2.8 Auto Transformer

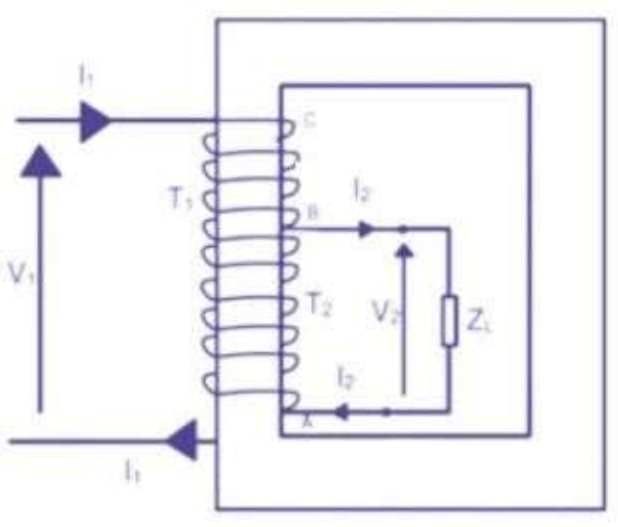


Figure 2.8.1 Auto Transformer Physical Arrangement

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 192]*

The primary and secondary windings of a two winding transformer have induced emf in them due to a common mutual flux and hence are in phase. The currents drawn by these two windings are out of phase by 180°. This prompted the use of a part of the primary as secondary. This is equivalent to fusing the secondary turns into primary turns. The fused section need to have a cross sectional area of the conductor to carry $(I_2 - I_1)$ ampere! This ingenious thought led to the invention of an auto transformer. Fig. 28 shows the physical arrangement of an auto transformer. Total number of turns between A and C are T1. At point B a connection is taken. Section AB has T2 turns. As the volts per turn, which is proportional to the flux in the machine, is the same for the whole winding,

$$V_1 : V_2 = T_1 : T_2$$

For simplifying analysis, the magnetizing current of the transformer is neglected. When the secondary winding delivers a load current of I2 ampere the demagnetizing ampere turns is I2T2 . This will be countered by a current I1 flowing from the source through the T1 turns such that,

$$I_1T_1 = I_2T_2$$

A current of I_1 ampere flows through the winding between B and C . The current in the winding between A and B is $(I_2 - I_1)$ ampere. The cross section of the wire to be selected for AB is proportional to this current assuming a constant current density for the whole winding. Thus some amount of material saving can be achieved compared to a two winding transformer. The magnetic circuit is assumed to be identical and hence there is no saving in the same. To quantify the saving the total quantity of copper used in an auto transformer is expressed as a fraction of that used in a two winding transformer as,

$$\begin{aligned}\frac{copper\ in\ auto\ transformer}{copper\ in\ two\ winding\ transformer} &= \frac{(T_1 - T_2)I_1 + T_2(I_2 - I_1)}{T_1I_1 + T_2I_2} \\ &= 1 - \frac{2T_2I_1}{T_1I_1 + T_2I_2} \\ \text{But } T_1I_1 &= T_2I_2 \\ \therefore \text{The Ratio} &= 1 - \frac{2T_2I_1}{2T_1I_1} = 1 - \frac{T_2}{T_1}\end{aligned}$$

This means that an auto transformer requires the use of lesser quantity of copper given by the ratio of turns. This ratio therefore denotes the savings in copper. As the space for the second winding need not be there, the window space can be less for an auto transformer, giving some saving in the lamination weight also. The larger the ratio of the voltages, smaller is the savings. As T2 approaches T_1 the savings become significant. Thus auto transformers become ideal choice for close ratio transformations. The savings in material is obtained, however, at a price. The electrical isolation between primary and secondary.

Tap Changing Transformer

Regulating the voltage of a transformer is a requirement that often arises in a power application or power system. In an application it may be needed

1.To supply a desired voltage to the load.

2. To counter the voltage drops due to loads.

3. To counter the input supply voltage changes on load.

On a power system the transformers are additionally required to perform the task of regulation of active and reactive power flows.

The voltage control is performed by changing the turns ratio. This is done by provision of taps in the winding. The volts per turn available in large transformers is quite high and hence a change of even one turn on the LV side represents a large percentage change in the voltage. Also the LV currents are normally too large to take out the tapping from the windings. LV winding being the inner winding in a core type transformer adds to the difficulty of taking out of the taps. Hence irrespective of the end use for which tapping is put to, taps are provided on the HV winding. Provision of taps to control voltage is called tap changing. In the case of power systems, voltage levels are sometimes changed by injecting a suitable voltage in series with the line.

This may be called buck-boost arrangement. In addition to the magnitude, phase of buck boost arrangement with phase shift are shown in Fig. 42. Tap changing can be effected when a) the transformers is on no- load and b) the load is still remains connected to the transformer. These are called off load tap changing and on load tap changing. The Off load taps changing relatively costs less. The tap positions are changed when the transformer is taken out of the circuit and reconnected. The on-load tap changer on the other hand tries to change the taps without the interruption of the load current.

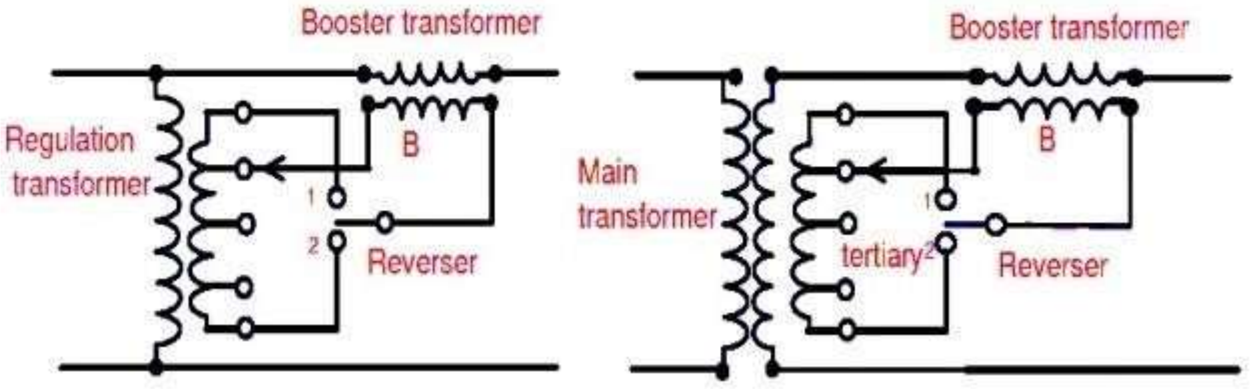
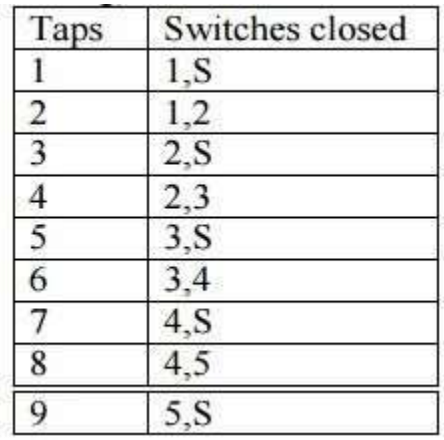


Figure 2.8.2 Tap Changing and Buck-Boost arrangement

[Source: “‘Electric Machinery Fundamentals” by Stephen J. Chapman, *Page: 213]*

In view of this requirement it normally costs more. A few schemes of on-load tap changing are now discussed. Reactor method The diagram of connections is shown in Fig. 43. This method employs an auxiliary reactor to assist tap changing. The switches for the taps and that across the reactor(S) are connected as shown. The reactor has a center tapped winding on a magnetic core. The two ends of the reactor are connected to the two bus bars to which tapping switches of odd/even numbered taps are connected. When only one tap is connected to the reactor the shorting switch S is closed minimizing the drop in the reactor. The reactor can also be worked with both ends connected to two successive taps. In that case the switch ’S’ must be kept open. The reactor limits the circulating current between the taps in such a situation. Thus a four step tapped winding can be used for getting seven step voltage on the secondary(see the table of switching).



Taps	Switches closed
1	1,S
2	1,2
3	2,S
4	2,3
5	3,S
6	3,4
7	4,S
8	4,5
9	5,S

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step voltage on the secondary (see the table of switching). The advantage of this type of tap changer is

1. Load need not be switched.
2. More steps than taps are obtained.
3 Switches need not interrupt load current as a alternate path is always provided.

The major objection to this scheme seems to be that the reactor is in the circuit always generating extra loss. Parallel winding, transformer method In order to maintain the continuity of supply the primary winding is split into two parallel circuits each circuit having the taps. As Two circuit breakers A and B are used in the two circuits. Initially tap 1a and 1b are closed and the transformer is energized with full primary voltage. To change the tap the circuit breaker A is opened momentarily and tap is moved from 1a to 2a. Then circuit breaker A is closed. When the circuit A is opened whole of the primary current of the transformer flows through the circuit B. A small difference in the number of turns between the two circuit exists. This produces a circulating current between them. Next, circuit breaker B is opened momentarily, the tap is changed from 1b to 2b and the breaker is closed. In this position the two circuits are similar and there is no circulating current. The circulating current is controlled by careful selection of the leakage reactance.

Generally, parallel circuits are needed in primary and secondary to carry the large current in a big transformer. Provision of taps switches and circuit breakers are to be additionally provided to achieve tap changing in these machines. Series booster method in this case a separate transformer is used to buck/boost the voltage of the main transformer. The main transformer need not be having a tapped arrangement. This arrangement can be added to an existing system also. It shows the booster arrangement for a single phase supply. The reverser switch reverses the polarity of the injected voltage and hence a boost is converted into a buck and vice versa. The power rating of this transformer need be a small fraction of the main transformer as it is required to handle only the power associated with the injected voltage.

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This arrangement can be added to an existing system also. It shows the booster arrangement for a single phase supply. The reverser switch reverses the polarity of the injected voltage and hence a boost is converted into a buck and vice versa. The power rating of this transformer need be a small fraction of the main transformer as it is required to handle only the power associated with the injected voltage. One precaution to be taken with this arrangement is that the winding must output side. In smaller ratings this is highly cost effective. Two winding arrangements are also possible. The two winding arrangement provides electrical isolation. Not be open circuited. If it gets open circuited the core (B in fig) gets highly saturated.

In spite of the small ratings and low voltages and flexibility, this method of voltage control costs more mainly due to the additional floor space it needs. The methods of voltage regulation discussed so far basically use the principle of tap changing and hence the voltage change takes place in steps. Applications like a.c. and D.C. motor speed control, illumination control by dimmers, electro-chemistry and voltage stabilizers need continuous control of voltage. This can be obtained with the help of moving coil voltage regulators. Moving coil voltage regulator shows the physical arrangement of one such transformer. a, b are the two primary windings wound on a long core, wound in the opposite sense. Thus the flux produced by each winding takes a path through the air to link the winding. These fluxes link their secondaries a2 and b2. A short circuited moving coil s is wound on the same limb and is capable of being held at any desired position. This moving coil alters the inductances of the two primaries. The sharing of the total applied voltage thus becomes different and also the induced emf in the secondaries a2 and b2. The total secondary voltage in the present case varies from 10 percent to 20 percent of the input in a continuous manner. The turn's ratios of a1: a2 and b1: b2 are 4.86 and 10.6 respectively. 5 4.86 + 95 10.6 = 10% when s is in the top position. In the bottom position it becomes 95 4.86 + 5 10.6 = 20%. By selecting proper ratios for the secondaries a2 and b2 one can get the desired voltage variation. Sliding contact regulators these have two winding or auto transformer like construction. The winding from which the output is taken is bared and a sliding contact taps the voltage. The minimum step size of voltage change obtainable is the voltage across a single turn. The conductor is chosen on the basis of the maximum load current on the output side. In smaller ratings this is highly cost effective. Two winding arrangements are also possible. The two winding arrangement provides electrical isolation also.

Tertiary Winding

When the transformer is make of three winding is called Tertiary winding. It is generally connected in delta. generally Transformer have primary and secondary winding, if transformer constructed with third winding is called tertiary winding or three winding transformer.

When the fault or short circuit is occur, high circulating current will flow through the primary and secondary winding. Because of that current , overheating of transformer is produced. By using tertiary winding we can avoid that overheating. Reactant of tertiary winding is such that which can limit over current.

