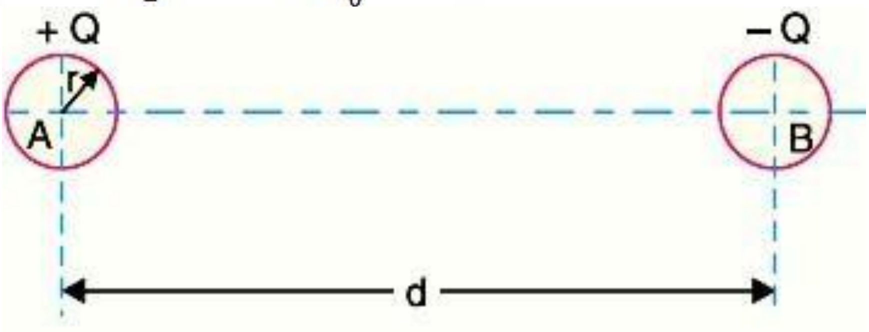
1.3 CAPACITANCE OF A SINGLE PHASE TWO-WIRE LINE

Consider a single phase overhead transmission line consisting of two parallel conductors A and B spaced d metres apart in air. Suppose that radius of each conductor is r metres. Let their respective charge be + Q and − Q coulombs per metre length. The total p.d. between conductor A and neutral "infinite" plane is

$$V_A = \int_r^{\infty} \frac{Q}{2\pi x \varepsilon_0} dx + \int_d^{\infty} \frac{-Q}{2\pi x \varepsilon_0} dx$$

$$= \frac{Q}{2\pi \varepsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right] \text{volts} = \frac{Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$



Similarly, p.d. between conductor B and neutral "infinite" plane is

$$V_B = \int_r^{\infty} \frac{-Q}{2\pi x \varepsilon_0} dx + \int_d^{\infty} \frac{Q}{2\pi x \varepsilon_0} dx$$

$$= \frac{-Q}{2\pi \varepsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right] = \frac{-Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Both these potentials are w.r.t. the same neutral plane. Since the unlike charges attract each other, the potential difference between the conductors is

$$V_{AB} = 2V_A = \frac{2Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

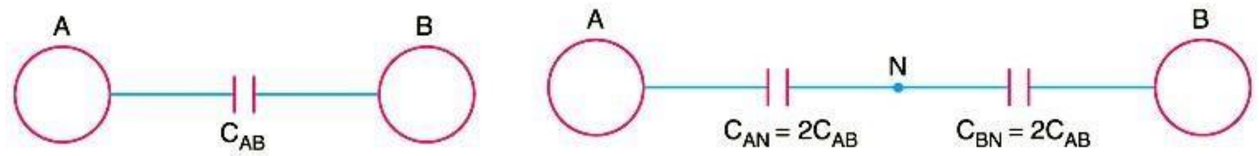
Capacitance,

$$C_{AB} = Q/V_{AB} = \frac{Q}{\frac{2Q}{2\pi \varepsilon_0} \log_e \frac{d}{r}} \text{ F/m}$$

$$C_{AB} = \frac{\pi \varepsilon_0}{\log_e \frac{d}{r}} \text{ F / m}$$

Capacitance to neutral

Equation (i) gives the capacitance between the conductors of a two-wire line Often it is desired to know the capacitance between one of the conductors and a neutral point between them. Since potential of the mid-point between the conductors is zero, the potential difference between each conductor and the ground or neutral is half the potential difference between the conductors. Thus the capacitance to ground or capacitance to neutral for the two-wire line is twice the line-to-line capacitance



Capacitance to neutral, $C_N = C_{AN} = C_{BN} = 2C_{AB}$

$$C_N = \frac{2\pi\varepsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

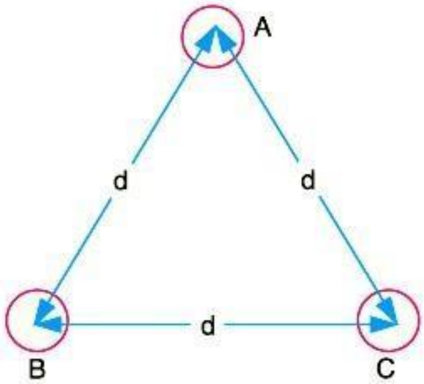
The reader may compare eq. (ii) to the one for inductance. One difference between the equations for capacitance and inductance should be noted carefully. The radius in the equation for capacitance is the actual outside radius of the conductor and not the GMR of the conductor as in the inductance formula. Note that eq. (ii) applies only to a solid round conductor.

1.3.1 CAPACITANCE OF A 3-PHASE OVERHEAD LINE

In a 3-phase transmission line, the capacitance of each conductor is considered instead of capacitance from conductor to conductor. Here, again two cases arise viz., symmetrical spacing and unsymmetrical spacing.

