3.1 ENERGY IN MAGNETIC SYSTEMS

It is often necessary in today's computer controlled industrial setting to convert an electrical signal into a mechanical action. To accomplish this, the energy in the electrical signal must be converted to mechanical energy. A variety of devices exist that can convert electrical energy into mechanical energy using a magnetic field. One such device, often referred to as a reluctance machine, produces a translational force whenever the electrical signal is applied. There are several variations of the reluctance machine but all operate on the same basic electromechanical principles.

The principles of electromechanical energy conversion are investigated. The motivation for this investigation is to show how the governing equations of an electromechanical device can be derived from a magnetic circuit analysis. An expression for the mechanical force will be derived in terms of the magnetic system parameters.

Electromechanical Energy Conversion Principles

Electromechanical-energy-conversion process takes place through the medium of the electric or magnetic field of the conversion device of which the structures depend on their respective functions.

Transducers: microphone, pickup, sensor, loudspeaker Force producing devices: solenoid, relay, electromagnet Continuous energy conversion equipment: motor, generator This chapter is devoted to the principles of electromechanical energy conversion and the analysis of the devices accomplishing this function. Emphasis is placed on the analysis of systems that use magnetic fields as the conversion medium. The concepts and techniques can be applied to a wide range of engineering situations involving electromechanical energy conversion. Based on the energy method, we are to develop expressions for forces and torques in magnetic field based electromechanical systems.

Forces and Torques in Magnetic Field Systems

The Lorentz Force Law gives the force on a particle of charge in the presence of electric and magnetic fields. *F*: newtons, : coulombs, : volts/meter, *qEB*: telsa , :

meters/second In a pure electric-field system, F=qE

In pure magnetic-field systems, F=q*(v*B)

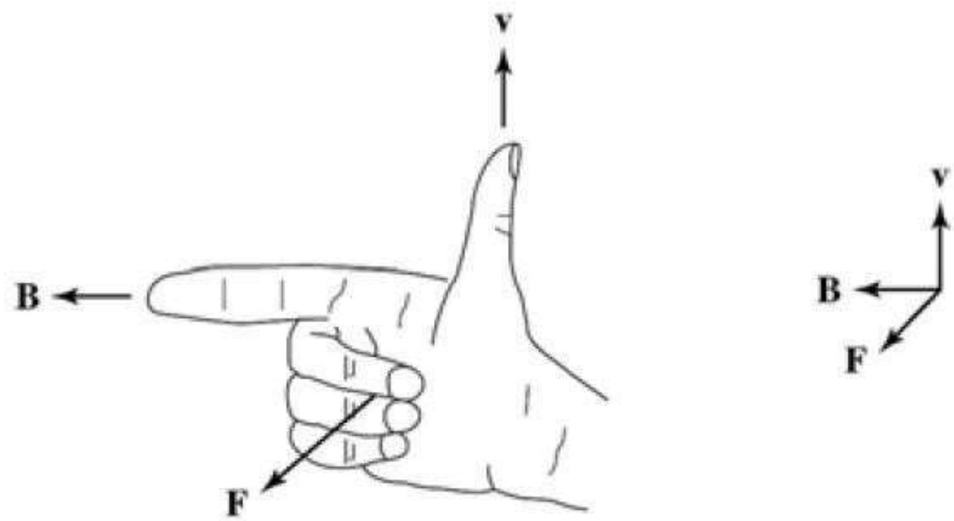


Figure 3.1.1 Right Hand rule

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 241]*

For situations where large numbers of charged particles are in motion F=J*V most electromechanical-energy-conversion devices contain magnetic material. Forces act directly on the magnetic material of these devices which are constructed of rigid, non deforming structures. The performance of these devices is typically determined by the net force, or torque, acting on the moving component. It is rarely necessary to calculate the details of the internal force distribution. Just as a compass needle tries to align with the earth's magnetic field, the two sets of fields associated with the rotor and the stator of rotating machinery attempt to align, and torque is associated with their displacement from alignment. In a motor, the stator magnetic field rotates ahead of that of the rotor, pulling on it and performing work. For a generator, the rotor does the work on the stator.

The Field Energy

Based on the principle of conservation of energy: energy is neither created nor destroyed; it is merely changed in form.

Energy Balance

A magnetic-field-based electromechanical-energy-conversion device. A lossless magnetic-energy-storage system with two terminals The electric terminal has two terminal variables: (voltage), (current). The mechanical terminal has two terminal variables: (force), (position) The loss mechanism is separated from the energy-storage mechanism.

- Electrical losses: ohmic losses.
- Mechanical losses: friction, windage.

A simple force-producing device with a single coil forming the electric terminal, and a movable plunger serving as the mechanical terminal.

