Analysis of unsymmetrical faults

Introduction:

The unsymmetrical faults will have faulty parameters at random. They can be analyzed by using the symmetrical components. The standard types of unsymmetrical faultsconsidered for analysis include the following (in the order of their severity):

Fault Analysis

The normal mode of operation of a power system is balanced three-phase AC. However, there are undesirable but unavoidable incidents that may temporarily disrupt normal conditions, as when the insulation of the system fails at any point or when a conducting material comes in contact with a bare conductor. Then we say a fault has occurred. A fault may be caused by lightning, trees falling on the electric wires, vehicular collision with the poles or towers, vandalism, and so forth.Faults may be classified into four types. The different types of fault are listed here in the order of the frequency of their occurrence.

- Line–to–Ground (L-G) Fault
- Line–to–Line (L-L) Fault
- Double Line–to–Ground (L-L-G)Fault and
- Three-Phase–to–Ground (LLL-G) Fault.
-

Further the neutrals of various equipment may be grounded or isolated, the faults can occur at any general point F of the given system, the faults can be through a fault impedance, etc. Of the various types of faults as above, the 3-phase fault involving the ground is the most severe one. Here the analysis is considered in two stages as under:

(i) Fault at the terminals of a Conventional (Unloaded) Generator and

(ii) (ii) Faults at any point F, of a given Electric Power System (EPS).

Three-Phase Fault Analysis

Sufficient accuracy in fault studies can be obtained with certain simplifications in the model of the power system. These assumptions include the following:

1. Shunt elements in the transformer model are neglected; that is, magnetizing currents and core losses are omitted.
2. Shunt capacitances in the transmission line model are neglected.
3. Transformers are set at nominal tap positions.
4. All internal voltage sources are set equal to $1.0\angle 0°$. This is equivalent to neglecting pre-fault load currents.

Three-phase fault calculations can be performed on a per-phase basis because the power system remains effectively balanced, or symmetrical, during a three-phase fault. Thus, the various power system components are represented by single-phase equivalent circuits wherein all three-phase connections are assumed to be converted to their equivalent connections. Calculations are performed using impedances per phase, phase currents, and line-to-neutral voltages.

Consider now the symmetrical component relational equations derived from the three sequence networks corresponding to a given unsymmetrical system as a function of sequence impedances and the positive sequence voltage source in the form as under:

$$\begin{aligned} V_{a0} &= -I_{a0}Z_0 \\ V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \end{aligned}$$

These equations are refered as the sequence equations. In matrix Form the sequence equations can be considered as:

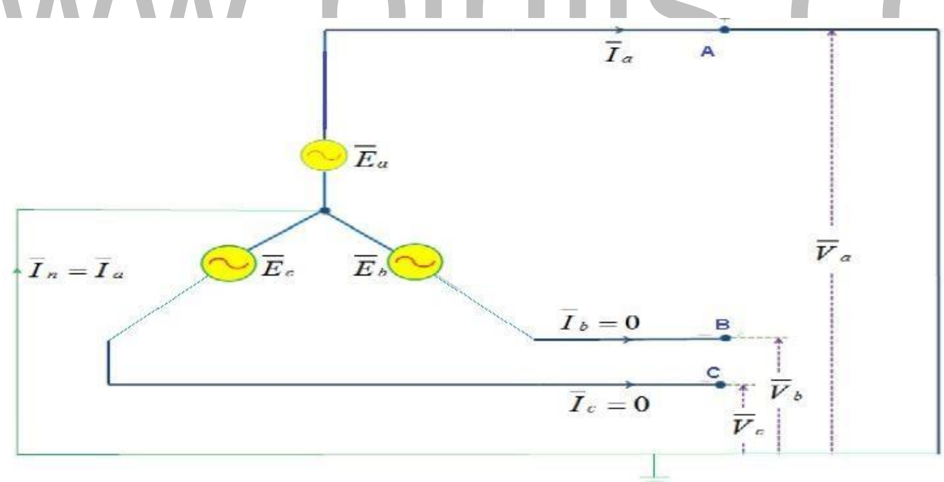
$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix}$$

This equation is used along with the equations i.e., conditions under fault (c.u.f.), derived to describe the fault under consideration, to determine the sequence current Ia1 and hence the fault current If, in terms of Ea and the sequence impedances, Z1, Z2 and Z0. Thus during unsymmetrical fault analysis of any given type of fault, two sets of equations as follows are considered for solving them simultaneously to get the required fault parameters:

- Equations for the conditions under fault (c.u.f.)
- Equations for the sequence components (sequence equations)

SINGLE LINE TO GROUND FAULT ON A CONVENTIONAL (UNLOADED) GENERATOR:

A conventional generator is one that produces only the balanced voltages. Let Ea, nd Ec be the internally generated voltages and Zn be the neutral impedance. The fault is assumed to be on the phase'a' as shown in figure Consider now the conditions under fault as under:



c.u.f.:

$$I_b = 0; I_c = 0; \text{ and } V_a = 0.$$

Now consider the symmetrical components of the current Ia with Ib=Ic=0, given by:

$$\begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} = (1/3)\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} I_a \\ 0 \\ 0 \end{vmatrix}$$

Solving the equation we get,

$I_{a1} = I_{a2} = I_{a0} = (Ia/3)$

Further, using above equation, we get,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a3} \end{bmatrix}$$

Pre-multiplying equation throughout by [1 1 1], we get,

$V_{a1}+V_{a2}+V_{a0} = - I_{a1}Z_0 + Ea – I_{a1}Z_1 –$

$I_{a2}Z_2$ i.e., $V_a = E_a – I_a1 (Z_1 + Z_2 +$

Z_0) = zero, Or in other words,

$I_{a1} = [E_a/(Z_1 + Z_2 + Z_0)]$

The equation (4.7) derived as above implies that the three sequence networks are

connected in series to simulate a LG fault, as shown in figure 4.2. Further we have the

following relations satisfied under the fault

conditions: 1. $I_{a1} = I_{a2} = I_{a0} = (I_a/3) = [E_a/(Z_1 + Z_2 +$

$Z_0)]$

2. Fault current If = $I_a = 3I_{a1} = [3E_a/(Z_1 + Z_2$

$+ Z_0)]$ 3. $V_{a1} = E_a - I_{a1}Z_1 =$

$E_a(Z_2+Z_0)/(Z_1+Z_2+Z_0)$

4. V_{a2} = - $E_aZ_2/(Z_1+Z_2+Z_0)$

5. V_{a0} = - $E_aZ_0/(Z_1+Z_2+Z_0)$

6. Fault phase voltage V_a = 0,

7. Sound phase voltages V_b =

a $2V_{a1}+aV_{a2}+V_{a0}$; V_c =

$aV_{a1}+a_2V_{a2}+V_{a0}$

8. Fault phase power: V_aI_a* = 0, Sound pahse

powers: V_bI_b* = 0, and V_cI_c* = 0,

9. If Z_n = 0, then Z_0 = Z_{g0},

10 If Z_n = α then Z_o = α.e., the zero sequence network is open so

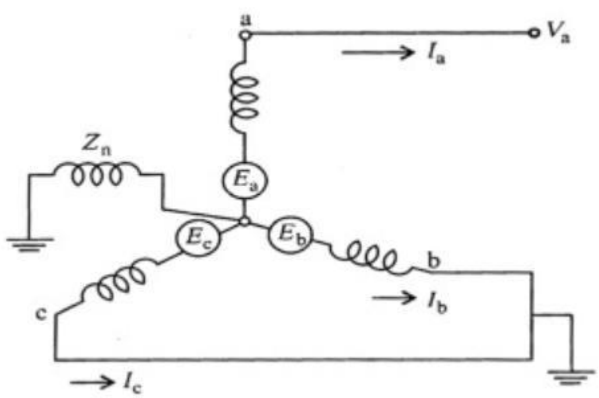
that then, then Z_0 = 10. If Z_n = I_f=I_a=0.

Double Line-to-Ground Fault:

A double line-to-ground (2LG) fault involves a short circuit between two phase conductors b and c and ground. As with the line-to-line fault, there is symmetry with respect to the principal phase a.

In double line-to-ground fault, the two lines contact with each other along with the ground. The probability of such types of faults is nearly 10 %. The symmetrical and unsymmetrical fault mainly occurs in the terminal of the generator, and the open circuit and short circuit fault occur on the distribution system.