(i) Symmetrical Spacing

Fig shows the three conductors A, B and C of the 3-phase overhead transmission line having charges QA , QB and QC per meter length respectively. Let the conductors be equidistant (d meters) from each other. We shall find the capacitance from line conductor to neutral in this symmetrically spaced line. Referring to Fig,



Overall potential difference between conductor A and infinite neutral plane is given by

$$V_A = \int_r^\infty \frac{Q_A}{2\pi x \varepsilon_0}dx + \int_d^\infty \frac{Q_B}{2\pi x \varepsilon_0}dx + \int_d^\infty \frac{Q_C}{2\pi x \varepsilon_0}dx$$

$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\frac{1}{r} + Q_B \log_e\frac{1}{d} + Q_C \log_e\frac{1}{d}\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\frac{1}{r} + (Q_B + Q_C) \log_e\frac{1}{d}\right]$$

Assuming balanced supply, we have, $Q_A + Q_B + Q_C = 0$

$\therefore \quad Q_B + Q_C = -Q_A$

$$\therefore \quad V_A = \frac{1}{2\pi\varepsilon_0}\left[Q_A \log_e\frac{1}{r} - Q_A \log_e\frac{1}{d}\right] = \frac{Q_A}{2\pi\varepsilon_0}\log_e\frac{d}{r} \text{ volts}$$

∴ Capacitance of conductor A w.r.t neutral,

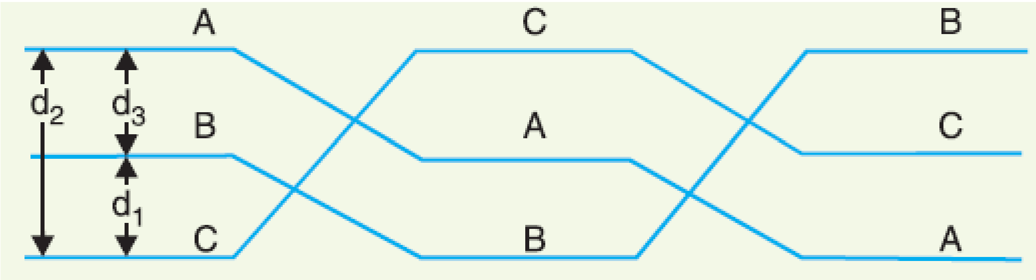
$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi\varepsilon_0}\log_e\frac{d}{r}} \text{ F/m} = \frac{2\pi\varepsilon_0}{\log_e\frac{d}{r}} \text{ F/m}$$

$$C_A = \frac{2\pi\varepsilon_0}{\log_e\frac{d}{r}} \text{ F/m}$$

Note that this equation is identical to capacitance to neutral for two-wire line. Derived in a similar manner, the expressions for capacitance are the same for conductors B and C.

(ii) Unsymmetrical spacing.

Fig. shows a 3-phase transposed line having unsymmetrical spacing. Let us assume balanced conditions i.e. $Q_A + Q_B + Q_C = 0$.



Considering all the three sections of the transposed line for phase A**,**

Potential of 1st position, $V_1 = \frac{1}{2\pi\varepsilon_0}\left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_3} + Q_C \log_e \frac{1}{d_2}\right)$

Potential of 2nd position, $V_2 = \frac{1}{2\pi\varepsilon_0}\left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_3}\right)$

Potential of 3rd position, $V_3 = \frac{1}{2\pi\varepsilon_0}\left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_2} + Q_C \log_e \frac{1}{d_1}\right)$

Average voltage on condutor A is

$$V_A = \frac{1}{3}(V_1 + V_2 + V_3)$$

$$= \frac{1}{3 \times 2\pi\varepsilon_0} * \left[Q_A \log_e \frac{1}{r^3} + (Q_B + Q_C) \log_e \frac{1}{d_1 d_2 d_3}\right]$$

As $Q_A + Q_B + Q_C = 0$, therefore, $Q_B + Q_C = -Q_A$

$$\therefore \quad V_A = \frac{1}{6\pi\varepsilon_0}\left[Q_A \log_e \frac{1}{r^3} - Q_A \log_e \frac{1}{d_1 d_2 d_3}\right]$$

$$= \frac{Q_A}{6\pi\varepsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}$$

$$= \frac{1}{3} \times \frac{Q_A}{2\pi\varepsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}$$

$$= \frac{Q_A}{2\pi\varepsilon_0} \log_e\left(\frac{d_1 d_2 d_3}{r^3}\right)^{1/3}$$

$$= \frac{Q_A}{2\pi\varepsilon_0} \log_e \frac{(d_1 d_2 d_3)^{1/3}}{r}$$

$$C_A = \frac{Q_A}{V_A} = \frac{2\pi\varepsilon_0}{\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}} \ F/m$$

Problems :

1. Determine the capacitance and the charging current per km when the transmission line of example 2.2 is operating at 132 kV.

Solution: The radius of conductor = 0.4 cm.

The mutual GMD of conductors, D_m = 2.015 metres.

$$\therefore \quad \text{Capacitance per phase per metre} = \frac{2\pi\varepsilon_0}{\ln \frac{2.015}{0.4} \times 10^2} \text{ F/metre}$$

$$= \frac{10^{-9}}{18 \times \ln \frac{2015}{0.4}} = 8.928 \text{ pF/metre}$$

$$= 8.928 \times 10^{-12} \times 10^3 \text{ F/km}$$

$$= 8.928 \times 10^{-9} \text{ F/km}$$

$$\text{The charging current} = \frac{132 \times 1000}{\sqrt{3}} \times 8.928 \times 10^{-9} \times 314$$

$$= 0.2136 \text{ amp/km.} \quad \textbf{Ans.}$$

2. Determine the capacitance and the charging current per km when the transmission line of example 2.5 operates at 220 kV, dia of conductor = 2.5 cm.

