Cylindrical Fiber

1. Modes

The exact solution of Maxwell's equations for a cylindrical homogeneous core dielectric waveguide* involves much algebra and yields a complex result. Although the presentation of this mathematics is beyond the scope of this text, it is useful to consider the resulting modal fields. In common with the planar guide TE (where $E_z = 0$) and TM (where $H_z = 0$) modes are obtained within the dielectric cylinder. The cylindrical waveguide, however, is bounded in two dimensions rather than one. Thus two integers, l and m, are necessary in order to specify the modes, in contrast to the single integer (m) required for the planar guide.

For the cylindrical waveguide we therefore refer to TE_{lm} and TM_{lm} modes. These modes correspond to meridional rays (see Section 1.2.1) traveling within the fiber. However, hybrid modes where E_z and H_z are nonzero also occur within the cylindrical waveguide. These modes, which result from skew ray propagation (see Section 1.2.4) within the fiber, are designated HE_{lm} and EH_{lm} depending upon whether the components of **H** or **E** make the larger contribution to the transverse (to the fiber axis) field. Thus an exact description of the modal fields in a step index fiber proves somewhat complicated.

Fortunately, the analysis may be simplified when considering optical fibers for com-munication purposes. These fibers satisfy the weakly guiding approximation where the relative index difference Δ1. This corresponds to small grazing angles θ in Eq. (1.34). In fact is usually less than 0.03 (3%) for optical communications fibers. For weakly guiding structures with dominant forward propagation, mode theory gives dominant transverse field components. Hence approximate solutions for the full set of HE, EH, TE and TM modes may be given by two linearly polarized components.

These linearly polarized (LP) modes are not exact modes of the fiber except for the fundamental (lowest order) mode. However, as in weakly guiding fibers is very small, then HE– EH mode pairs occur which have almost identical propagation constants. Such modes are said to be degenerate. The superpositions of these degenerating modes characterized by a common propagation constant correspond to particular LP modes regardless of their HE, EH, TE or TM field configurations. This linear combination of degenerate modes obtained from the exact solution produces a useful simplification in the analysis of weakly guiding fibers.

The relationship between the traditional HE, EH, TE and TM mode designations and the LP_{lm} mode designations is shown in Table 1.1. The mode subscripts *l* and *m* are related to the electric field intensity profile for a particular LP mode (see Figure 1.11(d)). There are in general 2*l* field maxima around the circumference of the fiber core and *m* field maxima along a radius vector. Furthermore, it may be observed from Table 1.1 that the notation for labeling the HE and EH modes has changed from that specified for the exact solution in the cylindrical waveguide mentioned previously.

Table 1.1 Correspondence between the lower order in linearly polarized modes and the traditional exact modes from which they are formed

Linearly polarized	*Exact*
LP_{01}	HE_{11}
LP_{11}	HE_{21}, TE_{01}, TM_{01}
LP_{21}	HE_{31}, EH_{11}
LP_{02}	HE_{12}
LP_{31}	HE_{41}, EH_{21}
LP_{12}	HE_{22}, TE_{02}, TM_{02}
LP_{lm}	HE_{2m}, TE_{0m}, TM_{0m}
LP_{lm} ($l \neq 0$ or 1)	$HE_{l+1,m}$, $EH_{l-1,m}$

[Source: htpp://img.brainkart.com]

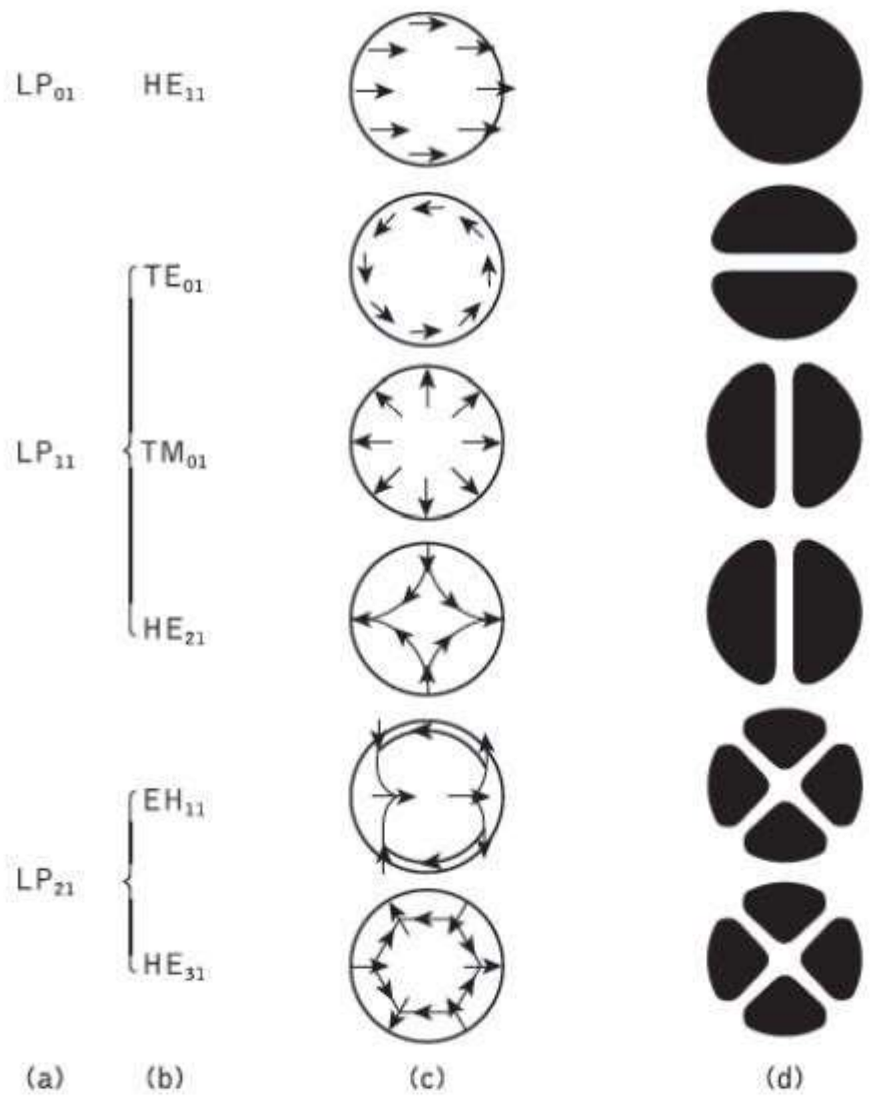

Figure 1.11 The electric field configurations for the three lowest LP modes illustrated in terms of their constituent exact modes: (a) LP mode designations; (b) exact mode designations; (c) electric field distribution of the exact modes; (d) intensity distribution of Ex for the exact modes indicating the electric field intensity profile for the corresponding LP modes

[Source: htpp://img.brainkart.com]

The subscript l in the LP notation now corresponds to HE and EH modes with labels $l + 1$ and $l - 1$ respectively. The electric field intensity profiles for the lowest three LP modes, together with the electric field distribution of their constituent exact modes, are shown in Figure 1.11. It may be observed from the field configurations of the exact modes that the field strength in the transverse direction (E_x or E_y) is identical for the modes which belong to the same LP mode. Hence the origin of the term ‘linearly polarized’. Using Eq. (1.31) for the cylindrical homogeneous core waveguide under the weak guidance conditions outlined above, the scalar wave equation can be written in the form

$$\frac{d^2\psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} + \frac{1}{r^2}\frac{d^2\psi}{d\phi^2} + (n_1^2k^2 - \beta^2)\psi = 0 \tag{1.41}$$

where ψ is the field (**E** or **H**), n_1 is the refractive index of the fiber core, *k* is the propagation constant for light in a vacuum, and *r* and φ are cylindrical coordinates. The propagation constants of the guided modes β lie in the range:

$$n_2k < \beta < n_1k \tag{1.42}$$

where n_2 is the refractive index of the fiber cladding. Solutions of the wave equation for the cylindrical fiber are separable, having the form:

$$\psi = E(r)\left[\begin{matrix}\cos l\phi \\ \sin l\phi\end{matrix}\right]\exp(\omega t - \beta z) \tag{1.43}$$

where in this case ψ represents the dominant transverse electric field component. The periodic dependence on φ following cos*l*φ or sin *l*φ gives a mode of radial order *l*. Hence the fiber supports a finite number of guided modes of the form of Eq. (1.43).Introducing the solutions given by Eq. (1.43) into Eq. (1.41) results in a differential equation of the form:

$$\frac{d^2\mathbf{E}}{dr^2} + \frac{1}{r}\frac{d\mathbf{E}}{dr} + \left[(n_1k^2 - \beta^2) - \frac{l^2}{r^2}\right]\mathbf{E} = 0 \tag{1.45}$$

For a step index fiber with a constant refractive index core, Eq. (1.43) is a Bessel differential equation and the solutions are cylinder functions. In the core region the solutions are Bessel functions denoted by J_l.