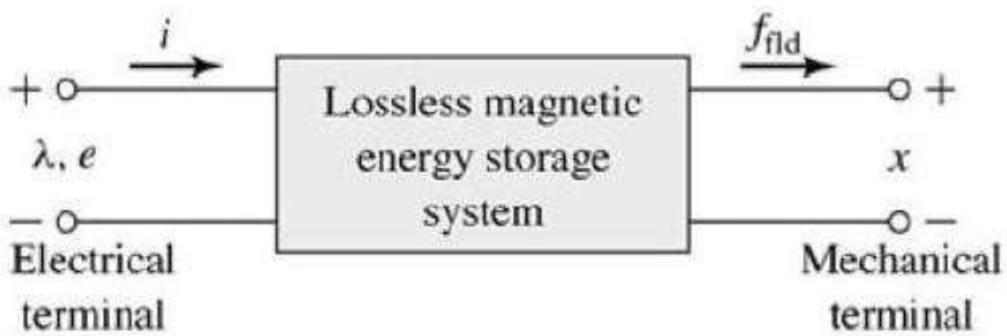


Figure 3.1.2 Schematic magnetic field

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 219]*

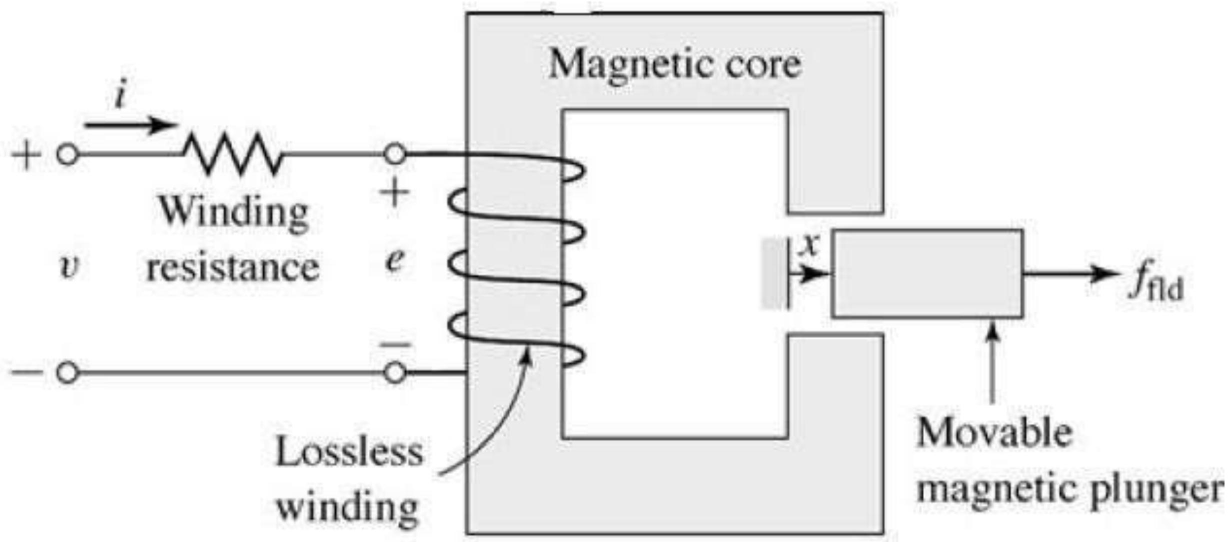


Figure 3.1.3 Simple force producing device

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 220]*

The interaction between the electric and mechanical terminals, i.e. the electromechanical energy conversion, occurs through the medium of the magnetic stored energy.

Consider the electromechanical systems whose predominant energy-storage mechanism is in magnetic fields. For motor action, we can account for the energy transfer. The ability to identify a lossless-energy-storage system is the essence of the energy method. This is done mathematically as part of the modeling process. For the lossless magnetic-energy-storage system gives the expression as

$$dWelec=dmech+dfld$$

Here E is the voltage induced in the electric terminals by the changing magnetic stored energy. It is through this reaction voltage that the external electric circuit supplies power to the coupling magnetic field and hence to the mechanical output terminals. The basic energy-conversion process is one involving the coupling field and its action and reaction on the electric and mechanical systems.

$$dWelec=Ei.dt=dmech+dfld$$

The Co Energy

The magnetic stored energy is a state function, determined uniquely by the values of the independent state variables λ and *x*

Coenergy:

Here the force can be obtained directly as a function of the current. The selection of energy or coenergy as the state function is purely a matter of convenience.

For a magnetically-linear system, the energy and coenergy (densities) are numerically equal:

Wfld+Wfld=λi

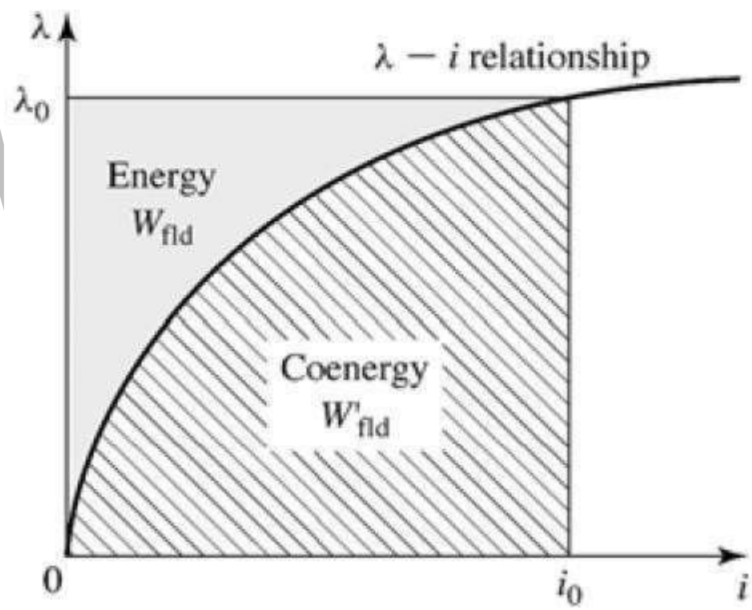


Figure 3.1.4 Energy and co energy in single excited system

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 221]*

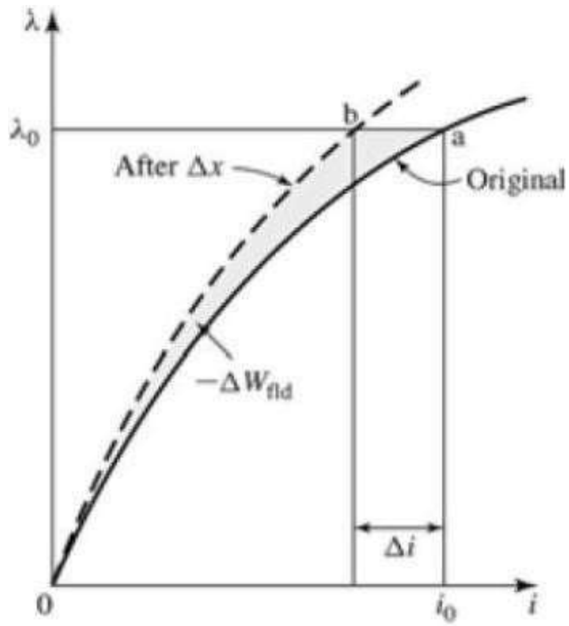


Figure 3.1.5 (a) Change of energy with λ

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 223]*

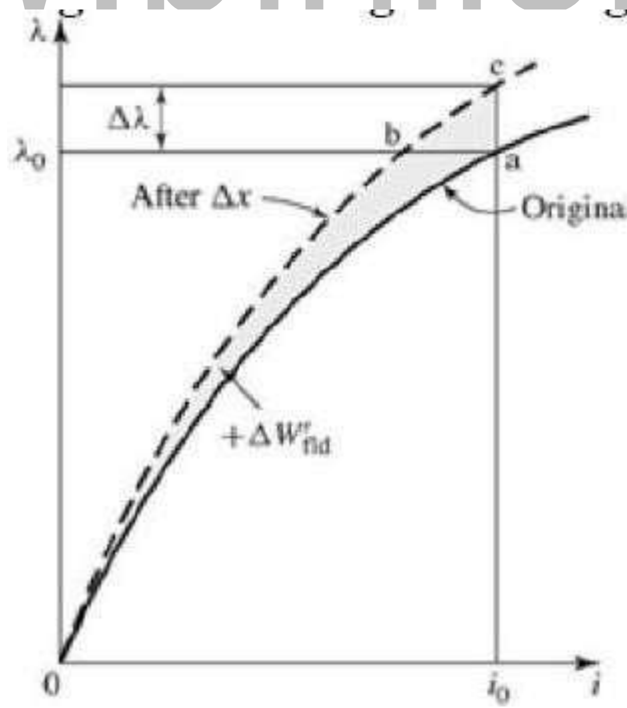


Figure 3.1.5 (b) Change of co energy with λ

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 223]*

The force acts in a direction to decrease the magnetic field stored energy at constant flux or to increase the coenergy at constant current. In a singly-excited device, the force acts to increase the inductance by pulling on members so as to reduce the reluctance of the magnetic path linking the winding.

3.3 MMF OF DISTRIBUTED WINDINGS

Alternating Field Distribution

Spatial field distribution and zero crossings remain the same, where as the field strength amount changes periodically with current frequency. This kind of field is called alternating field.

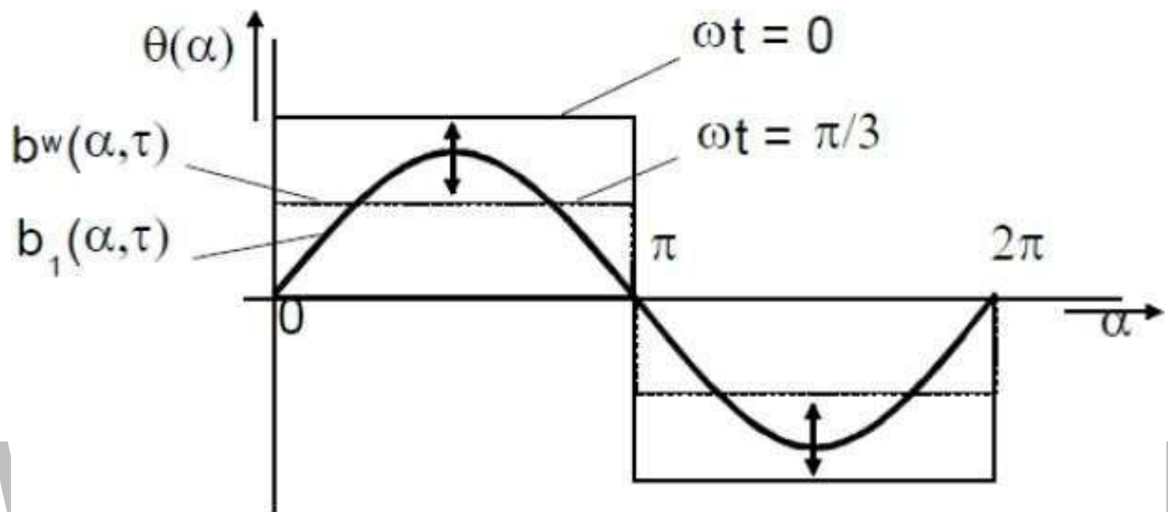


Figure 3.3.1 Alternating field distribution

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 236]*

The fundamental wave of the square-wave function can be determined by Fourier analysis. This results in an infinite count of single waves of odd ordinal numbers and anti-proportional decreasing amplitude with ordinary numbers. The amplitudes off fundamental waves and harmonics show proportional dependency to the current, zero crossings remain the same. These are called standing wave. The existence of harmonics is to be attributed to the spatial distributions of the windings. The generating current is of pure sinusoidal form, not containing harmonics. it necessarily needs to be distinguished between wave: spatiotemporal behavior , oscillation: pure time dependent behavior.

Rotating field

Rotating fields appear as spatial distributed fields of constant form and amount, revolving with angular speed *w*1:

A sinusoidal alternating field can be split up into two sinusoidal rotating fields. Their peak value is of half the value as of the according alternating field, their angular speeds are oppositely signed

Three-phase winding

Most simple arrangement of a three-phase stator consist of:

Core stack composed of laminations with approximately 0,5 mm thickness, mutual insulation for a reduction of eddy currents 2. The number of pole pairs is $p=1$ in Fig.138. In case of $p>1$, the configuration repeats p-times along the circumference.

Determination of slot mmf for different moments (temporal)

Quantity of slot mmf is applied over the circumference angle. line integrals provide enveloped mmf, dependent on the circumference angle. total mmf is shaped like a staircase step function, being constant between the slots. At slot edges, with slots assumed as being narrow, the total mmf changes about twice the amount of the slot mmf,the air gap field results from the total mmf

Magnetic Fields In Rotating Machines

Winding factor Winding factor

If *w* windings per phase are not placed in two opposing slots, but are moreover spread over more than one slot (zone winding) and return conductors are returned under an electric angle smaller than < *180°*, the effective number of windings appears smaller than it is in real

This means is utilized for a supression of harmonics, which cause parasitic torques and losses, influencing proper function of a machine.. Actually there is no machine with *q*

Rotating Magnetic Field

A symmetric rotating magnetic field can be produced with as few as three coils. The three coils will have to be driven by a symmetric 3-phase AC sine current system, thus each phase will be shifted 120 degrees in phase from the others. For the purpose of this example, the magnetic field is taken to be the linear function of the coil's current.

Sine wave current in each of the coils produces sine varying magnetic field on the rotation axis. Magnetic fields add as vectors. Vector sum of the magnetic field vectors of the stator coils produces a single rotating vector of resulting rotating magnetic field.

The result of adding three 120-degrees phased sine waves on the axis of the motor is a single rotating vector. The rotor has a constant magnetic field. The N pole of the rotor will move toward the S pole of the magnetic field of the stator, and vice versa. This magneto-mechanical attraction creates a force which will drive rotor to follow the rotating magnetic field in a synchronous manner.

