BUS CLASSIFICATION

INTRODUCTION

Load flow studies are one of the most important aspects of power system planning and operation. The load flow gives us the sinusoidal steady state of the entire system - voltages, real and reactive power generated and absorbed and line losses. Since the load is a static quantity and it is the power that flows through transmission lines, the purists prefer to call this Power Flow studies rather than load flow studies. We shall however stick to the original nomenclature of load flow.

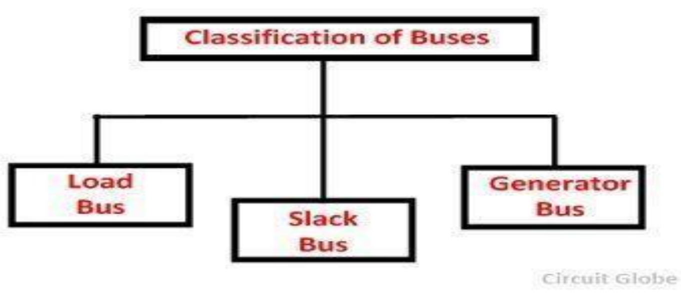
Through the load flow studies we can obtain the voltage magnitudes and angles at each bus in the steady state. This is rather important as the magnitudes of the bus voltages are required to be held within a specified limit. Once the bus voltage magnitudes and their angles are computed using the load flow, the real and reactive power flow through each line can be computed. Also based on the difference between power flow in the sending and receiving ends, the losses in a particular line can also be computed. Furthermore, from the line flow we can also determine the over and under load conditions.

The steady state power and reactive power supplied by a bus in a power network are expressed in terms of nonlinear algebraic equations. We therefore would require iterative methods for solving these equations. In this chapter we shall discuss two of the load flow methods. We shall also delineate how to interpret the load flow results.

CLASSIFICATION OF BUSES

A bus in a power system is defined as the vertical line at which the several components of the power system like generators, loads, and feeders, etc., are connected. The buses in a power system are associated with four quantities. These quantities are the magnitude of the voltage, the phase angle of the voltage, active or true power and the reactive power.

In the load flow studies, two variables are known, and two are to be determined. Depends on the quantity to be specified the buses are classified into three categories generation bus, load bus and slack bus.



The table shown below shows the types of buses and the associated known and unknown value.

Type of Buses	Know or Specified Quantities	Unknown Quantities or Quantities to be determined.
Generation or P-V Bus	P, |V|	Q, δ
Load or P-Q Bus	P, Q	| V |, δ
Slack or Reference Bus	| V |, δ	P, Q

LOAD BUSES:

This is also called the P-Q bus and at this bus, the active and reactive power is injected into the network. Magnitude and phase angle of the voltage are to be computed. Here the active power P and reactive power Q are specified, and the load bus voltage can be permitted within a tolerable value, i.e., 5%. The phase angle of the voltage, i.e.δ is not very important for the load.

In these buses no generators are connected and hence the generated real power PGi and reactive power QGi are taken as zero. The load drawn by these buses are defined by real power -PLi and reactive power -QLi in which the negative sign accommodates for the power flowing out of the bus. This is why these buses are sometimes referred to as P-Q bus. The objective of the load flow is to find the bus voltage magnitude |Vi| and its angle δi

Now consider a typical load flow problem in which all the load demands are known. Even if the generation matches the sum total of these demands exactly, the mismatch between generation and load will persist because of the line I 2R losses. Since the I 2R loss of a line depends on the line current which, in turn, depends on the magnitudes and angles of voltages of the two buses connected to the line, it is rather difficult to estimate the loss without calculating the voltages and angles. For this reason a generator bus is usually chosen as the slack bus without specifying its real power. It is assumed that the generator connected to this bus will supply the balance of the real power required and the line losses.

VOLTAGE CONTROLLED BUSES

This bus is also called the P-V bus, and on this bus, the voltage magnitude corresponding to generate voltage and true or active power P corresponding to its rating are specified. Voltage magnitude is maintained constant at a specified value by injection of reactive power. The reactive power generation Q and phase angle δ of the voltage are to be computed.

These are the buses where generators are connected. Therefore the power generation in such buses is controlled through a prime mover while the terminal voltage is controlled through the generator excitation. Keeping the input power constant through turbine-governor control and keeping the bus voltage constant using automatic voltage regulator, we can specify constant PGi and ❙ Vi ❙ for these buses. This is why such buses are also referred to as P-V buses. It is to be noted that the reactive power supplied by the generator QGi depends on the system configuration and cannot be specified in advance. Furthermore we have to find the unknown angle δi of the bus voltage.

SLACK OR SWING BUS:

Slack bus in a power system absorbs or emits the active or reactive power from the power system. The slack bus does not carry any load. At this bus, the magnitude and phase angle of the voltage are specified. The phase angle of the voltage is usually set equal to zero. The active and reactive power of this bus is usually determined through the solution of equations.

The slack bus is a fictional concept in load flow studies and arises because the I2R losses of the system are not known accurately in advance for the load flow calculation. Therefore, the total injected power cannot be specified at every bus. The phase angle of the voltage at the slack bus is usually taken as reference or zero

Usually this bus is numbered 1 for the load flow studies. This bus sets the angular reference for all the other buses. Since it is the angle difference between two voltage sources that dictates the real and reactive power flow between them, the particular angle of the slack bus is not important. However it sets the reference against which angles of all the other bus voltages are measured. For this reason the angle of this bus is usually chosen as 0°. Furthermore it is assumed that the magnitude of the voltage of this bus is known.

Now consider a typical load flow problem in which all the load demands are known. Even if the generation matches the sum total of these demands exactly, the mismatch between generation and load will persist because of the line I 2R losses. Since the I 2R loss of a line depends on the line current which, in turn, depends on the magnitudes and angles of voltages of the two buses connected to the line, it is rather difficult to estimate the loss without calculating the voltages and angles. For this reason a generator bus is usually chosen as the slack bus without specifying its real power. It is assumed that the generator connected to this bus will supply the balance of the real power required and the line losses.

