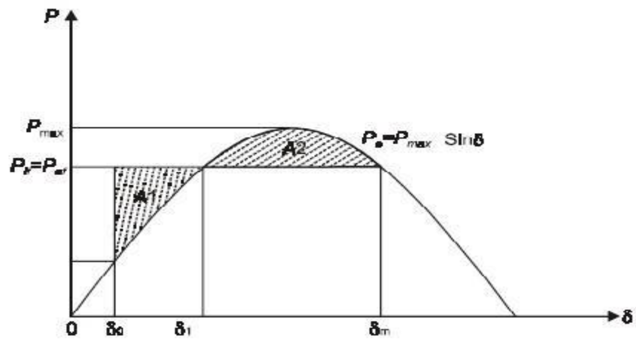
Example 3:-

A synchronous generator, capable of developing 500MW power per phase, operates at a power angle of 8^0. By how much can the input shaft power be increased suddenly without loss of stability? Assume that P_{max} will remain constant.

Solution:-

Initially, $\delta_0 = 8^0$

$$P_{e0} = P_{max}\sin\delta_0 = 500\sin 8^0 = 69.6\text{MW}$$



(Fig. 7 Power angle characteristics)

Let δ_m be the power angle to which the rotor can swing before losing synchronism. If this angle is exceeded, P_i will again become greater than P_e and the rotor will once again be accelerated and synchronism will be lost as shown in Fig. 7. Therefore, the equal area criterion requires that equation (57) be satisfied with δm replacing δ_2.

From Fig. 7 $\delta_m = \pi - \delta_1$. Therefore equation (59) becomes

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 + \cos(\pi - \delta_1) - \cos\delta_0 = 0$$

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 - \cos\delta_1 - \cos\delta_0 = 0 \quad \text{..............................} \text{(i)}$$

Substituting $\delta_0 = 8^0 = 0.139$ radian in equation (i) gives

$$(3 - \delta_1)\sin\delta_1 - \cos\delta_1 - 0.99 = 0 \quad \text{..} \text{(ii)}$$

Solving equation (ii) we get. $\delta_1 = 50^0$

Now $\quad P_{ef} = P_{max}\sin\delta_1 = 500\sin 50^0 = 383.02$ MW

Initial power developed by machine was 69.6MW. Hence without loss of stability, the system can accommodate a sudden increase of

$P_{ef} - P_{e0} = 383.02 - 69.6 = 313.42$ MW per phase
= 3x313.42 = 940.3 MW (3-φ) of input shaft power.

Step by Step Solution of Swing Equation:-

The swing equation is $\frac{d^2}{dt^2} = \frac{P_a}{M} = \frac{1}{M}(P_i - P_m \sin\delta)$ ....(69)

Its solution gives a plot of δ versus t. The swing equation indicates that δ starts decreasing after reaching maximum value, the system can be assumed to be stable. The swing equation is a non-linear equation and a formal solution is not feasible. The step by step solution is very simple and common method of solving this equation. In this method the change in δ during a small time interval Δt is calculated by assuming that the accelerating power P_a calculated at the beginning of the interval is constant from the middle of the preceding interval to the middle of the interval being considered.

Let us consider the nth time interval which begins at t = (n-1) Δt. The angular position of the rotor at this instant is δ_{n-1}(Fig. 20 c). The accelerating power $P_{a(n-1)}$ and hence, acceleration α_{n-1} as calculated at this instant is assumed to be constant from t = (n-3/2) Δt to (n-1/2) Δt.

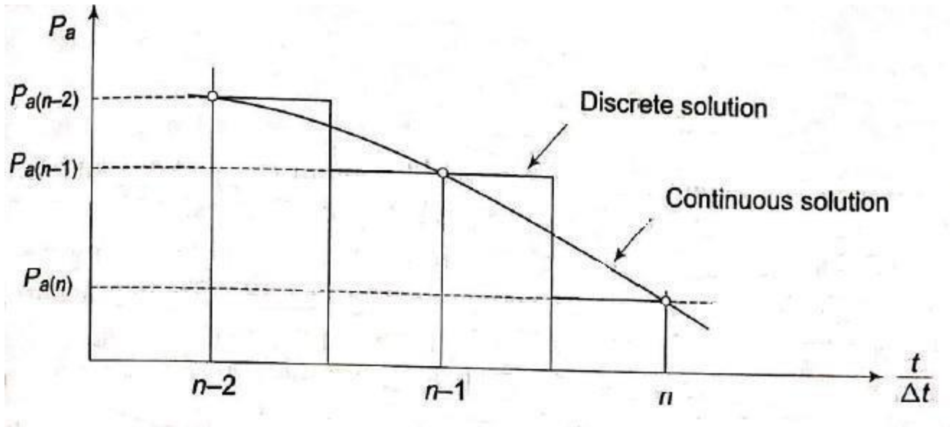
During this interval the change in rotor speed can be written as

$$\Delta \ _{-\frac{1}{2}} = (\Delta) \ _{-1} = \frac{\Delta}{} \ _{(\ -1)}$$ ....(70)

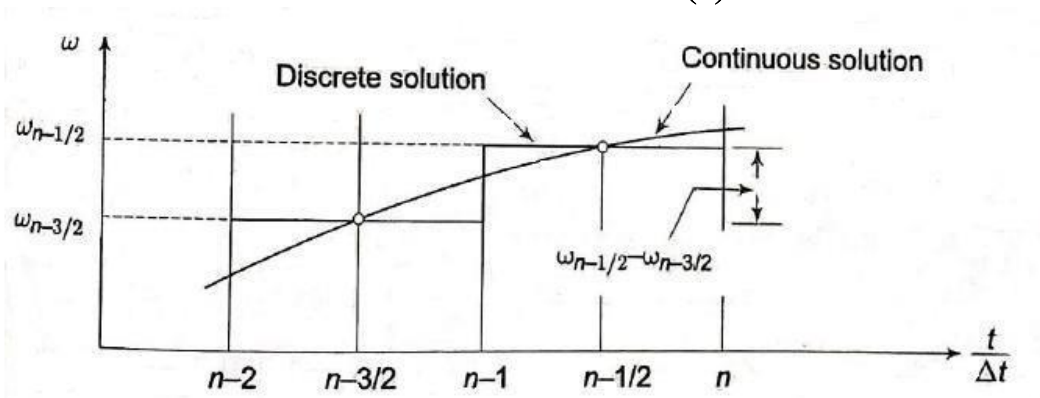
Thus, the speed at the end of nth interval is

$$_{-\frac{1}{2}} = \ _{-\frac{3}{2}} + \Delta \ _{-\frac{1}{2}}$$ ....(71)

Assume the change in speed occur at the middle of one interval, i.e., t=(n-1)Δt which is same the same instant for which the acceleration was calculated. Then the speed is assumed to remain constant till the middle of the next interval as shown in Fig. 18(b). In other words, the speed assumed to be constant at the value $_{-\frac{1}{2}}$ throughout the nth interval from t = (n-1) Δt to t = n Δt.