A permanent magnet in such a field will rotate so as to maintain its alignment with the external field. This effect was utilized in early alternating current electric motors. A rotating magnetic field can be constructed using two orthogonal coils with a 90 degree phase difference in their AC currents. However, in practice such a system would be supplied through a three-wire arrangement with unequal currents.

This inequality would cause serious problems in the standardization of the conductor size. In order to overcome this, three-phase systems are used where the three currents are equal in magnitude and have a 120 degree phase difference. Three similar coils having mutual geometrical angles of 120 degrees will create the rotating magnetic field in this case. The ability of the three phase system to create the rotating field utilized in electric motors is one of the main reasons why three phase systems dominate in the world electric power supply systems.

Rotating magnetic fields are also used in induction motors. Because magnets degrade with time, induction motors use short-circuited rotors (instead of a magnet) which follow the rotating magnetic field of a multi coiled stator. In these motors, the short circuited turns of the rotor develop eddy currents in the rotating field of stator which in turn move the rotor by Lorentz force. These types of motors are not usually synchronous, but instead necessarily involve a degree of 'slip' in order that the current may be produced due to the relative movement of the field and the rotor.

The single coil of a single phase induction motor does not produce a rotating magnetic field, but a pulsating 3-φmotor runs from 1-φ power, but does not start.

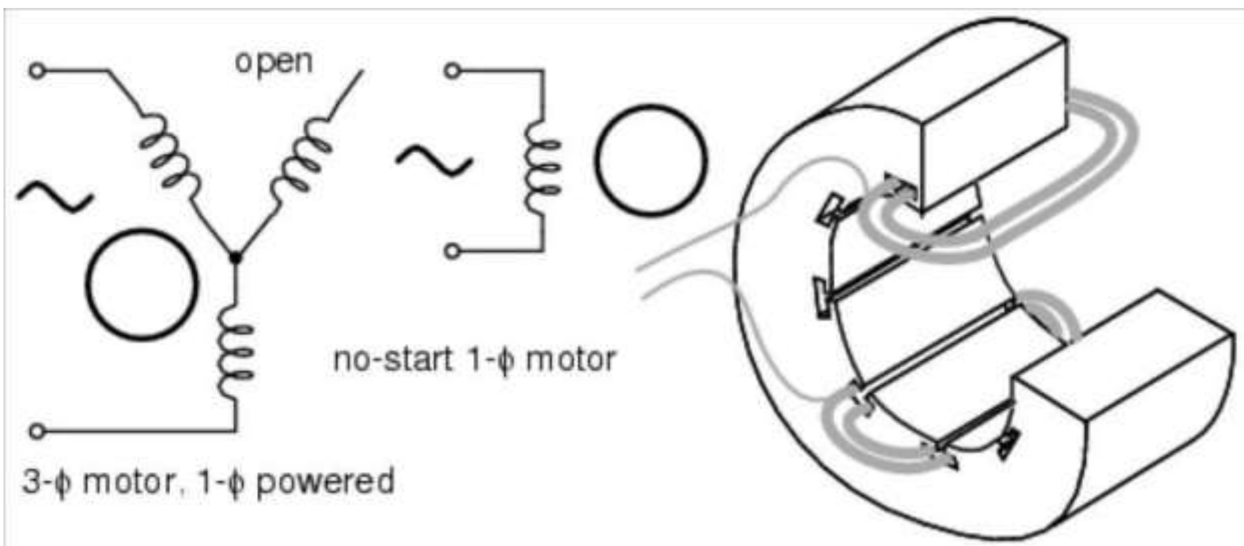


Figure 3.3.2 Non pulsating magnetic field

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 241]*

Another view is that the single coil excited by a single phase current produces two counter rotating magnetic field phasor.

From the above fig3.10 . When the phasor rotate to 90 deg and -90deg they canceled At 45 deg and -45deg they are partially additive along the +x axis and cancel along the y axis. An analogous situation exists in figure d. The sum of these two phasor is a phasor stationary in space, but alternating polarity in time. Thus, no starting torque is developed. However, if the rotor is rotated forward at a bit less than the synchronous speed, It will develop maximum torque at 10% slip with respect to the forward rotating phasor. Less torque will be developed above or below 10% slip. The rotor will see 200% - 10% slip with respect to the counter rotating magnetic field phasor. Little torque (see torque vs. slip curve) other than a double frequency ripple is developed from the counter rotating phasor. However, if the rotor is rotated forward at a bit less than the synchronous speed, It will develop maximum torque at 10% slip with respect to the forward rotating phasor. Less torque will be developed above or below 10% slip. The rotor will see 200% - 10% slip with respect to the counter rotating magnetic field phasor. Little torque (see torque vs. slip curve) other than a double frequency ripple is developed from the counter rotating phasor. Thus, the single phase coil will develop torque, once the rotor is started. If the rotor is started in the reverse direction, it will develop a similar large torque as it nears the speed of the backward rotating phasor. Single phase induction motors have a copper or aluminum squirrel cage embedded in a cylinder of steel laminations, typical of poly-phase induction motors.

Distribution factor

All w/p windings per pole and phase are distributed over q slots. Any of the w/pq conductors per slot show a spatial displacement

The resulting number of windings wrester phase is computed by geometric addition of all q partial windings w/pq. The vertices of all q phasors per phase, being displaced by Purpose: The purpose of utilizing zone winding is to aim slot mmf fundamental waves adding up harmonics compensating each other, s they suppose to do

Pitch factor

If windings are not implemented as diametric winding, but as chorded winding, return- actice the windings are distributed over two layers. Line conductors are placed into the bottom layer, whereas return conductors are integrated into the top layer. That arrangement complies with a superposition of two winding systems of halved number of windings

3.4 ROTATING MMF WAVES

The principle of operation of the induction machine is based on the generation of a rotating magnetic field. Let us understand this idea better.

Consider a cosine wave from 0 to 360◦. This sine wave is plotted with unit amplitude.

- Now allow the amplitude of the sine wave to vary with respect to time in a sinusoidal fashion with a frequency of 50Hz.Let the maximum value of the amplitude is, say, 10 units. This waveform is a pulsating sine wave.

Now consider a second sine wave, which is displaced by 120◦ from the first (lagging).and allow its amplitude to vary in a similar manner, but with a 120◦time lag. Similarly consider a third sine wave, which is at 240◦ lag. and allow its amplitude to change as well with a 240◦ time lag. Now we have three pulsating sine waves Let us see what happens if we sum up the values of these three sine waves at every angle. he result really speaks about Tesla's genius. What we get is a constant amplitude travelling sine wave.