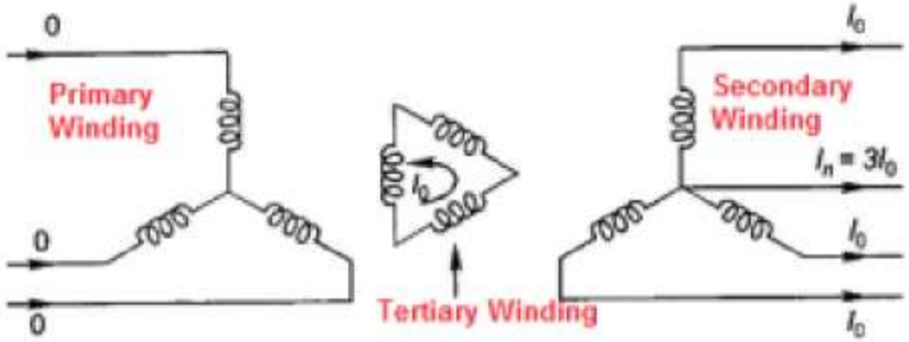


Figure 2.8.3 Tertiary Winding Connection

[Source: “‘Electric Machinery Fundamentals” by Stephen J. Chapman, *Page: 216]*

The main advantage of used thee winding transformer is that when any fault and short circuit is occur on primary and secondary side. a large unbalance of phase voltage may be produced, which is compensated by large tertiary winding circulating currents. Third winding is also known as stabilizing winding. because, A Delta connected tertiary winding reduced the impedance offered to zero sequence current. So large earth fault current flows for proper operations of protective devices.when the load is unbalanced, tertiary winding reduced third harmonic voltage and limits the unbalanced voltage. the magnetizing current is neglected.

2.1 Principle of Operation Transformer

A transformer is a device that transfers electrical energy from one circuit to another through inductively coupled conductor. A varying current in the first or primary winding creates a varying magnetic flux in the transformer core, and thus a varying magnetic field through the secondary winding. This varying magnetic field induces a varying electromotive force EMF or voltage in the secondary winding. This effect is called mutual induction.

If a load is connected to the secondary, an electric current will flow in the secondary winding and electrical energy will be transferred from the primary circuit through the transformer to the load. In an ideal transformer, the induced voltage in the secondary winding is in proportion to the primary voltage , and is given by the ratio of the number of turns in the secondary to the number of turns in the primary as follows:

By appropriate selection of the ratio of turns, a transformer thus allows an alternating current (AC) voltage to be "stepped up" by making greater than , or "stepped down" by making less than .

Construction

Figure 2.1.1 Laminated core Transformer

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 92]*

Laminated Steel Cores

Transformer use at power or audio frequencies typically have cores made of high permeability Si steel. The steel has permeability many times that of free and the core thus serves to greatly reduce the magnetizing current and confine the flux to a path which closely couples the windings. Early transformer developers soon realized that cores constructed from solid iron resulted in prohibitive eddy-current losses, and their designs mitigated this effect with cores consisting of bundles of insulated iron wires. Later designs constructed the core by stacking layers of thin steel laminations, a principle that has remained in use. Each lamination is insulated from its neighbors by a thin non-conducting layer of insulation. The universal transformer equation indicates a minimum cross-sectional area for the core to avoid saturation.

The effect of laminations is to confine eddy currents to highly elliptical paths that enclose little flux, and so reduce their magnitude. Thinner laminations reduce losses, but are more laborious and expensive to construct. Thin laminations are generally used on high frequency transformers, with some types of very thin steel laminations able to operate up to 10 kHz.

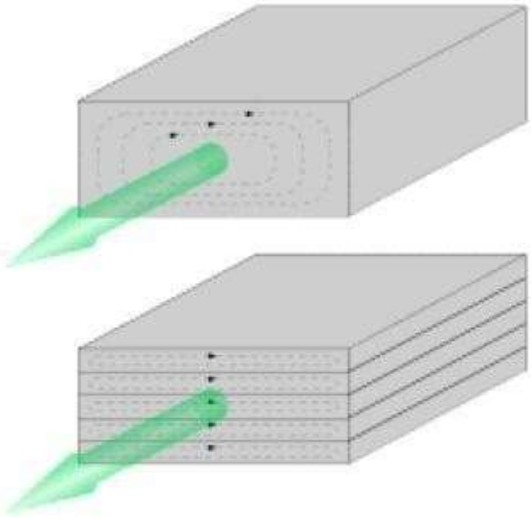


Figure 2.1.2 Laminated core reduces eddy current losses

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 94]*

One common design of laminated core is made from interleaved stacks of E-shaped steel sheets capped with shaped pieces, leading to its name of "E-I transformer”. Such a design tends to exhibit more losses, but is very economical to manufacture. The cut-core or C-core type is made by winding a steel strip around a rectangular form and then bonding the layers together. It is then cut in two, forming two C shapes, and the core assembled by binding the two C halves together with a steel strap. They have the advantage that the flux is always oriented parallel to the metal grains, reducing reluctance.

A steel core's permanence means that it retains a static magnetic field when power is removed. When power is then reapplied, the residual field will cause a high inrush until the effect of the remaining magnetism is reduced, usually after a few cycles of the applied alternating current. Over current protection devices such as fuses must be selected to allow this harmless inrush to pass. On transformers connected to long, overhead power transmission lines, induced currents due to geomagnetic disturbances during solar storms can cause saturation of the core and operation of transformer protection devices.

Distribution transformers can achieve low no-load losses by using cores made with low-loss high-permeability silicon steel or amorphous (non-crystalline) metal alloy. The higher initial cost of the core material is offset over the life of the transformer by its lower losses at light load.

Solid cores

Powdered iron cores are used in circuits such as switch-mode power supplies that operate above mains frequencies and up to a few tens of kilohertz. These materials combine high magnetic permeance high bulk electrical resistivity. For frequencies extending beyond the VHF band, cores made from non-conductive magnetic ceramic materials called ferrites are common.

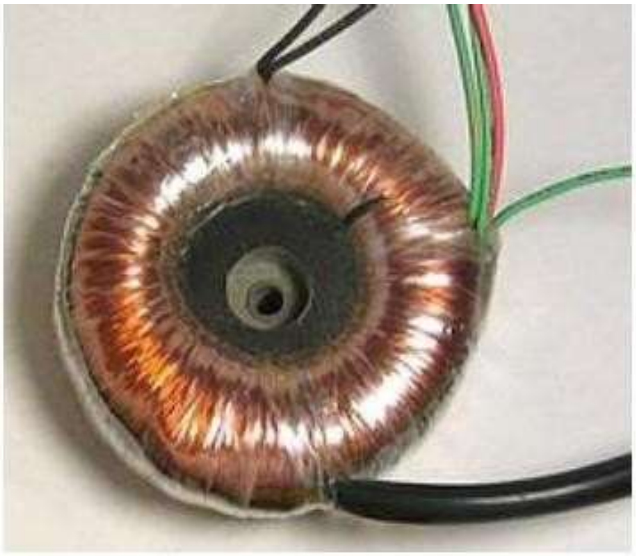


Figure 2.1.3 Toroidal Core Transformer

[Source: "“Electric Machinery Fundamentals” by Stephen J. Chapman, *Page: 94]*

Some radio-frequency transformers also have movable cores (sometimes called 'slugs') which allow adjustment of the coupling coefficient (and bandwidth) of tuned radio-frequency circuits. Toroidal transformers are built around a ring-shaped core, which, depending on operating frequency, is made from a long strip of silicon steel or perm alloy wound into a coil, powdered iron, or ferrite. A strip construction ensures that the grain boundaries are optimally aligned, improving the transformer's efficiency by reducing the core's reluctance. The closed ring shape eliminates air gaps inherent in the construction of an E-I core. The cross-section of the ring is usually square or rectangular, but more expensive cores with circular cross-sections are also available. The primary and secondary coils are often wound concentrically to cover the entire surface of the core. This minimizes the length of wire needed, and also provides screening to minimize the core's magnetic field from generating electromagnetic.

Toroidal transformers are more efficient than the cheaper laminated E-I types for a similar power level. Other advantages compared to E-I types, include smaller size (about half), lower weight (about half), less mechanical hum (making them superior in audio amplifiers), lower exterior magnetic field (about one tenth), low off-load losses (making them more efficient in standby circuits), single-bolt mounting, and greater choice of shapes. The main disadvantages are higher cost and limited power capacity (see "Classification" above). Because of the lack of a residual gap in the magnetic path, toroidal transformers also tend to exhibit higher inrush current, compared to laminated E-I types.

Ferrite toroidal cores are used at higher frequencies, typically between a few tens of kilohertz to hundreds of megahertz, to reduce losses, physical size, and weight of a switched-mode power supply. A drawback of toroidal transformer construction is the higher labor cost of winding. This is because it is necessary to pass the entire length of a coil winding through the core aperture each time a single turn is added to the coil. As a consequence, toroidal transformers are uncommon above ratings of a few kVA. Small distribution transformers may achieve some of the benefits of a toroidal core by splitting it and forcing it open, then inserting a bobbin containing primary and secondary windings.

Air Cores

A physical core is not an absolute requisite and a functioning transformer can be produced simply by placing the windings near each other, an arrangement termed an "air-core" transformer. The air which comprises the magnetic circuit is essentially lossless, and so an air-core transformer eliminates loss due to hysteresis in the core material.The leakage inductance is inevitably high, resulting in very poor regulation, and so such designs are unsuitable for use in power distribution. They have however very high bandwidth, and are frequently employed in radio-frequency applications, for which a satisfactory coupling coefficient is maintained by carefully overlapping the primary and secondary windings. They're also used for resonant transformers such as Tesla coils where they can achieve reasonablylow loss in spite of the high leakage inductance.

Windings

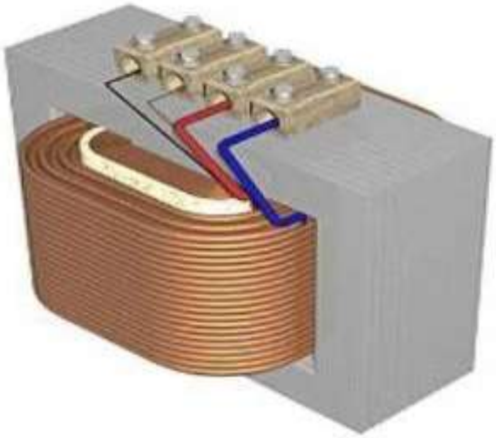


Figure 2.1.4 Arranged coil reduced flux leakage

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 93]*

The conducting material used for the windings depends upon the application, but in all cases the individual turns must be electrically insulated from each other to ensure that the current travels throughout every turn. For small power and signal transformers, in which currents are low and the potential difference between adjacent turns are shown in figure.

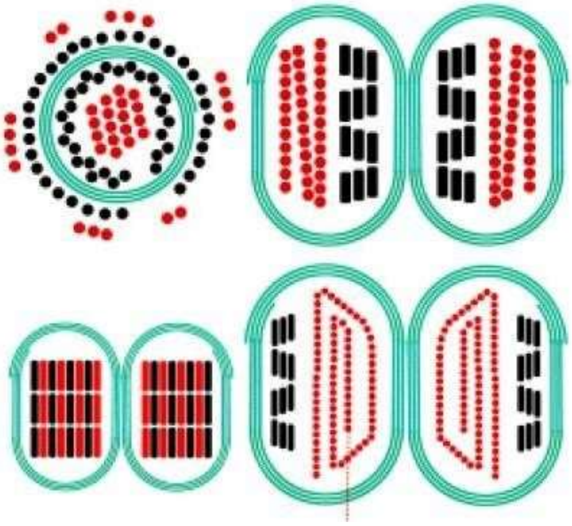


Figure 2.1.5 Windings Shapes

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 96]*

Cut view through transformer windings. White: insulator. Green spiral: Grain oriented silicon steel. Black: Primary winding made of oxygen-free copper. Red:

Secondary winding. Top left: Toroidal transformer. Right: C-core, but E-core would be similar. The black windings are made of film. Top: Equally low capacitance between all ends of both windings. Since most cores are at least moderately conductive they also need insulation. Bottom: Lowest capacitance for one end of the secondary winding needed for low power high voltage transformers. Bottom left: Reduction of leakage would lead to increase of capacitance.

Large power transformers use multiple-stranded conductors as well, since even at low power frequencies non-uniform distribution of current would otherwise exist in high-current windings. Each strand is individually insulated, and the strands are arranged so that at certain points in the winding, or throughout the whole winding, each portion occupies different relative positions in the complete conductor. The transposition equalizes the current flowing in each strand of the conductor, and reduces eddy current losses in the winding itself. The stranded conductor is also more flexible than a solid conductor of similar size, aiding manufacture.

For signal transformers, the windings may be arranged in a way to minimize leakage inductance and stray capacitance to improve high-frequency response. This can be done by splitting up each coil into sections, and those sections placed in layers between the sections of the other winding. This is known as a stacked type or interleaved winding.

Power transformers often have internal connections or taps at intermediate points on the winding, usually on the higher voltage winding side, for voltage regulation control purposes. Such taps are normally manually operated, automatic on-load tap changers being reserved, for cost and reliability considerations, to higher power rated or specialized transformers supplying transmission or distribution circuits or certain utilization loads such as furnace transformers. Audio-frequency transformers, used for the distribution of audio to public address loudspeakers, have taps to allow adjustment of impedance to each speaker. A center is often used in the output stage of an audio power amplifier in a push-pull circuit. Modulation transformers in AM transmitters are very similar Certain transformers have the windings protected by epoxy resin. By impregnating the transformer with epoxy under a vacuum, one can replace air spaces within the windings with epoxy, thus sealing the windings and helping to prevent the possible formation of corona and absorption of dirt or water. This produces transformers more suited to damp or dirty environments, but at increased manufacturing cost.