Consider line-to-line fault on phases b and c also grounded as shown in Fig.



$$I_a = 0$$
$$V_b = V_c = 0$$
$$I_b + I_c = I_F$$

$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2 V_c) = \frac{1}{3} V_a$$

$$V_{a2} = \frac{1}{3}(V_a + a^2 V_b + aV_c) = \frac{1}{3} V_a$$

Further

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c) = \frac{1}{3}V_a$$

Hence

$$V_{a1} = V_{a2} = V_{a0} = \frac{1}{3}V_a$$

But

$$V_{a1} = E_a - I_{a1}Z_1$$

$$V_{a2} = -I_{a2}Z_2$$

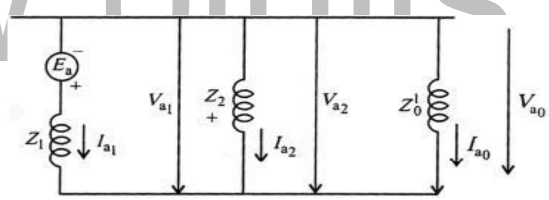
and

$$\begin{aligned} V_{a0} &= -I_{a0}Z_0 - I_F Z_n \\ &= -I_{a0}(Z_0 + 3Z_n) = -I_{a0}(Z_0^1) \end{aligned}$$

It may be noted that

$$\begin{aligned} I_F &= I_b + I_c = a^2 I_{a1} + aI_{a2} + I_{a0} + aI_{a1} + a^2 I_{a2} \\ &= (a + a^2)I_{a1}(a + a^2)I_{a2} + 2I_{a0} \\ &= -I_{a1} - I_{a2} + 2I_{a0} = -I_{a1} - I_{a2} - I_{a0} + 3I_{a0} \\ &= -(I_{a1} + I_{a2} + I_{a0}) + 3I_{a0} = 0 + 3I_{a0} = 3I_{a0} \end{aligned}$$

The sequence network connections are shown in Fig.



$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2 Z_0^1}{Z_2 + Z_0^1}}$$

$$= \frac{E_a (Z_2 + Z_0^1)}{Z_1 Z_2 + Z_2 Z_0^1 + Z_0^1 Z_1}$$

$$V_{a2} = V_{a1}$$

$$-I_{a1} Z_2 = E_a - I_{a1} Z_1$$

$$\begin{aligned} I_{a2} &= -\left(\frac{E_a - I_{a1} Z_1}{Z_2}\right) \\ &= -\left[E_a - \frac{E_a (Z_2 + Z_0^1) \cdot Z_1}{Z_1 Z_2 + Z_2 Z_0^1 + Z_0 Z_1}\right] \cdot \frac{1}{Z_2} \\ &= \frac{-E_a Z_0^1}{Z_1 Z_2 + Z_2 Z_0^1 + Z_0^1 Z_1} \end{aligned}$$

Similarly

$$-I_{a0} Z_0^1 = -I_{a2} Z_2$$

$$I_{a0} = -I_{a2} \frac{Z_2}{Z_0^1} = \frac{-E_a Z_2}{Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_1}$$

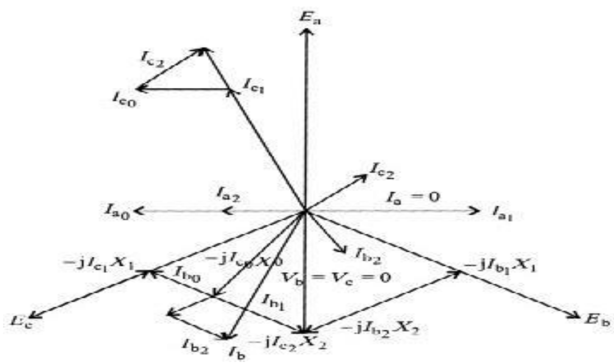
$$\begin{aligned} V_a &= V_{a1} + V_{a2} + V_{a0} \\ &= E_a - I_{a1} Z_1 - I_{a2} Z_2 - I_{a0} (Z_0 + 3Z_n) \\ &= E_a - \frac{E_a (Z_2 + Z_0)}{\Sigma Z_1 Z_2} Z_1 + \frac{E_a Z_0 Z_2}{\Sigma Z_1 Z_2} + \frac{E_a \cdot Z_2 (Z_0 + 3Z_n)}{\Sigma Z_1 Z_2} \\ &= E_a \frac{3Z_2 Z_0 + 3Z_2 Z_n}{\Sigma Z_1 Z_2} = 3E_a \left(\frac{Z_2 (Z_0 + Z_n)}{Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_1}\right) \end{aligned}$$