(a)



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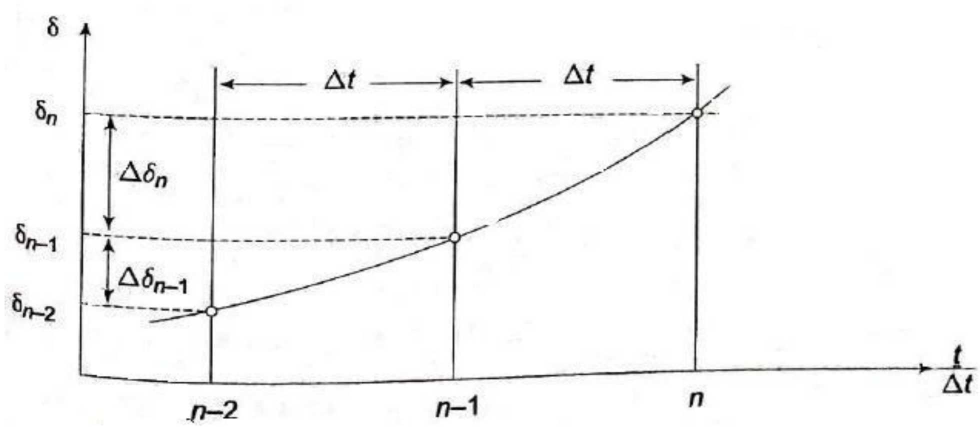


(c)

(Fig. 20 Step by step solution of swing equation)

The change in angular position of rotor during nth time interval is

$$\Delta\delta_n = (\Delta t)\omega_{n-\frac{3}{2}} \qquad \ldots\ldots(72)$$

And the value of δ at the end of nth interval is

$$\delta_n = \delta_{n-1} + \Delta\delta_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

This is shown in Fig. 20 (c). Substituting equation (70) into equation (71) and the result in equation (72) leads to

$$\Delta\delta_n = (\Delta t)\omega_{n-\frac{3}{2}} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \ldots\ldots\ldots\ldots\ldots (74)$$

By analogy with equation (72)

$$\Delta\delta_{n-1} = (\Delta t)\omega_{n-\frac{3}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

Substituting the value of $\omega_{n-\frac{3}{2}}$ from equation (75) into equation (74)

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \ldots\ldots\ldots\ldots\ldots (76)$$

Equation (76) gives the increment in angle δ during any interval (say nth) in terms of the increment during (n-1) th interval.

During the calculations, a special attention has to be paid to the effects of discontinuities in the accelerating power P_a which occur when a fault is applied or cleared or when a switching operation takes place. If a discontinuity occurs at the beginning of an interval then the average of the values of P_a before and after the discontinuity must be used. Thus, for calculating the increment in δ occuring in the first interval after a fault is applied at t=0, equation (76) becomes

$$\Delta\delta_1 = \frac{(\Delta t)^2}{M} \cdot \frac{P_{a0+}}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

Where P_{a0+}, the accelerating power immediately after occurrence of the fault. Immediately before the occurrence of fault, the system is in steady state with P_{a0-} = 0 and the previous increment in rotor angle is zero.

Multimachine stability Studies:-

The equal-area criterion cannot be used directly in systems where three or more machines are represented, because the complexity of the numerical computations increases with the number of machines considered in a transient stability studies. To ease the system complexity of system modeling, and thereby computational burden, the following assumptions are commonly made in transient stability studies:

1. The mechanical power input to each machine remains constant.
2. Damping power is negligible.
3. Each machine may be represented by a constant transient reactance in series with a constant transient internal voltage.

4. The mechanical rotor angle of each machine coincides with δ.
5. All loads may be considered as shunt impedances to ground with values determined by conditions prevailing immediately prior to the transient conditions.

   The system stability model based on these assumptions is called the **classical stability model**, and studies which use this model are called **classical stability studies**.

Consequently, in the multi-machine case two preliminary steps are required.

1. The steady-state prefault conditions for the system are calculated using a production-type power flow program.
2. The prefault network representation is determined and then modified to account for the fault and for the postfault conditions.

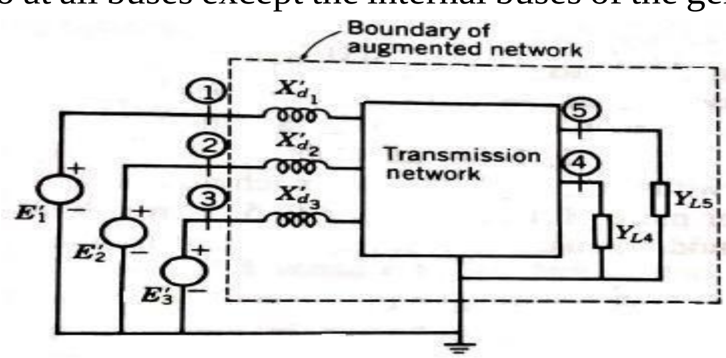
The transient internal voltage of each generator is then calculated using the equation

$$= \quad + jX' \qquad \text{.......(80)}$$

Where V_t is the corresponding terminal voltage and I is the output current. Each load is converted into a constant admittance to ground at its bus using the equation

$$Y_L = \frac{P_L - jQ_L}{|V|^2} \qquad \text{.......(81)}$$

Where $P_L - jQ_L$the load and |VL| is is the magnitude of the corresponding bus voltage. The bus admittance matrix which is used for the prefault power-flow calculation is now augmented to include the transient reactance of each generator and the shunt admittance of each load, as shown in Fig. 21. Note that the injected current is zero at all buses except the internal buses of the generators.



(Fig. 21 Augmented network of a power system)

In the second preliminary step the bus admittance matrix is modified to correspond to the faulted and post fault conditions. During and after the fault the power flow into the network from each generator is calculated by the corresponding power angle equation. For example, in Fig. 2 the power output of generator 1 is given by

$$P_{e1} = |E'_1|^2 G_{11} + |E'_1||E'_2||Y_{12}| \cos(\delta_{12} - \theta_{12}) + |E'_1||E'_3||Y_{13}| \cos(\delta_{13} - \theta_{13}) \quad \text{.....................(82)}$$

Where δ_{12} equals $\delta_1 - \delta_2$. Similar equations are written for P_{e2} and P_{e3} using the Y_{ij} elements of the 3X3 bus admittance matrices appropriate to the fault or postfault condition. The P_{ei} expressions form part of the equations

$$\frac{2H_i}{\omega_s}\frac{d^2\delta_i}{dt^2} = P_{ii} - P_{ei} \quad i=1, 2, 3 \text{ ....(83)}$$

Which represent the motion of each rotor during the fault and post fault periods. The solutions depend on the location and duration of the fault, and Y_{bus} resulting when the faulted line is removed.