Cooling

Figure 2.1.6 Cooling

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 102]*

Cut away view of oil-filled power transformer. The conservator (reservoir) at top provides oil-to-atmosphere isolation. Tank walls' cooling fins provide required heat dissipation balance as shown in above figure .

Though it is not uncommon for oil-filled transformers to have today been in operation for over fifty years high temperature damages winding insulation, the accepted rule of thumb being that transformer life expectancy is halved for every 8 degree C increase in operating temperature. At the lower end of the power rating range, dry and liquid-immersed transformers are often self-cooled by natural convection and radiation heat dissipation. As power ratings increase, transformers are often cooled by such other means as forced-air cooling, force-oil cooling, water- cooling, or a combinations of these. The dialectic coolant used in many outdoor utility and industrial service transformers is transformer oil that both cools and insulates the windings. Transformer oil is a highly refined mineral oil that inherently helps thermally stabilize winding conductor insulation, typically paper, within acceptable insulation temperature rating limitations. However, the heat removal problem is central to all electrical apparatus such that in the case of high

value transformer assets, this often translates in a need to monitor, model, forecast and manage oil and winding conductor insulation temperature conditions under varying, possibly difficult, power loading conditions. Indoor liquid-filled transformers are required by building regulations in many jurisdictions to either use a non-flammable liquid or to be located in fire-resistant rooms. Air-cooled dry transformers are preferred for indoor applications even at capacity ratings where oil-cooled construction would be more economical, because their cost is offset by the reduced building construction cost.

The oil-filled tank often has radiators through which the oil circulates by natural convection. Some large transformers employ electric-operated fans or pumps for forced-air or forced-oil cooling or heat exchanger-based water-cooling. Oil-filled transformers undergo prolonged drying processes to ensure that the transformer is completely free of water before the cooling oil is introduced. This helps prevent electrical breakdown under load. Oil-filled transformers may be equipped with Buchholz relays, which detect gas evolved during internal arcing and rapidly de- energize the transformer to avert catastrophic failure. Oil-filled transformers may fail, rupture, and burn, causing power outages and losses. Installations of oil-filled transformers usually include fire protection measures such as walls, oil containment, and fire-suppression sprinkler systems.

Insulation drying

Construction of oil-filled transformers requires that the insulation covering the windings be thoroughly dried before the oil is introduced. There are several different methods of drying. Common for all is that they are carried out in vacuum environment.

The vacuum makes it difficult to transfer energy (heat) to the insulation. For this there are several different methods. The traditional drying is done by circulating hot air over the active part and cycle this with periods of hot-air vacuum (HAV) drying. More common for larger transformers is to use evaporated solvent which condenses on the colder active part. The benefit is that the entire process can be carried out at lower pressure and without influence of added oxygen. This process is commonly called vapor-phase drying (VPD).

For distribution transformers, which are smaller and have a smaller insulation weight, resistance heating can be used. This is a method where current is injected in the windings to heat the insulation. The benefit is that the heating can be controlled very well and it is energy efficient. The method is called low-frequency heating (LFH) since the current is injected at a much lower frequency than the nominal of the grid, which is normally 50 or 60 Hz. A lower frequency reduces the effect of theinductance in the transformer, so the voltage needed to induce the current can be reduced. The LFH drying method is also used for service of older transformers.

Terminals

Very small transformers will have wire leads connected directly to the ends of the coils, and brought out to the base of the unit for circuit connections. Larger transformers may have heavy bolted terminals, bus bars or high-voltage insulated bushings made of polymers or porcelain.

An ideal Transformer

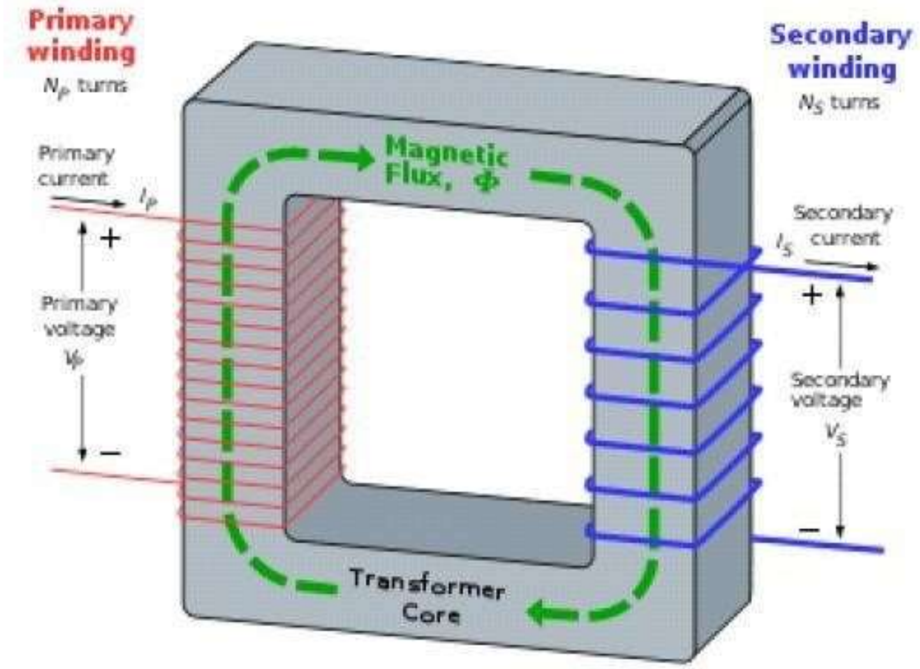


Figure 2.1.7 Basic principle of operation

[Source: ""*Electric Machinery Fundamentals" by* Stephen J. Chapman, *Page: 104]*

An ideal transformer. The secondary current arises from the action of the secondary EMF on the (not shown) load impedance. The transformer is based on two principles: first, that an electric current can produce a magnetic field (electromagnetism) and second that a changing magnetic field within a coil of wire induces a voltage across the ends of the coil (electromagnetic induction). Changing the current in the primary coil changes the magnetic flux that is developed. The changing magnetic flux induces a voltage in the secondary coil An ideal transformer is shown in the adjacent figure. Current passing through the primary coil creates a magnetic field. The primary and secondary coils are wrapped around a core of very high magnetic, such as iron, so that most of the magnetic flux passes through both the primary and secondary coils. If a load is connected to the secondary winding, the load current and voltage will be in the directions indicated, given the primary current and voltage in the directions indicated.

Induction Law

The voltage induced across the secondary coil may be calculated from Faraday's law of induction, which states that:

$$V_s = N_s \frac{d\Phi}{dt},$$

where *V*s is the instantaneous voltage, *N*s is the number of turns in the secondary coil and Φ is the magnetic flux through one turn of the coil. If the turns of the coil are oriented perpendicularly to the magnetic field lines, the flux is the product of the magnetic flux

$$V_p = N_p \frac{d\Phi}{dt}.$$

density *B* and the area *A* through which it cuts. The area is constant, being equal to the cross-sectional area of the transformer core, whereas the magnetic field varies with time according to the excitation of the primary. Since the same magnetic flux passes through both the primary and secondary coils in an ideal transformer, the instantaneous voltage across the primary winding equals

Taking the ratio of the two equations for *V*s and *V*p gives the basic equation for stepping up or stepping down the voltage

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

*N*p/*N*s is known as the *turns ratio*, and is the primary functional characteristic of any transformer. In the case of step-up transformers, this may sometimes be stated as the reciprocal, *N*s/*N*p. *Turns ratio* is commonly expressed as an irreducible fraction or ratio: for example, a transformer with primary and secondary windings of, respectively, 100 and 150 turns is said to have a turns ratio of 2:3 rather than 0.667 or 100:150.

An elementary transformer consists of a soft iron or silicon steel core and two windings, placed on it. The windings are insulated from both the core and each other. The core is built up of thin soft iron or low reluctance to the magnetic flux. The winding connected to the magnetic flux. The winding connected to the supply main is called the primary and the winding connected to the load circuit is called the secondary.

When primary winding is connected to an ac supply mains, current flows through it. Since this winding links with an iron core, so current flowing through this winding produces an alternating flux in the core. Since this flux is alternating and links with the secondary winding also, so induces an emf in the secondary winding.

The frequency of induced emf in secondary winding is the same as that of the flux or that of the s supply voltage. The induced emf in the secondary winding enables it to deliver current to an external load connected across it. Thus the energy is transformed from primary winding to the secondary winding by means of electro- magnetic induction without any change in frequency. The flux of the iron core links not only with the secondary winding but also with the primary winding, so produces self-induced emf in the primary winding

This induced in the primary winding opposes the applied voltage and therefore sometimes it is known as back emf of the primary. In fact the induced emf in the primary winding limits the primary current in much the same way that the back emf in a dc motor limits the armature current.

Transformation ratio.

The ratio of secondary voltage to primary voltage is known as the voltage transformation ratio and is designated by letter K. i.e. Voltage transformation ratio. Current ratio**.** The ratio of secondary current to primary current is known as current ratio and is reciprocal of voltage transformation ratio in an ideal transformer.

2.2 Equivalent Circuit Parameters

The electrical circuit for any electrical engineering device can be drawn if the equations describing its behavior are known. The equivalent circuit for electromagnetic device is a combination of resistances, inductances, capacitances, voltages etc. In the equivalent circuit, (R1+jX1) and (R2+jX2) are the leakage impedances of the primary and secondary windings respectively. The primary current I1 consists of two components. One component, I1´ is the load component and the second is no-load current Io which is composed of Ic and Im. The current Ic is in phase with E1 and the product of these two gives core loss. Ro represents the core loss and is called core-loss resistance. The current Im is represented by a reactance Xo and is called magnetizing reactance. The transformer magnetization curve is assumed linear, since the effect of higher order harmonics can’t be represented in the equivalent circuit. In transformer analysis, it is usual to transfer the secondary quantities to primary side or primary quantities to secondary side.