A graph of these gradually damped oscillatory functions (with respect to r) is shown in Figure 1.12(a). It may be noted that the field is finite at $r = 0$ and may be represented by the zero-order Bessel function $J0$. However, the field vanishes as r goes to infinity and the solutions in the cladding are therefore modified Bessel functions denoted by K_l. These modified functions decay exponentially with respect to r, as illustrated in Figure 1.12(b). The electric field may therefore be given by:

$$\begin{aligned} \mathbf{E}(r) &= GJ_l(UR) \qquad \text{for } R < 1 \text{ (core)} \\ &= GJ_l(U)\frac{K_l(WR)}{K_l(W)} \qquad \text{for } R > 1 \text{ (cladding)} \end{aligned} \tag{1.45}$$

Where G is the amplitude coefficient and $R = r/a$ is the normalized radial coordinate when a is the radius of the fiber core; U and W, which are the eigenvalues in the core and cladding respectively,* are defined as:

$$U = a(n_1^2k^2 - \beta^2)^{\frac{1}{2}} \tag{1.46}$$

$$W = a(\beta^2 - n_2^2k^2)^{\frac{1}{2}} \tag{1.47}$$

2. Mode Coupling

We have thus far considered the propagation aspects of perfect dielectric waveguides. However, waveguide perturbations such as deviations of the fiber axis from straightness, variations in the core diameter, irregularities at the core–cladding interface and refractive index variations may change the propagation characteristics of the fiber. These will have the effect of coupling energy traveling in one mode to another depending on the specific perturbation. Ray theory aids the understanding of this phenomenon, as shown in Figure 1.13, which illustrates two types of perturbation. It may be observed that in both cases the ray no longer maintains the same angle with the axis. In electromagnetic wave theory this corresponds to a change in the propagating mode for the light. Thus individual modes do not normally propagate throughout the length of the fiber without large energy transfers to adjacent modes, even when the fiber is exceptionally good quality and is not strained or bent by its surroundings. This mode conversion is known as mode coupling or mixing. It is usually analyzed using coupled mode equations which can be obtained directly from Maxwell's equations.

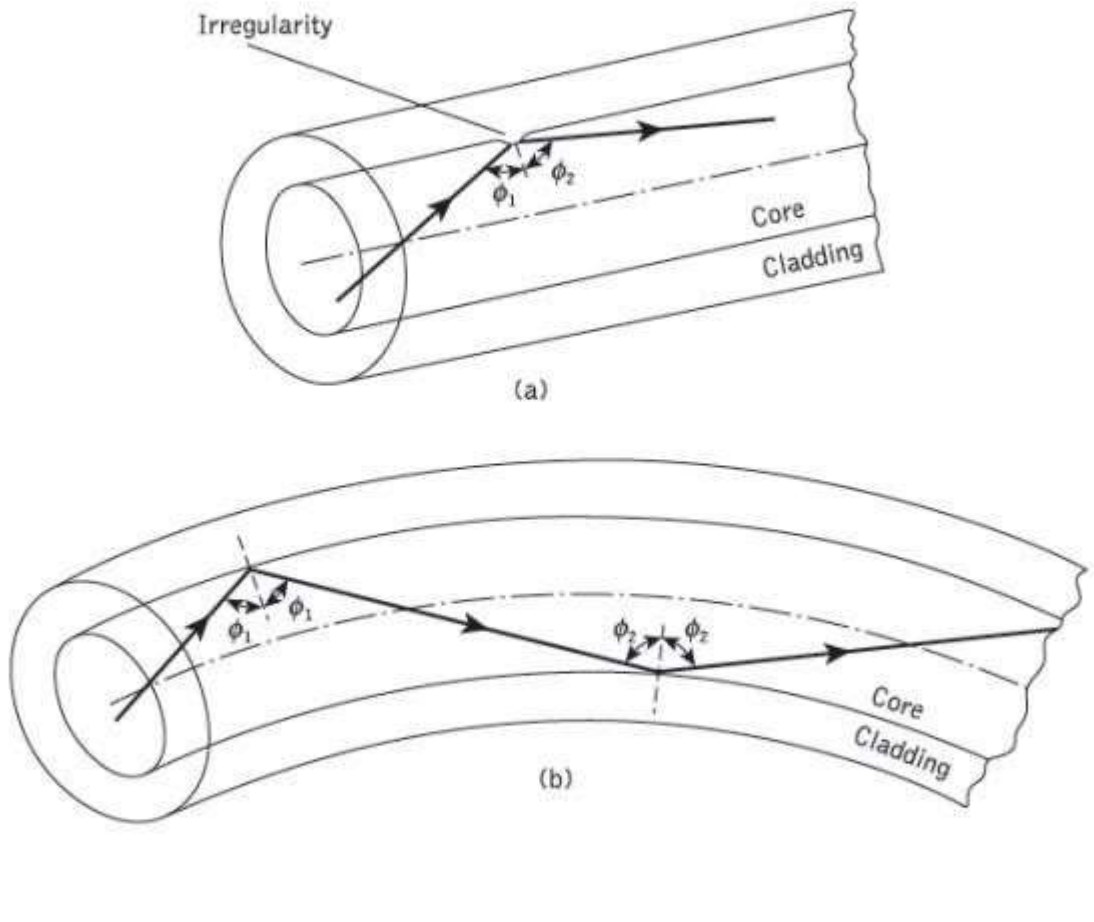

Figure 1.13 Ray theory illustrations showing two of the possible fiber perturbations which give mode coupling: (a) irregularity at the core–cladding interface; (b) fiber bend

[Source: htpp://img.brainkart.com]

3. Step Index Fibers

The optical fiber considered in the preceding sections with a core of constant refractive index n_1 and a cladding of a slightly lower refractive index n_2 is known as step index fiber. This is because the refractive index profile for this type of fiber makes a step change at the core–cladding interface, as indicated in Figure 1.14, which illustrates the two major types of step index fiber.

The refractive index profile may be defined as:

$$n(r) = \begin{cases} n_1 & r < a \quad \text{(core)} \\ n_2 & r \geq a \quad \text{(cladding)} \end{cases} \tag{1.48}$$

in both cases.

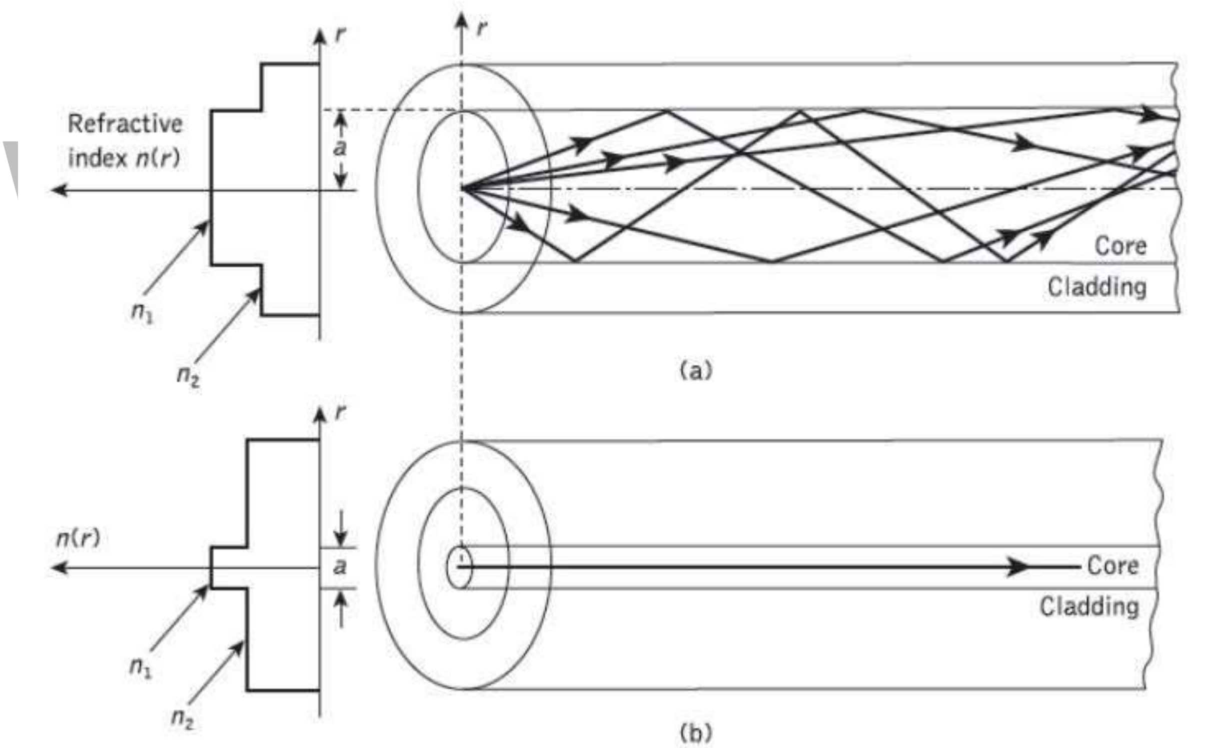

Figure 1.14 The refractive index profile and ray transmission in step index fibers: (a) multimode step index fiber; (b) single-mode step index fiber

[Source: htpp://img.brainkart.com]

Figure 1.14(a) shows a multimode step index fiber with a core diameter of around 50μm or greater, which is large enough to allow the propagation of many modes within the fiber core. This is illustrated in Figure 1.14(a) by the many different possible ray paths through the fiber. Figure 1.14(b) shows a single-mode or mono-mode step index fiber which allows the propagation of only one transverse electromagnetic mode (typically HE11), and hence the core diameter must be of the order of 2 to 10µm. The propagation of a single mode is illustrated in Figure 1.14(b) as corresponding to a single ray path only (usually shown as the axial ray) through the fiber.