Factors Affecting Transient Stability:-

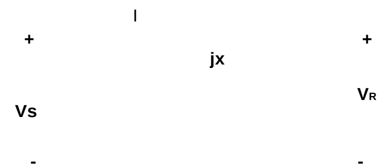
Various methods which improve power system transient stability are

1. Improved steady-state stability
   a) Higher system voltage levels
   b) Additional transmission line
   c) Smaller transmission line series reactance
   d) Smaller transfer leakage reactance
   e) Series capacitive transmission line compensation
   f) Static var compensators and flexible ac transmission systems (FACTs)
2. High speed fault clearing
3. High speed reclosuer of circuit breaker
4. Single pole switching
5. Large machine inertia, lower transient reactance
6. Fast responding, high gain exciter
7. Fast valving
8. Breaking resistor

Power Flow under Steady State:-

Consider a short transmission line with negligible resistance.

V_S = per phase sending end voltage
V_R = per phase receiving end voltage
V_s leads V_R by an angle δ
x = reactance of per transmission line



(Fig.3-A short transmission line)

On the per phase basis power on sending end,

$$S_S = P_S + j\,Q_S = V_S I^* \quad \text{.......... (29)}$$

From Fig.3 I is given as

$$I = \frac{V_S - V_R}{jX}$$

or $$I^* = \frac{V_S^* - V_R^*}{-jX} \quad \text{.......... (30)}$$

From equation (29) and (30), we get

$$S_S = \frac{V_S(V_S^* - V_R^*)}{-jX} \quad \text{.......... (31)}$$

Now $V_R = |V_R| \angle 0^0$ so, $V_R = V_R^* = |V_R|$

$V_S = |V_S| \angle \delta = |V_S| e^{j\delta}$

Equation (31) becomes

$$S_S = P_S + jQ_S = \frac{|V_S||V_R|}{X}\sin\delta + j\frac{1}{X}(|V_S|^2 - |V_S||V_R|\cos\delta)$$

So $$P_S = \frac{|V_S||V_R|}{X}\sin\delta \quad \text{.......... (32)}$$

and $$Q_s = \frac{|V_S|^2 - |V_S||V_R|\cos\delta}{X} \quad \text{.......... (33)}$$

Similarly, at the receiving end we have

$$S_R = P_R + j\,Q_R = V_R I^* \quad \text{.......... (34)}$$

Proceeding as above we finally obtain

$$Q_R = \frac{V_S||V_R| \cos \delta - |V_R|^2}{x} \quad \text{.......................................} (35)$$

$$P_R = \frac{V_S||V_R|}{x} \sin\delta \quad \text{.......................................} (36)$$

Therefore for lossless transmission line,

$$P_S = P_R = \frac{V_S||V_R|}{x} \sin\delta \quad \text{.......................................} (37)$$

In a similar manner, the equation for steady-state power delivered by a lossless synchronous machine is given by

$$P_e = P_d = \frac{E_g||V_t|}{x_d} \sin \delta$$

$$= P_{max} \sin \delta \qquad (38)$$

Where $|E_g|$ is the rms internal voltage, $|V_t|$ is the rms terminal voltage, x_d is the direct axis reactance (or the synchronous reactance in a round rotor machine) and δ is the electrical power angle.

<u>Steady-state Stability</u>:-

The steady state stability limit of a particular circuit of a power system defined as the maximum power that can be transmitted to the receiving end without loss of synchronism.

Now consider equation (1 ng fixed at P_i ca ... operating point (... P) ......(39)

$P_e = (\frac{P_e}{\ldots})_0 \Delta\delta \ldots$ e

Where $= \frac{H}{\pi f}$

δ are th (P)

And $\frac{\delta}{2}$ = e $\frac{|V_t|}{x_d}$ + Δ) = max sin δ...... (40)

Let the system be operating ... $\frac{\delta}{}$ w $\Delta\delta = [\frac{6P_e}{6\delta}]$ over transfer of $P_{e0} = P_i$ with torque angle δ_0. Assume a small increment ... electric power with the input from the prime mover remaining fixed ($\frac{P_e}{2}$)]$_i$ $\delta =$ ... causing the torque angle to change to ($\delta_0 + \delta$). Linearizing the operating point 0, 0](δ_0) can write

$\Delta P_e = (\frac{d}{dt})_0 \Delta \ldots$ (41)

The excursions of Δ are then described by

$$M \frac{d^2\Delta}{dt^2} = P_i - (P_{e0} + \Delta P_e) = -\Delta P_e \qquad (42)$$

or

$$M \frac{d^2\Delta}{dt^2} + [\frac{6}{6\delta}]_0 \Delta = 0 \qquad (43)$$

or

$$[Mp^2 + (\frac{6}{6\delta})_0]\Delta = 0 \qquad (44)$$

Where p =

The system stability to small changes is determined from the characteristic equation

$$Mp^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_0 = 0 \quad \text{.......... (45)}$$

Where two roots are
$$p = \pm\left[-\frac{\left(\partial P_e / \partial \delta\right)_0}{M}\right]^{\frac{1}{2}} \quad \text{.......... (46)}$$

As long as $\left(\partial P_e / \partial \delta\right)_0$ is positive, the roots are purely imaginary and conjugate and system behavior is oscillatory about δ_0. The resistance and damper windings of machine cause the system oscillations to decay. The system is therefore stable for a small increment in power so long as $\left(\partial P_e / \partial \delta\right)_0 > 0$.

When $\left(\partial P_e / \partial \delta\right)_0$ is negative, the roots are real, one positive and the other negative but of equal magnitude. The torque angle therefore increases without bound upon occurrence of a small power increment and the synchronism is soon lost. The system is therefore unstable for $\left(\partial P_e / \partial \delta\right)_0 < 0$.

$\left(\partial P_e / \partial \delta\right)_0$ is known as **synchronizing coefficient**. This is also called **stiffness** of synchronous machine. It is denoted as S_p. This coefficient is given by

$$S_p = \left.\frac{\partial P_e}{\partial \delta}\right|_{\delta = \delta_0} = P_{max} \cos \delta_0 \quad \text{.......... (47)}$$

If we include damping term in swing equation, then equation (43) becomes

$$M\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + \left[\frac{\partial P_e}{\partial \delta}\right]_0 \Delta\delta = 0$$

or
$$\frac{d^2\Delta\delta}{dt^2} + \frac{D}{M}\frac{d\Delta\delta}{dt} + \frac{1}{M}\left[\frac{\partial P_e}{\partial \delta}\right]_0 \Delta\delta = 0$$

or
$$\frac{d^2\Delta\delta}{dt^2} + \frac{D\pi f}{H}\frac{d\Delta\delta}{dt} + \frac{S_p \pi f}{H}\Delta\delta = 0$$

or
$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0 \quad \text{.......... (48)}$$

Where $m_n = \sqrt{\frac{fS_p}{H}}$ and $r = \frac{D}{2}\sqrt{\frac{f}{HS_p}}$ .... (49)