$$\begin{aligned} V_b &= V_{a0} + a^2 V_{a1} + a V_{a2} \\ &= -I_{a0} (Z_0 + 3Z_n) + a^2 \left[E_a - I_{a1} Z_1\right] + a\left[-I_{a2} Z\right. \\ &= \frac{E_a (Z_2)(Z_0 + 3Z_n)}{\Sigma Z_1 Z_2} (Z_0 + 3Z_n) + a^2 \left[E_a - \frac{(E_a Z_2 + }{\Sigma Z_1 Z}\right. \\ &= \frac{E_a [Z_2 Z_0 \quad 3Z_2 Z_n] + a^2 E_a [Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_1 \quad Z_2 Z_1 \quad Z_0 Z_1}{\Sigma Z_1 Z_2 + a E_a Z_0 Z_2} \end{aligned}$$

$$V_b = \frac{E_a\left[Z_0 Z_2 + a^2 Z_0 Z_2 + a Z_0 Z_2 - 3 Z_2 Z_n\right]}{\Sigma Z_1 Z_2}$$

$$= \frac{E_a\left[Z_0 Z_2 (1 + a + a^2) + 3 Z_2 Z_n\right]}{Z_1 Z_2 + Z_2 Z_0 + Z_0 Z_4} = \frac{3 \cdot E_a \cdot Z_2 Z_n}{Z_1 Z_2 - Z_2 Z_0 + Z_0 Z_1}$$

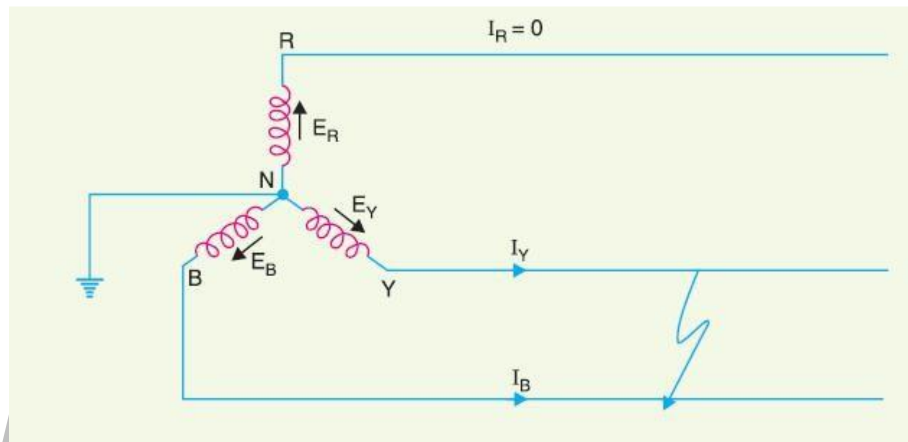
If $Z_n = 0;\ V_b = 0$



LINE TO LINE FAULT ON A CONVENTIONAL GENERATOR

A line-to-line (L-L) fault involves a short circuit between two phase conductors that are assumed to be phases b and c. Therefore, there is symmetry with respect to the principal phase a. A line-to-line fault is illustrated in Fig

Consider a line to line fault between phase 'b' and phase 'c' as shown in figure, at the terminals of a conventional generator, whose neutral is grounded through a reactance.



Consider now the conditions under fault as under:

c.u.f.:

Ia = 0; Ib = - Ic; and Vb = Vc (4.8)

Now consider the symmetrical components of the voltage Va with Vb=Vc, given by:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} V_a \\ V_b \\ V_b \end{vmatrix}$$

Solving the equation we get,

Va1 = Va2

Further, consider the symmetrical components of current Ia with Ib=-Ic, and Ia=0; given by

$$\begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} = (1/3)\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} 0 \\ I_b \\ -I_b \end{vmatrix}$$

Solving above equation we get,

Ia0 = 0; and Ia2 = -Ia1

Using equation above equation, and since V_{a0} = 0 (I_{a0} being 0), we get

$$\begin{vmatrix} 0 \\ V_{a1} \\ V_{a1} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{vmatrix}$$

Pre-multiplying above equation throughout by [0 1 -1], we get,

$$V_{a1} - V_{a1} = E_a - I_{a1}Z_1 - I_{a1}Z_2 = 0$$

Or in other words,

$$I_{a1} = [E_a/(Z_1 + Z_2)]$$

The above equation derived implies that the three sequence networks are connected such that the zero sequence network is absent and only the positive and negative sequence networks are connected in series-opposition to simulate the LL fault, as shown in figure.Further we have the following relations satisfied under the fault conditions:

1. $Ia_1 = - I_{a2} = [E_a/(Z_1 + Z_2)]$ and $I_{a0} = 0$,

2. Fault current $If = I_b = -Ic = [3E_a/(Z_1 + Z_2)]$ (since $I_b = (a_2 - a)I_{a1} = 3I_{a1}$)

3. $V_{a1} = E_a - I_{a1}Z_1 = E_aZ_2/(Z_1+Z_2)$

4. $V_{a2} = V_{a1} = E_aZ_2/(Z_1+Z_2)$

5. $V_{a0} = 0$,

6. Fault phase voltages; $V_b = V_c = aV_{a1}+a_2V_{a2}+V_{a0} = (a+a_2)V_{a1} = - V_{a1}$

7. Sound phase voltage; $V_a = V_{a1}+V_{a2}+V_{a0} = 2V_{a1}$

8. Fault phase powers are V_bI_b * and V_cI_c*

,9. Sound phase power: V_aI_a* = 0,

10. Since Ia0=0, the presence of absence of neutral impedance does not make any difference in the analysis.

Symmetrical components

Introduction

Installment 3 of these notes dealt primarily with networks that are balanced, in which the three voltages (and three currents) are identical but for exact 120◦ phase shifts. Unbalanced conditions may arise from unequal voltage sources or loads. It is possible to analyze some simple types of unbalanced networks using straightforward solution techniques and wye-delta transformations. However, power networks can be come quite complex and many situations would be very difficult to handle using ordinary network analysis. For this reason, a technique which has come to be called symmetrical components has been developed.

Symmetrical components, in addition to being a powerful analytical tool, are also conceptually useful. The symmetrical components themselves, which are obtained from a transformation of the ordinary line voltages and currents, are useful in their own right. Symmetrical components have become accepted as one way of describing the properties of many types of network elements such as transmission lines, motors and generators

Symmetrical components of a 3 phase system

In a 3 phase system, the unbalanced vectors (either currents or voltage) can be resolved into three balanced system of vectors.

They are Positive sequence components

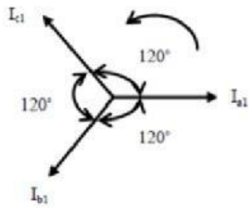
Negative sequence components

Zero sequence components

Unsymmetrical fault analysis can be done by using symmetrical components.

Positive sequence components

It consists of three components of equal magnitude, displaced each other by 120˚ in phase and having the phase sequence abc .



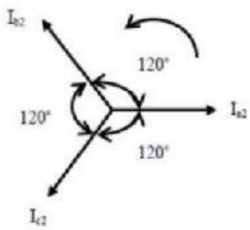
$I_{a1} = I_{a1}\angle 0^{\circ}$

$I_{b1} = I_{a1}\angle 240^{\circ} = I_{a1}\angle -120^{\circ}$

$I_{c1} = I_{a1}\angle 120^{\circ}$

Negative sequence components

It consists of three components of equal magnitude, displaced each other by 120˚ in phase and having the phase sequence acb .



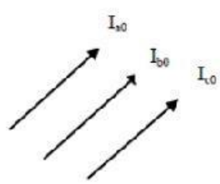
$I_{a2} = I_{a2}\angle 0^{\circ}$

$I_{b2} = I_{a2}\angle 120^{\circ}$

$I_{c1} = I_{a2}\angle 240^{\circ} = I_{a2}\angle -120^{\circ}$

Zero sequence components

It consists of three phasors equal in magnitude and with zero phase displacement from each other.



$I_{a0} = I_{b0} = I_{c0}$

Sequence operator:

In unbalanced problem, to find the relationship between phase voltages and phase currents, we use sequence operator ‘a’.

a = 1∠120˚ == - 0.5+j0.86

$$a^2 = 1\angle 240^\circ = -0.5 - j0.866$$

$$1 + a + a^2 = 0$$

Unbalanced currents from symmetrical currents

Let, Ia, Ib, Ic be the unbalanced phase currents

Let, Ia0, Ia1, Ia2 be the symmetrical components of phase a

$$\begin{bmatrix} I_a \\ I_b \\ I_b \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Determination of symmetrical currents from unbalanced currents.

Let, Ia, Ib, Ic be the unbalanced phase currents

Let, Ia0, Ia1, Ia2 be the symmetrical components of phase a

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_b \end{bmatrix}$$

The Symmetrical Component Transformation

The basis for this analytical technique is a transformation of the three voltages and three currents

In to a second set of voltages and currents. This second set is known as the symmetrical components.