The single-mode step index fiber has the distinct advantage of low intermodal dispersion (broadening of transmitted light pulses), as only one mode is transmitted, whereas with multimode step index fiber considerable dispersion may occur due to the differing group velocities of the propagating modes. This in turn restricts the maximum bandwidth attainable with multimode step index fibers, especially when com- pared with single-mode fibers. However, for lower bandwidth applications multimode fibers have several advantages over single-mode fibers. These are:

a) The use of spatially incoherent optical sources (e.g. most light-emitting diodes) which cannot be efficiently coupled to single-mode fibers.

b) Larger numerical apertures, as well as core diameters, facilitating easier coupling to optical sources

c) Lower tolerance requirements on fiber connectors

Multimode step index fibers allow the propagation of a finite number of guided modes along the channel. The number of guided modes is dependent upon the physical para- meters (i.e. relative refractive index difference, core radius) of the fiber and the wavelengths of the transmitted light which are included in the normalized frequency V for the fiber. Mode propagation does not entirely cease below cutoff. Modes may propagate as unguided or leaky modes which can travel considerable distances along the fiber. Nevertheless, it is the guided modes which are of paramount importance in optical fiber communications as these are confined to the fiber over its full length. that the total number of guided modes or mode volume M_s for a step index fiber is related to the V value for the fiber by the approximate expression

$$M_s \simeq \frac{V^2}{2} \tag{1.49}$$

Which allows an estimate of the number of guided modes propagating in a particular multimode step index fiber.

4. Graded index fibers

Graded index fibers do not have a constant refractive index in the core* but a decreasing core index $n(r)$ with radial distance from a maximum value of n_1 at the axis to a constant value n_2 beyond the core radius a in the cladding. This index variation may be represented as:

$$n(r) = \begin{cases} n_1(1 - 2\Delta(r/a)^{\alpha})^{\frac{1}{2}} & r < a \quad \text{(core)} \\ n_1(1 - 2\Delta)^{\frac{1}{2}} = n_2 & r \geq a \quad \text{(cladding)} \end{cases} \tag{1.50}$$

where Δ is the relative refractive index difference and α is the profile parameter which gives the characteristic refractive index profile of the fiber core. Equation (1.50) which is a convenient method of expressing the refractive index profile of the fiber core as a variation of α, allows representation of the step index profile when $\alpha = \infty$, a parabolic profile when $\alpha = 2$ and a triangular profile when $\alpha = 1$. This range of refractive index profiles is illustrated in Figure 1.15

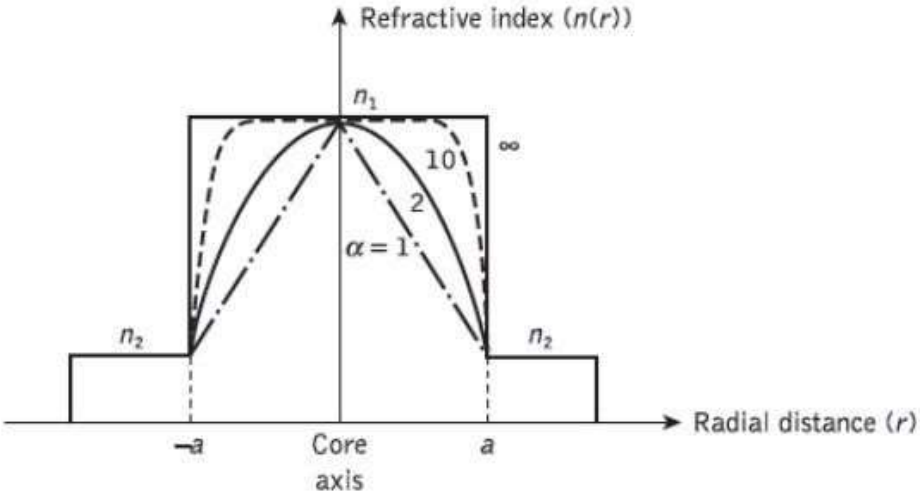

Figure 1.15 Possible fiber refractive index profiles for different values of α (given in Eq. (1.50)

[Source: htpp://img.brainkart.com]

The graded index profiles which at present produce the best results for multimode optical propagation have a near parabolic refractive index profile core with α ~~2. Fibers with such core index profiles are well established and consequently when the term 'graded index' is used without qualification it usually refers to a fiber with this profile.

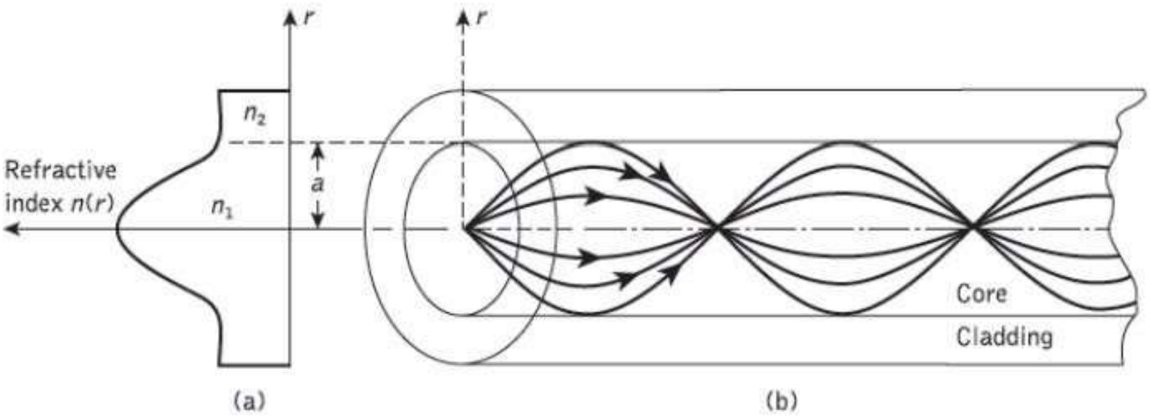

Figure 1.16 The refractive index profile and ray transmission in a multimode graded index fiber

[Source: htpp://img.brainkart.com]

For this reason in this section we consider the wave guiding properties of graded index fiber with a parabolic refractive index profile core.

A multimode graded index fiber with a parabolic index profile core is illustrated in Figure 1.16. It may be observed that the meridional rays shown appear to follow curved paths through the fiber core. Using the concepts of geometric optics, the gradual decrease in refractive index from the centre of the core creates many refractions of the rays as they are effectively incident on a large number or high to low index interfaces. This mechanism is illustrated in Figure 1.17 where a ray is shown to be gradually curved, with an ever- increasing angle of incidence, until the conditions for total internal reflection are met, and the ray travels back towards the core axis, again being continuously refracted.

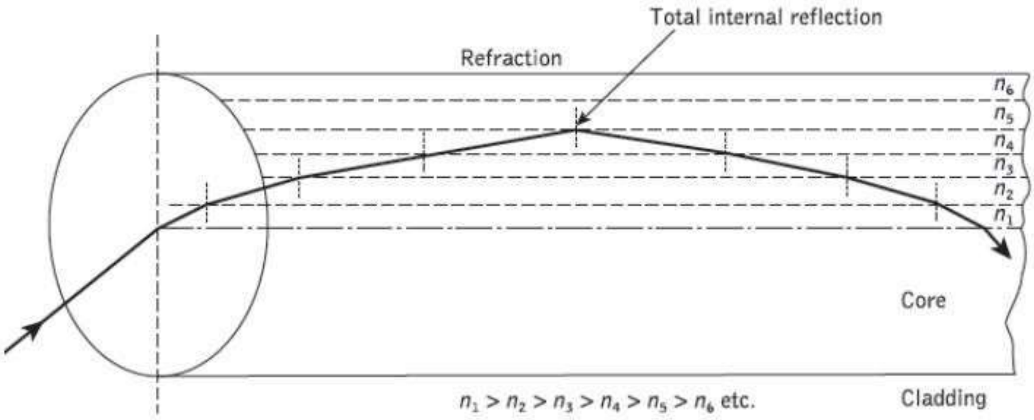

Figure 1.17 An expanded ray diagram showing refraction at the various high to low index interfaces within a graded index fiber, giving an overall curved ray path into the outer regions of the core.