Solution:

Mutual GMD = 6.61 metres

$$D_{s_1} = \sqrt{1.25 \times 10^{-2} \times 10.965} = 0.3702 \text{ metre} = D_{s_3}$$

$$D_{s_2} = \sqrt{1.25 \times 10^{-2} \times 9} = 0.3354 \text{ metre}$$

$$\therefore \quad D_s = \sqrt[3]{D_{s_1} D_{s_2} D_{s_3}} = \sqrt[3]{0.045965899} = 0.3582 \text{ metre}$$

$$\therefore \quad \text{Capacitance per km} = \frac{10^{-6}}{18 \ln \frac{6.61}{0.3582}} = 0.019056 \ \mu\text{F/km}$$

$$\therefore \quad \text{Charging current per km} = \frac{220 \times 1000}{\sqrt{3}} \times 314 \times 0.01905 \times 10^{-6}$$

$$= 0.76 \text{ amp/km.} \quad \textbf{Ans.}$$

3. Determine the capacitance and charging current per km of the line of example 2.7 if the line operates at 220 kV, dia = 4.5 cms.

Solution:

$$D_s = \sqrt{2.25 \times 10^{-2} \times 0.4} = 0.094868 \text{ metre}$$

$$\therefore \quad \text{Capacitance per km} = \frac{10^{-6}}{18 \ln \frac{8.19}{0.094868}} = 0.01246 \ \mu\text{F}$$

$$\text{The charging current per km} = \frac{220 \times 1000}{\sqrt{3}} \times 314 \times 0.01246 \times 10^{-6}$$

$$= 0.497 \text{ amp.} \quad \textbf{Ans.}$$

1.4 CONCEPT OF SELF-GMD AND MUTUAL-GMD

The use of self geometrical mean distance (abbreviated as self-GMD) and mutual geometrical mean distance (mutual-GMD) simplifies the inductance calculations, particularly relating to multi conductor arrangements. The symbols used for these are respectively Ds and Dm. We shall briefly discuss these terms.

(i) Self-GMD (D_s)

In order to have concept of self-GMD (also sometimes called Geometrical mean radius;

GMR), consider the expression for inductance per conductor per metre already derived in Art. Inductance/conductor/m

$$= 2 \times 10^{-7}\left(\frac{1}{4} + \log_e \frac{d}{r}\right)$$
$$= 2 \times 10^{-7} \times \frac{1}{4} + 2 \times 10^{-7} \log_e \frac{d}{r}$$

In this expression, the term $2 \times 10^{-7} \times (1/4)$ is the inductance due to flux within the solid conductor. For many purposes, it is desirable to eliminate this term by the introduction of a concept called self-GMD or GMR. If we replace the original solid conductor by an equivalent hollow cylinder with extremely thin walls, the current is confined to the conductor surface and internal conductor flux linkage would be almost zero. Consequently, inductance due to internal

flux would be zero and the term $2 \times 10^{-7} \times (1/4)$ shall be eliminated. The radius of this equivalent hollow cylinder must be sufficiently smaller than the physical radius of the conductor to allow room for enough additional flux to compensate for the absence of internal flux linkage. It can be proved mathematically that for a solid round conductor of radius r, the self-GMD or GMR = 0·7788 r. Using self-GMD, the eqn. becomes

$$\text{Inductance/conductor/m} = 2 \times 10^{-7} \log_e d/D_s$$

$$D_s = \text{GMR or self-GMD} = 0{\cdot}7788\ r$$

It may be noted that self-GMD of a conductor depends upon the size and shape of the conductor and is independent of the spacing between the conductors.

(ii) Mutual-GMD

The mutual-GMD is the geometrical mean of the distances form one conductor to the other and, therefore, must be between the largest and smallest such distance. In fact, mutual- GMD simply represents the equivalent geometrical spacing.

(a) The mutual-GMD between two conductors (assuming that spacing between conductors is large compared to the diameter of each conductor) is equal to the distance between their centres i.e. D_m = spacing between conductors = d

(b) For a single circuit 3-φ line, the mutual-GMD is equal to the equivalent equilateral spacing i.e.,

$$(d_1 d_2 d_3)^{1/3}$$

1.2 PARAMETERS OF SINGLE AND THREE PHASE TRANSMISSION LINES WITH SINGLE AND DOUBLE CIRCUITS

1.2.1 CONSTANTS OF A TRANSMISSION LINE

A transmission line has resistance, inductance and capacitance uniformly distributed along the whole length of the line. Before we pass on to the methods of finding these constants for a transmission line, it is profitable to understand them thoroughly.

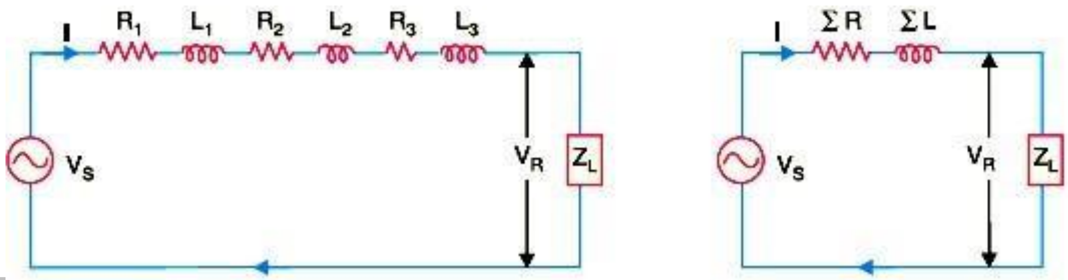


Figure 1.2.1 Resistance and Inductance

[Source: "Principles of Power System" by V.K.Mehta Page: 203]

(*i*) Resistance. It is the opposition of line conductors to current flow. The resistance is distributed uniformly along the whole length of the line as shown in Fig. However, the performance of a transmission line can be analysed conveniently if distributed resistance is considered as lumped as shown in Fig.1.2.1