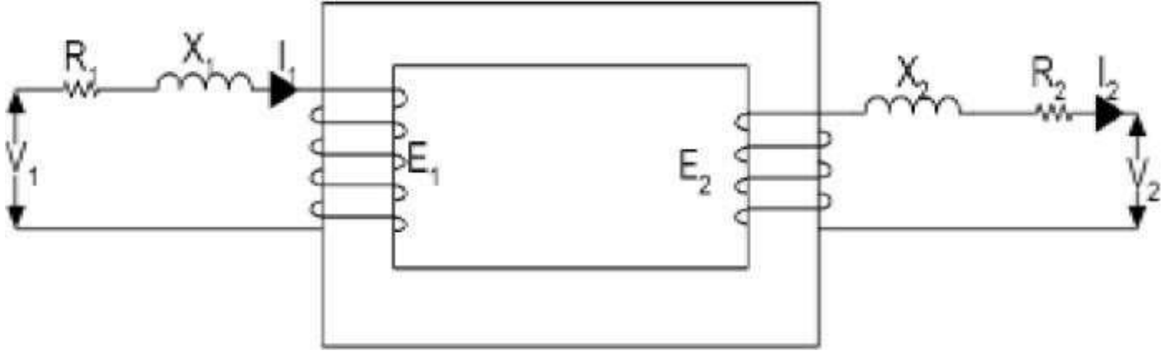


Fig 2.2.1 (a) Exciting current neglected

[Source: “‘Electric Machinery Fundamentals” by Stephen J. Chapman, *Page: 112]*

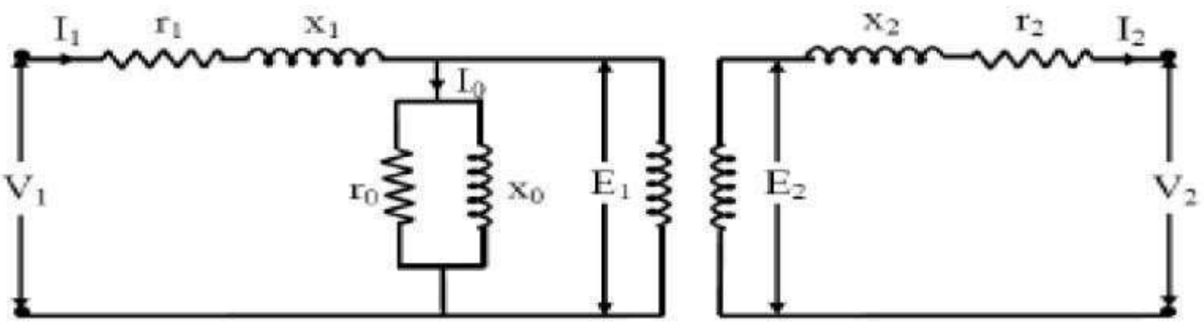


Fig 2.2.1 (b) Exact Equivalent Circuit

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 113]*

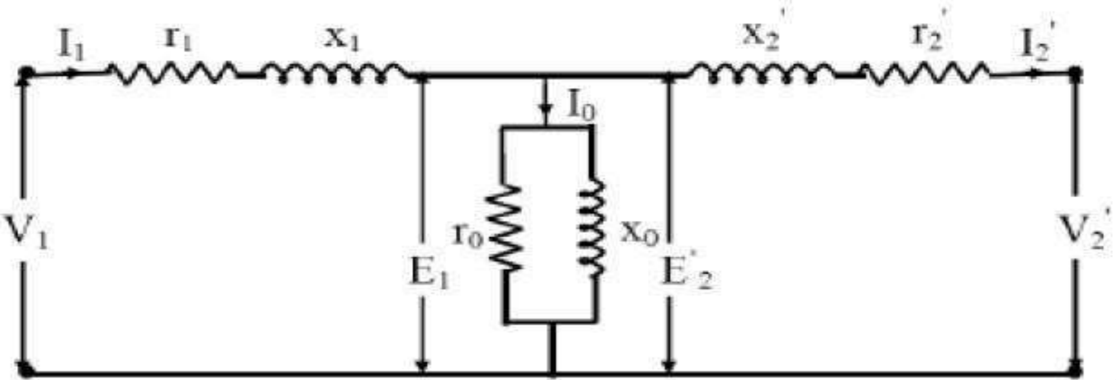


Fig 2.2.1 (c) Equivalent Circuit in general form

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 113]*

Transformer Losses

1. Primary and Secondary copper loss
2. Iron loss
3. Dielectric loss
4. Stray load loss

Primary and secondary copper losses take place in the respective winding resistances due to the flow of the current in them. The primary and secondary resistances differ from their D.C values due to skin effect and the temperature rise of the windings. While the average temperature rise can be approximately used, the skin effect is harder to get analytically. The short circuit test gives the value of Re taking into account the skin effect.

The iron losses contain two components - Hysteresis loss and Eddy current loss. The Hysteresis loss is a function of the material used for the core. Ph = KhB1.6f For constant voltage and constant frequency operation this can be taken to be constant. The eddy current loss in the core arises because of the induced emf in the steel lamination sheets and the eddies of current formed due to it. This again produces a power loss Pe in the lamination. Where t is the thickness of the steel lamination used. As the lamination thickness is much smaller than the depth of penetration of the field, the eddy current loss can be reduced by reducing the thickness of the lamination. Present day laminations are of 0.25 mm thickness and are capable of operation at 2 Tesla. These reduce the eddy current losses in the core. This loss also remains constant due to constant voltage and frequency of operation. The sum of hysteresis and eddy current losses can be obtained by the open circuit test.

The dielectric losses take place in the insulation of the transformer due to the large electric stress. In the case of low voltage transformers this can be neglected. For constant voltage operation this can be assumed to be a constant. The stray load losses arise out of the leakage fluxes of the transformer. These leakage fluxes link the metallic structural parts, tank etc. and produce eddy current losses in them. Thus they take place ’all round’ the transformer instead of a definite place , hence the name ’stray’. Also the leakage flux is directly proportional to the load current unlike the mutual flux which is proportional to the applied voltage. Hence this loss is called ’stray load’ loss. This can also be estimated experimentally.

It can be modeled by another resistance in the series branch in the equivalent circuit. The stray load losses are very low in air-cored transformers due to the absence of the metallic tank. Thus, the different losses fall in to two categories Constant losses (mainly voltage dependent) and Variable losses (current dependent). The expression for the efficiency of the transformer operating at a fractional load x of its rating, at a load power factor of 2, can be written as losses and Pvar the variable losses at full load. For a given power factor an expression for in terms of the variable x is thus obtained. By differentiating with respect to x and equating the same to zero, the condition for maximum efficiency is obtained.

The maximum efficiency it can be easily deduced that this maximum value increases with increase in power factor and is zero at zero power factor of the load. It may be considered a good practice to select the operating load point to be at the maximum efficiency point. Thus if a transformer is on full load, for most part of the time then the max can be made to occur at full load by proper selection of constant and variable losses. However, in the modern transformers the iron losses are so low that it is practically impossible to reduce the full load copper losses to that value. Such a design wastes lot of copper. This point is illustrated with the help of an example below. Two 100 kVA transformers And B are taken. Both transformers have total full load losses to be 2 kW. The break up of this loss is chosen to be different for the two transformers. Transformer A: iron loss 1 kW, and copper loss is 1 kW. The maximum efficiency of 98.04%occurs at full load at unity power factor. Transformer B: Iron loss =0.3 kW and full load copper loss =1.7 kW. This also has a full load of 98.04%. Its maximum occurs at a fractional load of q0.31.7 = 0.42. The maximum efficiency at unity power factor being at the corresponding point the transformer A has an efficiency of Transformer A uses iron of more loss per kg at a given flux density, but transformer B uses lesser quantity of copper and works at higher current density.

When the primary of a transformer is connected to the source of an ac supply and the secondary is open circuited, the transformer is said to be on no load. Which will create alternating flux. No-load current, also known as excitation or exciting current has two components the magnetizing component Im and the energy component Ie.

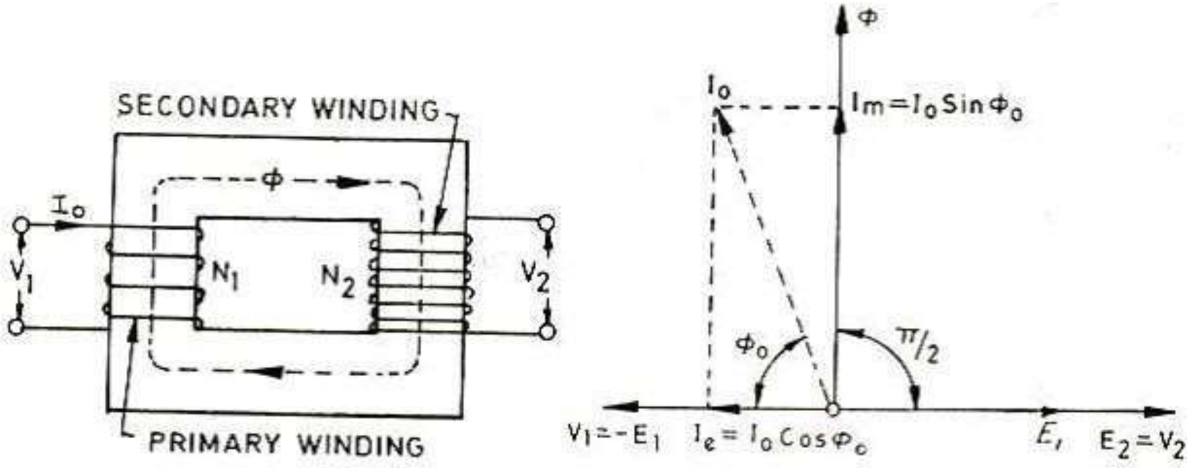


Figure 2.2.2 Transformer on No load

[Source: “‘Electric Machinery Fundamentals” by Stephen J. Chapman, *Page: 116]*

I_m is used to create the flux in the core and Ie is used to overcome the hysteresis and eddy current losses occurring in the core in addition to small amount of copper losses occurring in the primary only (no copper loss occurs in the secondary, because it carries no current, being open circuited.) From vector diagram shown in above it is obvious that

1. Induced emfs in primary and secondary windings, and lag the main flux by and are in phase with each other.
2. Applied voltage to primary and leads the main flux by and is in phase opposition to Secondary voltage is in phase and equal to since there is no voltage drop in secondary.
3. is in phase with and so lags
4. is in phase with the applied voltage .
5. Input power on no load

Transformer on Load

The transformer is said to be loaded, when its secondary circuit is completed through an impedance or load. The magnitude and phase of secondary current (i.e. current flowing through secondary) with respect to secondary terminals depends upon the characteristic of the load i.e. current will be in phase, lag behind and lead the terminal voltage respectively when the load is non-inductive, inductive and capacitive. The net flux passing through the core remains almost constant from no-load to full load irrespective of load conditions and so core losses remain almost constant from no- load to full load.

Secondary windings Resistance and Leakage Reactance In actual practice, both of the primary and have got some ohmic resistance causing voltage drops and copper losses in the windings. In actual practice, the total flux created does not link both of the primary and secondary windings but is divided into three components namely the main or mutual flux linking both of the primary and secondary windings, primary leakage flux linking with primary winding only and secondary leakage flux linking with secondary winding only.

2.5 Per Unit representation of a Transformer

It has been said in Section 4.2 that a three-phase transformer forming part of a three-phase system can be represented by a single-phase transformer in obtaining per phase solution of the system. The delta connected winding of the transformer is replaced by an equivalent star so that the transformation ratio of the equivalent single-phase transformer is always the line-to-line voltage ratio of the three-phase transformer.

Figure represents a single-phase transformer in terms of primary and secondary leakage reactance Z_p and Z_s and an ideal transformer of ratio 1 : a. The magnetizing impedance is neglected. Let us choose a volt ampere base of $(VA)_B$ and voltage bases on the two sides of the transformer in the ratio of transformation,

i.e.

$$=$$

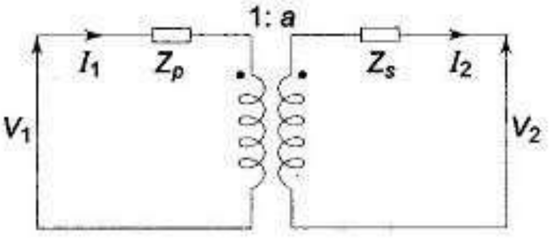


Figure 2.5.1(a) Representation of single phase transformer (Magnetizing impedance Neglected)

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 154]*

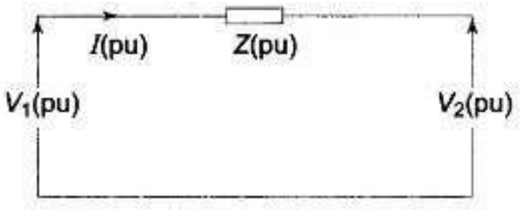


Figure 2.5.1(b) Per Unit quivalent circuit of a single phase transformer

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 154]*

Therefore, $$\frac{I_{1B}}{I_{2B}} = a \text{ (as } (VA)_B \text{ is common)} \tag{4.11b}$$

$$Z_{1B} = \frac{V_{1B}}{I_{1B}},\; Z_{2B} = \frac{V_{2B}}{I_{2B}} \tag{4.11c}$$

From Fig. 4.7a we can write

$$V_2 = (V_1 - I_1 Z_p)\, a - I_2 Z_S \tag{4.12}$$

We shall convert Eq. (4.12) into per unit form

$$V_2(\text{pu})V_{2B} = [V_1(\text{pu})V_{1B} - I_1(\text{pu})I_{1B}Z_p(\text{pu})Z_{1B}]a$$
$$-I_2(\text{pu})I_{2B}Z_s(\text{pu})Z_{2B}$$

Dividing by V_{2B} throughout and using base relations (4.11 a, b, c), we get

$$V_2(\text{pu}) = V_1(\text{pu}) - I_1(\text{pu})Z_p(\text{pu}) - I_2(\text{pu})Z_s(\text{pu}) \tag{4.13}$$

Now $$\frac{I_1}{I_2} = \frac{I_{1B}}{I_{2B}} = a$$

or $$\frac{I_1}{I_{1B}} = \frac{I_2}{I_{2B}}$$

$\therefore$ $$I_1(\text{pu}) = I_2(\text{pu}) = I\,(\text{pu})$$

Equation (4.13) can therefore be written as

$$V_2(\text{pu}) = V_1(\text{pu}) - I(\text{pu})Z(\text{pu}) \tag{4.14}$$
$$Z(\text{pu}) = Z_p(\text{pu}) + Z_s(\text{pu})$$

Equation (4.14) can be represented by the simple equivalent circuit of Fig. 4.7b which does not require an ideal transformer. Considerable simplification has therefore been achieved by the Per Unit System Definition method with a common voltampere base and voltage bases on the two sides in the ratio of transformation.

Z(pu) can be determined directly from the equivalent impedance on primary secondary side of a transformer by using the appropriate impedance base.