In a three phase induction machine, there are three sets of windings, phase A winding, phase B and phase C windings. These are excited by a balanced three-phase voltage supply.

This would result in a balanced three phase current. Note that they have a 120◦ time lag between them. Further, in an induction machine, the windings are not all located in the same place .They are distributed in the machine 120◦ away from each other (more about this in the section on alternators). The correct terminology would be to say that the windings have their axes separated in space by 120◦. This is the reason for using the phase A, B and C since waves separated in space as well by 120◦. When currents flow through the coils, they generate mmfs. Since mmf is proportional to current, these waveforms also represent the mmf generated by the coils and the total mmf. Further, due to magnetic material in the machine (iron), these mmfs generate magnetic flux, which is proportional to the mmf (we may assume that iron is infinitely permeable and non-linear effects such as hysteresis are neglected). Thus the waveforms seen above would also represent the flux generated within the machine. The net result as we have seen is a travelling flux wave. The x-axis would represent the space angle in the machine as one travels around the air gap. The first pulsating waveform seen earlier would then represent the a-phase flux, the second represents the b-phase flux and the third represents the c-phase. This may be better visualized in a polar plot. The angles of the polar plot represent the space angle in the machine, i.e., angle as one travels around the stator bore of the machine.

This plot shows the pulsating wave at the zero degree axes. The amplitude is maximum at zero degree axes and is zero at 90◦ axis. Positive parts of the waveform are shown in red while negative in blue. Note that the waveform is pulsating at the 0−180◦ axis and red and blue alternate in any given side. This corresponds to the sinewave current changing polarity. Note that the maximum amplitude of the sinewave is reached only along the 0−180◦ axis. At all other angles, the amplitude does not reach a maximum of this value. It however reaches a maximum value which is less than that of the peak occurring at the 0 − 180◦ axis. More exactly, the maximum reached at any space angle would be equal to cos times the peak at the 0 − 180◦ axis. Further, at any space angle ,the time variation is sinusoidal with the frequency and phase lag being that of the excitation, and amplitude being that corresponding to the space angle.

- This plot shows the pulsating waveforms of all three cosines. Note that the first is pulsating about the 0 − 180◦ axis, the second about the120◦− 300◦axis and the third at 240◦− 360◦axis.
- This plot shows the travelling wave in a circular trajectory. Note that while individual pulsating waves have maximum amplitude of 10, the resultant has amplitude of 15. If f1 is the amplitude of the flux waveform in each phase .It is worthwhile pondering over the following points.

1. what is the interpretation of the pulsating plots of the animation? If one wants to know the ‘a’ phase flux at a particular angle for all instants of time, how can it be obtained?

What will this time variation look like? It is obviously periodic. What will be the amplitude and frequency?

Voltage induction caused by influence of rotating field

Voltage in three-phase windings revolving at variable speed, induced by a rotating field is subject to computation in the following:

Spatial integration of the air gap field results in the flux linkage of a coil. Induced voltage ensues by derivation of the flux linkage with respect to time. Using the definition of slip and a transfer onto three-phase windings, induced voltages in stator and rotor can be discussed. The following considerations are made only regarding the fundamental wave.

Flux linkage

The air gap field is created in the three-phase winding of the stator, characterized by the number of windings *w1* and current *I1*:

First of all, only one single rotor coil with number of windings *w2* and arbitrary from spatial integration of the air gap flux density over one pole pitch.

Induced voltage, slip

Induced voltage in a rotor coil of arbitrary angle of twist *a(t)*, which is flowed through by the air gap flux density, computes from variation of the flux linkage with time. Described variation of flux linkage can be caused by both variation of currents *iu(t), iv(t), iw(t)* with time, inside the exciting three-phase winding and also by rotary motion *a(t)* of the coil along the air gap circumference.

Some aspects regarding induced voltage dependencies are listed below:

The amplitude of the induced voltage is proportional to the line frequency of the stator and to the according slip.frequency of induced voltage is equal to slip frequency at rotor standstill (*s=1*), frequency of the induced voltage is equal to line frequency. when rotating (*s* fundamental wave of the stator windings no voltage is induced into the rotor at synchronous speed (*s=0*).phase displacement of voltages to be induced into the rotor is only dependent from the spatial position of the coil, represented by the (elec.) angle *p R a*

Is a rotor also equipped with a three-phase winding, instead of a single coil similar to the stator arrangement with phases being greater than 1 (*q*>follows for the induced voltage of single rotor phases

Torque In Ac And Dc Machines

As fulfilled previous considerations, only the fundamental waves of the effects caused by the air gap field are taken into account. Rotating mmf, caused in stator windings,

is revolving. An according rotating mmf is evoked in the rotor windings. Initially no assumptions are made for the number of pole pairs, angular frequency and phase angle of rotating magneto-motive forces of stator- and rotor. With appliance of Ampere's law, the resulting air gap field calculates from superimposing of both rotating magneto-motive forces of stator and rotor

3.2 FORCE IN A SINGLY EXCITED MAGNETIC FIELD SYSTEM

Model & Analysis

The conversion of electrical energy to mechanical energy follows the law of conservation of energy. In general, the law of conservation of energy states that energy is neither created nor destroyed. Equation (1) describes the process of electromechanical energy conversion for a differential time interval dt, where dWe is the change in electrical energy, dWm is the change in mechanical energy, and dWf is the change in magnetic field energy. Energy losses in the form of heat are neglected.

dWe = dWm + dWf _(1)

If the electrical energy is held constant, the dWe term is zero for Equation (1). The differential mechanical energy, in the form of work, is the force multiplied by the differential distance moved. The force due to the magnetic field energy is shown in Equation (2). The negative sign implies that the force is in a direction to decrease the reluctance by making the air gap smaller

$$f_m = \frac{-dW_f}{dx} \qquad (2)$$

An expression for the energy stored in the magnetic field can be found in terms of the magnetic system parameters. This expression is then substituted into Equation (2) for Wf to get an expression for the force. This derivation is shown in Appendix A. The result is Equation (3), in terms of the current, i, the constant for the permeability of free space, m0, the cross-sectional area of the air gap, Ag, the number of turns, N, and the air gap distance, x

$$f_m = \frac{i^2 \mu_0 A_g N^2}{2x^2} \quad (3)$$

To verify this relationship in the lab, it is convenient to have an expression for the current necessary to hold some constant force. In a design, the dimensions and force are often known. So, the user of the reluctance machine needs to know how much current to supply.