[Source: htpp://img.brainkart.com]

Multimode graded index fibers exhibit far less intermodal dispersion than multimode step index fibers due to their refractive index profile. Although many different modes are excited in the graded index fiber, the different group velocities of the modes tend to be normalized by the index grading. Again considering ray theory, the rays traveling close to the fiber axis have shorter paths when compared with rays which travel

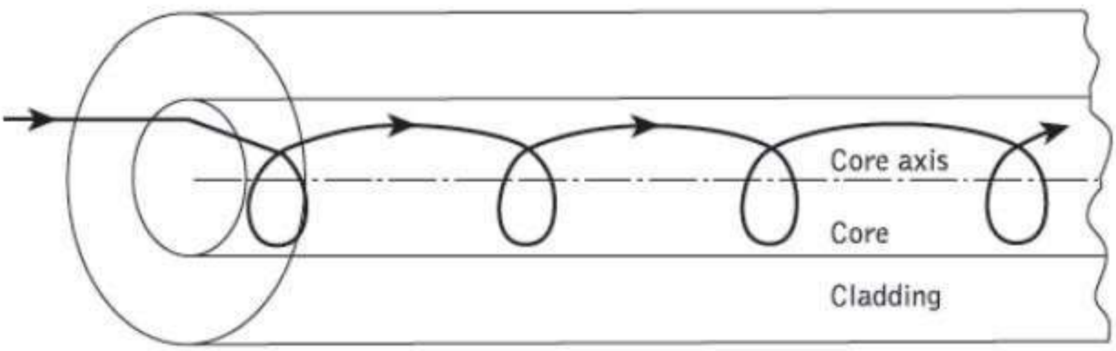

Figure 1.18 A helical skew ray path within a graded index fiber

[Source: htpp://img.brainkart.com]

However, the near axial rays are transmitted through a region of higher refractive index and therefore travel with a lower velocity than the more extreme rays. This compensates for the shorter path lengths and reduces dispersion in the fiber. A similar situation exists for skew rays which follow longer helical paths, as illustrated in Figure 1.18. These travel for the most part in the lower index region at greater speeds, thus giving the same mechanism of mode transit time equalization. Hence, multi- mode graded index fibers with parabolic or near-parabolic index profile cores have trans- mission bandwidths which may be orders of magnitude greater than multimode step index fiber bandwidths. Consequently, although they are not capable of the bandwidths attain- able with single-mode fibers, such multimode graded index fibers have the advantage of large core diameters (greater than 30 μm) coupled with bandwidths suitable for long- distance communication.

The parameters defined for step index fibers (i.e. NA, Δ, V) may be applied to graded index fibers and give a comparison between the two fiber types. However, it must be noted that for graded index fibers the situation is more complicated since the numerical aperture is a function of the radial distance from the fiber axis. Graded index fibers, therefore, accept less light than corresponding step index fibers with the same relative refractive index difference.

Electromagnetic Mode Theory for Optical Propagation

1. Electromagnetic Waves

In order to obtain an improved model for the propagation of light in an optical fiber, electromagnetic wave theory must be considered. The basis for the study of electromagnetic wave propagation is provided by Maxwell's equations. For a medium with zero conductivity these vector relationships may be written in terms of the electric field **E**, magnetic field **H**, electric flux density **D** and magnetic flux density **B** as the curl equations:

I

$$\Delta \times \mathbf{E} = -\frac{9\backslash}{9\mathbf{B}} \qquad (1.18)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \qquad (1.19)$$

and the divergence conditions:

$$\nabla \cdot \mathbf{D} = 0 \quad \text{(no free charges)} \qquad (1.20)$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{(no free poles)} \qquad (1.21)$$

where ∇ is a vector operator.

The four field vectors are related by the relations:

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad (1.22)$$

$$\mathbf{B} = \mu \mathbf{H}$$

where ε is the dielectric permittivity and μ is the magnetic permeability of the medium. Substituting for **D** and **B** and taking the curl of Eqs (1.18) and 1.19) gives:

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad (1.23)$$

$$\nabla \times (\nabla \times \mathbf{H}) = -\mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} \qquad (1.24)$$

Then using the divergence conditions of Eqs (1.20) and (1.21) with the vector identity:

$$\nabla \times (\nabla \times \mathbf{Y}) = \nabla(\nabla \cdot \mathbf{Y}) - \nabla^2(\mathbf{Y})$$

We obtain the nondispersive wave equations:

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad (1.25)$$

And

$$\nabla^2 \mathbf{H} = \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} \qquad (1.26)$$

where∇2 is the Laplacian operator. For rectangular Cartesian and cylindrical polar coordinates the above wave equations hold for each component of the field vector, every component satisfying the scalar wave equation

$$\nabla^2 \psi = \frac{1}{\upsilon_p^2} \frac{\partial^2 \psi}{\partial t^2} \qquad (1.27)$$

Where ψ may represent a component of the **E** or **H** field and up is the phase velocity (velocity of propagation of a point of constant phase in the wave) in the dielectric medium. It follows that:

$$\upsilon_p = \frac{1}{(\mu\varepsilon)^{\frac{1}{2}}} = \frac{1}{(\mu_r \mu_0 \varepsilon_r \varepsilon_0)^{\frac{1}{2}}} \qquad (1.28)$$

Where μ_r and ε_r are the relative permeability and permittivity for the dielectric medium and μ_0 and ε_0 are the permeability and permittivity of free space. The velocity of light in free space c is therefore:

$$c = \frac{1}{(\mu_0 \varepsilon_0)^{\frac{1}{2}}} \tag{1.29}$$

If planar waveguides, described by rectangular Cartesian coordinates (*x*, *y*, *z*), or circular fibers, described by cylindrical polar coordinates (*r*, φ, *z*), are considered, then the Laplacian operator takes the form:

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \tag{1.30}$$

Or

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2} \tag{1.31}$$

respectively. It is necessary to consider both these forms for a complete treatment of optical propagation in the fiber, although many of the properties of interest may be dealt with using Cartesian coordinates.The basic solution of the wave equation is a sinusoidal wave, the most important form of which is a uniform plane wave given by

$$\psi = \psi_0 \exp[\mathrm{j}(\omega t - \mathbf{k} \cdot \mathbf{r})] \tag{1.32}$$

where ω is the angular frequency of the field, *t* is the time, **k** is the propagation vector which gives the direction of propagation and the rate of change of phase with distance, while the components of **r** specify the coordinate point at which the field is observed. When λ is the optical wavelength in a vacuum, the magnitude of the propagation vector or the vacuum phase propagation constant *k* (where *k* = |**k**|) is given by:

$$k = \frac{2\pi}{\lambda} \tag{1.33}$$

It should be noted that in this case *k* is also referred to as the free space wave number.

2. Modes in a Planar Guide

The planar guide is the simplest form of optical waveguide. We may assume it consists of a slab of dielectric with refractive index $n1$ sandwiched between two regions of lower refractive index $n2$. In order to obtain an improved model for optical propagation it is useful to consider the interference of plane wave components within this dielectric waveguide.

The conceptual transition from ray to wave theory may be aided by consideration of a plane monochromatic wave propagating in the direction of the ray path within the guide (see Figure 1.8(a)). As the refractive index within the guide is n_1, the optical wavelength in this region is reduced to λ/n_1, while the vacuum propagation constant is increased to n_1k. When θ is the angle between the wave propagation vector or the equivalent ray and the guide axis, the plane wave can be resolved into two component plane waves propagating in the z and x directions, as shown in Figure 1.8(a).

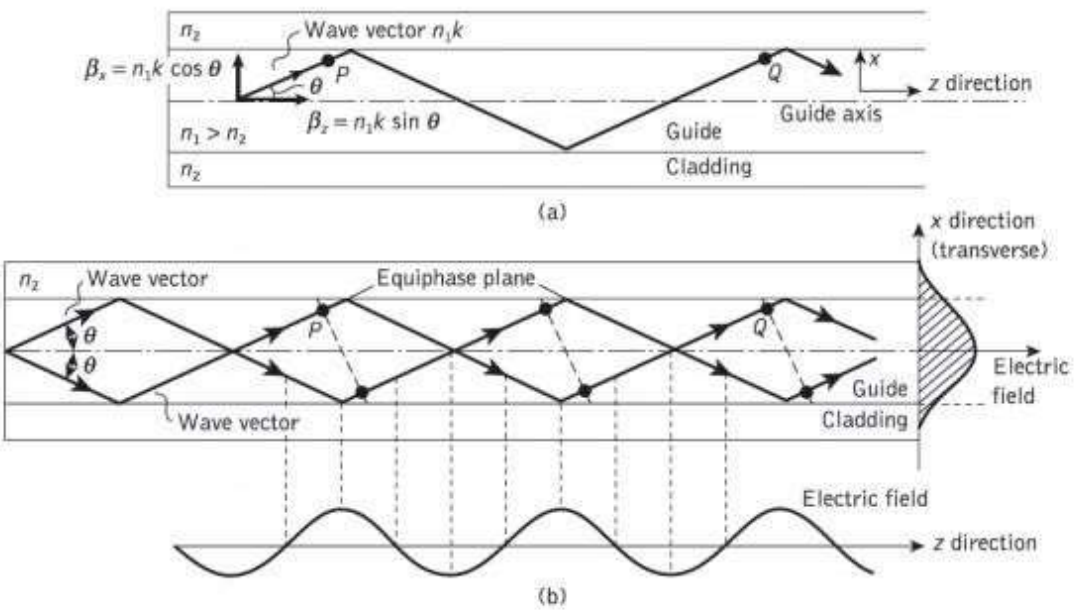

Figure 1.8 The formation of a mode in a planar dielectric guide: (a) a plane wave propagating in the guide shown by its wave vector or equivalent ray – the wave vector is resolved into components in the *z* and *x* directions; (b) the interference of plane waves in the guide forming the lowest order mode ($m = 0$)