REAL AND REACTIVE POWER INJECTED IN A BUS

For the formulation of the real and reactive power entering a b us, we need to define the following quantities. Let the voltage at the i^{th} bus be denoted by

$$V_i = |V_i|\angle\delta_i = |V_i|(\cos\delta_i + j\sin\delta_i)$$

Also let us define the self admittance at bus- *i* as

$$Y_{ii} = |Y_{ii}|\angle\theta_{ii} = |Y_{ii}|(\cos\theta_{ii} + j\sin\theta_{ii}) = G_{ii} + jB_{ii}$$

Similarly the mutual admittance between the buses *i* and *j* can be written as

$$Y_{ij} = |Y_{ij}| \angle \theta_{ij} = |Y_{ij}| (\cos\theta_{ij} + j\sin\theta_{ij}) = G_{ij} + jB_{ij}$$

Let the power system contains a total number of *n* buses. The current injected at bus- *i* is given as

$$I_i = Y_{i1}V_1 + Y_{i2}V_2 + \cdots + Y_{in}V_n$$
$$= \sum_{k=1}^{n} Y_{ik}V_k$$

It is to be noted we shall assume the current entering a bus to be positive and that leaving the bus to be negative. As a consequence the power and reactive power entering a bus will also be assumed to be positive. The complex power at bus- *i* is then given by

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{k=1}^{n} Y_{ik}V_k$$
$$= |V_i|(\cos\delta_i - j\sin\delta_i) \sum_{k=1}^{n} |Y_{ik}V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k)$$
$$= \sum_{k=1}^{n} |Y_{ik}V_iV_k| (\cos\delta_i - j\sin\delta_i)(\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k)$$

Note that

$$(\cos\delta_i - j\sin\delta_i)(\cos\theta_{ik} + j\sin\theta_{ik})(\cos\delta_k + j\sin\delta_k)$$
$$= (\cos\delta_i - j\sin\delta_i)[\cos(\theta_{ik} + \delta_k) + j\sin(\theta_{ik} + \delta_k)]$$
$$= \cos(\theta_{ik} + \delta_k - \delta_i) + j\sin(\theta_{ik} + \delta_k - \delta_i)$$

Therefore substituting in $P_I - jQ_i$ we get the real and reactive power as

$$P_i = \sum_{k=1}^{n} |Y_{ik}V_iV_k| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$Q_i = -\sum_{k=1}^{n} |Y_{ik}V_iV_k| \sin(\theta_{ik} + \delta_k - \delta_i)$$

FORMULATION OF POWER FLOW PROBLEM IN POLAR COORDINATES

The power flow problem can also be solved by using Newton-Raphson method. In fact, among the numerous solution methods available for power flow analysis, the Newton-Raphson method is considered to be the most sophisticated and important. Many advantages are attributed to the Newton-Raphson (N-R) approach.

Gauss-Seidel (G-S) is a simple iterative method of solving n number load flow equations by iterative method. It does not require partial derivatives. Newton-Raphson method is based on Taylor's series and partial derivatives.

The N-R method is recent, needs less number of iterations to reach convergence, takes less computer time hence computation cost is less and the convergence is certain. The N-R method is more accurate, and is insensitive to factors like slack bus selection, regulating transformers etc. and the number of iterations required in this method is almost independent of the system size.

The drawbacks of this method are difficult solution technique, more calculations involved in each iteration resulting in large computer time per iteration and the large requirement of computer memory but the last drawback has been overcome through a compact storage scheme.

Convergence can be considerably speeded up by performing the first iteration through the G-S method and using the values of voltages so obtained for starting the N-R iterations. These voltages are used to compute active power P at every bus except the swing bus and also reactive power Q wherever reactive power is specified.

The difference between the specified and calculated values is used to determine the correction of bus voltages. The process of iteration is continued till the difference in the specified and calculated values of P, Q and V are within the given permissible limit.

Before explaining the application of N-R method to the power flow problems, it is useful to review this method in its general form.

Quantities associated with each bus in a system

Each bus in a power system is associated with four quantities and they are real power (P), reactive power (Q), magnitude of voltage (V), and phase angle of voltage (δ).

Work involved (or) to be performed by a load flow study

(i). Representation of the system by a single line diagram

(ii). Determining the impedance diagram using the information in single line diagram

(iii). Formulation of network equation

(iv). Solution of network equations

Iterative methods to solve load flow problems

The load flow equations are non linear algebraic equations and so explicit solution as not possible. The solution of non linear equations can be obtained only by iterative numerical techniques.

Mainly used for solution of load flow study

The Gauss seidal method, Newton Raphson method and Fast decouple method

Flat voltage start

In iterative method of load flow solution, the initial voltages of all buses except slack bus assumed as 1+j0 p.u. This is referred to as flat voltage start

Classification of Buses

Bus

The meeting point of various components in a power system is called a bus. The bus is a conductor made of copper or aluminum having negligible resistance .At some of the buses power is being injected into the network, whereas at other buses it is being tapped by the system loads.

Bus admittance matrix

The matrix consisting of the self and mutual admittance of the network of the power system is called bus admittance matrix (Ybus).

Methods available for forming bus admittance matrix

Direct inspection method.

Singular transformation method.(Primitive network)

Different types of buses in a power system

Types of bus	Known or specified quantities	Unknown quantities or quantities to be determined
Slack or Swing or Reference bus	V, δ	P,Q
Generator or Voltage control or PV bus	P, V	Q, δ
Load or PQ bus	P, Q	V, δ

Need for slack bus

The slack bus is needed to account for transmission line losses. In a power system the total power generated will be equal to sum of power consumed by loads and losses. In a power system only the generated power and load power are specified for buses. The slack bus is assumed to generate the power required for losses. Since the losses are unknown the real and reactive power are not specified for slack bus.

Effect of acceleration factor in load flow study

Acceleration factor is used in gauss seidal method of load flow solution to increase the rate of convergence. Best value of A.F=1.6

Generator buses are treated as load bus

If the reactive power constraint of a generator bus violates the specified limits then the generator is treated as load bus.

Formulation of Load flow Equatio

The complex power injected by the generating source into the ith bus of a power system is given as:

$S_i = P_i + j\,Q_i = V_i\,I_i^*$ $\qquad$ $i = 1, 2, \ldots, n$

where Vi is the voltage at the ith bus with respect to ground and Ii* is the complex conjugate of source current Ii injected into the bus.