So damped frequency of oscillation, $m_l = m_n\sqrt{1 - r^2}$ .... (50)

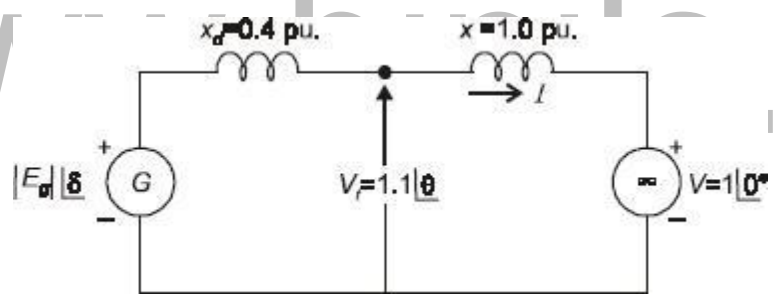
And Time Constant, $T = \frac{1}{c\omega_n} = \frac{2H}{\pi f D}$ .... (51)

Example2:-

Find the maximum steady-state power capability of a system consisting of a generator equivalent reactance of 0.4pu connected to an infinite bus through a series reactance of 1.0 p.u. The terminal voltage of the generator is held at1.10 p.u. and the voltage of the infinite bus is 1.0 p.u.

Solution:-

Equivalent circuit of the system is shown in Fig.4.



(Fig.4 Equivalent circuit of example2)

$$|E_g|L\delta = V_t + jx_d I \quad \text{...(i)}$$

$$I = \frac{V_t - V}{jx} = \frac{1.1L\theta - 1L0°}{j1} \quad \text{...(ii)}$$

Using equation (i) and (ii)

$$|E_g|L\delta = 1.1L\theta + j0.4\left(\frac{1.1L\theta - 1L0°}{j1}\right)$$

$$\therefore |E_g|L\delta = 1.1\cos\theta + j1.1\sin\theta + 0.4x1.1L\theta - 0.4$$

$$\therefore |E_g|L\delta = (1.4\cos\theta - 0.4) + j1.4\sin\theta \quad \text{.........................(iii)}$$

Maximum steady-state power capability is reached when δ = 90°, i.e., real part of equation is zero. Thus

$$1.54 \cos \theta - 0.4 = 0$$

$$\therefore \theta = 74.9°$$

$$\therefore |E_g| = 1.54 \sin 74.9° = 1.486 \text{ pu.}$$

$$\therefore V_t = 1.1 \angle 74.9°$$

$$\therefore P_{max} = \frac{|E_g||V|}{(x_d} \quad 1.48x1.0$$

Rotor Angle Stability:

Rotor angle stability is the ability of the interconnected synchronous machines running in the power system to remain in the state of synchronism. Two synchronous generators running parallel and delivering active power to the load depends on the rotor angle of the generator (load sharing between alternators depends on the rotor angle).

During normal operation of the generator, rotor magnetic field and stator magnetic field rotates with the same speed, however there will be an angular separation between the rotor magnetic field and stator magnetic field which depends on the electrical torque (power) output of the generator.

An increase in the prime mover speed (turbine speed) will result in the advancement of the rotor angle to a new postion realtive to the rotating magnetic field of the stator. On the other hand reduction in the mechanical torque will result in the fall back of the rotor angle relative to the stator field.

In equilibrium condition there will be equilibrium between the input mechanical torque and output electrical torque of each machine (generator) in the power system and speed of the machines will remain same. If the equilibrium is upset which results in the acceleration or deceleration of rotors of the machines.

If one of the inter connected generator moves faster temporarily with respect to the other machine. Rotor angle of the machine will advance with respect to slow machine. This results in the load deliver by faster generator increases and load delivered by slow machine decreases. This tends to reduce the speed difference between the two generators and also the angular separation between the slow generator and fast generator.

Beyond certain point the increase in the angular separation will result in decrease of power transfer by the fast machine. This increases the angular separation further and also may lead to instability and synchronous generators fall out of synchronism.

Power system stability involves the study of the dynamics of the power system under disturbances. Power system stability implies that its ability to return to normal or stable operation after having been subjected to some form of disturbances.

From the classical point of view power system instability can be seen as loss of synchronism (i.e., some synchronous machines going out of step) when the system is subjected to a particular disturbance. Three type of stability are of concern: Steady state, transient and dynamic stability.

Steady-state Stability:-

Steady-state stability relates to the response of synchronous machine to a gradually increasing load. It is basically concerned with the determination of the upper limit of machine loading without losing synchronism, provided the loading is increased gradually.

Dynamic Stability:-

Dynamic stability involves the response to small disturbances that occur on the system, producing oscillations. The system is said to be dynamically stable if theses oscillations do not acquire more than certain amplitude and die out quickly. If these oscillations continuously grow in amplitude, the system is dynamically unstable. The source of this type of instability is usually an interconnection between control systems.

Transient Stability:-

Transient stability involves the response to large disturbances, which may cause rather large changes in rotor speeds, power angles and power transfers. Transient stability is a fast phenomenon usually evident within a few second.

Power system stability mainly concerned with rotor stability analysis. For this various assumptions needed such as:

- For stability analysis balanced three phase system and balanced disturbances are considered.
- Deviations of machine frequencies from synchronous frequency are small.
- During short circuit in generator, dc offset and high frequency current are present. But for analysis of stability, theses are neglected.
- Network and impedance loads are at steady state. Hence voltages, currents and powers can be computed from power flow equation.

Dynamics of a Synchronous Machine :-

The kinetic energy of the rotor in synchronous machine is given as:

$$KE = 1/2 J w_s^2 \times 10^{-6} \text{ MJoule} \quad \ldots\ldots (1)$$

Where J = rotor moment of inertia in kg-m^2

w_s = synchronous speed in mechanical radian/sec.

Speed in electrical radian is

$$w_{se} = (P/2)\, w_s = \text{rotor speed in electrical radian/sec} \quad \ldots\ldots (2)$$

Where P = no. of machine poles

From equation (1) and (2) we get

$$KE = \frac{1}{2}\left[J \left(\frac{2}{P}\right)^2 . w_{se} \times 10^{-6}\right] . w_{se}^2 \text{ MJ} \quad \ldots\ldots (3)$$

or

$$KE = \frac{1}{2} M w_{se}^2 \text{ MJ}$$

Where

$$M = \left[J \left(\frac{2}{P}\right)^2 . w_{se} \times 10^{-6}\right] = \text{moment of inertia in MJ.sec/elect. radian} \quad \ldots\ldots (4)$$

We shall define the inertia constant H, such that

$$GH = KE = \frac{1}{2} M w_{se} \text{ MJ} \quad \ldots\ldots (5)$$

Where G = three-phase MVA rating (base) of machine

H = inertia constant in MJ/MVA or MW.sec/MVA

From equation (5), we can write,

$$M = \frac{2GH}{w_{se}} = \frac{2GH}{2\ f} = \frac{GH}{f}\ \text{MJ.sec/elect. radian} \quad \text{.......................... (6)}$$

or $$M = \frac{GH}{180f}\ \text{MJ.sec/elect. degree} \quad \text{.. (7)}$$

M is also called the inertia constant.