[Source: htpp://img.brainkart.com]

The component of the phase propagation in the Z direction is given by:

$$\beta_z = n_1 k \cos\theta \tag{1.34}$$

The component of the phase propagation constant in the *x* direction β_x is:

$$\beta_x = n_1 k \sin\theta \tag{1.35}$$

The component of the plane wave in the *x* direction is reflected at the interface between the higher and lower refractive index media. When the total phase change* after two successive reflections at the upper and lower interfaces (between the points *P* and *Q*) is equal to $2m\pi$ radians, where *m* is an integer, then constructive interference occurs and a standing wave is obtained in the *x* direction. This situation is illustrated in Figure 1.8(b), where the interference of two plane waves is shown. In this illustration it is assumed that the interference forms the lowest order (where $m = 0$) standing wave, where the electric field is a maximum at the center of the guide decaying towards zero at the boundary between the guide and cladding. However, it may be observed from Figure 1.8(b) that the electric field penetrates some distance into the cladding, a phenomenon which is discussed.

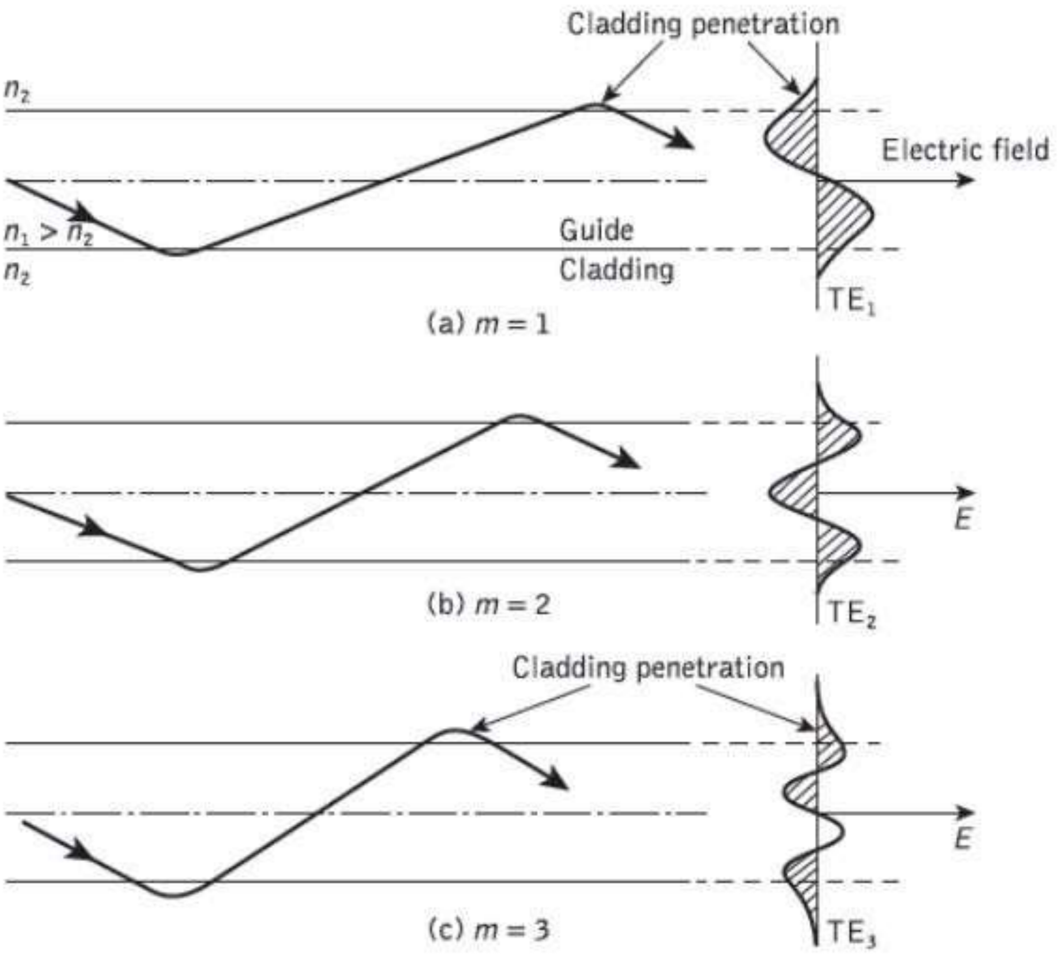

Figure 1.9 Physical model showing the ray propagation and the corresponding transverse electric (TE) field patterns of three lower order models (m =1, 2, 3) in the planar dielectric guide

[Source: htpp://img.brainkart.com]

Nevertheless, the optical wave is effectively confined within the guide and the electric field distribution in the *x* direction does not change as the wave propagates in the *z* direction. The sinusoidally varying electric field in the *z* direction is also shown in Figure 1.8(b). The stable field distribution in the *x* direction with only a periodic *z* dependence is known as a mode. A specific mode is obtained only when the angle between the propagation vectors or the rays and the interface have a particular value, as indicated in Figure 1.8(b). In effect, Eqs (1.34) and (1.35) define a group or congruence of rays which in the case described represents the lowest order mode. Hence the light propagating within the guide is formed into discrete modes, each typified by a distinct value of θ.

To visualize the dominant modes propagating in the *z* direction we may consider plane waves corresponding to rays at different specific angles in the planar guide. These plane waves give constructive interference to form standing wave patterns across the guide following a sine or cosine formula. Figure 2.9 shows examples of such rays for $m = 1, 2, 3$, together with the electric field distributions in the *x* direction. It may be observed that *m* denotes the number of zeros in this transverse field pattern. In this way *m* signifies the order of the mode and is known as the mode number.

When light is described as an electromagnetic wave it consists of a periodically varying electric field **E** and magnetic field **H** which are orientated at right angles to each other. The transverse modes shown in Figure 1.9 illustrate the case when the electric field is perpendicular to the direction of propagation and hence $E_z = 0$, but a corresponding component of the magnetic field **H** is in the direction of propagation. In this instance the modes are said to be transverse electric (TE). Alternatively, when a component of the **E** field is in the direction of propagation, but $H_z = 0$, the modes formed are called transverse magnetic (TM). The mode numbers are incorporated into this nomenclature by referring to the Te_m and TM_m modes, as illustrated for the transverse electric modes shown in Figure 1.9. When the total field lies in the transverse plane, transverse electromagnetic (TEM) waves exist where both E_z and H_z are zero. However, although TEM waves occur in metallic conductors (e.g. coaxial cables) they are seldom found in optical waveguides.

3. Phase and Group Velocity

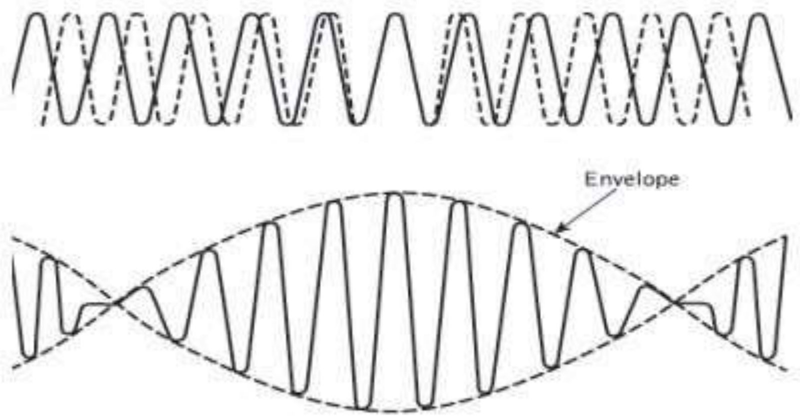

Figure 1.10 The formation of a wave packet from the combination of two waves with nearly equal frequencies.