On primary side:

$$Z_1 = Z_p + Z_s/a^2$$

$$Z_1(\text{pu}) = \frac{Z_1}{Z_{1B}} = \frac{Z_p}{Z_{1B}} + \frac{Z_s}{Z_{1B}} \times \frac{1}{a^2}$$

$$a^2 Z_{1B} = Z_{2B}$$

$$Z_1(\text{pu}) = Z_p(\text{pu}) + Z_s(\text{pu}) = Z(\text{pu}) \tag{4.15}$$

On secondary side:

$$Z_2 = Z_s + a^2 Z_p$$

$$Z_2(\text{pu}) = \frac{Z_2}{Z_{2B}} = \frac{Z_s}{Z_{2B}} + a^2 \frac{Z_p}{Z_{2B}}$$

$$Z_2(\text{pu}) = Z_s(\text{pu}) + Z_p(\text{pu}) = Z(\text{pu}) \tag{4.16}$$

Thus the per unit impedance of a transformer is the same whether computed from primary or secondary side so long as the voltage bases on the two sides are in the ratio of transformation (equivalent per phase ratio of a three-phase transformer which is the same as the ratio of line-to-line voltage rating).

The pu transformer impedance of a three-phase transformer is conveniently obtained by direct use of three-phase MVA base and line-to-line kV base in relation (4.9). Any other impedance on either side of a transformer is converted to pu value just like Z_p or Z_s.

Inrush Current

When a transformer is switched on from the primary side while keeping its secondary circuit open, it acts as a simple inductance. When the electrical power transformer runs normally, the flux produced in the core is in quadrature with applied voltage as shown in the figure below.

The flux wave will reach its maximum value, 1/4 cycle or π/2 angle later, reaching the maximum value of the voltage wave. As per the waves shown in the figure below, at the instant when the voltage is zero, the corresponding steady state value of flux should be the negative maximum (i.e. minimum value). But it is not practically possible to have flux the instant you switch on the supply to the transformer. This is because there will be no flux linked to the core prior to switching on the supply. The steady state value of flux will not be reached instantly.

This is because the rate of energy transfer to a circuit cannot be infinity. So the flux in the core also will start from its zero value at the time of switching on the transformer. According to Faraday's law of electromagnetic induction the voltage induced across the winding is given as e = dφ/dt. Where φ is the flux in the core. Hence the flux will be integral of the voltage wave, which can be calculated using the formula

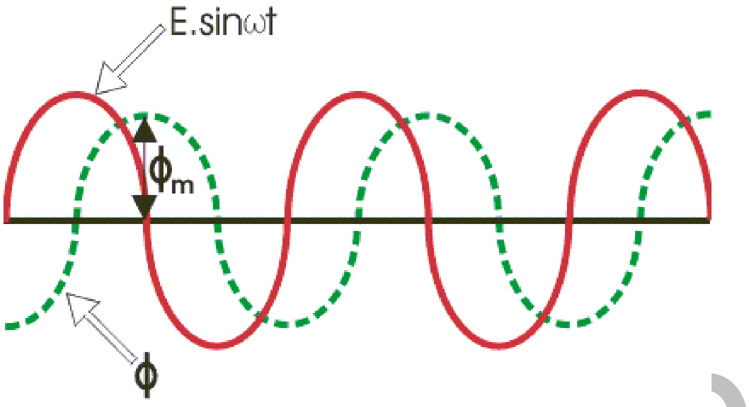


Figure 2.5.2 Inrush Current wavefprm

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 168]*

$$\boxed{e = E.\sin\omega t = d\varphi/dt \Rightarrow \varphi = \int e.dt = E\int \sin\omega t.dt}$$

2.7 Parallel Operation of Transformers

By parallel operation we mean two or more transformers are connected to the same supply bus bars on the primary side and to a common bus bar/load on the secondary side. Such requirement is frequently encountered in practice. The reasons that necessitate parallel operation are as follows.

1. Non-availability of a single large transformer to meet the total load requirement.
2. The power demand might have increased over a time necessitating augmentation of the capacity. More transformers connected in parallel will then be pressed into service.
3. To ensure improved reliability. Even if one of the transformers gets into a fault or is taken out for maintenance/repair the load can continued to be serviced.
4. To reduce the spare capacity. If many smaller size transformers are used one machine can be used as spare. If only one large machine is feeding the load, a spare of similar rating has to be available. The problem of spares becomes more acute with fewer machines in service at a location.
5. When transportation problems limit installation of large transformers at site, it may be easier to transport smaller ones to site and work them in parallel. Fig. 37 shows the physical arrangement of two single phase transformers working in parallel on the primary side. Transformer A and Transformer B are connected to input voltage bus bars. After ascertaining the polarities they are connected to output/load bus bars. Certain conditions have to be met before two or more transformers are connected in parallel and share a common load satisfactorily. They are,

1. The voltage ratio must be the same.
2. The per unit impedance of each machine on its own base must be the same.
3. The polarity must be the same, so that there is no circulating current between the transformers.
4. The phase sequence must be the same and no phase difference must exist between the voltages of the two transformers.

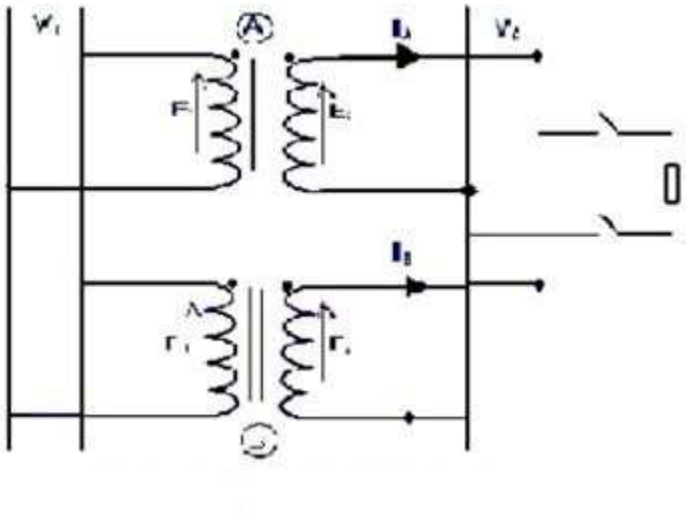


Figure 2.7.1 Parallel Operation of Transformer

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 181]*

Where,

V_1=Load bus voltage

V_2=Supply voltage

These conditions are examined first with reference to single phase transformers and then the three phase cases are discussed. Same voltage ratio generally the turns ratio and voltage ratio are taken to be the same. If the ratio is large there can be considerable error in the voltages even if the turns ratios are the same. When the primaries are connected to same bus bars, if the secondary do not show the same voltage, paralleling them would result in a circulating current between the secondary. Reflected circulating current will be there on the primary side also. Thus even without connecting a load considerable current can be drawn by the transformers and they produce copper losses. In two identical transformers with percentage impedance of 5 percent, a no-load voltage difference of one percent will result in a circulating current of 10 percent of full load current. This circulating current gets added to the load current when the load is connected resulting in unequal sharing of the load. In such cases the combined full load of the two transformers can never be met without one transformer getting overloaded.

Per unit impedance Transformers of different ratings may be required to operate in parallel. If they have to share the total load in proportion to their ratings the larger machine has to draw more current. The voltage drop across each machine has to be the same by virtue of their connection at the input and the output ends. Thus the larger machines have smaller impedance and smaller machines must have larger ohmic impedance. Thus the impedances must be in the inverse ratios of the ratings. As the voltage drops must be the same the per unit impedance of each transformer on its own base, must be equal. In addition if active and reactive powers are required to be shared in proportion to the ratings the impedance angles also must be the same. Thus we have the requirement that per unit resistance and per unit reactance of both the transformers must be the same for proper load sharing. Polarity of connection The polarity of connection in the case of single phase transformers can be either same or opposite. Inside the loop formed by the two secondaries the resulting voltage must be zero.

If wrong polarity is chosen the two voltages get added and short circuit results. In the case of poly phase banks it is possible to have permanent phase error between the phases with substantial circulating current. Such transformer banks must not be connected in parallel. The turn's ratios in such groups can be adjusted to give very close voltage ratios but phase errors cannot be compensated. Phase error of 0.6 degree gives rise to one percent difference in voltage. Hence poly phase transformers belonging to the same vector group alone must be taken for paralleling. Transformers having −30degree angle can be paralleled to that having +30 angle by reversing the phase sequence of both primary and secondary terminals of one of the transformers.

This way one can overcome the problem of the phase angle error. Phase sequence the phase sequence of operation becomes relevant only in the case of poly phase systems. The poly phase banks belonging to same vector group can be connected in parallel. A transformer with +30◦ phase angle however can be paralleled with the one with −30· phase angle; the phase sequence is reversed for one of them both at primary and secondary terminals. If the phase sequences are not the same then the two transformers cannot be connected in parallel even if they belong to same vector group.

cases arise in these problems. Case A: when the voltage ratio of the two transformers is the same and Case B: when the voltage ratios are not the same. These are discussed now in sequence The phase sequence can be found out by the use of a phase sequence indicator. Performance of two or more single phase transformers working in parallel can be computed using their equivalent circuit. In the case of poly phase banks also the approach is identical and the single phase equivalent circuit of the same can be used. Basically two

2.9 Solved Problems

Example 1:

A source which can be represented by a voltage source of 8 V rms in series with an internal resistance of 2 kΩ is connected to a 50-Ω load resistance through an ideal transformer. Calculate the value of turns ratio for which maximum power is supplied to the load and the corresponding load power? Using MATLAB, plot the the power in milli watts supplied to the load as a function of the transformer ratio, covering ratios from 1.0 to 10.0.

Solution:

For maximum power transfer, the load resistance (referred to the primary) must be equal to the source resistance.

$$R_L' = \left(\frac{N_1}{N_2}\right)^2 R_L = n^2 R_L = 2000 \quad \Rightarrow \quad n = \sqrt{\frac{2000}{50}} = 6.32$$

The primary current:

$$I_1 = \frac{V_s}{2R_s} \quad \Rightarrow \text{Power supplied to the load: } P_{load} = R_L' I_1^2 = R_s \frac{V_s^2}{4R_s^2} = \frac{V_s^2}{4R_s} = 8 \text{ mW}$$

For a general turns ratio n:

$$I_1 = \frac{V_s}{R_s + R_L'} = \frac{V_s}{R_s + n^2 R_L} \quad \Rightarrow \quad P_{load} = R_L' I_1^2 = n^2 R_L \left(\frac{V_s}{R_s + n^2 R_L}\right)^2$$

Example 2

A 460-V:2400-V transformer has a series leakage reactance of 37.2 Ω as referred to the high-voltage side. A load connected to the low-voltage side is observed to be absorbing 25 kW, unity power factor, and the voltage is measured to be 450 V. Calculate the corresponding voltage and power factor as measured at the high-voltage terminals.

Solution:

Secondary current:

$$I_2 = \frac{P_{load}}{V_{load}} = \frac{25000}{450} = 55.55\text{ A} \quad \Rightarrow \quad \text{Primary current } I_1 = \frac{460}{2400} \times 55.55 = 10.65\text{ A}$$

Primary voltage: $V_1 = j37.2\ I_1 + V_2' \qquad V_2' = \frac{2400}{460} \times 450 = 2347.8\text{ V}$

$$\Rightarrow V_1 = j37.2\ I_1 + V_2' = j37.2 \times 10.65 + 2347.8 = 2347.8 + j396.18 = 2381.0\angle 9.58^\circ\text{ V}$$

Power factor at primary terminals: cos(9.58) = 0.9861 lagging

Example 3:

The resistances and leakage reactances of a 30-kVA, 60-Hz, 2400-V:240-V distribution transformer are $R_1 = 0.68\ \Omega$, $R_2 = 0.0068\ \Omega$, $X_{l1} = 7.8\ \Omega$, $X_{l2} = 0.0780\ \Omega$ where subscript 1 denotes the 2400-V winding and subscript 2 denotes the 240-V winding. Each quantity is referred to its own side of the transformer.

a. Draw the equivalent circuit referred to (i) the high- and (ii) the low-voltage sides. Label the impedances numerically.

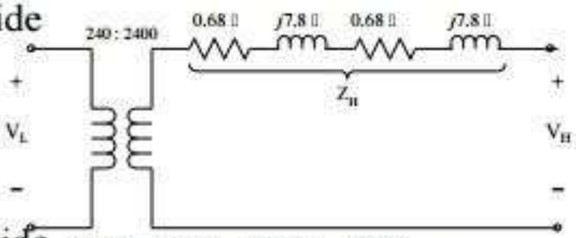
b. Consider the transformer to deliver its rated kVA to a load on the low-voltage side with 230 V across the load. (i) Find the high-side terminal voltage for a load power factor of 0.85 lagging. (ii) Find the high-side terminal voltage for a load power factor of 0.85 leading.

c. Consider a rated-kVA load connected at the low-voltage terminals operating at 240V. Use MATLAB to plot the high-side terminal voltage as a function of the power-factor angle as the load power factor varies from 0.6 leading through unity power factor to 0.6 pf lagging.