It is convenient to handle load flow problem by using Ii rather than Ii*. So, taking the complex conjugate of above equation), we have

$S_i^* = P_i - j\,Q_i = V_i^* I_i$; $\qquad$ n = 1, 2, 3, …., n

Substituting $I_i = \sum_{k=1}^{n} Y_{ik} V_k$ form Eq. (6.8) in above equation, we have –

$$S_i^* = P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k \; ; \quad i = 1, 2, \ldots, n$$

Equating real and imaginary parts, we have

$$\text{Real power} = P_i = R_e \left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\}$$

$$\text{Reactive power} = Q_i = -I_m \left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\}$$

$$\text{In polar form } V_i = V_i \angle\delta_i,\ V_i^* = V_i \angle{-\delta_i}$$

$$\text{and } Y_{ik} = Y_{ik} \angle\theta_{ik}.$$

So real and reactive power can now be expressed as

$$\text{Real power, } P_i = V_i \sum_{k=1}^{n} V_k Y_{ik} \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$\text{Reactive power, } Q_i = -V_i \sum_{k=1}^{n} V_k Y_{ik} \sin(\theta_{ik} + \delta_k - \delta_i)$$

Above Equations are known as static load flow equations. (SLFE). These equations are nonlinear equations and, therefore, only a numerical solution is possible. For each of the n system buses we have two such equations giving a total of 2n equations (n real flow power equations and n reactive power flow equations).

Each bus is characterized by four variables Pi, Qi, Vi and δi giving a total of 4n variables. To obtain a solution it is necessary to specify two variables at each bus so that the number of unknowns is reduced to 2n.

NEWTON RAPHSON METHOD

Newton Raphson Method is an iterative technique for solving a set of various nonlinear equations with an equal number of unknowns. There are two methods of solutions for the load flow using Newton Raphson Method. The first method uses rectangular coordinates for the variables while the second method uses the polar coordinate form. Out of these two methods the polar coordinate form is used widely.

Procedure of Newton Raphson Method

The computational procedure for Newton Raphson Method using polar coordinate is given below.

The Newton-Raphson procedure is as follows:

Step-1:

Choose the initial values of the voltage magnitudes $|V|^{(0)}$ of all n_p load buses and $n - 1$ angles $\delta^{(0)}$ of the voltages of all the buses except the slack bus.

Step-2:

Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to calculate a total $n - 1$ number of injected real power $P_{calc}^{(0)}$ and equal number of real power mismatch $\Delta P^{(0)}$.

Step-3:

Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to calculate a total n_p number of injected reactive power $Q_{calc}^{(0)}$ and equal number of reactive power mismatch $\Delta Q^{(0)}$.

Step-3:

Use the estimated $|V|^{(0)}$ and $\delta^{(0)}$ to formulate the Jacobian matrix $J^{(0)}$.

Step-4:

Solve equation for $\delta^{(0)}$ and $\Delta |V|^{(0)} \div |V|^{(0)}$.

Step-5:

Obtain the updates from

$$\delta^{(1)} = \delta^{(0)} + \Delta\delta^{(0)}$$

$$|V|^{(1)} = |V|^{(0)}\left[1 + \frac{\Delta|V|^{(0)}}{|V|^{(0)}}\right]$$

Step-6:

Check if all the mismatches are below a small number. Terminate the process if yes. Otherwise go back to step-1 to start the next iteration with the updates given by the equation

We shall now discuss the formation of the sub matrices of the Jacobian matrix. To do that **we shall use the real and reactive power equations .**Let us rewrite them with the help of equation as

A. Formation of J11

Let us define J11 as

$$Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k\neq i}}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$P_i = |V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$J_{11} = \begin{bmatrix} L_{22} & \cdots & L_{2n} \\ \vdots & \ddots & \vdots \\ L_{n2} & \cdots & L_{nn} \end{bmatrix}$$

It can be seen from the equation that L_{ik}'s are the partial derivatives of P_i with respect to δ_k. The derivative P_i equation with respect to k for $i \neq k$ is given by

$$L_{ik} = \frac{\partial P_i}{\partial \delta_k} = -|Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i), \quad i \neq k$$

Similarly the derivative P_i with respect to k for $i = k$ is given by

$$L_{ii} = \frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i)$$

Comparing the above equation we can write

$$L_{ii} = \frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 B_{ii}$$

B.* Formation of J*21

Let us define J_{21} as

$$J_{21} = \begin{bmatrix} M_{11} & \cdots & M_{1n} \\ \vdots & \ddots & \vdots \\ M_{n_p 1} & \cdots & M_{n_p n} \end{bmatrix}$$

it is evident that the elements of J_{21} are the partial derivative of Q with respect to δ . From the equation we can write

$$M_{ik} = \frac{\partial Q_i}{\partial \delta_k} = -|Y_{ik}V_iV_k|\cos(\theta_{ik} + \delta_k - \delta_i) \quad i \neq k$$

Similarly for i = k we have

$$M_{ii} = \frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}V_iV_k|\cos(\theta_{ik} + \delta_k - \delta_i) = P_i - |V_i|^2 G_{ii}$$

C. Formation of J12

Let us define J12 as

$$J_{12} = \begin{bmatrix} N_{11} & \cdots & N_{1n_q} \\ \vdots & \ddots & \vdots \\ N_{n1} & \cdots & N_{nn_q} \end{bmatrix}$$

As evident from equation the elements of J21 involve the derivatives of real power P with respect to magnitude of bus voltage |V| . For i ≠ k , we can write from equation

$$N_{ik} = |V_k|\frac{\partial P_i}{\partial |V_k|} = |Y_{ik}V_iV_k|\cos(\theta_{ik} + \delta_k - \delta_i) = -M_{ik} \quad i \neq k$$

For $i = k$ we have

$$N_{ii} = |V_i|\frac{\partial P_i}{\partial |V_i|} = |V_i|\left[2|V_i|G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}V_k|\cos(\theta_{ik} + \delta_k - \delta_i)\right]$$
$$= 2|V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}V_iV_k|\cos(\theta_{ik} + \delta_k - \delta_i) = 2|V_i|^2 G_{ii} + M_{ii}$$

Formation of J_{22}

For the formation of J_{22} let us define

$$J_{22} = \begin{bmatrix} O_{22} & \cdots & O_{2n_e} \\ \vdots & \ddots & \vdots \\ O_{n_e 2} & \cdots & O_{n_e n_e} \end{bmatrix}$$

For $i \neq k$ we can write from equation

$$O_{ik} = |V_i| \frac{\partial Q_i}{\partial |V_k|} = -|V_i| |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) = L_{ik}, \quad i \neq k$$

Finally for $i = k$ we have

$$O_{ii} = |V_i| \frac{\partial Q_i}{\partial |V_i|} = V_i \left[-2|V_i| B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \right]$$

$$= -2|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) = -2|V_i|^2 B_{ii} - L_{ii}$$

We therefore see that once the sub matrices J_{11} and J_{21} are computed, the formation of the sub matrices J_{12} and J_{22} is fairly straightforward. For large system this will result in considerable saving in the computation time.