(*ii*) Inductance. When an alternating current flows through a conductor, a changing flux is set up which links the conductor. Due to these flux linkages, the conductor possesses inductance. Mathematically, inductance is defined as the flux linkages per ampere *i.e.*,

$$L = \frac{\psi}{I} \text{ henry}$$

where ψ = flux linkages in weber-turns

I = current in amperes

The inductance is also uniformly distributed along the length of the line as show in Fig. Again for the convenience of analysis, it can be taken to be lumped as shown in Fig1.2.1

(iii) **Capacitance.** We know that any two conductors separated by an insulating material consti-tute a capacitor. As any two conductors of an overhead transmission line are separated by air which acts as an insulation, therefore, capacitance exists between any two overhead line conductors. The capacitance between the conductors is the charge per unit potential difference *i.e.*,

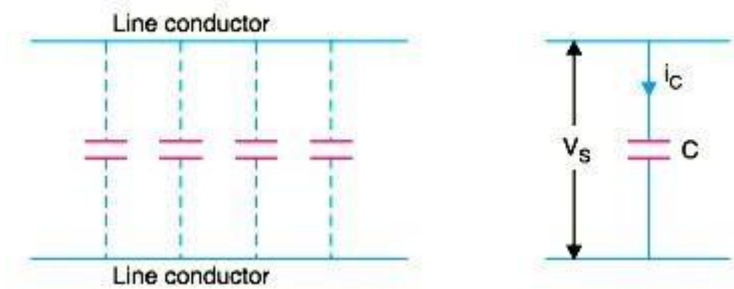


Figure 1.2.2 Capacitance

[Source: "Principles of Power System" by V.K.Mehta Page: 203]

C = q/v

q = charge on the line in coulomb

v = p.d. between the conductors in volts

The capacitance is uniformly distributed along the whole length of the line and may regarded as a uniform series of capacitors connected between the conductors as shown in Fig. 9.2(*i*). When an alternating voltage is impressed on a transmission line, the charge on the conductors at any point increases and decreases with the increase and decrease of the instantaneous value of the voltage between conductors at that point. The result is that a current (known as *charging current*) flows between the conductors [See Fig. 1 . 2 . 2]. This charging current flows in the line even when it is open-circuited *i.e.*, supplying no load. It affects the voltage drop along the line as well as the efficiency and power factor of the line.

1.2.2 RESISTANCE OF A TRANSMISSION LINE

The resistance of transmission line conductors is the most important cause of power loss in a transmission line. The resistance *R* of a line conductor having resistivity ρ, length *l* and area of cross-section *a* is given by ;

$R = \rho\, l/a$

The variation of resistance of metallic conductors with temperature is practically linear over the normal range of operation. Suppose R_1 and R_2 are the resistances of a conductor at t_1 ºC and t_2 ºC

($t_2 > t_1$) respectively. If α_1 is the temperature coefficient at t_1 °C, then,

$$□_2 = □_1[1 + □_1(□_2 - □_2)]$$

$$□_1 = \frac{□_0}{1 + □_0□_1}$$

$$□_0 = □□□□□□□□□□□□ □□□□□□□□□□□ □□ 0^{□}□$$

1.2.3 INDUCTANCE OF A SINGLE PHASE TWO-WIRE LINE

A single phase line consists of two parallel conductors which form a rectangular loop of one turn.

When an alternating current flows through such a loop, a changing magnetic flux is set up. The changing flux links the loop and hence the loop (or single phase line) possesses inductance. It may appear that inductance of a single phase line is negligible because it consists of a loop of one turn and the flux path is through air of high reluctance. But as the X -sectional area of the loop is very large, even for a small flux density, the total flux linking the loop is quite large and hence the line has appreciable inductance.

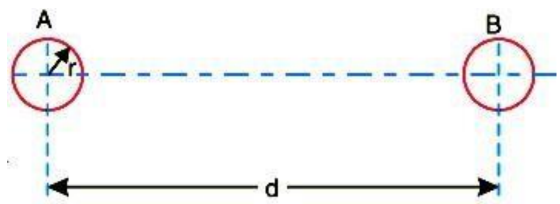


Figure 1.2.3 Single phase Line

[Source: "Principles of Power System" by V.K.Mehta Page: 208]

Consider a single phase overhead line consisting of two parallel conductors A and B spaced d metres apart as shown in Fig. 9.7. Conductors A and B carry the same amount of current (i.e. $I_A = I_B$), but in the opposite direction because one forms the return circuit of the other.

$$I_A + I_B = 0$$

In order to find the inductance of conductor A (or conductor B), we shall have to consider the flux linkages with it. There will be flux linkages with conductor A due to its own current I_A and also A due to the mutual inductance effect of current I_B in the conductor B Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right)$$

Flux linkages with conductor A due to current I

$$= \frac{\mu_0 I_B}{2\pi}\int_d^{\infty} \frac{dx}{x}$$

Total flux linkages with conductor A is

$$\psi_A = \text{exp. }(i) + \text{exp }(ii)$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_d^\infty \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) I_A + I_B \int_d^\infty \frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \log_e \infty - \log_e r\right) I_A + \left(\log_e \infty - \log_e d\right) I_B\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{I_A}{4} + \log_e \infty \left(I_A + I_B\right) - I_A \log_e r - I_B \log_e d\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} - I_A \log_e r - I_B \log_e d\right] \quad (\because I_A + I_B = 0)$$