[Source: htpp://img.brainkart.com]

The envelope of the wave package or group of waves travels at a group velocity υg With in all electromagnetic waves, whether plane or otherwise, there are points of constant phase. For plane waves these constant phase points form a surface which is referred to as a wave front. As a monochromatic light wave propagates along a waveguide in the *z* direction these points of constant phase travel at a phase velocity υp given by

$$\upsilon_p = \frac{\omega}{\beta} \qquad (1.36)$$

where ω is the angular frequency of the wave. However, it is impossible in practice to produce perfectly monochromatic light waves, and light energy is generally composed of a sum of plane wave components of different frequencies. Often the situation exists where a group of waves with closely similar frequencies propagate so that their resultant forms a packet of waves. The formation of such a wave packet resulting from the combination of two waves of slightly different frequency propagating together is illustrated in

Figure 1.10. This wave packet does not travel at the phase velocity of the individual waves but is observed to move at a group velocity υg given by

$$\upsilon_g = \frac{\delta\omega}{\delta\beta} \tag{1.37}$$

The group velocity is of greatest importance in the study of the transmission character- istics of optical fibers as it relates to the propagation characteristics of observable wave groups or packets of light. If propagation in an infinite medium of refractive index n_1 is considered, then the propagation constant may be written as:

$$\beta = n_1 \frac{2\pi}{\lambda} = \frac{n_1\omega}{c} \tag{1.38}$$

Where c is the velocity of light in free space. Equation (1.38) follows from Eqs (1.33) and (1.34) where we assume propagation in the z direction only and hence cos θ is equal to unity. Using Eq. (1.36) we obtain the following relationship for the phase velocity:

$$\upsilon_p = \frac{c}{n_1} \tag{1.39}$$

Similarly, employing Eq. (1.37), where in the limit δω/δβ becomes dω/dβ, the group velocity:

$$\begin{aligned}\upsilon_g &= \frac{d\lambda}{d\beta}\cdot\frac{d\omega}{d\lambda} = \frac{d}{d\lambda}\left(n_1\frac{2\pi}{\lambda}\right)^{-1}\left(\frac{-\omega}{\lambda}\right)\\ &= \frac{-\omega}{2\pi\lambda}\left(\frac{1}{\lambda}\frac{dn_1}{d\lambda} - \frac{n_1}{\lambda^2}\right)^{-1}\\ &= \frac{c}{\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right)} = \frac{c}{N_g}\end{aligned} \tag{1.40}$$

The parameter N_g is known as the group index of the guide.

Fiber Materials

- In selecting material for optical fiber, it satisfied a number of requirements

1. It must be possible to make long, thin, flexible fiber
2. The material must be transparent, in order to guide the light efficiently
3. The materials that have slightly different refractive indices

- The materials that satisfy the above requirements are glasses and plastics
- The basic raw material for fiber is silica or silicate (sand)
- The glass fiber are widely used and it ranges from high loss with large core area used for short distance transmission. low loss with small core area used for long distance transmission.
- The plastic fiber are less widely used because of higher attenuation and used for short distance application for several hundred meters, but is have greater mechanical strength over glass fiber.

Glass fiber

- The principal raw material for silica is sand, glass composed of pure silica (silica glass, fused silica, vitreous silica)
- Glass is made by fusing mixtures of metal oxide, sulfides, selenides. These material is randomly connected to found a crystalline materials.
- When glass is heated at temp. as high as 1000°C it becomes viscious liquid

To produce two similar materials that have slightly different indicies adding silica with fluorine or various dopants such as B_2O_3 , GeO_2, P_2 O_5

The fiber compositions are

1. GeO_2 – Sio_2 core; Sio_2 cladding
2. P_2 O_5 - Sio_2 core; Sio_2 cladding
3. Sio_2 core; B_2O_3 - Sio_2 cladding
4. GeO_2 - B_2O_3 - Sio_2 core; B_2O_3 - Sio_2 cladding

If high melting temp. is a disadvantages if the glass is prepared from a molten state its avoided by vapor deposition tech.

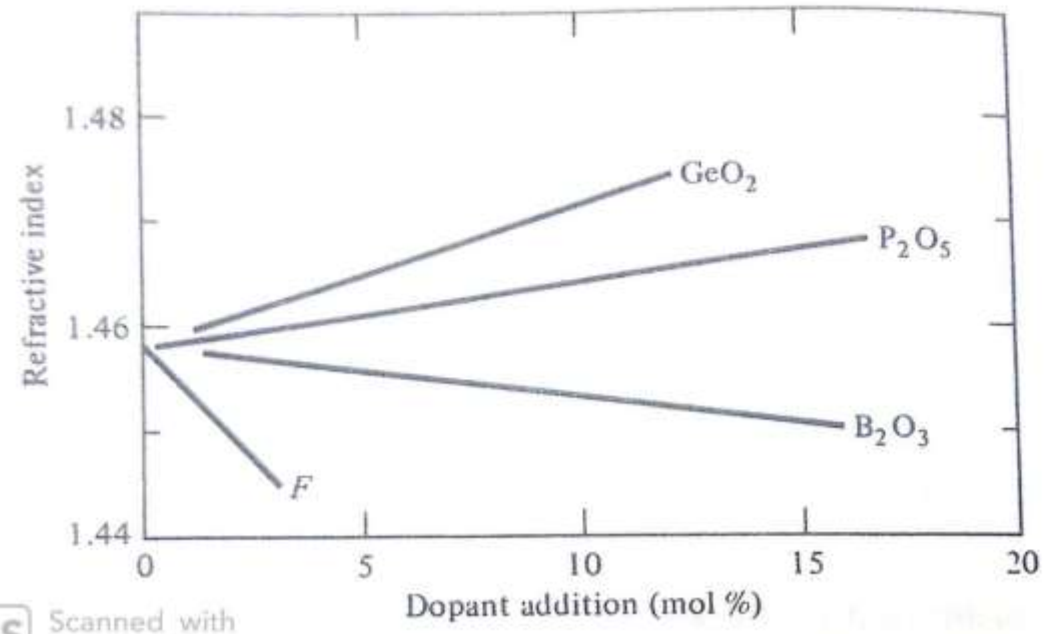

Figure 1.24 Fiber Compositions

[Source: Gred Keiser,"Optical Fiber Communication]

Halide glass fiber.

- Fluoride glasses belong to a general family of halide glasses.
- This element is group VII of the periodic table namely fluorine, chlorine, bromine, iodine.
- The materials that have a heavy metal fluoride glass as the major component and several other elements added to make the glass crystallization.

- The material forms the core of a fiber referred to as ZBLAN (ZrF_4, BaF_2, LaF_3, AlF_3, NaF)
- The material forms the cladding of a fiber referred to as ZHBLAN (ZrF_4, HaF_4, BaF_2, LaF_3, AlF_3, NaF).

Active glass fiber.

- Its rare earth element of a passive glass
- It give new optical and magnetic properties.
- These new properties allow the material to perform amplification, attenuation and phase retardation.
- Doping can be carried out for both silica and halide glasses.
- Due to rare earth element it avoid clustering effect.
- The commonly used materials are erbium and neodymium.

Chalgenide glass fiber.

- The non linear properties of glass fiber in optical amplifier such as optical switches.
- Due to its high optical non linearity and its long interaction length, chalgenide glass is one
- These glasses contain at least one chalcogen element (S, Se, Te). As_2S_3 is one of the most well known material
- The other element such as P, I, Cl,Br, Cd, Ba, Si, Tl are used for thermal, mechanical and optical properties of the glass.

Plastic Optical Fiber (POF).

- In the growing demand for delivering high speed services, the fiber developers to create high bandwidth graded index Polymer(plastic) Optical Fiber.

- The core of this fiber is either Poly Methyl Meth Acrylate (PMMA) POF or Per Fluorinated Polymer (PFP) POF.
- Compare with silica fiber the core diameter of plastic fiber are 10-20 times larger.
- It exhibit greater optical signal attenuations than glass fiber

Fiber Fabrication.

- In the fabrication of all glass optical waveguide two basic technique

Vapor phase oxidation process

Direct melt method

- In direct melt method follows traditional glass making procedure, here the fiber is made directly from the molten state of purified components of silicate glasses.
- In vapor phase oxidation process, a highly pure vapor of metal halides react with oxygen to form white powder of sio2 particles.
- Then particles are collected on the surface of a bulk glass and are collected to form a clear glass rod or tube.
- This rod or tube is called preform. Its around 10-25mm in diameter and 60-120cm long.
- Then its placed in an equipment, Fiber drawing tower.