Assuming G as base, the inertia constant in per unit is

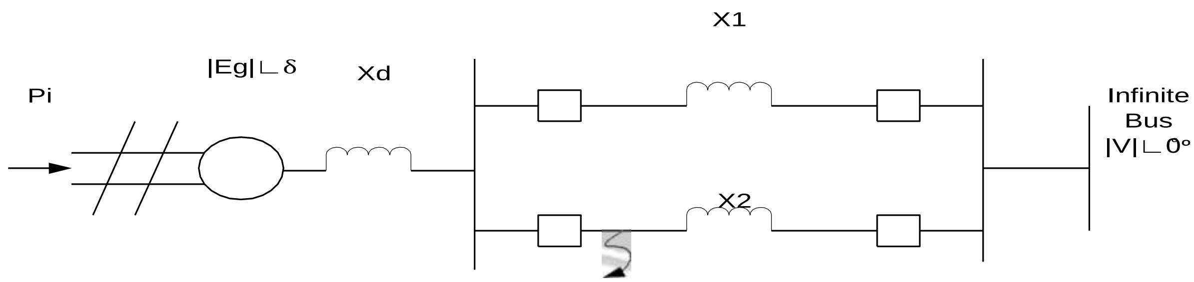
$$M(pu) = \frac{H}{f}\ \text{Sec}^2\text{/elect.radian} \quad \text{.. (8)}$$

or $$M(pu) = \frac{H}{180f}\ \text{Sec}^2\text{/elect.degree} \quad \text{.. (9)}$$

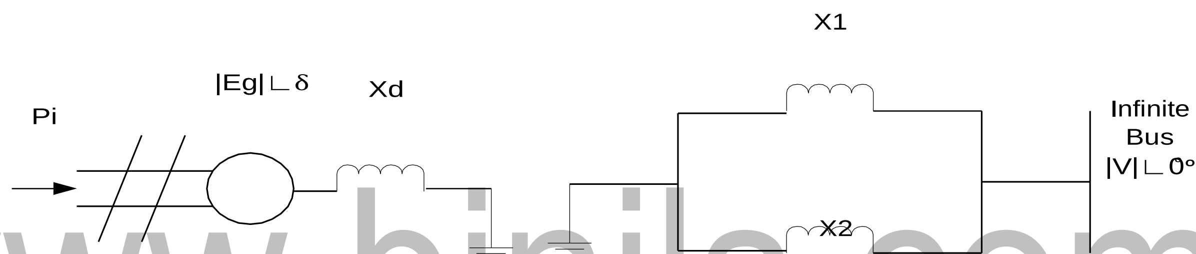
Sudden Short Circuit on One of Parallel Lines:-

(1) Short circuit at one end of line:-

Let us a temporary three phase bolted fault occurs at the sending end of one of the line.



(a)



(b)

(Fig.13 Short circuit at one of the line)

Before the occurrence of a fault, the power angle curve is given by

$$P_{eI} = \frac{|E_g||V|}{X_d + X_1||X_2} \sin\delta = P_{maxI} \sin\delta$$

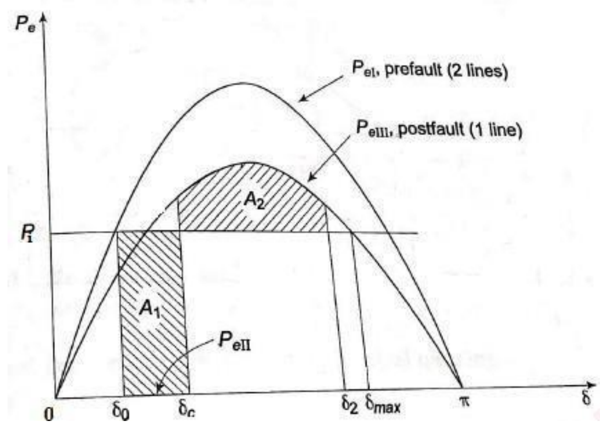
This is plotted in Fig. 12.

Upon occurrence of a three-phase fault at the generator end of line 2 , generator gets isolated from the power system for purpose of power flow as shown Fig. 13 (b). Thus during the period the fault lasts.

$$P_{eII} = 0$$

The rotor therefore accelerates and angles δ increases. Synchronism will be lost unless the fault is cleared in time. The circuit breakers at the two ends of the faulted line open at time t_c (corresponding to angle δ_c), the clearing time, disconnecting the faulted line. The power flow is now restored via the healthy line (through higher line reactance X_2 in place of $(X_1||X_2)$, with power angle curve

$$P_{eII} = \frac{|E_g||V|}{X_d + X_1} \sin\delta = P_{maxIII} \sin\delta$$



(Fig. 14 Equal area criterion applied to the system)

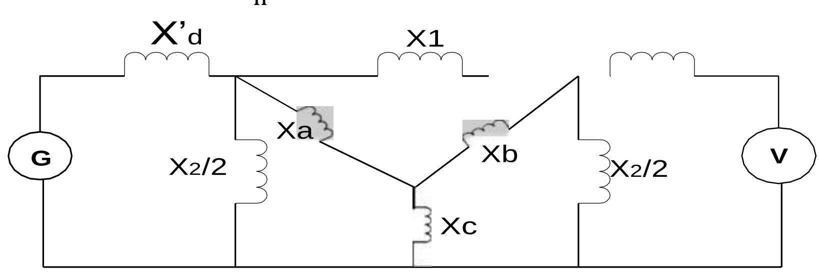
Obviously, $P_{maxIII} < P_{maxI}$. The rotor now starts decelerate as shown in Fig 14. The system will be stable if a decelerating area A2 can be found equal to accelerating area A1before δ reaches the maximum allowable value δ_{max}. As area A1 depends upon clearing time t_c (corresponding to clearing angle δ_c), clearing time must be less than a certain value (critical clearing time) for the system to be stable.

(2) Short circuit at the middle of a line:-

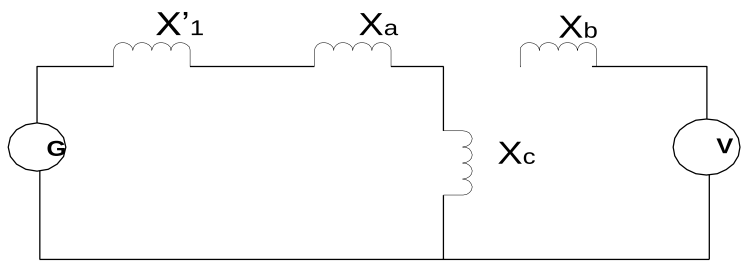
When fault occur at the middle of a line or away from line ends, there is some power flow during the fault through considerably reduced. Circuit model of the system during the fault is shown in fig. 15 (a). This circuit reduces to fig. 15 (c) through one delta-star and star-delta conversion.