Now. $I_A + I_B = 0 \quad \text{or} \quad -I_B = I_A$

$\therefore \quad -I_B \log_e d = I_A \log_e d$

$$\therefore \quad \psi_A = \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \log_e d - I_A \log_e r\right] \text{ wb-turns/m}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \log_e \frac{d}{r}\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ wb-turns/m}$$

$$\text{Inductance of conductor } A, L_A = \frac{\Psi_A}{I_A}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H/m} = \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H/m}$$

$$L_A = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{d}{r}\right] \text{H/m}$$

$$\text{Loop inductance} = 2L_A \text{ H/m} = 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right] \text{H/m}$$

$$\text{Loop inductance} = 10^{-7}\left[1 + 4\log_e \frac{d}{r}\right] \text{H/m}$$

Note that eqn. is the inductance of the two-wire line and is sometimes called loop inductance. However, inductance given by eqn. is the inductance per conductor and is equal to half the loop inductance.

1.2.4 INDUCTANCE OF A 3-PHASE OVERHEAD LINE

Fig. shows the three conductors A, B and C of a 3-phase line carrying currents I_A , I_B and I_C respectively. Let d_1 , d_2 and d_3 be the spacings between the conductors as shown. Let us further assume that the loads are balanced i.e. $I_A + I_B + I_C = 0$. Consider the flux linkages with conductor There will be flux linkages with conductor A due to its own current and also due to the mutual inductance effects of I_B and I_C

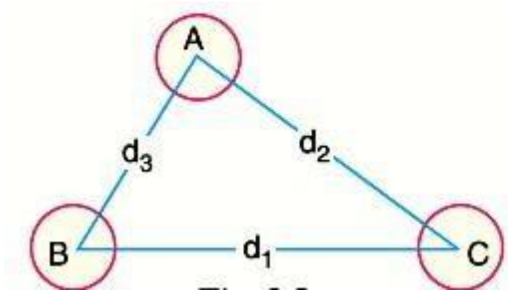


Figure 1.2.4 Three phase Line

[Source: "Principles of Power System" by V.K.Mehta Page: 208]

Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) \qquad \ldots(i)$$

Flux linkages with conductor A due to current I_B

$$= \frac{\mu_0 I_B}{2\pi}\int_{d_3}^\infty \frac{dx}{x}$$

Flux linkages with conductor A due to current I_C

$$= \frac{\mu_0 I_C}{2\pi}\int^\infty \frac{dx}{x}$$

Total flux linkages with conductor A is

$$\psi_A = (i) + (ii) + (iii)$$

$$= \frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) + \frac{\mu_0 I_B}{2\pi}\int_{d_3}^\infty \frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^\infty \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right) I_A + I_B \int_{d_3}^\infty \frac{dx}{x} + I_C \int_{d_2}^\infty \frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2 + \log_e \infty \left(I_A + I_B + I_C\right)\right]$$

As $\quad I_A + I_B + I_C = 0,$

$$\therefore \quad \psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

1.2.4.1 SYMMETRICAL SPACING

If the three conductors A, B and C are placed symmetrically at the corners of an equilateral triangle of side d, then, d1 = d2 = d3 = d. Under such conditions, the flux Derived in a similar way, the expressions for inductance are the same for conductors B and C.

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d - I_C \log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - (I_B + I_C)\log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A + I_A \log_e d\right] \qquad (\because I_B + I_C = -I_A)$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{ werber-turns/m}$$

$$L_A = \frac{\psi_A}{I_A}\,\text{H/m} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H/m}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right] \text{H/m}$$

$$L_A = 10^{-7}\left[0.5 + 2\log_e \frac{d}{r}\right] \text{H/m}$$

1.2.4.2 UNSYMMETRICAL SPACING

When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical. Under such conditions, the flux linkages and inductance of each phase are not the same. A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced. Therefore, the voltage at the receiving end will not be the same for all phases. In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as transposition. Fig. shows the transposed line. The phase conductors are designated as A, B and C and the positions occupied are numbered 1, 2 and 3. The effect of transposition is that each conductor has the same average inductance.

Fig. shows a 3-phase transposed line having unsymmetrical spacing. Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions i.e.,

$$I_A + I_B + I_C = 0$$

Let the line currents be :

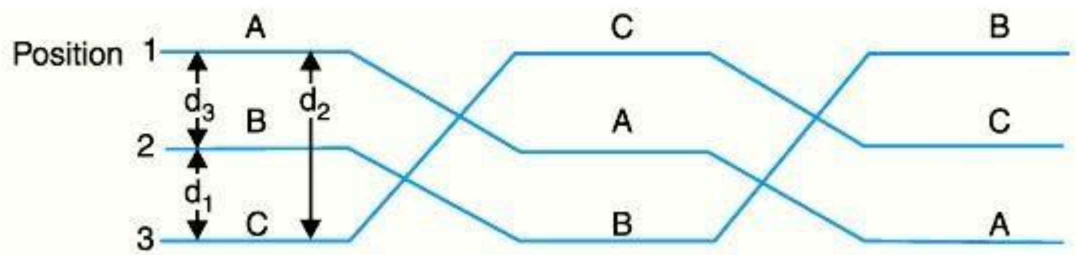


Figure 1.2.5 Unsymmetrical Spacing

[Source: "Principles of Power System" by V.K.Mehta Page: 211]

As proved above, the total flux linkages per metre length of conductor A is

$$I_A = I(1 + j\,0)$$
$$I_B = I(-0{\cdot}5 - j\,0{\cdot}866)$$
$$I_C = I(-0{\cdot}5 + j\,0{\cdot}866)$$

$$\psi_A = \frac{\mu_0}{2z}\left[\left(\frac{1}{4} - \log_e r\right)I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