Newton Raphson Method Flow Chart

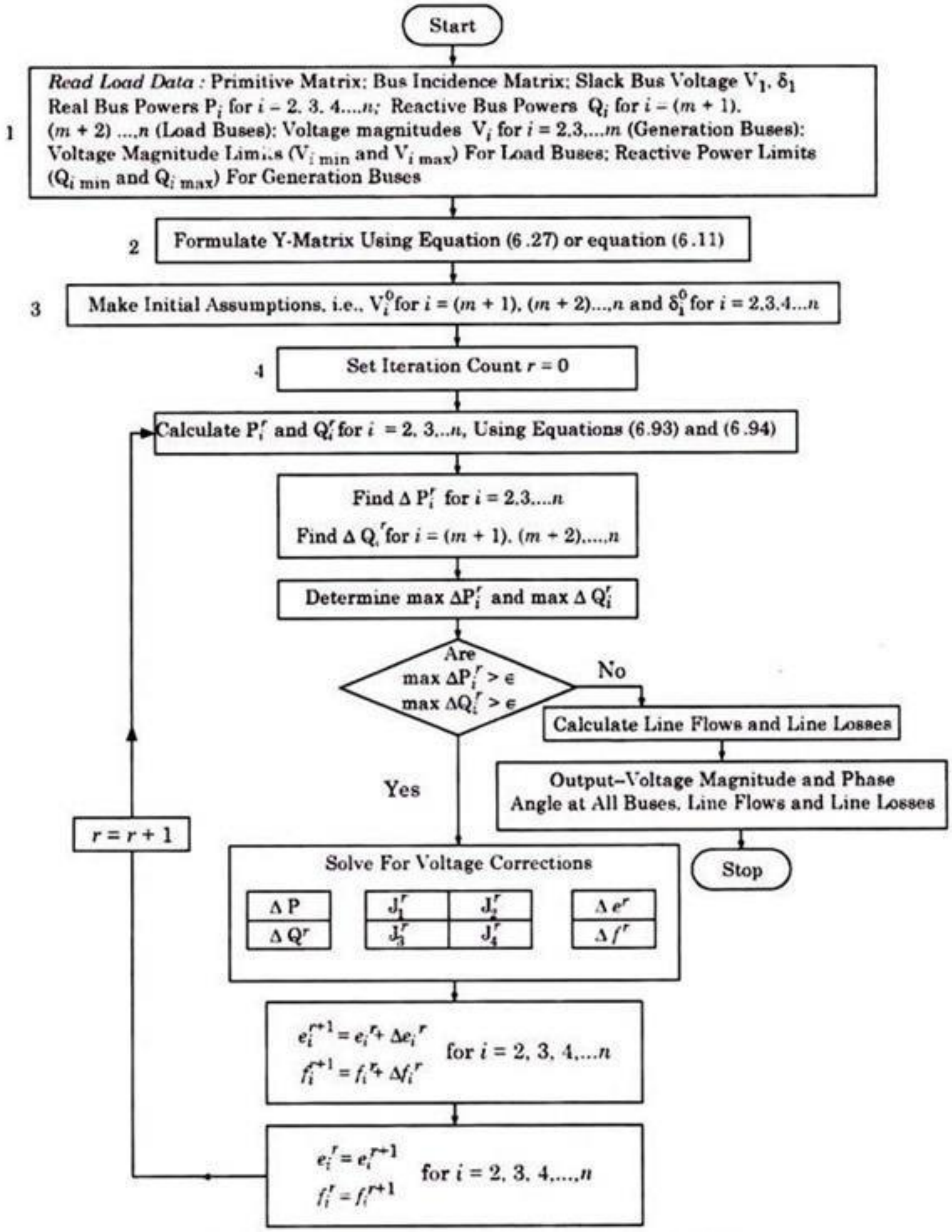
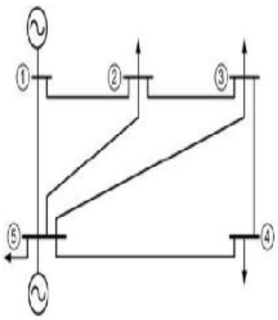


Fig. 6.21. *Flow Chart For Power Flow Solution Using N–R Method*

Solution of Newton-Raphson Load Flow

The Newton-Raphson load flow program is tested on the system of Fig. with the system data and initial conditions given in Tables we can write



Line (bus to bus)	Impedance	Line charging (Y/2)
1-2	$0.02 + j\,0.10$	$j\,0.030$
1-5	$0.05 + j\,0.25$	$j\,0.020$
2-3	$0.04 + j\,0.20$	$j\,0.025$
2-5	$0.05 + j\,0.25$	$j\,0.020$
3-4	$0.05 + j\,0.25$	$j\,0.020$
3-5	$0.08 + j\,0.40$	$j\,0.010$
4-5	$0.10 + j\,0.50$	$j\,0.075$

$$L_{23}^{(0)} = -\left|Y_{23}V_2^{(0)}V_3^{(0)}\right|\sin\left(\theta_{23} + \delta_3 - \delta_2\right) = -\left|Y_{23}\right|\sin\theta_{23} = -B_{23} = -4.8077$$

Similarly from the equation we have

$$Q_2^{(0)} = -\left|V_2^{(0)}\right|^2 B_{22} - \sum_{\substack{k=1\\k\neq 2}}^{n}\left|Y_{2k}V_2^{(0)}V_k^{(0)}\right|\sin\left(\theta_{2k} + \delta_k - \delta_2\right)$$

$$= -B_{22} - 1.05B_{21} - B_{23} - B_{24} - 1.02B_{25} = -0.6327$$

Hence from the equation we get

$$L_{22}^{(0)} = -Q_2^{(0)} - \left|V_2^{(0)}\right|^2 B_{22} = -0.6327 - B_{22} = 18.8269$$