The power angle curve during fault is given by

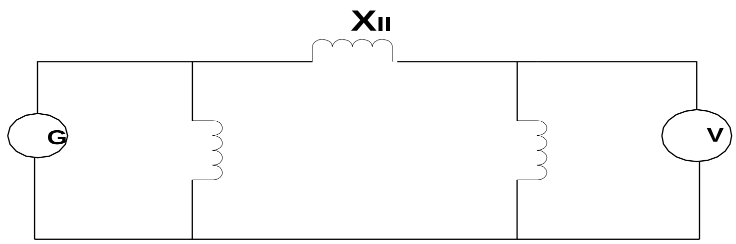
$$P_{eII} = \frac{|E_g||V|}{X_{II}} \sin\delta = P_{maxII} \sin\delta$$



(a)



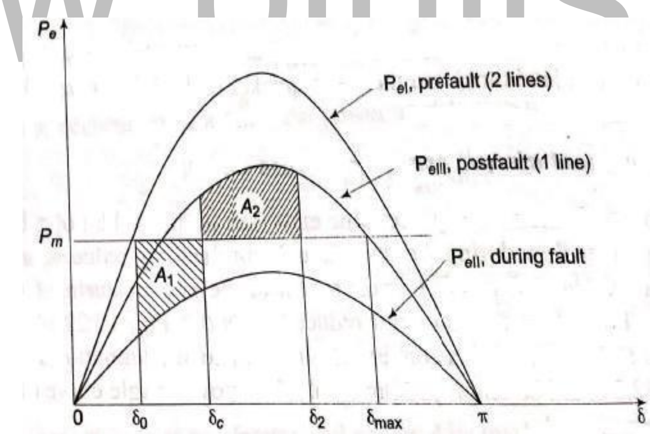
(b)



(c)

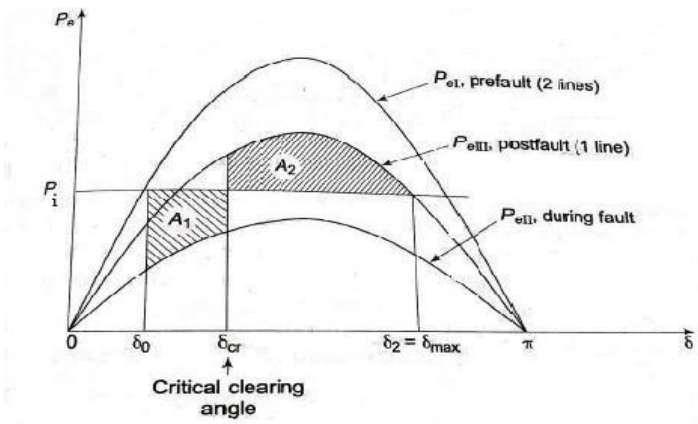
(Fig.15 Circuit Model)

P_{eI} and P_{eIII} as in Fig. 12 and P_{eII}as obtained above are all plotted in Fig. 16.



(Fig. 16 Fault on middle of one line of the system with $\delta_c < \delta_{cr}$)

Accelerating area A_1 corresponding to a given clearing angle δ_c is less in this case. Stable system operation is shown in Fig. 16, wherein it is possible to find an area A_2 equal to A_1 for$\delta_2 < \delta_{max}$. As the clearing angle δ_c is increased, area A_1 increases and to find $A_2 = A_1$, δ_2 increases till it has a valueδ_{max}, the maximum allowable for stability. This case of critical clearing angle is shown in Fig. 17.



(Fig. 17 Fault on middle on one line of the system)

Applying equal area criterion to the case of critical clearing angle of Fig. 17, we can write

$$\int_{\delta_0}^{\delta_{cr}} (P_i - P_{maxII}\sin\delta)\,d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{maxIII}\sin\delta - P_i)\,d\delta$$

Where

$$\delta_{max} = \pi - \sin^{-1}\frac{P_i}{P_{maxIII}} \quad \text{.......(68)}$$

Integrating we get

$$(P_i\delta + P_{maxII}\cos\delta)\Big|_{\delta_0}^{\delta_{cr}} + (P_{maxIII}\cos\delta + P_i\delta)\Big|_{\delta_{cr}}^{\delta_{max}} = 0$$

$$\text{or } P_i(\delta_{cr} - \delta_0) + P_{maxII}(\cos\delta_{cr} - \cos\delta_0) + P_i(\delta_{max} - \delta_{cr}) + P_{maxIII}(\cos\delta_{max} - \cos\delta_{cr}) = 0$$

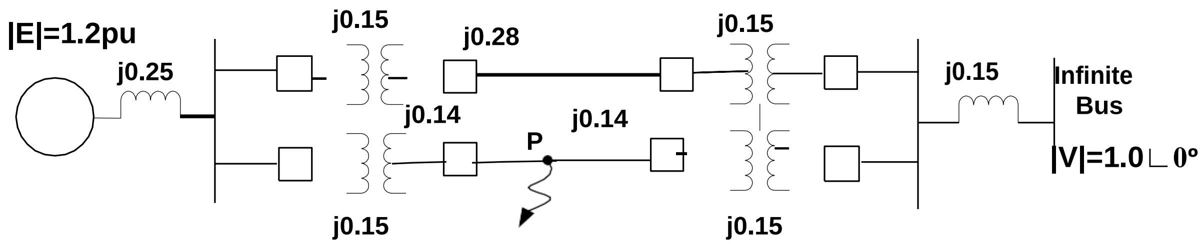
$$\cos\delta_{cr} = \frac{P_i(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$$

This critical clearing angle is in radian. The equation modifies as below if the angles are in degree

$$\cos\delta_{cr} = \frac{\frac{\pi}{180}P_i(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$$

Example 4:-

Find the critical clearing angle for the system shown in Fig. 18 for a three phase fault at point P. The generator is delivering 1.0 pu. Power under prefault conditions.