Putting the values of/q, I_B and Ip , we get,

$$\psi_A \quad \frac{\mu_0}{2z}\left|\left(\frac{1}{4} \quad \log_e r\right)I — /(— 0\ 5 — y\ 0\text{-}866) 1ogp\ JJ — /(— 0\text{-}5 + J\ 0\ 866)\log_e d\right]$$

$$= \frac{\mu_0}{2\pi}\left[_4 \quad f — / 1Og_e r + 0\ 5\ I \log JJ + y\ 0\ 866\ lob_e If + 0\ 5\ Hogg\ J_2 — J\ 0\ 866\ I \log I,\right]$$

$$\frac{\mu_0}{2\pi}\left[\frac{—}{4} I — I \log_e r + 0\ 1 I (\log, J_3 + \log, I,) + io - as6\ clogs \quad — logo \quad \right)$$

$$\frac{\mu_0}{2}\left[\frac{—}{4} I — I \log, r + \quad "logo \sqrt{d_2 d_3} + y\ 0\ 866 \quad \log_e \frac{3}{d_2}\right]$$

Ac

Download Binils Android App in Playstore Download Photoplex App

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j\, 0 \cdot 866 \log_e \frac{d_3}{d_2}\right]$$

∴ Inductance of conductor A is

$$L_A = \frac{\psi_A}{I_A} = \frac{\psi_A}{I}$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2\, d_3}}{r} + j\, 0 \cdot 866 \log_e \frac{d_3}{d_2}\right]$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2\, d_3}}{r} + j\, 0 \cdot 866 \log_e \frac{d_3}{d_2}\right] \text{ H/m}$$

$$= 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_2\, d_3}}{r} + j\, 1 \cdot 732 \log_e \frac{d_3}{d_2}\right] \text{ H/m}$$

Similarly inductance of conductors B and C will be :

$$L_B = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_3\, d_1}}{r} + j\, 1 \cdot 732 \log_e \frac{d_1}{d_3}\right] \text{ H/m}$$

$$L_C = 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{\sqrt{d_1\, d_2}}{r} + j\, 1 \cdot 732 \log_e \frac{d_2}{d_1}\right] \text{ H/m}$$

Inducance of each line conductor

$$= \frac{1}{3}(L_A + L_B + L_C)$$

$$= {}^{*}\left[\frac{1}{2} + 2\log_e \frac{\sqrt[3]{d_1\, d_2\, d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

$$= \left[0 \cdot 5 + 2\log_e \frac{\sqrt[3]{d_1\, d_2\, d_3}}{r}\right] \times 10^{-7} \text{ H/m}$$

If we compare the formula of inductance of an un symmetrically spaced transposed line with that of symmetrically spaced line, we find that inductance of each line conductor in the two cases will be equal if $d = \sqrt[3]{d_1 d_2 d_3}$ The distance d is known as equivalent equilateral spacing for un symmetrically transposed line.

1. A single phase transmission line has two parallel conductors 3 m apart, the radius of each conductor being 1 cm. Calculate the loop inductance per km length of the line if the material of the conductor is (i) copper (ii) steel with relative permeability of 100.

   Spacing of conductors, d = 300 cm

   Radius of conductor, r = 1 cm

   Loop inductance = 10^{-7} (μ_r + 4 loge d/r) H/m

   (i) With copper conductors, $\mu_r = 1$

   ∴ Loop inductance/m = 10^{-7} (1 + 4 loge d/r) H = 10^{-7} (1 + 4 loge 300/1) H

   = $23{\cdot}8 \times 10^{-7}$ H

   Loop inductance/km = $23{\cdot}8 \times 10^{-7} \times 1000 = 2{\cdot}38 \times 10^{-3}$ H = 2·38 mH

   (ii) With steel conductors, μr = 100

   ∴ Loop inductance/m = 10^{-7} (100 + 4 loge 300/1) H = $122{\cdot}8 \times 10^{-7}$ H

   Loop inductance/km = $122{\cdot}8 \times 10^{-7} \times 1000 = 12{\cdot}28 \times 10^{-3}$ H = 12·28 mH

2. The three conductors of a 3-phase line are arranged at the corners of a triangle of sides 2 m, 2·5 m and 4·5 m. Calculate the inductance per km of the line when the conductors are regularly transposed. The diameter of each conductor is 1·24 cm.

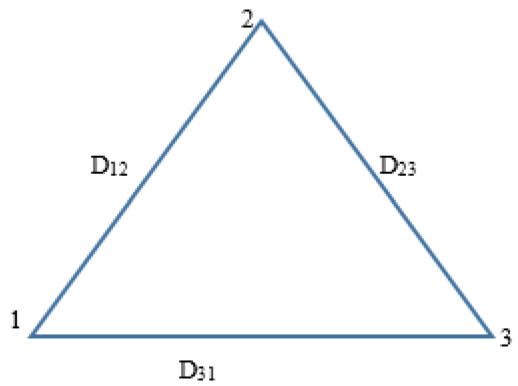


Figure 1.2.6

Fig.1.2.6 shows three conductors of a 3-phase line placed at the corners of a triangle of sides D_{12} = 2 m, D_{23} = 2·5 m and D_{31} = 4·5 m.

The conductor radius r = 1·24/2 = 0·62 cm.