Working in complex amplitudes:

$$
\begin{aligned}
v_a &= Re\left(\underline{V}_a e^{j\omega t}\right) \\
v_b &= Re\left(\underline{V}_b e^{j\left(\omega t-\frac{2\pi}{3}\right)}\right) \\
v_c &= Re\left(\underline{V}_c e^{j\left(\omega t+\frac{2\pi}{3}\right)}\right)
\end{aligned}
$$

The transformation is defined as:

$$
\begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix}
$$

where the complex number a is:

$$
\begin{aligned}
\underline{a} &= e^{j\frac{2\pi}{3}} &&= -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\
\underline{a}^2 &= e^{j\frac{4\pi}{3}} = e^{-j\frac{2\pi}{3}} &&= -\frac{1}{2} - j\frac{\sqrt{3}}{2} \\
\underline{a}^3 & &&= 1
\end{aligned}
$$

This transformation may be used for both voltage and current, and works for variables in

ordinary form as well as variables that have been normalized and are in per-unit form. The inverse

of this transformation is:

$$
\begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \underline{a}^2 & \underline{a} & 1 \\ \underline{a} & \underline{a}^2 & 1 \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_0 \end{bmatrix}
$$

The three component variables V 1, V 2, V 0 are called, respectively, positive sequence, negative

sequence and zero sequence. They are called symmetrical components because, taken separately,

they transform into symmetrical sets of voltages. The properties of these components can be

demonstrated by tranforming each one back into phase variables.

Consider first the positive sequence component taken by itself:

$$\underline{V}_1 = V$$
$$\underline{V}_2 = 0$$
$$\underline{V}_0 = 0$$

yields:

$$\underline{V}_a = V \quad \text{or} \quad v_a = V\cos\omega t$$
$$\underline{V}_b = \underline{a}^2 V \quad \text{or} \quad v_b = V\cos(\omega t - \frac{2\pi}{3})$$
$$\underline{V}_c = \underline{a} V \quad \text{or} \quad v_c = V\cos(\omega t + \frac{2\pi}{3})$$

This is the familiar balanced set of voltages: Phase b lags phase a by 120◦, phase c lags phase

b and phase a lags phase c.

The same transformation carried out on a negative sequence voltage:

$$\underline{V}_1 = 0$$
$$\underline{V}_2 = V$$
$$\underline{V}_0 = 0$$

yields:

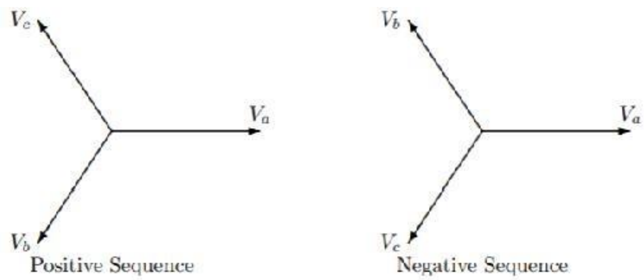
$$\underline{V}_a = V \quad \text{or} \quad v_a = V\cos\omega t$$
$$\underline{V}_b = \underline{a} V \quad \text{or} \quad v_b = V\cos(\omega t + \frac{2\pi}{3})$$
$$\underline{V}_c = \underline{a}^2 V \quad \text{or} \quad v_c = V\cos(\omega t - \frac{2\pi}{3})$$

This is called negative sequence because the sequence of voltages is reversed: phase b now leads

phase a rather than lagging. Note that the negative sequence set is still balanced in the sense

that the phase components still have the same magnitude and are separated by 120◦ . The only

difference between positive and negative sequence is the phase rotation. This is shown in Figure



The third symmetrical component is zero sequence. If:

$$\underline{V}_1 = 0$$
$$\underline{V}_2 = 0$$
$$\underline{V}_0 = V$$

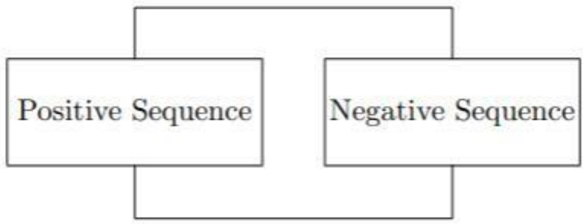
Then:

$$\underline{V}_a = V \quad \text{or} \quad v_a = V\cos\omega t$$
$$\underline{V}_b = V \quad \text{or} \quad v_b = V\cos\omega t$$
$$\underline{V}_c = V \quad \text{or} \quad v_c = V\cos\omega t$$