(Fig. 18)

Solution:-

1. **Prefault Operation:-** Transfer reactance between generator and infinite bus is

$$X_I = 0.25+0.17 + \frac{0.15+0.28+0.15}{2} = 0.71$$

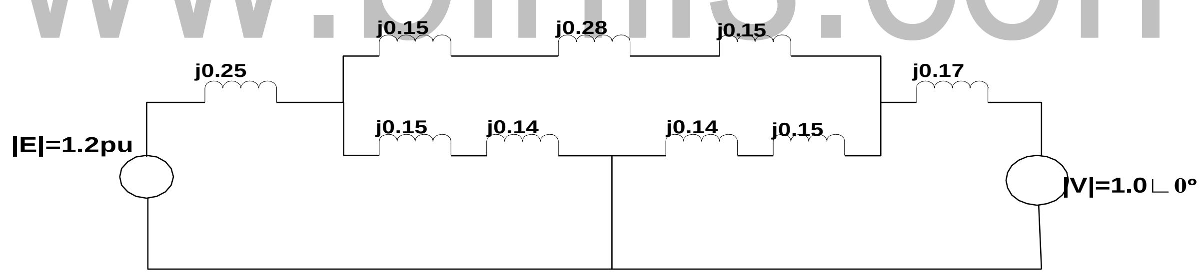
$$P_{eI} = \frac{1.2X1}{0.71} \sin\delta = 1.69 \sin\delta$$

The operating power angle is given by

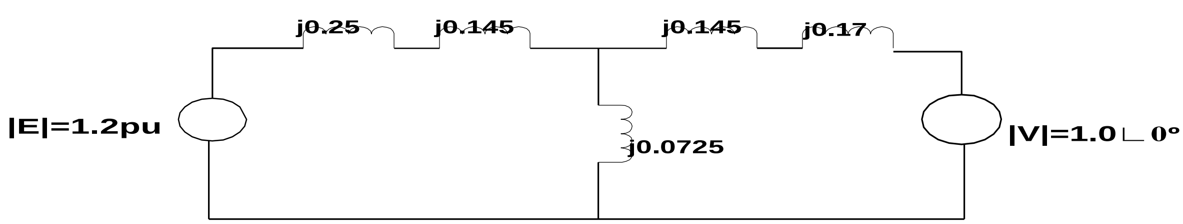
$$1.0 = 1.69 \sin\delta$$

$$\text{or } \delta_0 = 0.633\text{rad}$$

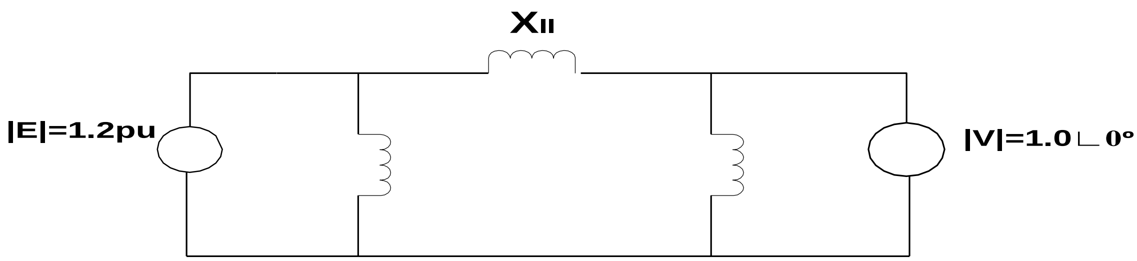
2. **During Fault:-** The positive sequence reactance diagram during fault is presented in Fig. 17.



(a) Positive sequence reactance diagram during fault



(b) Network after delta-star conversion



(c) Network after star- delta conversion

(Fig.19)

Converting delta to star, the reactance network is changed to that Fig. 19 (b). Further upon converting star to delta, we obtain the reactance network of Fig. 19(c). The transfer reactance is given by

$$X_{II} = \frac{(0.25 + 0.145)0.0725 + (0.145 + 0.17)0.0725 + (0.25 + 0.145)(0.145 + 0.17)}{0.075}$$

$$= 2.424$$

$$P_{eII} = \frac{1.2 \times 1}{2.424}\sin\delta = 0.495\sin\delta$$

3. Post fault operation(faulty line switched off):-

$$X_{III} = 0.25 + 0.15 + 0.28 + 0.15 + 0.17 = 1.0$$

$$P_{eIII} = \frac{1.2 \times 1}{1}\sin\delta = 1.2\sin\delta$$

With reference to Fig. 16 and equation (68), we have

$$\delta_{max} = \pi - \sin^{-1}\frac{1}{2.1} = \text{rad } 1.2$$

To find critical clearing angle, areas A1 and A2 are to be equated.

$$A_1 = 1.0(\delta_{cr} - 0.633) - \int_{\delta_0}^{\delta_{cr}} 0.495\sin\delta\, d\delta$$

And $$A_2 = \int_{\delta_{cr}}^{\delta_{max}} 1.2\sin\delta\, d\delta - 1.0(2.155 - \delta_c)$$

Now $A_1 = A_2$

or $$\delta_{cr} - 0.633 - \int_{0.633}^{\delta_{cr}} 0.495\sin\delta\, d\delta = \int_{\delta_{cr}}^{2.155} 1.2\sin\delta\, d\delta - 2.155 + \delta_{cr}$$

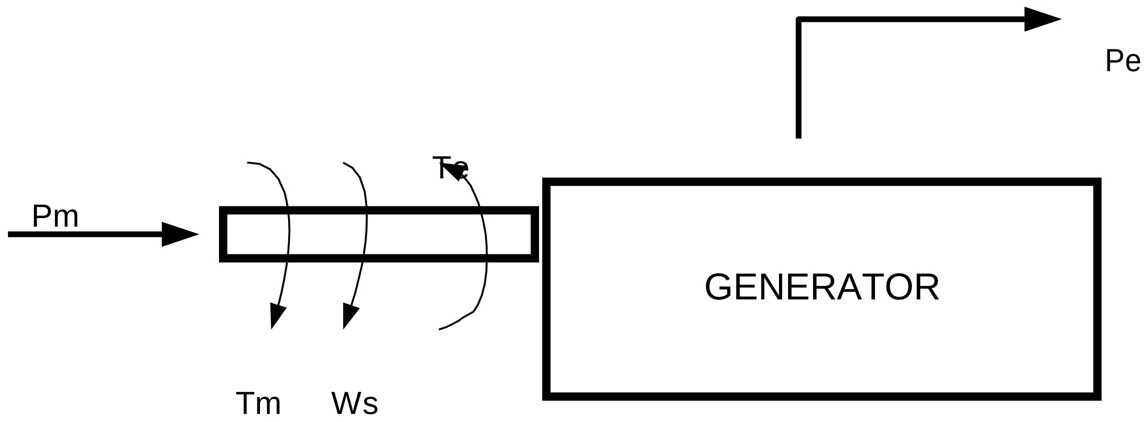
or $$-0.633 + 0.495\cos\delta\,\Big|_{0.633}^{\delta_{cr}} = -1.2\cos\delta\,\Big|_{\delta_{cr}}^{2.155} - 2.155$$

or $$-0.633 + 0.495\cos\delta_{cr} - 0.399 = 0.661 - 1.2\cos\delta_{cr} - 2.155$$

or $$\cos\delta_{cr} = 0.655$$

or $\delta_{cr} = 49.1°$

`Swing Equation:-



(Fig.-1 Flow of power in a synchronous generator)