In a similar way the rest of the components of the matrix $J_{11}^{(0)}$ are calculated. This matrix is given by

$$J_{11}^{(0)} = \begin{bmatrix} 18.8269 & -4.8077 & 0 & -3.9231 \\ -4.8077 & 11.1058 & -3.8462 & -2.4519 \\ 0 & -3.8462 & 5.8077 & -1.9615 \\ -3.9231 & -2.4519 & -1.9615 & 12.4558 \end{bmatrix}$$

For forming the off diagonal elements of J_{21} we note from equation that

$$M_{23}^{(0)} = -\left|Y_{23}V_2^{(0)}V_3^{(0)}\right|\cos\left(\theta_{23} + \delta_2 - \delta_3\right) = -G_{23} = 0.9615$$

Also from equation the real power injected at bus-2 is calculated as

$$P_2^{(0)} = \left|V_2^{(0)}\right|^2 G_{22} + \sum_{\substack{k=1 \\ k\neq 2}}^{n} \left|Y_{2k} V_2^{(0)} V_k^{(0)}\right| \cos\left(\theta_{2k} + \delta_k - \delta_2\right)$$
$$= G_{22} + 1.05G_{21} + G_{23} + G_{24} + 1.02G_{25} = -0.1115$$

Hence from equation we have

$$M_{22} = P_2^{(0)} - \left|V_2^{(0)}\right|^2 G_{22} = -3.7654$$

Similarly the rest of the elements of the matrix J_{21} are calculated. This matrix is then given as

$$J_{21}^{(0)} = \begin{bmatrix} -3.7654 & 0.9615 & 0 & 0.7846 \\ 0.9615 & -2.2212 & 0.7692 & 0.4904 \\ 0 & 0.7692 & -1.1615 & 0.3923 \end{bmatrix}$$

For calculating the off diagonal elements of the matrix J12 we note from (4.47) that they are negative of the off diagonal elements of J21 . However the size of J21 is (3 X 4) while the size of J12 is (4 X 3). Therefore to avoid this discrepancy we first compute a matrix M that is given by

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}$$

The elements of the above matrix are computed in accordance with equation . We can then define

$$J_{21} = M(1:3, 1:4) \text{ and } J_{12} = -M(1:4, 1:3)$$

Furthermore the diagonal elements of J_{12} are overwritten in accordance with equation. This matrix is then given by

$$J_{12}^{(0)} = \begin{bmatrix} 3.5423 & -0.9615 & 0 \\ -0.9615 & 2.2019 & -0.7692 \\ 0 & -0.7692 & 1.1462 \\ 0.7846 & -0.4904 & -0.3923 \end{bmatrix}$$

Finally it can be noticed from equation that $J_{22} = J_{11}$ (1:3, 1:3). However the diagonal elements of J_{22} are then overwritten in accordance with equation.This gives the following matrix

$$J_{22}^{(0)} = \begin{bmatrix} 17.5615 & -4.8077 & 0 \\ -4.8077 & 10.8996 & -3.8462 \\ 0 & -3.8462 & 5.5408 \end{bmatrix}$$

From the initial conditions the power and reactive power are computed as

$$P_{calc}^{(0)} = [-0.1115 \quad -0.0096 \quad -0.0077 \quad -0.0098]^T$$
$$Q_{calc}^{(0)} = [-0.6327 \quad -0.1031 \quad -0.1335]^T$$

Consequently the mismatches are found to be

$$\Delta P^{(0)} = [-0.8485 \quad -0.3404 \quad -0.1523 \quad 0.2302]^T$$
$$\Delta Q^{(0)} = [0.0127 \quad -0.0369 \quad 0.0535]^T$$

Then the updates at the end of the first iteration are given as

$$\begin{bmatrix} \delta_2^{(1)} \\ \delta_3^{(1)} \\ \delta_3^{(1)} \\ \delta_4^{(1)} \end{bmatrix} = \begin{bmatrix} -4.91 \\ -6.95 \\ -7.19 \\ -3.09 \end{bmatrix} \text{deg} \quad \begin{bmatrix} |V_2|^{(0)} \\ |V_3|^{(0)} \\ |V_4|^{(0)} \end{bmatrix} = \begin{bmatrix} 0.9864 \\ 0.9817 \\ 0.9913 \end{bmatrix}$$

The load flow converges in 7 iterations when all the power and reactive power mismatches are below 10−6.

Load Flow Results

It is to be noted here that both Gauss-Seidel and Newton-Raphson methods yielded the same result. However the Newton-Raphson method converged faster than the Gauss-Seidel method. The bus voltage magnitudes, angles of each bus along with power generated and consumed at each bus are given in Table 4.4. It can be seen from this table that the total power generated is 174.6 MW whereas the total load is 171 MW. This indicates that there is a line loss of about 3.6 MW for all the lines put together. It is to be noted that the real and reactive power of the slack bus and the reactive power of the P-V bus are computed from equation after the convergence of the load flow.

Bus no.	Bus voltage		Power generated		Load	
	Magnitude (pu)	Angle (deg)	P (MW)	Q (MVAr)	P (MW)	P (MVAr)
1	1.05	0	126.60	57.11	0	0
2	0.9826	– 5.0124	0	0	96	62
3	0.9777	– 7.1322	0	0	35	14
4	0.9876	– 7.3705	0	0	16	8
5	1.02	– 3.2014	48	15.59	24	11

The current flowing between the buses *i* and *k* can be written as

$$I_{ik} = -Y_{ik}\left(V_i - V_k\right) \quad i \neq k$$

Therefore the complex power leaving bus- *i* is given by

$$P_i + jQ_i - V_i I_i^*$$

Similarly the complex power entering bus- *k* is

$$P_k + jQ_k - V_k I_k^*$$

Therefore the $I^2 R$ loss in the line segment *i*-*k* is

$$P_{loss,i-k} = P_i - P_k$$

The real power flow over different lines is listed in Table 4.5. This table also gives the $I^2 R$ loss along various segments. It can be seen that all the losses add up to 3.6 MW, which is the net difference between power generation and load. Finally we can compute the line I^2X drops in a similar fashion. This drop is given by

$$Q_{drop,i-k} = Q_i - Q_k$$

However we have to consider the effect of line charging separately.