That is, all three phases are varying together. Positive and negative sequence sets contain those parts of the three-phase excitation that represent balanced normal and reverse phase sequence. Zero sequence is required to make up the difference between the total phase variables and the two rotating components. The great utility of symmetrical components is that, for most types of network elements, the symmetrical components are independent of each other. In particular, balanced impedances and rotating machines will draw only positive sequence currents in response to positive sequence voltages. It is thus possible to describe a network in terms of sub-networks, one for each of the symmetrical components. These are called sequence networks. A completely balanced network will have three entirely separate sequence networks. If a network is unbalanced at a particular spot, the sequence networks will be interconnected at that spot. The key to use of symmetrical components in handling unbalanced situations is in learning how to formulate those interconnections.

Sequence Impedances:

Many different types of network elements exhibit different behavior to the different symmetrical components. For example, as we will see shortly, transmission lines have one impedance for positive and negative sequence, but an entirely different impedance to zero sequence. Rotating machines have different impedances to all three sequences.



To illustrate the independence of symmetrical components in balanced networks, consider the transmission line illustrated back in Figure 20 of Installment 3 of these notes. The expressions for voltage drop in the lines may be written as a single vector expression:

$$\underline{V}_{ph1} - \underline{V}_{ph2} = j\omega \underline{\underline{L}}_{ph} \underline{I}_{ph}$$

Where

$$\underline{V}_{ph} = \begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix}$$

$$\underline{I}_{ph} = \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix}$$

$$\underline{\underline{L}}_{ph} = \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix}$$

Note that the symmetrical component transformation (4) may be written in compact form:

$$\underline{V}_s = \underline{\underline{T}} \underline{V}_p$$

Where

$$\underline{\underline{T}} = \frac{1}{3}\begin{bmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{bmatrix}$$

and V s is the vector of sequence voltages:

$$\underline{V}_s = \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_0 \end{bmatrix}$$

$$\underline{\underline{T}}^{-1}\underline{V}_{s1} - \underline{\underline{T}}^{-1}\underline{V}_{s2} = j\omega\underline{\underline{L}}_{ph}\underline{\underline{T}}^{-1}\underline{I}_s$$

Then transforming to get sequence voltages:

$$\underline{V}_{s1} - \underline{V}_{s2} = j\omega\underline{\underline{T}}\underline{\underline{L}}_{ph}\underline{\underline{T}}^{-1}\underline{I}_s$$

The sequence inductance matrix is defined by carrying out the operation indicated:

$$\underline{\underline{L}}_s = \underline{\underline{T}}\underline{\underline{L}}_{ph}\underline{\underline{T}}^{-1}$$

which is:

$$\underline{L}_s = \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L+2M \end{bmatrix}$$

Thus the coupled set of expressions which described the transmission line in phase variables becomes an uncoupled set of expressions in the symmetrical components:

$$\begin{aligned} \underline{V}_{11} - \underline{V}_{12} &= j\omega(L-M)\underline{I}_1 \\ \underline{V}_{21} - \underline{V}_{22} &= j\omega(L-M)\underline{I}_2 \\ \underline{V}_{01} - \underline{V}_{02} &= j\omega(L+2M)\underline{I}_0 \end{aligned}$$

The positive, negative and zero sequence impedances of the balanced transmission line are then:

$$\begin{aligned} \underline{Z}_1 = \underline{Z}_2 &= j\omega(L-M) \\ \underline{Z}_0 &= j\omega(L+2M) \end{aligned}$$

So, in analysis of networks with transmission lines, it is now possible to replace the lines with three independent, single- phase networks. Consider next a balanced three-phase load with its neutral connected to ground through an impedance as shown in Figure. The symmetrical component voltage-current relationship for this network is found simply, byassuming positive, negative and zero sequence currents and finding the corresponding voltages. Ifthis is done, it is found that the symmetrical components are independent, and that the voltage current relationships are:

$$\underline{V}_1 = \underline{Z}\underline{I}_1$$
$$\underline{V}_2 = \underline{Z}\underline{I}_2$$
$$\underline{V}_0 = (\underline{Z} + 3\underline{Z}_g)\underline{I}_0$$