Consider a synchronous generator developing an electromagnetic torque T_e(and a corresponding electromagnetic power P_e) while operating at the synchronous speed w_s. If the input torque provided by the prime mover, at the generator shaft is T_i, then under steady state conditions (i.e., without any disturbance).

$$T_e = T_i \quad \text{.......................} (10)$$

Here we have neglected any retarding torque due to rotational losses. Therefore we have

$$T_e\ w_s = T_i\ w_s \quad \text{.......................} (11)$$

And $$T_e\ w_s - T_i\ w_s = P_i - P_e = 0 \quad \text{.......................} (12)$$

When a change in load or a fault occurs, then input power Pi is not equal to Pe. Therefore left side of equation is not zero and an accelerating torque comes into play. If Pa is the accelerating (or decelerating) power, then

$$Pi - Pe = M. \frac{d^2\ _e}{dt^2} + D \frac{d\ _e}{dt} = P_a \quad \text{.......................} (13)$$

Where D = damping coefficient

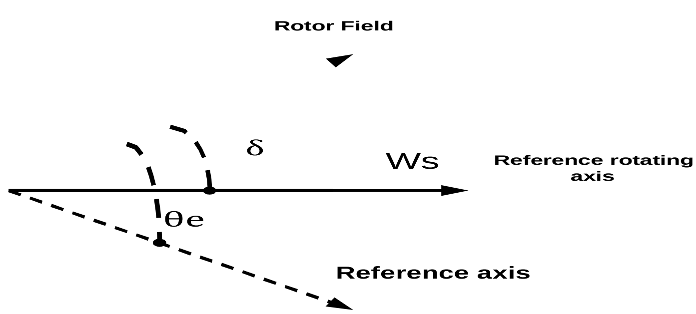
θ_e = electrical angular position of the rotor

It is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference. Let

$$\delta = \theta_e - w_s.t \quad \text{.......................} (14)$$

So $$\frac{d^2\ _e}{dt^2} = \frac{d^2}{dt^2} \quad \text{.......................} (15)$$

Where δ is power angle of synchronous machine.



(Fig.2 Angular Position of rotor with respect to reference axis)

Neglecting damping (i.e., D = 0) and substituting equation (15) in equation (13) we get

$$M.\frac{d^2\delta}{dt^2} = P_i - P_e \text{ MW} \quad \text{...(16)}$$

Using equation (6) and (16), we get

$$\frac{GH}{\pi f}.\frac{d^2\delta}{dt^2} = P_i - P_e \text{ MW} \quad \text{...(17)}$$

Dividing throughout by G, the MVA rating of the machine,

$$M_{(pu)}.\frac{d^2}{dt^2} = (P_i - P_e)\text{ pu} \quad \text{...(18)}$$

Where $M_{(pu)} = \frac{}{f}$ ....(19)

or $\frac{H}{\pi f}.\frac{d^2}{dt^2} = (P_i - P_e)\text{ pu}$ ....(20)

Equation (20) is called **Swing Equation.** It describes the rotor dynamics for a synchronous machine. Damping must be considered in dynamic stability study.

Multi Machine System:-

In a multi machine system a common base must be selected. Let

$G_{machine}$ = machine rating (base)

G_{system} = system base

Equation (20) can be written as:

$$\frac{G_{machine}}{G_{system}}\left(\frac{H_{machine}}{f}\right)\frac{d^2}{dt^2} = (P_i - P_e).\frac{G_{machine}}{G_{system}} \quad \text{.....................(21)}$$

So $\left(\frac{H_{system}}{f}\right)\frac{d^2}{dt^2} = (P_i - P_e)$ pu on system base .... (22)

Where $H_{system} = \frac{G_{machine}}{G_{system}}.H_{machine}$ ....(23)

= machine inertia constant in system base

Machines Swinging in Unison (Coherently) :-

Let us consider the swing eqtions of two machines on a common system base, i.e.,

$$\frac{H_1}{\pi f}.\frac{d\delta_1}{dt^2} = (P_{i1} - P_{e1}) \quad \text{..(24)}$$

$$\frac{H_2}{\pi f} \cdot \frac{d^2\delta_2}{dt^2} = (P_{i2} - P_{e2}) \quad \text{.. (25)}$$

Since the machines rotor swing in unison,

$$\delta_1 = \delta_2 = \delta \quad \text{.. (26)}$$

Adding equations (24) and (25) and substituting equation (26), we get

$$\frac{H_{eq}}{f} \cdot \frac{d^2\delta}{dt^2} = (P_i - P_e) \quad \text{...(27)}$$

Where $P_i = P_{i1} + P_{i2}$

$P_e = P_{e1} + P_{e2}$

$H_{eq} = H_1 + H_2$

Equivalent inertia H_{eq} can be expressed as:

$$H_{eq} = \left(\frac{G_{1,machine}}{G_{System}}\right) . H_{1,machine} + \left(\frac{G_{2,machine}}{G_{system}}\right) . H_{2,machine} \quad \text{................ (28)}$$

Example1:-

A 60 Hz, 4 pole turbo-generator rated 100MVA, 13.8 KV has inertia constant of 10 MJ/MVA.

(a) Find stored energy in the rotor at synchronous speed.
(b) If the input to the generator is suddenly raised to 60 MW for an electrical load of 50 MW, find rotor acceleration.
(c) If the rotor acceleration calculated in part (b) is maintained for 12 cycles, find the change in torque angle and rotor speed in rpm at the end of this period.
(d) Another generator 150 MVA, having inertia constant 4 MJ/MVA is put in parallel with above generator. Find the inertia constant for the equivalent generator on a base 50 MVA.

Solution:-

(a) Stored energy = GH
= 100MVA x 10MJ/MVA
= 1000MJ

(b) $P_a = P_i - P_e = 60\text{-}50 = 10\text{MW}$

We know, $M = \frac{GH}{180f} = \frac{100X10}{180X60} = \frac{5}{54}$ MJ.sec/elect.deg.\

Now $M.\frac{d^2\delta}{dt^2} = P_i - P_e = P_a$

$\Rightarrow \frac{5}{54}\frac{d^2\delta}{dt^2} = 10$

$\Rightarrow \frac{d^2\delta}{dt^2} = \frac{10X54}{5} = 108$ elect.deg./sec^2

So, α = acceleration = 108 elect.deg./sec^2

(c) 12 cycles = 12/60 = 0.2sec.

Change in δ = ½ α.(Δt)^2 = ½.108.(0.2)^2 =2.16 elect.deg

Now α = 108 elect.deg./sec^2

= 60 x (108/360^0) rpm/sec

= 18 rpm/sec

Hence rotor speed at the end of 12 cycles

$= \frac{120f}{P} + \alpha.\Delta t$

$= (\frac{120X60}{4} + 18X0.2)$ rpm

= 1803.6 rpm.

(d) $H_{eq} = \frac{H1G1}{G_b} + \frac{H_2G_2}{G_b} = \frac{10x100}{50} + \frac{4x150}{50}$ =32MJ/MVA