Equivalent equilateral spacing, $D_{eq} = (D_{12}\, D_{23}\, D_{31})^{1/3}$

$= (2 \times 2·5 \times 4·5)^{1/3}$

= 2·82 m

= 282 cm

Inductance/phase/m $= 10^{-7}(0·5 + 2 \log_e D_{eq}/r)$ H

$= 10^{-7}(0·5 + 2 \log_e 282/0·62)$ H

$= 12·74 \times 10^{-7}$ H

Inductance/phase/km $= 12·74 \times 10^{-7} \times 1000$

$= 1·274 \times 10^{-3}$ H

= **1·274 mH**

1.5 SKIN EFFECT

The phenomena arising due to unequal distribution of electric current over the entire cross section of the conductor being used for long distance power transmission is referred as the skin effect in transmission lines. Such a phenomena does not have much role to play in case of a very short line, but with increase in the effective length of the conductors, skin effect increases considerably. So the modifications in line calculation needs to be done accordingly. The distribution of electric current over the entire cross section of the conductor is quite uniform in case of a DC system. But what we are using in the present era of power system engineering is predominantly an alternating electric current system, where the electric current tends to flow with higher density through the surface of the conductors (i.e skin of the conductor), leaving the core deprived of necessary number of electrons.

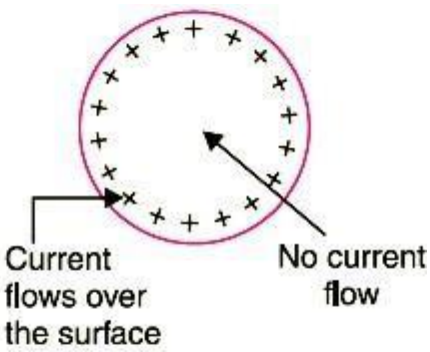


Figure 1.1 Structure of Power System

[Source: "Principles of Power System" by V.K.Mehta Page: 204]

In fact there even arises a condition when absolutely no electric current flows through the core, and concentrating the entire amount on the surface region, thus resulting in an increase in the effective electrical resistance of the conductor. This particular trend of an AC transmission system to take the surface path for the flow of electric current depriving the core is referred to as the skin effect in transmission lines.

PROXIMITY EFFECT

Proximity means nearness in space or time, so as the name suggests, proximity effect in transmission lines indicates the effect in one conductor for other neighboring conductors. When the alternating current is flowing through a conductor, alternating magnetic flux is generated surrounding the conductor. This magnetic flux associates with the neighboring wires and generates a circulating current (it can be termed as'eddy current' also). This circulating current increases the resistance of the conductor

and push away the flowing current through the conductor, which causes the crowding effect.

1.1 STRUCTURE OF POWER SYSTEM

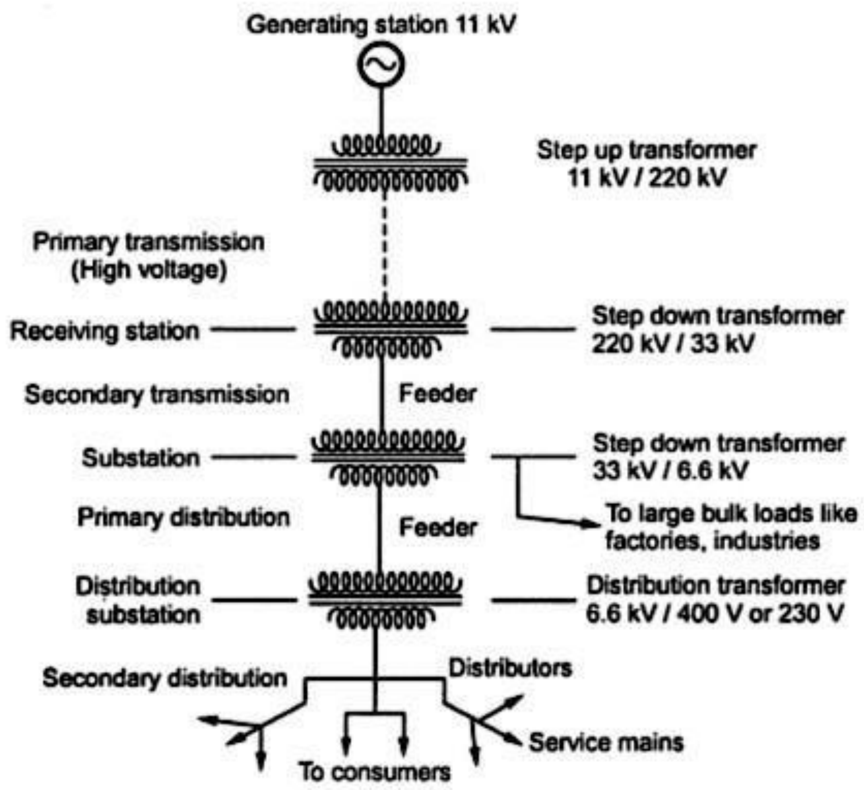


Figure 1.1 Structure of Power System

[Source: "Principles of Power System" by V.K.Mehta Page: 128]

COMPONENTS OF POWER SYSTEM

Power transformers:

Power transformers are used generation and transmission network for stepping-up the voltage at generating station and stepping-down the voltage for distribution. Auxiliary transformers supply power to auxiliary equipments at the substations.

Current transformers: (CT):

The lines in substations carry currents in the order of thousands of amperes. The measuring instruments are designed for low value of currents. Current transformers are connected in lines to supply measuring instruments and protective relays.