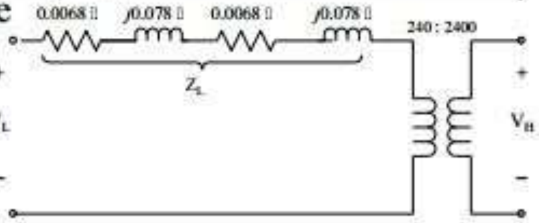
Solution:

(a)

(i) referred to the HV side



(ii) referred to the LV side



(b) Using the equivalent circuit referred to the HV side, $V_L = 230\angle 0^\circ$ V

Load current: $I_{load} = \dfrac{30000}{230}\angle\phi = 93.8\angle\phi$ A where ϕ is the pf angle ($\phi > 0$ for leading pf).

Referred to the HV side:
$I_H = 9.38\angle\phi$ A $\Rightarrow$ $V_H = V_L' + Z_H I_H = 2300\angle 0^\circ + (1.36 + j15.6)9.38\angle\phi$

$$V_H = 2300 + 12.7568\cos\phi - 146.328\sin\phi + j(146.328\cos\phi + 12.7568\sin\phi)$$

pf = 0.85 leading $\phi = 31.79^\circ$ $\Rightarrow$ $V_H = 2233.76 + j131.1 = 2237.6\angle 3.36^\circ$ V

pf = 0.85 lagging $\phi = -31.79^\circ$ $\Rightarrow$ $V_H = 2387.93 + j117.66 = 2390.83\angle 2.82^\circ$ V

Example 4:

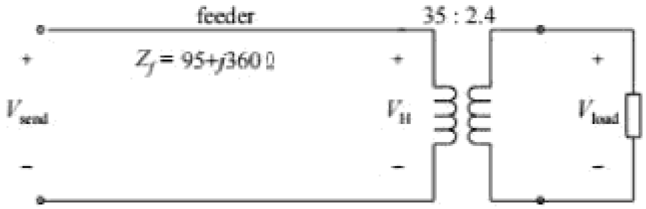
A single-phase load is supplied through a 35-kV feeder whose impedance is 95 + *j*360 Ω and a 35-kV:2400-V transformer whose equivalent impedance is (0.23 + *j*1.27) Ω referred to its low-voltage side. The load is 160 kW at 0.89 leading power factor and 2340 V.

a. Compute the voltage at the high-voltage terminals of the transformer.

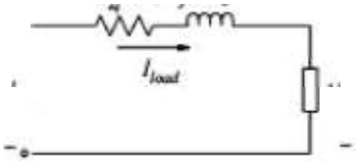
b. Compute the voltage at the sending end of the feeder.

Compute the power and reactive power input at the sending end of the feeder.

Solution:



(a) Equivalent circuit for the transformer and load.



$P_L = |V_{load}|.|I_{load}|.\cos\theta \Rightarrow |I_{load}| = \frac{hx1o'}{2340x0.89} = 76.83$ A

8 = cos*‘ 0 89 = 27 13 leading I = 76. fi3 27 1 T A

The HV side collage referred to the LV side:

$$V'_H = Z_{eq} I_{load} + V_L = (0.23 + j1.27)(76.83\angle 27.13^\circ) + 2340 = 2311.2 + j94.9 \text{ V}$$

$$\Rightarrow V_H = \frac{35}{2.4} \times V'_H = 33.71 + j1.384 \text{ kV} \quad \Rightarrow \quad |V_H| = 33.734 \text{ kV}$$

(b) Load current referred to the HV side:

$$I'_{load} = I_{feed} = \frac{2.4}{35} \times 76.83\angle 27.13^\circ = 5.2683\angle 27.13^\circ \text{ A}$$

$$V_{send} = Z_f I_{feed} + V_H = (95 + j360) \times 5.2683\angle 27.13^\circ + 3371 + j1384 = 33.286 + j3.3 \text{ kV}$$

$$|V_{send}| = 33.45 \quad \text{kV}$$

Example 5:

The following data were obtained for a 20-kVA, 60-Hz, 2400:240-V distribution transformer tested at 60 Hz:

	Voltage, V	Current, A	Power, W
With high-voltage winding open-circuited	240	1.038	122
With low-voltage terminals short-circuited	61.3	8.33	257

a. Compute the efficiency at full-load current and the rated terminal voltage at 0.8 power factor.

b. Assume that the load power factor is varied while the load current and secondary terminal voltage are held constant. Use a phasor diagram to determine the load power factor for which the regulation is greatest. What is this regulation?

Solution:

(a) Rated current on the HV side = 20 kVA / 2400 = 8.33 A. Therefore, total power loss at full load current:

P_L= 122 + 257 = 379 W. Load power at full load, 0.8 pf = 0.8 × 20 kW = 16 kW. Therefore, input power = 16 + 0.379 = 16.379 kW $\Rightarrow$ efficiency = (16 / 16.379) × 100% = 97.7 %.

(b) The equivalent impedance of the transformer: $Z_{eq,H} = R_{eq,H} + jX_{eq,H}$

$$R_{eq,H} = \frac{P_{sc}}{I_{sc}^2} = \frac{257}{8.33^2} = 3.7\ \Omega \quad |Z_{eq,H}| = \frac{V_{sc}}{I_{sc}} = \frac{61.3}{8.33} = 7.36\ \Omega$$

$$\Rightarrow \quad X_{eq,H} = \sqrt{(Z_{eq,H})^2 - (R_{eq,H})^2} = 6.36\ \Omega$$

Let load current and voltage referred to the HV side: $V_{lH} = V\angle 0^{\circ} \quad I_{lH} = I\angle\theta$

$$V_s = V_{IH} + Z_{eq} I_{IH} = V + \left|Z_{eq} I_{IH}\right| \angle(\theta + \phi_Z) = V + V_d \angle \alpha \qquad V_d \square \left|Z_{eq} I_{IH}\right| \text{ and } \alpha \square \theta + \phi_Z$$

$$= (V + V_d \cos\alpha) + jV_d \sin\alpha$$

$$\left|V_s\right| = \sqrt{(V + V_d \cos\alpha)^2 + (V_d \sin\alpha)^2} = \sqrt{V^2 + 2VV_d \cos\alpha + V_d^2}$$

$$\text{Regulation } = \frac{\left|V_s\right| - \left|V_{IH}\right|}{\left|V_{IH}\right|} = \frac{\sqrt{V^2 + 2VV_d \cos\alpha + V_d^2} - V}{V} = \sqrt{1 + \frac{2V_d \cos\alpha}{V} + \frac{V_d^2}{V^2}} - 1$$

Therefore, regulation is maximum when $\cos\alpha$ is maximum

$$\Rightarrow \cos\alpha = 1 \;\Rightarrow\; \alpha = \theta + \phi_Z = 0 \;\Rightarrow\; \theta = -\phi_Z = -\tan^{-1}\left(\frac{X_{eqH}}{R_{eqH}}\right) = -59.81^\circ$$

Maximum regulation:

$$V_d = 7.36 \times 8.33 = 61.31 \text{ V}$$

When $\alpha = 0^\circ \quad V_s = V + V_d \;\Rightarrow\; \text{Regulation} = \frac{V_d}{V} = \frac{61.31}{2400} = 0.026 = 2.6\%$

Example 6:

A three-phase generator step-up transformer is rated 26-kV:345-kV, 850 MVA and has a series impedance of 0.0035 + j0.087 per unit on this base. It is connected to a 26-kV, 800-MVA generator, which can be represented as a voltage source in series with a reactance of *j*1.57 per unit on the generator base.

(a) Convert the per unit generator reactance to the step-up transformer base.

(b) The unit is supplying 700 MW at 345 kV and 0.95 power factor lagging to the system at the transformer high-voltage terminals.

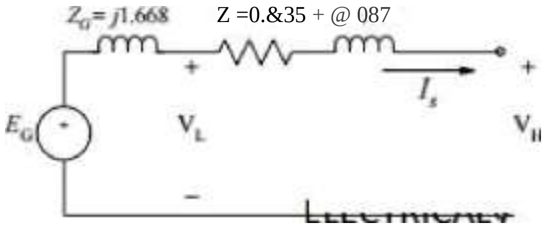
(i) Calculate the transformer low-side voltage and the generator internal voltage behind its reactance in kV.

(ii) Find the generator output power in MW and the power factor.

Solution:

(a) On the transformer base $X_{gen} = 1.57 \times \left(\frac{850}{800}\right)$=1.668pu

(b) Per-unit equivalent circuit.



(i) Transformer how-side voltage and genemtor internal voltage.

$$V_{base,H} = 345\ \text{kV}, \quad V_{base,L} = 26\ \text{kV}, \quad VA_{base} = 850\ \text{MVA}$$

$$V_H = 1.0\angle 0^\circ\ \text{pu.}$$

$$|I_s| = \frac{700}{\sqrt{3}\times 345\times 0.95}\ \text{kA} = 1.233\ \text{kA} \qquad I_{base,H} = \frac{VA_{base}}{\sqrt{3}\times V_{base,H}} = \frac{850}{\sqrt{3}\times 345}\ \text{kA} = 1.4225\ \text{kA}$$

$$|I_s| = \frac{1.233}{1.4225} = 0.8668\ \text{pu.} \qquad I_s = 0.8668\angle -18.2^\circ\ \text{pu.}$$

$$\text{OR} \qquad P = \frac{700}{850} = 0.8235\ \text{pu.} \ \Rightarrow\ |I_s|_{pu} = \frac{P}{|V_H|_{pu}\cos\theta} = \frac{0.8235}{1\times 0.95} = 0.8668\ \text{pu.}$$

$$V_L = V_H + (0.0035 + j0.087)I_s = 1.0264 + j0.071\ \text{pu.} = 1.0289\angle 3.94^\circ$$

$$\Rightarrow\ |V_L| = 1.0289\ \text{pu.} = 26.75\ \text{kV}$$

$$E_G = V_L + (j1.668)I_s = 1.478 + j1.4442\ \text{pu.} = 2.0664\angle 44.34^\circ\ \text{pu.} \quad |E_G| = 26\times 2.0664 = 53.73\ \text{kV}$$

(ii) Generator output power (at its terminals)

$$S_G = V_L I_s^* = 1.0289\angle 3.94^\circ \times 0.8668\angle 18.2^\circ = 0.8261 + j0.3361 \text{ pu.}$$
$$P_G = 0.8261 \times 850 = 702.19 \text{ MW}$$
$$\text{power factor } = \cos(\tan^{-1}\frac{0.3361}{0.8261}) = 0.9263 \text{ lagging}$$

2.4 Sumpner Test

Sumpner's test or back to back test can be employed only when two Identical transformers are available. Both transformers are connected to supply such that one transformer is loaded on another. Primaries of the two identical transformers are connected in parallel across a supply. Secondaries are connected in series such that emf's of them are opposite to each other. Another low voltage supply is connected in series with secondary's to get the readings, as shown in the circuit diagram shown below.

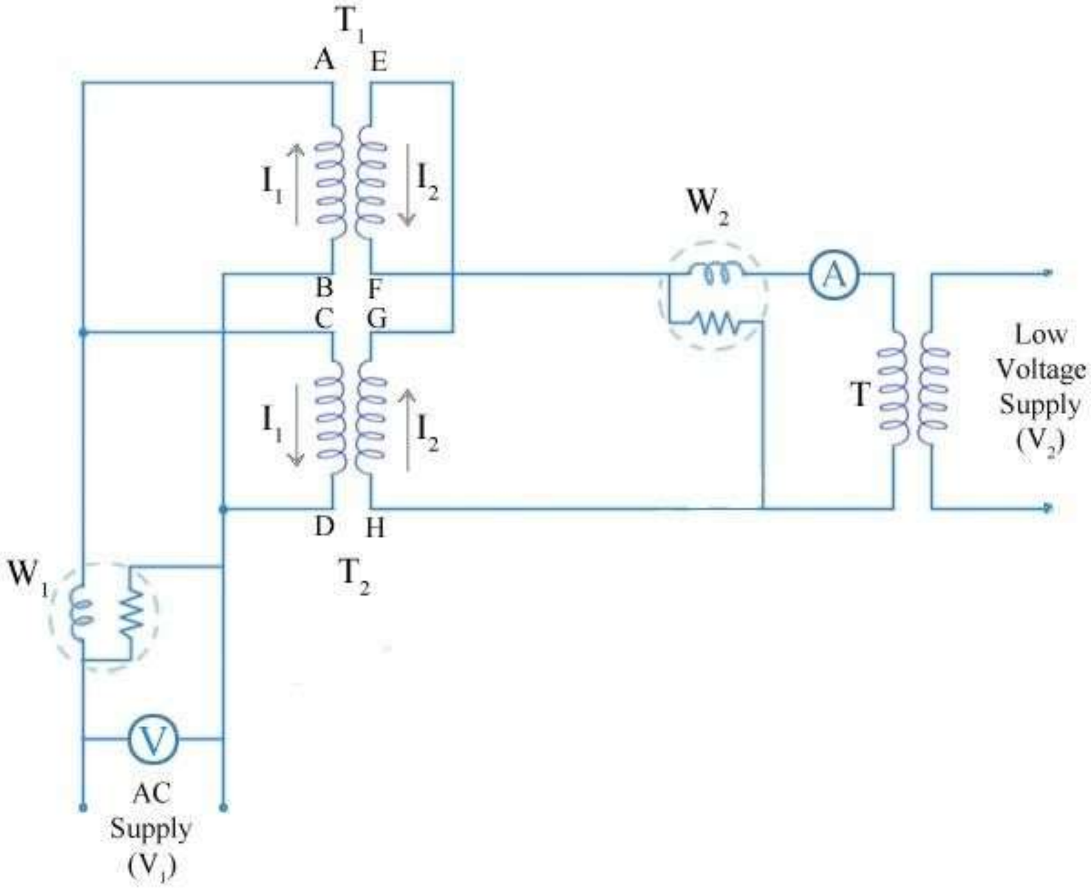


Figure 2.4.1 Sumpner Test

[Source: “‘Electric Machinery Fundamentals” by Stephen J. Chapman, *Page: 135]*

In above diagram, T_1 and T_2 are identical transformers. Secondary of them are connected in voltage opposition, i.e. E_{EF} and E_{GH}. Both the emf's cancel each other, as transformers are identical. In this case, as per superposition theorem, no current flows through secondary. And thus the no load test is simulated. The current drawn from V_1 is

$2I_0$, where I_0 is equal to no load current of each transformer. Thus input power measured by wattmeter W_1 is equal to iron losses of both transformers.

i.e. iron loss per transformer $Pi = W_1/2$.