Rearranging terms in Equation (3) yields Equation (4).

$$i(x) = \sqrt{\frac{f_m \cdot 2x^2}{\mu_0 A_g N^2}}$$

(4)

Sample Calculations

For the simple magnetic system of Figure 3.7, the current necessary to suspend the armature can be calculated using Equation (4).

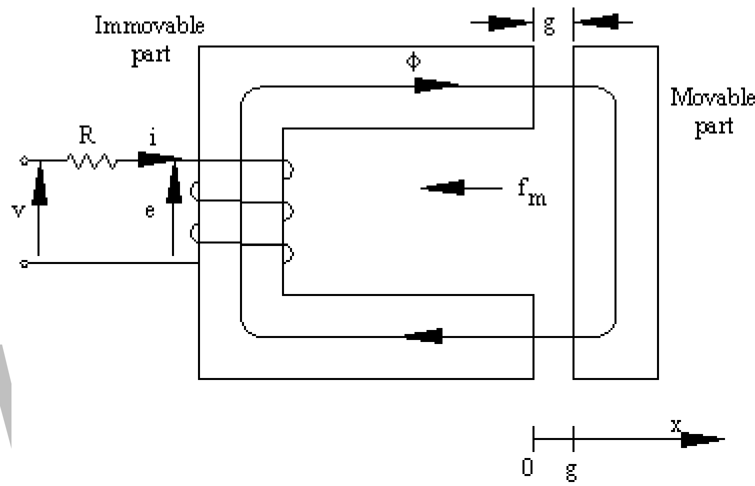


Figure 3.2.1 Electromechanical System

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 231]*

For an air gap length of 0.12 mm, an air gap cross sectional area of 1092 mm2, and a 230 turn coil the current required to just suspend the 12.5 newton armature is

$$i(0.12\text{mm}) = \sqrt{\frac{(12.5\text{newton})\cdot 2\cdot(0.00012\text{m})^2}{\left(4\cdot\pi\cdot 10^{-7}\,\frac{\text{Henry}}{\text{m}}\right)\cdot(1.092\cdot 10^{-3}\,\text{m}^2)\cdot 230^2}} = 100\text{mA}$$

5

Derivation of Magnetic Field Energy and Magnetic Force

Let Wf be the energy stored in a magnetic field

$$W_f = \int e\cdot i\,dt$$

$$e = \frac{d\lambda}{dt}$$

where l is flux linkages

$$W_f = \int \frac{d\lambda}{dt} \cdot idt = \int i d\lambda = \int \frac{\lambda}{L} \cdot d\lambda = \frac{1}{2} \cdot \frac{\lambda^2}{L} = \frac{1}{2} \cdot i^2 \cdot L(x)$$

$$L(x) = \frac{N^2}{\Re} = \frac{N^2}{\frac{x}{\mu_0 \cdot A_g}} = \frac{\mu_0 \cdot A_g \cdot N^2}{x}$$

L(x) is the inductance as a function of the air gap length, x.

where Ag is the area of the air gap. The magnetic force is

$$f_m = -\frac{1}{2} i^2 \cdot \frac{dL(x)}{dx} = \frac{i^2 \mu_0 A_g N^2}{2x^2}$$

Force in A Multiply Excited Magnetic Field System

For continuous energy conversion devices like Alternators, synchronous motors etc., multiply excited magnetic systems are used. In practice ,doubly excited systems are very much in use.

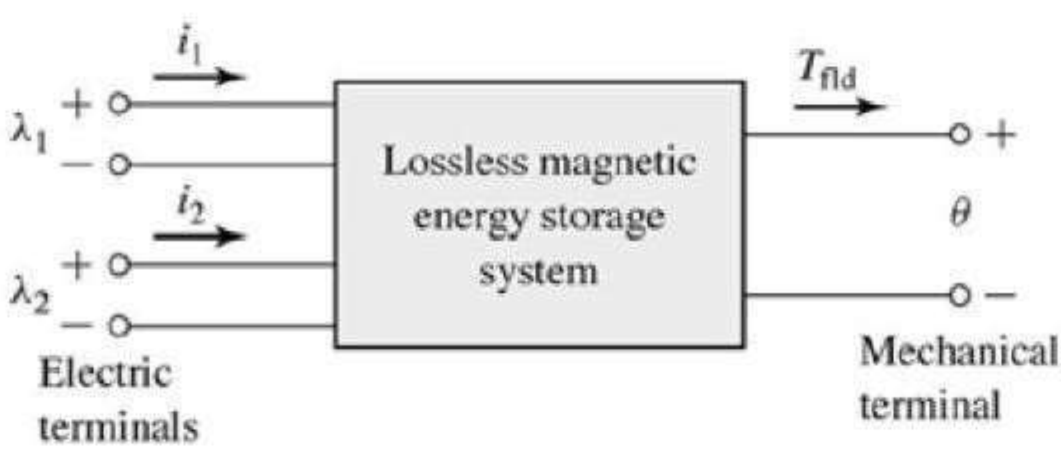


Figure 3.2.2 Electromechanical System

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 232]*

The Figure shows doubly excited magnetic system. This system has two independent sources of excitations. One source is connected to coil on stator while other is connected to coil on rotor.

Let i1 = Current due to source 1 i2 = Current due to source 2

= Flux linkages due to i1

= Flux linkages due to i2

= Angular displacement of rotor

Tf = Torque developed

Due to two sources, there are two sets of three independent variables

i.e. (,) or (i1,i2,)

Case:1 Independent Variables , i.e. i1,i2, From the easier analysis it is known,

Tf = ….Currents are Variables ....(1)

While the field energy is,

Wf(,) = +................ (2)

Now let L11 = Self inductance of stator L22 = Self inductance of rotor

L12= L21 = Mutual inductance between stator and rotor

= L11 i1 + L12 i2......... (3)

And = L12 i1 + L22 i2.........(4)

Solve equation (3) and (4) to express i1 and i2interms of and as and are independent variables. Multiply equation (3) by L12 and equation (4) by L11,

L12 =L11L12 i1+L12 i2

and L11 =L11L12 i1+L11L22 i2

Subtracting the two ,

L12 - L11 = L12 i2 - L11L22 i2

= [L12 - L11L22]i2

Example 1:

An actuator with a rotating vane is shown in Fig. 3.26. You may assume that the permeability of both the core and the vane are infinite (μ -> ¥). The total air-gap length is 2g and shape of the vane is such that the effective area of the air gap can be assumed to be of the form

$$A_g = A_0\left[1-\left(\frac{4\theta}{\pi}\right)^2\right]$$

(valid only in the range $|\theta| \le \pi/6$). The actuator dimensions are $g = 0.8$ mm, $A_0 = 6.0$ mm^2, and $N = 650$ turns.