Power dispatched		Power received		Line loss (MW)
from (bus)	amount (MW)	in (bus)	amount (MW)	
1	101.0395	2	98.6494	2.3901
1	25.5561	5	25.2297	0.3264
2	17.6170	3	17.4882	0.1288
3	0.7976	4	0.7888	0.0089
5	15.1520	2	14.9676	0.1844
5	18.6212	3	18.3095	0.3117
5	15.4566	4	15.2112	0.2454
				Total = 3.5956

Consider the line segment 1-2. The voltage of bus-1 is $V_1 = 1.05 < 0°$ per unit while that of bus-2 is $V_2 = 0.9826 < -5.0124°$ per unit. From the equation we have

$$I_{12} = 0.9623 - j0.5187 = 1.0932\angle -28.33°$$

Therefore the complex power dispatched from bus-1 is

$$S_{12} = V_1 I_{12}^* \times 100 = -101.0395 - j54.4645$$

where the negative signal indicates the power is leaving bus-1. The complex power received at bus-2 is

$$S_{21} = V_2 I_{12}^* \times 100 = 98.6494 + j42.5141$$

Therefore out of a total amount of 101.0395 MW of real power is dispatched from bus-1 over the line segment 1-2, 98.6494 MW reaches bus-2. This indicates that the drop in the line segment is 2.3901 MW. Note that

$$|I_{12}|^2 \times R_{12} \times 100 = 1.0932^2 \times 0.02 \times 100 = 2.3901$$

where R_{12} is resistance of the line segment 1-2. Therefore we can also use this method to calculate the line loss.

Now the reactive drop in the line segment 1-2 is

$$|I_{12}|^2 \times X_{12} \times 100 = 1.0932^2 \times 0.1 \times 100 = 10.9508$$

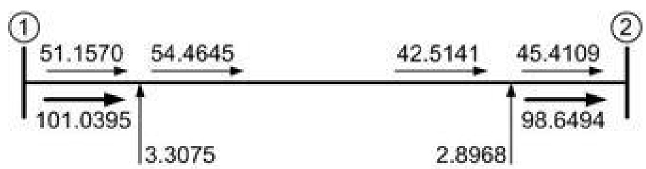
We also get this quantity by subtracting the reactive power absorbed by bus-2 from that supplied by bus-1. The above calculation however does not include the line charging. Note that since the line is modeled by an equivalent- p , the voltage across the shunt capacitor is the bus voltage to which the shunt capacitor is connected. Therefore the current *I* 12 flowing through line segment is not the current leaving bus-1 or entering bus-2 - it is the current flowing in between the two charging capacitors. Since the shunt branches are purely reactive, the real power flow does not get affected by the charging capacitors. Each charging capacitor is assumed to inject a reactive power that is the product of the half line charging admittance and square of the magnitude of the voltage of that at bus. The half line charging admittance of this line is 0.03. Therefore line charging capacitor will inject

$$0.03\times100\times|V_1|^2 = 3.3075$$

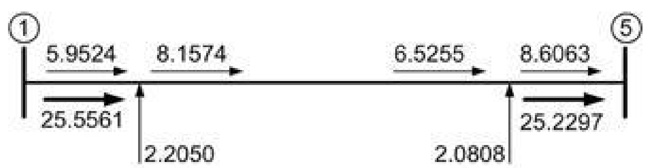
at bus-1. Similarly the reactive injected at bus-2 will be

$$0.03\times100\times|V_2|^2 = 2.8968$$

The power flow through the line segments 1-2 and 1-5 are shown in Fig..



(a)



(b)

Advantages of Newton Raphson Method

The various advantages of Newton Raphson Method are as follows:-

- It possesses quadratic convergence characteristics. Therefore, the convergence is very fast.
- The number of iterations is independent of the size of the system. Solutions to a high accuracy are obtained nearly always in two to three iterations for both small and large systems.
- The Newton Raphson Method convergence is not sensitive to the choice of slack bus.
- Overall, there is a saving in computation time since less number of iterations are required.

Limitations of Newton Raphson Method

The various limitations are given below.

- This solution technique is difficult.
- It takes longer time as the elements of the Jacobian are to be computed for each iteration.
- The computer memory requirement is large.

POWER FLOW SOLUTION USING GAUSS SEIDEL METHOD

Load Flow by Gauss-Seidel Method

The basic power flow equations (4.6) and (4.7) are nonlinear. In an n-bus power system, let the number of P-Q buses be np and the number of P-V (generator) buses be ng such that n = np + ng + 1. Both voltage magnitudes and angles of the P-Q buses and voltage angles of the P-V buses are unknown making a total number of 2np + ng quantities to be determined. Amongst the known quantities are 2np numbers of real and reactive powers of the P-Q buses, 2ng numbers of real powers and voltage magnitudes of the P-V buses and voltage magnitude and angle of the slack bus. Therefore there are sufficient numbers of known quantities to obtain a solution of the load flow problem. However, it is rather difficult to obtain a set of closed form equations from power flow equations. We therefore have to resort to obtain iterative solutions of the load flow problem.

At the beginning of an iterative method, a set of values for the unknown quantities are chosen. These are then updated at each iteration. The process continues till errors between all the known and actual quantities reduce below a pre-specified value. In the Gauss-Seidel load flow we denote the initial voltage of the i th bus by Vi(0) , i = 2, ... , n . This should read as the voltage of the i th bus at the 0th iteration, or initial guess. Similarly this voltage after the first iteration will be denoted by Vi(1) . In this Gauss-Seidel load flow the load buses and voltage controlled buses are treated differently. However in both these type of buses we use the complex power equation ifor updating the voltages. Knowing the real and reactive power injected at any bus we can expand.

$$P_{i,inj} - jQ_{i,inj} = V_i^* \sum_{k=1}^{n} Y_{ik} V_k = V_i^* \left[Y_{i1} V_1 + Y_{i2} V_2 + \cdots + Y_{ii} V_i + \cdots + Y_{in} V_n \right]$$