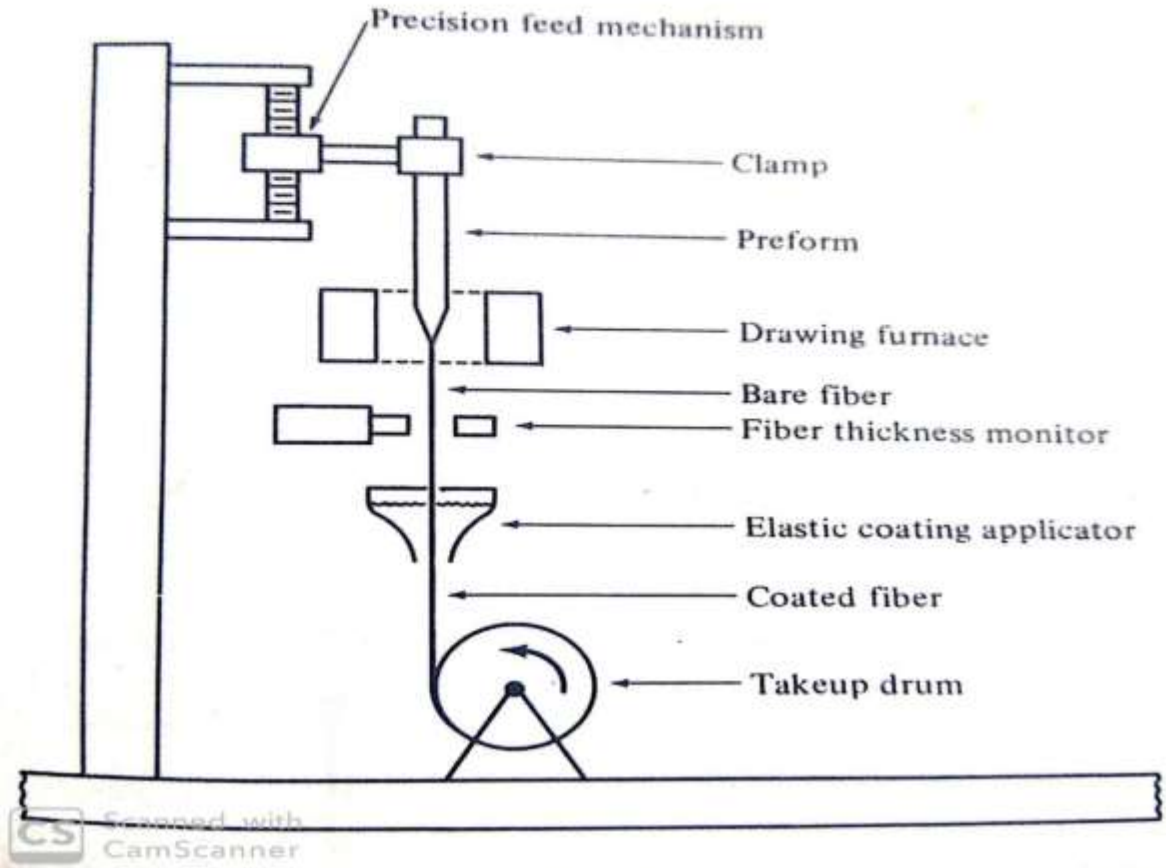

Figure 1.25 Fiber Drawing Apparatus

[Source: Gred Keiser,"Optical Fiber Communication]

Outside Vapor-Phase Oxidation.

- Is described as a corning glass work method
- First layer of sio2 particles called soot
- Burner onto a rotating graphite or ceramic mandrel.
- The glass soot is place on bait rod and layer by layer a cylindrical porous glass is built up.
- By properly controlling the metal halide vapor stream during deposition process
- The glass compositions are dimension for the core and cladding.

- When the process is completed the mandrel is removed and the porous tube is then vitrified(converted into glass or glass like substance)
- At high temp. of 1400°C a clear glass preform.
- Now the clear preform is mounted ina fiber drawing tower.

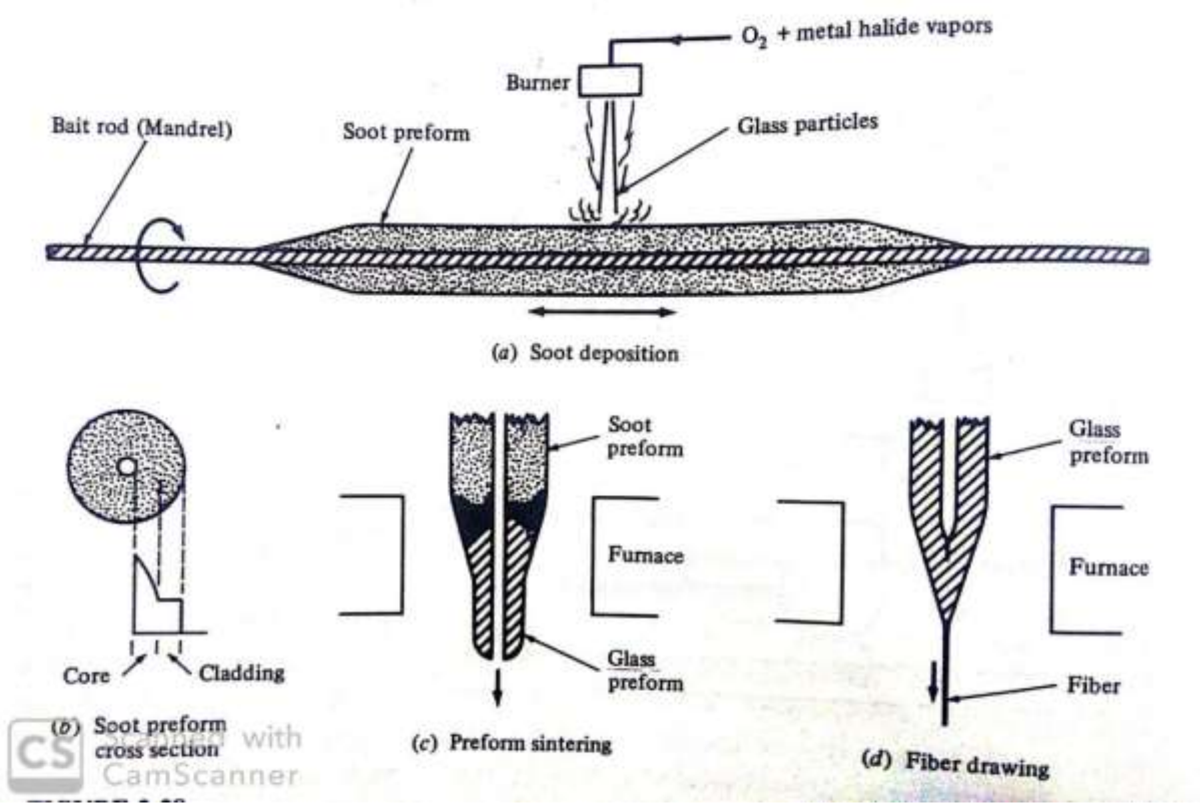

Figure 1.26 OVPO

[Source: Gred Keiser,"Optical Fiber Communication]

Vapor Phase Axial Deposition.

- Is described as lateral deposition method
- In this method sio2 particles are formed in the same way in OVPO process.
- As these particles emerge from the torches, they are deposited onto the end surface of a silica glass rod which act as seed.
- Now porous perform is grown in the axial direction by moving the rod upward.

- The rod is continuously rotated to maintain cylindrical symmetry of deposition.
- As the porous preform moves upwards, a solid transparent rod by zone melting with carbon ring heater
- Both step index and graded index in multi mode or single mode can be made by VPAD method.