Now, a small voltage V_2 is injected into secondary with the help of a low voltage transformer. The voltage V_2 is adjusted so that, the rated current I_2 flows through the secondary. In this case, both primaries and secondary carry rated current. Thus short circuit test is simulated and wattmeter W_2 shows total full load copper losses of both transformers.

i.e. copper loss per transformer $P_{Cu} = W_2/2$.

From above test results, the full load efficiency of each transformer can be given as -

$$\begin{array}{c}\text{\% full load efficiency} \\ \text{of each transformer}\end{array} = \frac{\text{output}}{\text{output} + \frac{W_1}{2} + \frac{W_2}{2}} \times 100$$

2.3 Transformer Testing

- Open-circuit or no-load test
- Short circuit or impedance test

Open-circuit or No-load Test.

In this test secondary (usually high voltage) winding is left open, all metering instruments (ammeter, voltmeter and wattmeter) are connected on primary side and normal rated voltage is applied to the primary (low voltage) winding, as illustrated below.

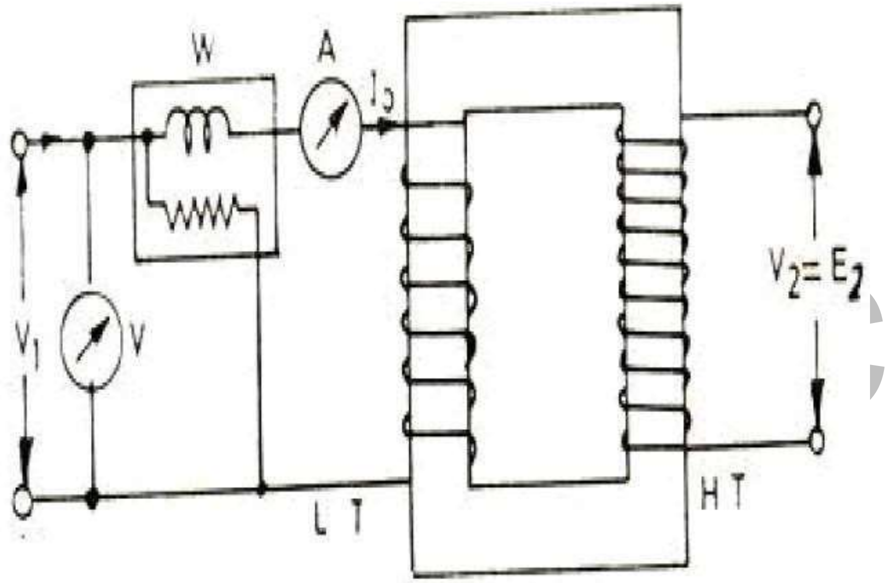


Figure. 2.3.1 Open Circuit

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 124]*

Iron loss = Input power on no-load W0 watts (wattmeter reading) No-load current = 0 amperes (ammeter reading).

no load current I0 is very small, therefore, pressure coils of watt meter and the volt meter should be connected such that the current taken by them should not flow through the current taken by them should not flow through the current coil of the watt meter.

Short-circuit or Impedance Test

This test is performed to determine the full-load copper loss and equivalent resistance and reactance referred to secondary side. In this test, the terminals of the secondary (usually the low voltage) winding are short circuited, all meters (ammeter, voltmeter and wattmeter) are connected on primary side and a low voltage, usually 5 to 10 % of normal rated primary voltage at normal frequency is applied to the primary, as shown in fig below. The applied voltage to the primary, say Vs' is gradually increased till the ammeter A indicates the full load current of the side in which it is connected. The reading Ws of the wattmeter gives total copper loss (iron losses being negligible due to very low applied voltage resulting in very small flux linking with the core) at full load. Let the ammeter reading be Is.

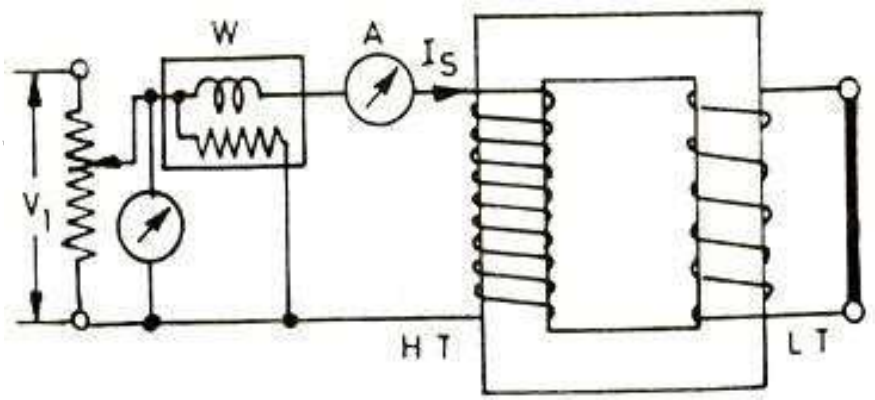


Figure 2.3.2 Short Circuit

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 126]*

Equivalent impedance referred to primary= Commercial Efficiency and All day Efficiency (a) Commercial Efficiency. Commercial efficiency is defined as the ratio of power output to power input in kilowatts.(b) All-day Efficiency**.** The all day efficiency is defined as the ratio of output in kwh to the input in kwh during the whole day. Transformers used for distribution are connected for the whole day to the line but loaded intermittently. Thus the core losses occur for the whole day but copper losses occur only when the transformer is delivering the load current. Hence if the transformer is not used to supply the load current for the whole day all day efficiency will be less than commercial efficiency. The efficiency (commercial efficiency) will be maximum when variable losses (copper losses) are equal to constant losses (iron or core losses).sign is for inductive load and sign is for capacitive load Transformer efficiency, Where x is the ratio of secondary current I2 and rated full load secondary current.

Efficiency

Transformers which are connected to the power supplies and loads and are in operation are required to handle load current and power as per the requirements of the load. An unloaded transformer draws only the magnetization current on the primary side, the secondary current being zero. As the load is increased the primary and secondary currents increase as per the load requirements. The volt amperes and wattage handled by the transformer also increases. Due to the presence of no load losses and I2R losses in the windings certain amount of electrical energy gets dissipated as heat inside the transformer.

This gives rise to the concept of efficiency. Efficiency of a power equipment is defined at any load as the ratio of the power output to the power input. Putting in the form of an expression, while the efficiency tells us the fraction of the input power delivered to the load, the deficiency focuses our attention on losses taking place inside transformer. As a matter of fact the losses heat up machine.

The temperature rise decides the rating of the equipment. The temperature rise of the machine is a function of heat generated the structural configuration, method of cooling and type of loading (or duty cycle of load). The peak temperature attained directly affects the life of the insulations of the machine for any class of insulation.

These aspects are briefly mentioned under section load test. The losses that take place inside the machine expressed as a fraction of the input is sometimes termed as deficiency. Except in the case of an ideal machine, a certain fraction of the input power gets lost inside the machine while handling the power. Thus the value for the efficiency is always less than one. In the case of a.c. machines the rating is expressed in terms of apparent power. It is nothing but the product of the applied voltage and the current drawn. The actual power delivered is a function of the power factor at which this current is drawn.

As the reactive power shuttles between the source and the load and has a zero average value over a cycle of the supply wave it does not have any direct effect on the efficiency. The reactive power however increases the current handled by the machine and the losses resulting from it. Therefore the losses that take place inside a transformer at any given load play a vital role in determining the efficiency.

Primary and secondary copper losses take place in the respective winding resistances due to the flow of the current in them. The primary and secondary resistances differ from their d.c. values due to skin effect and the temperature rise of the windings. While the average temperature rise can be approximately used, the skin effect is harder to get analytically. The short circuit test gives the value of Re taking into account the skin effect. The iron losses contain two components Hysteresis loss and Eddy current loss. The Hysteresis loss is a function of the material used for the core. For constant voltage(current varied) and constant frequency operation this can be taken to be constant. The eddy current loss in the core arises because of the induced emf in the steel lamination sheets and the eddies of current formed due to it. This again produces a power loss Pe in the lamination. Where t is the thickness of the steel lamination used. As the lamination thickness is much smaller than the depth of penetration of the field, the eddy current loss can be reduced by reducing the thickness of the lamination. Present day laminations are of 0.25 mm thickness and are capable of operation at 2 Tesla.

These reduce the eddy current losses in the core. This loss also remains constant due to constant voltage and frequency of operation. The sum of hysteresis and eddy current losses can be obtained by the open circuit test.

The dielectric losses take place in the insulation of the transformer due to the large electric stress. In the case of low voltage transformers this can be neglected. For constant voltage operation this can be assumed to be a constant. The stray load losses arise out of the leakage fluxes of the transformer. These leakage fluxes link the metallic structural parts, tank etc. and produce eddy current losses in them. Thus they take place ’all round’ the transformer instead of a definite place, hence the name ’stray’. Also the leakage flux is directly proportional to the load current unlike the mutual flux which is proportional to the applied voltage.

Hence this loss is called ’stray load’ loss. This can also be estimated experimentally. It can be modeled by another resistance in the series branch in the equivalent circuit. The stray load losses are very low in air-cored transformers due to the absence of the metallic tank. Thus, the different losses fall in to two categories Constant losses (mainly voltage dependent) and Variable losses (current dependent). The expression for the efficiency of the transformer operating at a fractional load x of its rating, at a load power factor of 2 can be written as losses and Pvar the variable losses at full load. For a given power factor an expression for _ in terms of the variable x is thus obtained. By differentiating with respect to x and equating the same to zero, the condition for maximum efficiency is obtained. The maximum efficiency it can be easily deduced that this Maximum value increases with increase in power factor and is zero at zero power factor of the load. It may be considered a good practice to select the operating load point to be at the maximum efficiency point.

Thus if a transformer is on full load, for most part of the time then the max can be made to occur at full load by proper selection of constant and variable losses. However, in the modern transformers the iron losses are so low that it is practically impossible to reduce the full load copper losses to that value. Such a design wastes lot of copper. This point is illustrated with the help of an example below. Two 100 kVA transformers A and B are taken. Both transformers have total full load losses to be 2 kW. The breakup of this loss is chosen to be different for the two transformers. Transformer A: iron loss 1 kW, and copper loss is 1 kW. The maximum efficiency of 98.04%occurs at full load at unity power factor. Transformer B: Iron loss =0.3 kW and full load copper loss =1.7 kW. This also has a full load of 98.04%. Its maximum occurs at a fractional load of Q0.31.7 = 0.42. The maximum efficiency at unity power factor being at the corresponding point the transformer A has an efficiency of Transformer A uses iron of more loss per kg at a given flux density, but transformer B uses lesser quantity of copper and works at higher current density.

All day Efficiency

Large capacity transformers used in power systems are classified broadly into Power transformers and Distribution transformers. The former variety is seen in generating stations and large substations. Distribution transformers are seen at the distribution substations. The basic difference between the two types arises from the fact that the power transformers are switched in or out of the circuit depending upon the load to be handled by them. Thus at 50% load on the station only 50% of the transformers need to be connected in the circuit. On the other hand a distribution transformer is never switched off. It has to remain in the circuit irrespective of the load connected. In such cases the constant loss of the transformer continues to be dissipated. Hence the concept of energy based efficiency is defined for such transformers. It is called ’all day’ efficiency.