We can rewrite as

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_{i,inj} - jQ_{i,inj}}{V_i^*} - Y_{i1} V_1 - Y_{i2} V_2 - \cdots - Y_{in} V_n \right]$$

ALGORITHM OF GAUSS SEIDAL METHOD

Step1:

Assume all bus voltage be 1+ j0 except slack bus. The voltage of the slack bus is a constant voltage and it is not modified at any iteration

Step 2:

Assume a suitable value for specified change in bus voltage which is used to compare the actual change in bus voltage between K th and (K+1) th iteration

Step 3:

Set iteration count K = 0 and the corresponding voltages are V10, V20, V30, …… Vno except slack bus

Step 4:

Set bus count P = 1

Step 5:

Check for slack bus. It is a slack bus then goes to step 12 otherwise go to next step

Step 6:

Check for generator bus. If it is a generator bus go to next step. Otherwise go to step 9 `

Step 7:

Set |VPK|= |VP| specified and phase of |VPK| as the K th iteration value if the bu generator bus where │VP│ specified is the specified magnitude of voltage for bus P. Calculate reactive power rating P-1 n QP K+1 Cal = (-1) Imag [(VPK)A (Σ Y pqVq k+1 +Σ Y pqVq K q=1 q =P

Step 8:

If calculated reactive power is within the specified limits then consider the bus as generator bus and then set QP = QPK+1 Cal for this iteration go to step 10

Step 9:

If the calculated reactive power violates the specified limit for reactive power then treat this bus as load bus.

If QPK+1 Cal < QPmin then QP = QPmin

QPK+1 Cal > QPmax then QP = QPmax

Step10:

For generator bus the magnitude of voltagedoes not change and so for all

Iterations the magnitude of bus voltage is the specified value. The phase of the

bus voltage can be calculated using

$V_{PK+}1$ temp = 1 / Y_{PP} [(P_P –jQ_P / VPK *) - ΣYpqVqK+1 - ΣYpqVqK]

Step 11:

For load bus the (k+1)th iteration value of load bus P voltage VPK+1 can be calculated using VPK+1 temp = 1 / YPP [(PP –jQP / VPK*) - ΣYpqVqK+1 - ΣYpqVqK]

Step 12:

An acceleration factor α can be used for faster convergence. If acceleration factor is specified then modify the (K+1)th iteration value of bus P using

VPaccK+1 = VPK + α (VPK+1 – VPK) then Set VPK+1 = VPaccK+1

Step 13:

Calculate the change in bus-P voltage using the relation

Δ VPK+1 = VPK+1 -VPK

Step 14:

Repeat step 5 to 12 until all the bus voltages have been calculated. For this increment the bus count by 1 go to step 5 until the bus count is n

Step 15:

Find the largest of the absolute value of the change in voltage

│ΔV1K+1│, │ΔV2K+1│, │ΔV3K+1│,.............. │ΔVnK+1│

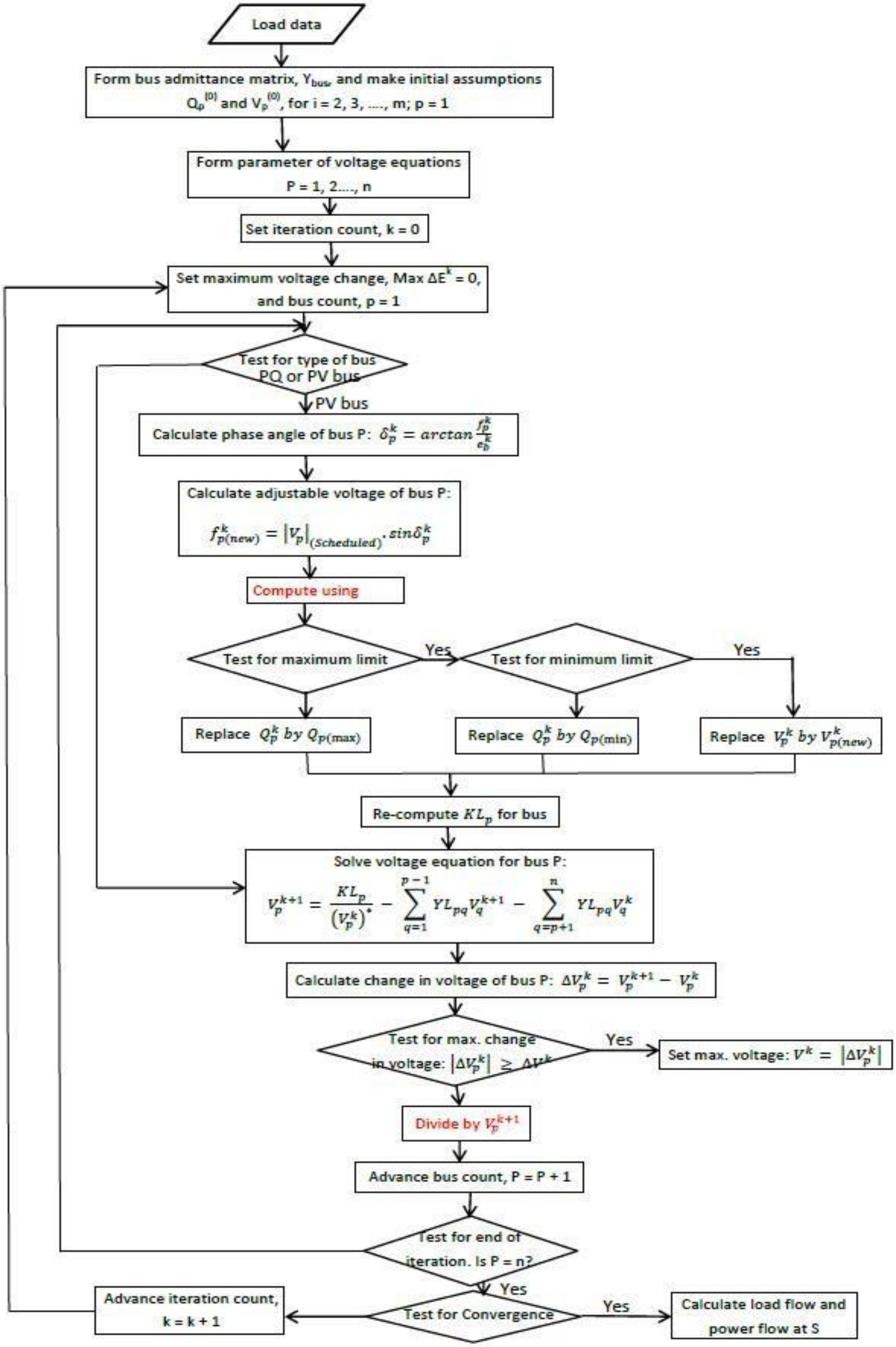
Let this largest value be the │ΔVmax│. Check this largest change