(a) Assuming the coil to be carrying current i, write an expression for the magnetic stored energy in the actuator as a function of angle θ for $|\theta| \le \pi/6$.

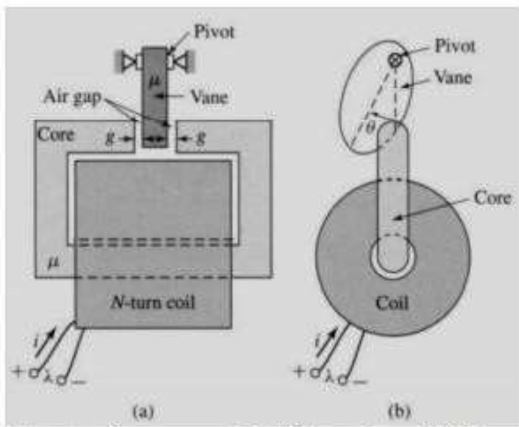


Figure 1 Actuator with rotating vane (a) Side view. (b) End view.

Solution

(a) Flux density in the air-gap: $B_g = \dfrac{\mu_0 N i}{2g}$

Magnetic energy density $= \dfrac{1}{2}\dfrac{B_g^2}{\mu_0}$

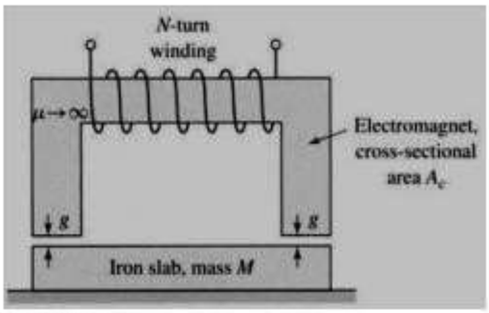
$$\Rightarrow W_{fld} = \left(\frac{1}{2}\frac{B_g^2}{\mu_0}\right)\times V_{ag} \qquad V_{ag} = 2gA_g \quad \Rightarrow \quad W_{fld} = \frac{\mu_0 N^2 i^2}{4g} A_0\left[1-\left(\frac{4\theta}{\pi}\right)^2\right]$$

(b)

$$W_{fld} = \frac{1}{2}L(\theta)i^2 \quad \Rightarrow \quad L(\theta) = \frac{2W_{fld}}{i^2} = \frac{\mu_0 N^2}{2g} A_0\left[1-\left(\frac{4\theta}{\pi}\right)^2\right]$$

Example 2:

As shown in Fig. 2, an N-turn (N = 100) electromagnet is to be used to lift a slab of iron of mass M. The surface roughness of the iron is such that when the iron and the electromagnet are in contact, there is a minimum air gap of g_{min} = 0.18 mm in each leg. The electromagnet cross-sectional area A_c = 32 cm^2 and coil resistance is 2.8 Ω. Calculate the minimum coil voltage which must be used to lift a slab of mass 95 kg against the force of gravity. Neglect the reluctance of the iron**.**



Solution

coil inductance: $L(g)=\dfrac{\mu_0 N^2 A_c}{2g} \Rightarrow f_{fld}=\dfrac{1}{2}i^2\dfrac{dL}{dg}=-\dfrac{\mu_0 N^2 A_c}{4g^2}i^2$

$|f_{fld}|=Mg_e \quad g_e=9.8\ \text{m/s}^2$: acceleration due to gravity $\Rightarrow |f_{fld}|=931\ \text{N}$

$$i_{min}=\frac{2g_{min}}{N}\sqrt{\frac{|f_{fld}|}{\mu_0 A_c}}=\frac{2\times0.18\times10^{-3}}{450}\sqrt{\frac{931}{4\pi\times10^{-7}\times32\times10^{-4}}}=0.385\ \text{A}$$

$$v_{min}=Ri_{min}=1.08\ \text{V}$$

Example :3

An inductor is made up of a 525-turn coil on a core of 14-cm^2 cross-sectional area and gap length 0.16 mm. The coil is connected directly to a 120-V 60-Hz voltage source. Neglect the coil resistance and leakage inductance. Assuming the coil reluctance to be negligible, calculate the time-averaged force acting on the core tending to close the air gap. How would this force vary if the air-gap length were doubled?

Solution

$$L = \frac{\mu_0 N^2 A_c}{g} \qquad f_{fld} = \frac{1}{2} i^2 \frac{dL}{dg} = -\frac{1}{2} i^2 \left(\frac{\mu_0 N^2 A_c}{g^2} \right) = -\frac{i^2 L}{2g}$$

Since coil resistance and leakage inductance are negligible, the current in the coil can be written as

$$i(t) = I_m \cos \omega t \quad \text{where } I_m = \frac{V_m}{\omega L}$$

$$f_{fld} = -\frac{i^2 L}{2g} = -\frac{I_m^2 L}{2g} \cos^2 \omega t \;\Rightarrow\; \langle f_{fld} \rangle = -\frac{I_{rms}^2 L}{2g} = -\frac{V_{rms}^2 L}{2g\omega^2 L^2} = -\frac{V_{rms}^2}{2\omega^2 \mu_0 N^2 A_c}$$

$$\Rightarrow \langle f_{fld} \rangle = -\frac{120^2}{2(120\pi)^2 525^2 \times 4\pi \times 10^{-7} \times 14 \times 10^{-4}} = -104.48 \text{ N}$$

The average force is independent of the air-gap length g.