The all day efficiency is thus the ratio of the energy output of the transformer over a day to the corresponding energy input. One day is taken as duration of time over which the load pattern repeats itself. This assumption, however, is far from being true. The power output varies from zero to full load depending on the requirement of the user and the load losses vary as the square of the fractional loads. The no-load losses or constant losses occur throughout the 24 hours. Thus, the comparison of loads on different days becomes difficult. Even the load factor, which is given by the ratio of the average load to rated load, does not give satisfactory results. The calculation of the all day efficiency is illustrated below with an example. The graph of load on the transformer, expressed as a fraction of the full load is plotted against time. In an actual situation the load on the transformer continuously changes. This has been presented by a stepped curve for convenience. For the same load factor different average loss can be there depending upon the values of xi and ti. Hence a better option would be to keep the constant losses very low to keep the all day efficiency high. Variable losses are related to load and are associated with revenue earned. The constant loss on the other hand has to be incurred to make the service available. The concept of all day efficiency may therefore be more useful for comparing two transformers subjected to the same load cycle. The concept of minimizing the lost energy comes into effect right from the time of procurement of the transformer. The constant losses and variable losses are capitalized and added to the material cost of the transformer in order to select the most competitive one, which gives minimum cost taking initial cost and running cost put together. Obviously the iron losses are capitalized more in the process to give an effect to the maximization of energy efficiency. If the load cycle is known at this stage, it can also be incorporated in computation of the best transformer.

Voltage Regulation

With the increase in load on the transformer, there is a change in its terminal voltage. The voltage falls if the load power factor is lagging. It increases if power is leading. The change in secondary terminal voltage from full load to no load, expressed as a percentage of full load voltage is called the percentage voltage regulation of the transformer

% Regulation (E- V)/V x 100

2.6 Three Phase Transformer Connections

Windings of a three phase transformer can be connected in various configurations as

(i) Star-star
(ii) Delta-delta
(iii) Star-delta
(iv) Delta-star
(v) Open delta
(vi) Scott connection.

These configurations are explained below.

Star-Star (Y-Y)

- Star-star connection is generally used for small, high-voltage transformers. Because of star connection, number of required turns/phase is reduced (as phase voltage in star connection is 1/√3 times of line voltage only). Thus, the amount of insulation required is also reduced.
- The ratio of line voltages on the primary side and the secondary side is equal to the transformation ratio of the transformers.
- Line voltages on both sides are in phase with each other.
- This connection can be used only if the connected load is balanced.

Delta-Delta (Δ-Δ)

- This connection is generally used for large, low-voltage transformers. Number of required phase/turns is relatively greater than that for star-star connection.
- The ratio of line voltages on the primary and the secondary side is equal to the transformation ratio of the transformers.
- This connection can be used even for unbalanced loading.
- Another advantage of this type of connection is that even if one transformer is disabled, system can continue to operate in open delta connection but with reduced available capacity.

Star-Delta OR Wye-Delta (Y-Δ)

- The primary winding is star star (Y) connected with grounded neutral and the secondary winding is delta connected.
- This connection is mainly used in step down transformer at the substation end of the transmission line.
- The ratio of secondary to primary line voltage is 1/√3 times the transformation ratio.
- There is 30° shift between the primary and secondary line voltages.

Delta-Star OR Delta-Wye (Δ-Y)

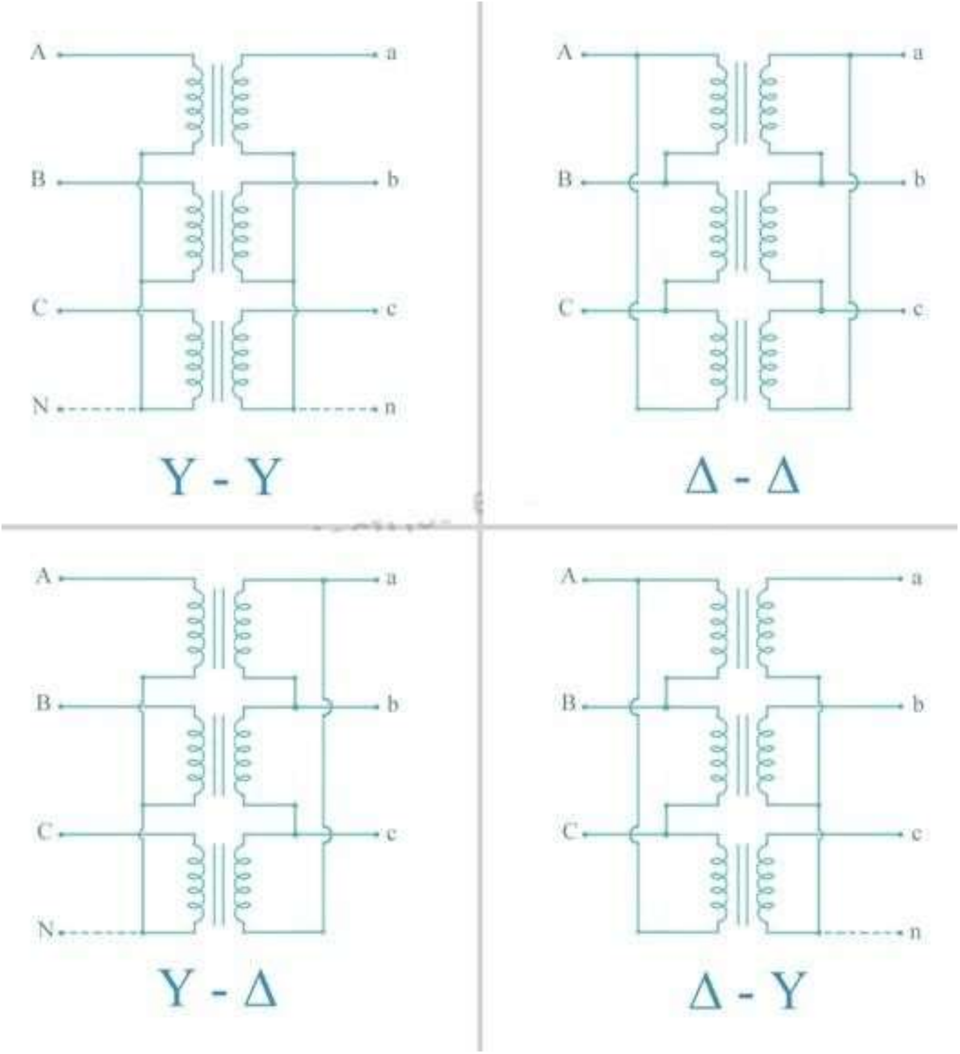


Figure 2.6.1 Three Phase Transformer Connections

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 168]*

- The primary winding is connected in delta and the secondary winding is connected in star with neutral grounded. Thus it can be used to provide 3-phase 4-wire service.
- This type of connection is mainly used in step-up transformer at the beginning of transmission line.
- The ratio of secodary to primary line voltage is √3 times the transformation ratio.
- There is 30° shift between the primary and secondary line voltages.

Above transformer connection configurations are shown in the following figure.

Open Delta (V-V) Connection

Two transformers are used and primary and secondary connections are made as shown in the figure below. Open delta connection can be used when one of the transformers in Δ-Δ bank is disabled and the service is to be continued until the faulty transformer is repaired or replaced. It can also be used for small three phase loads where installation of full three transformer bank is un-necessary. The total load carrying capacity of open delta connection is 57.7% than that would be for delta-delta connection.

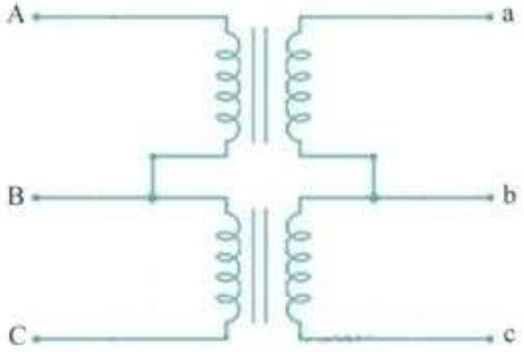


Figure 2.6.2 Open Delta (V-V) Connection

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 169]*

Scott (T-T) Connection

Two transformers are used in this type of connection. One of the transformers has center taps on both primary and secondary windings (which is called as main transformer). The other transformer is called as teaser transformer. Scott connection can also be used for three phase to two phase conversion. The connection is made as shown in the figure below.

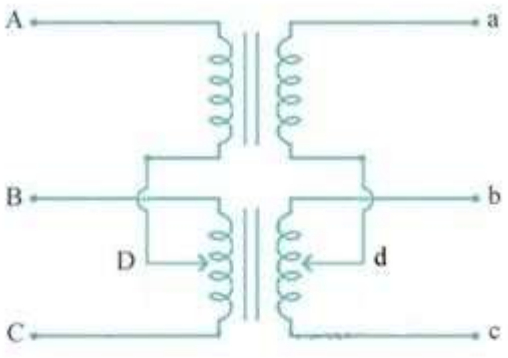


Figure 2.6.3 Scott (T-T) Connection

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 170]*

Transformer Phasing: The Dot Notation and Dot Convention

The Dot Notation

Generally, when we study about Transformers, we assume that the primary and secondary voltage and currents are in phase. But, such is not always the case. In Transformer, The phase relation between primary and secondary currents and

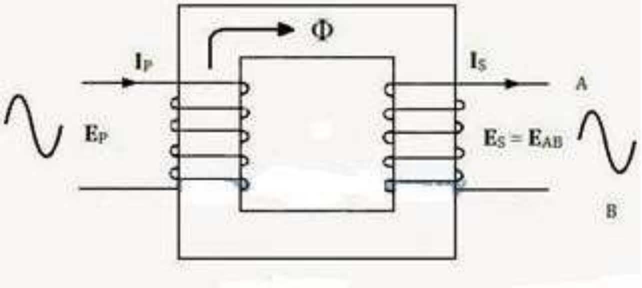


Figure 2.6.4 (a) Primary and Secondar

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 172]*

Refer to fig (1) and (2), you may see that the primary sides of both transformers are identical i.e. primary windings of both transformers wrapped in the same direction around the core.

But in fig (2) you may notice that the secondary winding is wound around the core in the opposite direction from the secondary winding in fig (1).

Consequently, the voltage induced in the Secondary winding in fig (2) is 180° out of phase as compared with the induced voltage in secondary in fig (1) and the direction of secondary current (I_S) is opposite from the primary current (I_P)

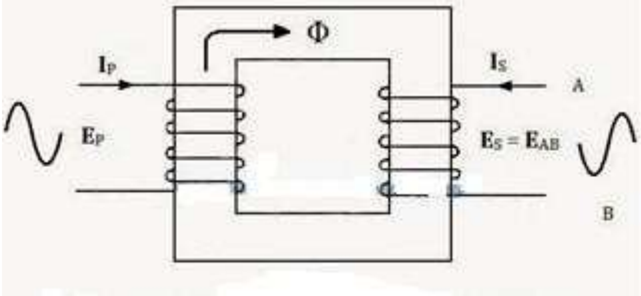


Figure 2.6.4 (b) Primary and Secondary voltage 180° out of Phase

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 173]*

So we see that

1. The primary and secondary voltage and current are in phase in fig (1)
2. The primary and secondary voltage and current are 180° out of phase in fig (2)

Dot Convention

To eliminate any confusion in the phase relation between primary and secondary voltage and current, a dot convention has been adopted for transformer schematic diagrams. Dots are placed on the top of primary and secondary terminals as shown in fig (3) and (4)

In fig (3), we see that dots are placed at the top in both primary and secondary terminals. It shows that the primary and secondary current and voltages are in phase. Moreover, the primary and secondary voltages (V_P and V_S) have similar sine wave, also the primary and secondary (I_P and I_S) currents are same in direction.

The story is opposite in fig (4). We can see that one dot is positioned at the top in primary terminal and the other one (dot) is placed at bottom of secondary terminal. It shows that the primary and secondary current and voltages are 180° out of phase. In addition, the primary and secondary voltages (V_P and V_S) sine waves are opposite to each other. Also the primary and secondary currents ($I_P$and I_S) are opposite in direction.

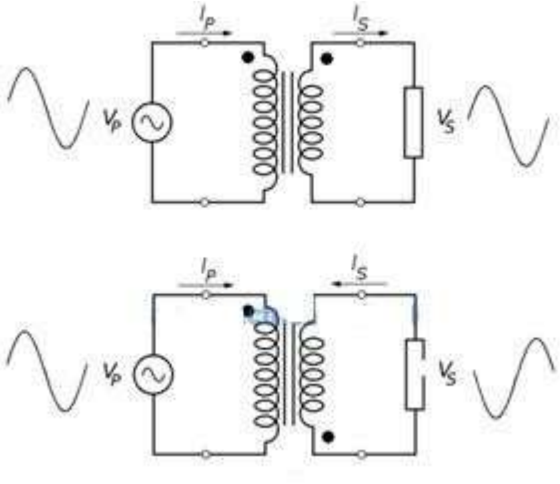


Figure 2.6.5 Phase relation between primary and secondary voltages and currents

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 175]*