Potential transformers (PT):

The lines in substations operate at high voltages. The measuring instruments are designed for low value of voltages. Potential transformers are connected in lines to supply measuring instruments and protective relays. These transformers make the low voltage instruments suitable for measurement of high voltages. For example a 11kV/110V PT is connected to a power line and the line voltage is 11kV then the secondary voltage will be 110V.

Circuit breaker (CB):

Circuit breakers are used for opening or closing a circuit under normal as well as abnormal (faulty) conditions. Different types of CBs which are generally used are oil circuit breaker, air-blast circuit breaker, and vacuum circuit breaker and SF6 circuit breaker.

Isolators or Isolating switches:

Isolators are employed in substations to isolate a part of the system for general maintenance. Isolator switches are operated only under no load condition. They are provided on each side of every circuit breaker Bus-bar: When number of lines operating at the same voltage levels needs to be connected electrically, bus-bars are used. Bus-bars are conductors made of copper or aluminum, with very low impedance and high current carrying capacity. Different types of bus-bar arrangements are single bus bar arrangements, single bus-bar with double bus-bar arrangements, sectionalized double bus-bar arrangement, double main and auxiliary bus-bar arrangement, breaker and a half scheme/1.5 Breaker scheme, and ring bus-bar scheme

1.6 TYPES OF CONDUCTOR

1. Copper

Copper is an ideal material for overhead lines owing to its high electrical conductivity and greater tensile strength. It is always used in the hard drawn form as stranded conductor. Although hard drawing decreases the electrical conductivity slightly yet it increases the tensile strength considerably. Copper has high current density *i.e.*, the current carrying capacity of copper per unit of Xsectional area is quite large. This leads to two advantages. Firstly, smaller X- sectional area of conductor is required and secondly, the area offered by the conductor to wind loads is reduced. Moreover, this metal is quite homogeneous, durable and has high scrap value. There is hardly any doubt that copper is an ideal material for transmission and distribution of electric power. However, due to its higher cost and non-availability, it is rarely used for these purposes. Now a days the trend is to use aluminium in place of copper.

2. Aluminium

Aluminium is cheap and light as compared to copper but it has much smaller conductivity and tensile strength. The relative comparison of the two materials is briefed below:

(i) The conductivity of aluminium is 60% that of copper. The smaller conductivity of aluminium means that for any particular transmission efficiency, the X-sectional area of conductor must be

larger in aluminium than in copper. For the same resistance, the diameter of aluminium conductor is about 1·26 times the diameter of copper conductor. The increased X-section of aluminium exposes a greater surface to wind pressure and, therefore, supporting towers must be designed for greater transverse strength. This often requires the use of higher towers with consequence of greater sag.

(ii) The specific gravity of aluminium (2·71 gm/cc) is lower than that of copper (8·9 gm/cc).Therefore, an aluminium conductor has almost one-half the weight of equivalent copper conductor. For this reason, the supporting structures for aluminium need not be made so strong as that of copper conductor.

(*iii*) Aluminium conductor being light, is liable to greater swings and hence larger cross-arms are required.

(*iv*) Due to lower tensile strength and higher co-efficient of linear expansion of aluminium, the sag is greater in aluminium conductors. Considering the combined properties of cost, conductivity, tensile strength, weight etc., aluminium has an edge over copper. Therefore, it is being widely used as a conductor material. It is particularly profitable to use aluminium for heavy-current transmission where the conductor size is large and its cost forms a major proportion of the total cost of complete installation.

3. Steel cored aluminium

Due to low tensile strength, aluminium conductors produce greater sag. This prohibits their use for larger spans and makes them unsuitable for long distance transmission. In order to increase the tensile strength, the aluminium conductor is reinforced with a core of galvanised steel wires. The composite conductor thus obtained is known as *steel cored aluminium* and is abbreviated as A.C.S.R. (aluminium conductor steel reinforced).

Steel-cored aluminium conductor consists of central core of galvanized steel wires surrounded by a number of aluminium strands. Usually, diameter of both steel and aluminium wires is the same. The X-section of the two metals are generally in the ratio of 1 : 6 but can be modified to 1 : 4 in order to get more tensile strength for the conductor. Fig. shows steel cored aluminium conductor having one steel wire surrounded by six wires of aluminium. The result of this composite conductor is that steel core takes greater percentage of mechanical strength while aluminium strands carry the bulk of current. The steel cored aluminium conductors have the following

Advantages:

(*i*) The reinforcement with steel increases the tensile strength but at the same time keeps the

composite conductor light. Therefore, steel cored aluminium conductors will produce smaller sag and hence longer spans can be used.

(*ii*) Due to smaller sag with steel cored aluminium conductors, towers of smaller heights can be used.

4. Galvanised steel

Steel has very high tensile strength. Therefore, galvanised steel conductors can be used for extremely long spans or for short line sections exposed to abnormally high stresses due to climatic conditions. They have been found very suitable in rural areas where cheapness is the main consideration. Due to poor conductivity and high resistance of steel, such conductors are not suitable for transmitting large power over a long distance. However, they can be used to advantage for transmitting a small power over a small distance where the size of the copper conductor desirable from economic considerations would be too small and thus unsuitable for use because of poor mechanical strength.

5. Cadmium copper

The conductor material now being employed in certain cases is copper alloyed with cadmium. An addition of 1% or 2% cadmium to copper increases the tensile strength by about 50% and the conductivity is only reduced by 15% below that of pure copper. Therefore, cadmium copper conductor can be useful for exceptionally long spans. However, due to high cost of cadmium, such conductors will be economical only for lines of small X-section i.e., where the cost of conductor material is comparatively small compared with the cost of supports.