│ΔVmax│ is less than pre specified tolerance. If │ΔVmax│ is less go to next step. Otherwise increment the iteration count and go to step 4

Step 16:

Calculate the line flows and slack bus power by using the bus voltages

GAUSS - SEIDAL METHOD FLOW CHART


Load data
Form bus admittance matrix, Ybus, and make initial assumptions Qp(0) and Vp(0), for i = 2, 3, ..., m; p = 1
Form parameter of voltage equations P = 1, 2...., n
Set iteration count, k = 0
Set maximum voltage change, Max ΔEk = 0, and bus count, p = 1
Test for type of bus PQ or PV bus
PV bus
Calculate phase angle of bus P: $\delta_p^k = arctan \frac{f_p^k}{e_p^k}$
Calculate adjustable voltage of bus P:
$f_{p(new)}^k = |V_p|_{(Scheduled)} . sin\delta_p^k$
Compute using
Test for maximum limit
Yes
Test for minimum limit
Yes
Replace Q_p^k by $Q_{p(max)}$
Replace Q_p^k by $Q_{p(min)}$
Replace V_p^k by $V_{p(new)}^k$
Re-compute KL_p for bus
Solve voltage equation for bus P:
$V_p^{k+1} = \frac{KL_p}{(V_p^k)^*} - \sum_{q=1}^{p-1} YL_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} YL_{pq} V_q^k$
Calculate change in voltage of bus P: $\Delta V_p^k = V_p^{k+1} - V_p^k$
Test for max. change in voltage: $|\Delta V_p^k| \geq \Delta V^k$
Yes
Set max. voltage: $V^k = |\Delta V_p^k|$
Divide by V_p^{k+1}
Advance bus count, P = P + 1
Test for end of iteration. Is P = n?
Yes
Test for Convergence
Yes
Calculate load flow and power flow at S
Advance iteration count, k = k + 1

Advantages and disadvantages of Gauss-Seidel method

Advantages:

- Calculations are simple and so the programming task is lessees.
- The memory requirement is less.
- Useful for small systems

Disadvantages:

- Requires large no. of iterations to reach converge .
- Not suitable for large systems.
- Convergence time increases with size of the system

Problems:1

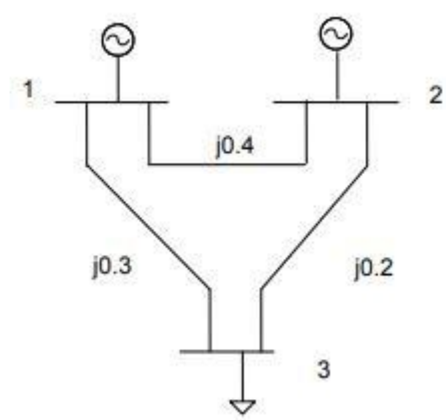
1) Fig. shows a three bus power system.

Bus 1 : Slack bus, V= 1.05/00 p.u.

Bus 2 : PV bus, V = 1.0 p.u. Pg = 3 p.u.

Bus 3 : PQ bus, PL = 4 p.u., QL = 2 p.u.

Carry out one iteration of load flow solution by Gauss Seidel method.



Neglect limits on reactive power generation.

Solution:

Admittance of each line

$$y_{12}=\frac{1}{Z_{12}}=\frac{1}{j0.4}=-j2.5\ p.u$$

$$y_{13}=\frac{1}{Z_{13}}=\frac{1}{j0.3}=-j3.333\ p.u$$

$$y_{23}=\frac{1}{Z_{23}}=\frac{1}{j0.2}=-j5\ p.u$$

$Y_{11}=y_{12}+y_{13}=-j2.5-j3.333=-j5.833\ p.u$

$Y_{22}=y_{12}+y_{23}=-j2.5-j5=-j7.5\ p.u$

$Y_{33}=y_{13}+y_{23}=-j3.333-j5=-j8.333\ p.u$

$Y_{12}=Y_{21}=-y_{12}=--j2.5=j2.5\ p.u$

$Y_{13}=Y_{31}=-y_{13}=--j3.33=j3.33\ p.u$

$Y_{23}=Y_{32}=-y_{23}=--j5=j5\ p.u$

The admittance matrix is given as

$$Y_{bus}=\begin{vmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21}+y_{23} & -y_{23} \\ -y_{31} & -y_{32} & y_{32}+y_{31} \end{vmatrix}$$

$$=\begin{vmatrix} -j5.833 & j2.5 & j3.33 \\ j2.5 & -j7.5 & j5 \\ j3.33 & j5 & -j8.333 \end{vmatrix}$$

Assume initial voltages to all buses

$V_1\ (0)= 1.05\angle 0^0 =1.05+j0$ p.u

$V_2\ (0)=1.0+j0$ p.u

$V_3\ (0)=1.0+j0$ p.u

Bus 1 is a slack bus

$V_1\ (1) = 1.05\angle 0^0 =1.05+j0$ p.u

Bus 2 is a generator bus

To calculate reactive power

$$Q_{p,cal}^{k+1}=(-1)\times Im\left\{\left(V_p^k\right)^*\left[\sum_{q=1}^{p-1}Y_{pq}V_q^{k+1}+\sum_{q=p}^{n}Y_{pq}V_q^k\right]\right\}$$

$$Q_{2cal}^{1}=(-1)\times Im\{(V_2^0)^*[Y_{21}V_1^1+Y_{22}V_2^0+Y_{23}V_3^0]\}$$

$$=(-1) \times Im(1 - j0)[(j2.5)(1.05 + j0) + (-j7.5)(1 + j0) + (j5)(1 + j0)]\}$$

$Q2cal1$ = − 0.125 p.u

The phase of bus-2 voltage in first iteration is given by phase of Vp,temp K+1

When p=3 $Q21$ = − 0.125 p.u and k=0