Example 4:

Two windings, one mounted on a stator and the other on a rotor, have self- and mutual inductances of L_{11}= 4.5 H L_{22}= 2.5 H L_{12} = 2.8cosθ H

where θ is the angle between the axes of the windings. The resistances of the windings may be neglected. Winding 2 is short-circuited, and the current in winding 1 as a function of time is i_1 = 10 sin ωt A.

a. Derive an expression for the numerical value in newton-meters of the instantaneous torque on the rotor in terms of the angle θ.

b. Compute the time-averaged torque in newton-meters when θ = 45°.

If the rotor is allowed to move, will it rotate continuously or will it tend to come to rest? If the latter, at what value of θ_0 ?

(a) $T_{fld} = i_1 i_2 \frac{dL(\theta)}{d\theta} = -2.8 i_1 i_2 \sin\theta$

Winding 2 short-circuited

$$e_2 = v_2 = 0 \;\Rightarrow\; \lambda_2 = L_{21} i_1 + L_{22} i_2 = 0 \;\Rightarrow\; i_2 = -\frac{L_{21}}{L_{22}} i_1 = -1.12 i_1 \cos\theta$$

$$T_{fld} = -2.8 i_1 i_2 \sin\theta = -3.14 i_1^2 \sin\theta\cos\theta = -314 \sin^2(\omega t) \sin\theta\cos\theta$$

(b) Time-averaged torque

$$\langle T_{fld} \rangle = -157 \sin\theta\cos\theta \quad \theta = 45^\circ \;\Rightarrow\; \langle T_{fld} \rangle = -157\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = -78.5 \text{ N-m}$$

Note: $\langle \sin^2(\omega t) \rangle = \frac{1}{\pi}\int_0^{\pi} \frac{1-\cos(2\omega t)}{2} d(\omega t) = \frac{1}{2}$

(c) The rotor will not rotate because the average torque with respect to θ is zero. It will come to rest when

$$\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta) = 0 \;\Rightarrow\; \theta = \frac{\pi}{2}$$

Example 5:

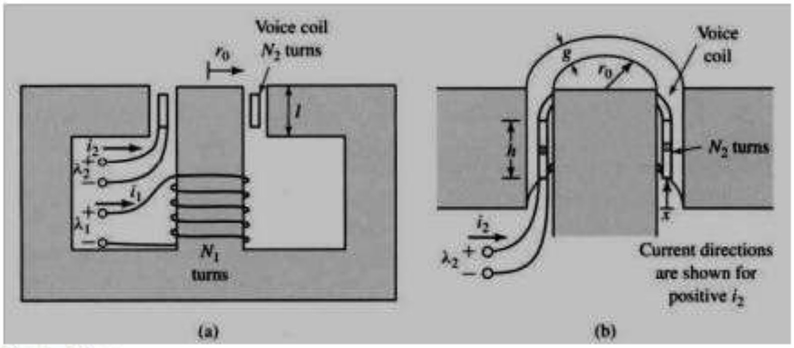
A loudspeaker is made of a magnetic core of infinite permeability and circular symmetry, as shown in Figs. 3.37a and b. The air-gap length g is much less than the radius r_0 of the central core. The voice coil is constrained to move only in the x direction and is attached to the speaker cone, which is not shown in the figure. A constant radial magnetic field is produced in the air gap by a direct current in coil 1, $i_1 = I_1$. An audio-frequency signal $i_2 = I_2 \cos(\omega t)$ is then applied to the voice coil. Assume the voice coil to be of negligible thickness and composed of N_2 turns uniformly distributed over its height h. Also assume that its displacement is such that it remains in the air gap (0 £ x £ l-h)

(a) Calculate the force on the voice coil, using the Lorentz Force Law (Eq. 3.1).

(b) Calculate the self-inductance of each coil.

(c) Calculate the mutual inductance between the coils. (Hint: Assume that current is applied to the voice coil, and calculate the flux linkages of coil 1. Note that these flux linkages vary with the displacement x.

(d) Calculate the force on the voice coil from the coenergy Wfld .



Solution

(a) Radial magnetic field intensity: $Ni_1 = H_{r,1}g \;\Rightarrow\; B_{r,1} = \mu_0 H_{r,1} = \dfrac{\mu_0 N i_1}{g}$

Lorentz force (directed upward):

$F = N_2 i_2 l_2 B_{r,1}$ where $l_2 = 2\pi r_0$ is the length of one turn of coil 2.

$$F = 2\pi r_0 N_2 i_2 B_{r,1} = \frac{2\pi r_0 \mu_0 N_1 N_2}{g} i_1 i_2$$

(b) Self-inductances:

$$L_{11} = \frac{\mu_0 N_1^2 A_g}{g} \qquad A_g = 2\pi r_0 l$$

To find the self-inductance of coil 2, apply Ampere's law to coil 2 at height z:

$$H_{r,2}g = \left(\frac{z-x}{h}\right) N_2 i_2 = \text{ total current enclosed bypath } C \text{ at height } z$$

$$\therefore B_{r,2} = \begin{cases} 0 & 0 \le z \le x \\ -\left(\dfrac{z-x}{gh}\right)\mu_0 N_2 i_2 & x \le z \le x+h \\ -\dfrac{\mu_0 N_2 i_2}{g} & x+h \le z \le l \end{cases}$$

(c) We can find the inductance of a section of coil 2 of length dz and then integrate with respect to z. At a height z

$$\Delta L_{22}(z) = \frac{\Delta\lambda_2(z)}{I_2} \qquad \Delta\lambda_2(z) = \Delta N_2 \int_z^l B_{r,2}(u)\, 2\pi r_0\, du \qquad z > x$$

where $\Delta N_2 = N_2 \Delta z$ is the number of turns of coil 2 in the section Δz.

$$\int_z^l B_{r,2}(u)\, du = \frac{\mu_0 N_2 I_2}{gh} \int_z^l (u - x) du = \frac{\mu_0 N_2 I_2}{gh} \left[\frac{1}{2} u^2 - ux \right]_z^l$$

$$\Delta L_{22}(z) = \frac{2\pi r_0 \mu_0 N_2^2}{gh} \left[\frac{1}{2} l^2 - lx - \frac{1}{2} z^2 + zx \right] \Delta z$$

$$L_{22} = \frac{2\pi r_0 \mu_0 N_2^2}{gh} \int_x^{x+h} \left[\frac{1}{2} l^2 - lx - \frac{1}{2} z^2 + zx \right] dz = \frac{\pi r_0 \mu_0 N_2^2}{g} (l - x)^2$$