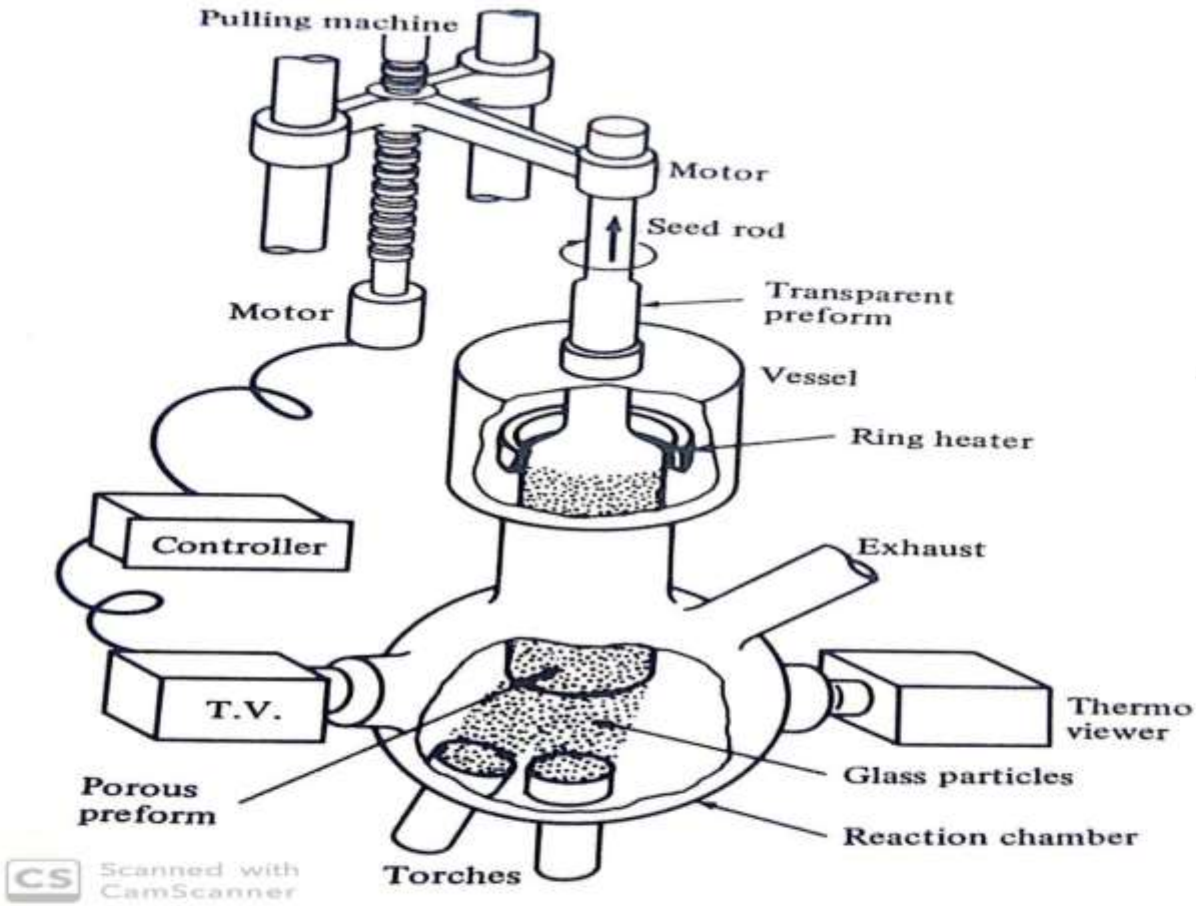

Figure 1.27 VPAD

[Source: Gred Keiser,"Optical Fiber Communication]

Modified Chemical Vapor Deposition.

- Is introduced in bell lab.
- To produce very low loss GI fiber.
- The glass vapor particles reaction with metal halide gases and oxygen flow through inside a revolving silica tube.
- As the sio2 particles are deposited they are sintered to a clear glass layer by an oxyhydrogen torch
- When the desired thickness of glass has be deposited the vapor flow is shut-off.
- Now the tube is heated strongly to cause it to be collapse in to solid rod preform.
- Now the preform is placed in fiber drawing tower.

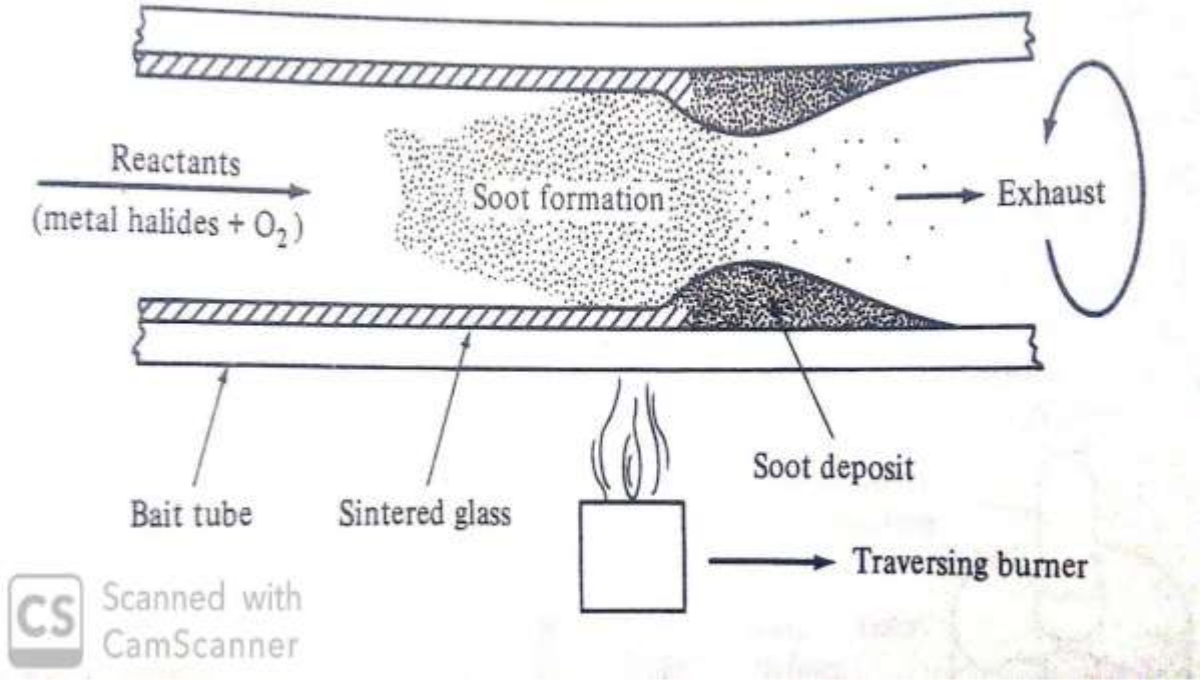

Figure 1.28 MCVD

[Source: Gred Keiser,"Optical Fiber Communication]

Plasma Activated Chemical Vapor Deposition.

- Scientists from philips research invented PACVD.
- Is similar to MCVD process in that deposition occur within a silica tube.
- A non-isothermal microwave plasma operating at low pressure initiates the chemical reaction.
- When the silica tube held at a temp. of 1000-1200°C
- To reduce mechanical stresses in the growing glass films, a moving microwave resonator which is operating at 2.45GHz
- Now a plasma inside the tube to activate the chemical reaction.
- This will give clear glass material directly on the tube wall then there is no soot formation.
- Thus no sintering is required.

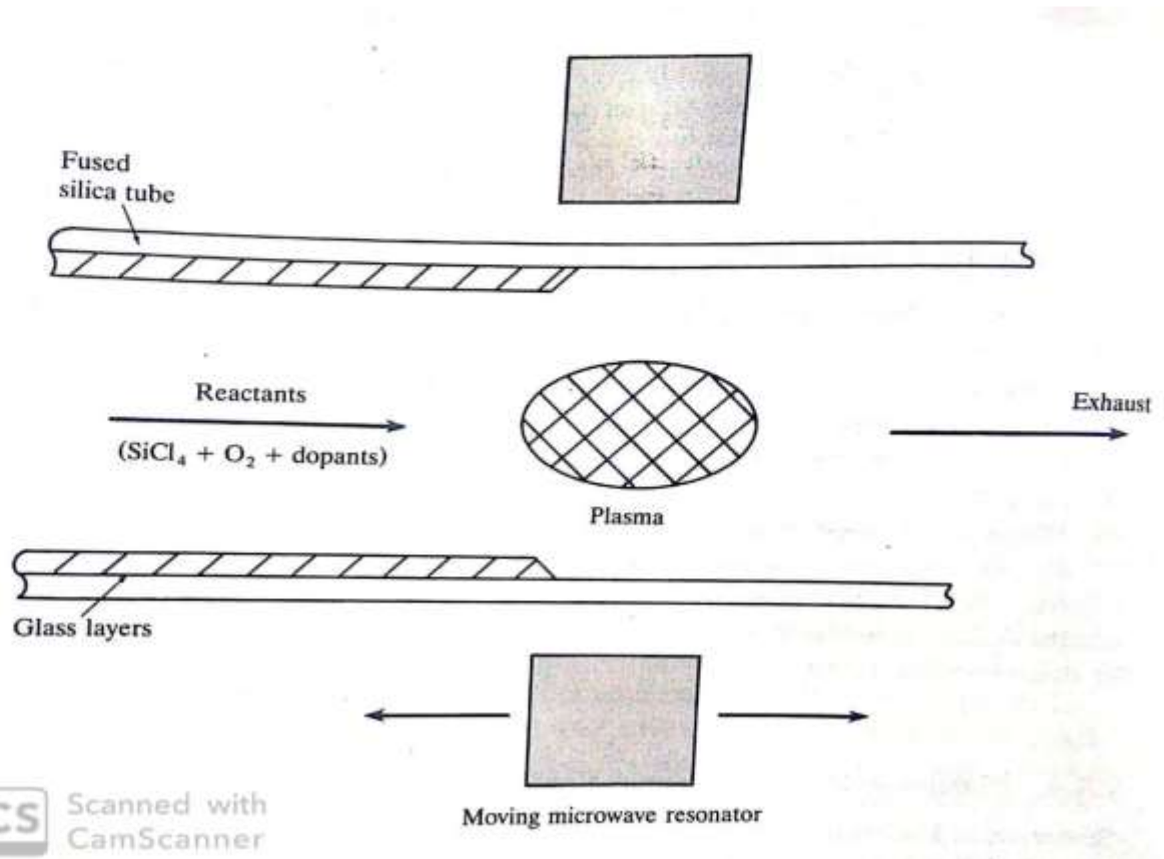

Figure 1.29 PACVD

[Source: Gred Keiser,"Optical Fiber Communication]

Double Crucible Method.

- Silica chalgenide halide glass fiber can be made using direct melt double crucible method.
- In this method glass rod for the core and cladding materials are first made separately by melting mixtures of purified powders to make glass composition.
- Now these rods are feed into two concentric crucible.
- The inner crucible contains molten core glass and the outer crucible contains cladding glass.
- Now the fibers are drawn from the molten state through opening in the bottom of the two concentric crucibles in continuous production process.

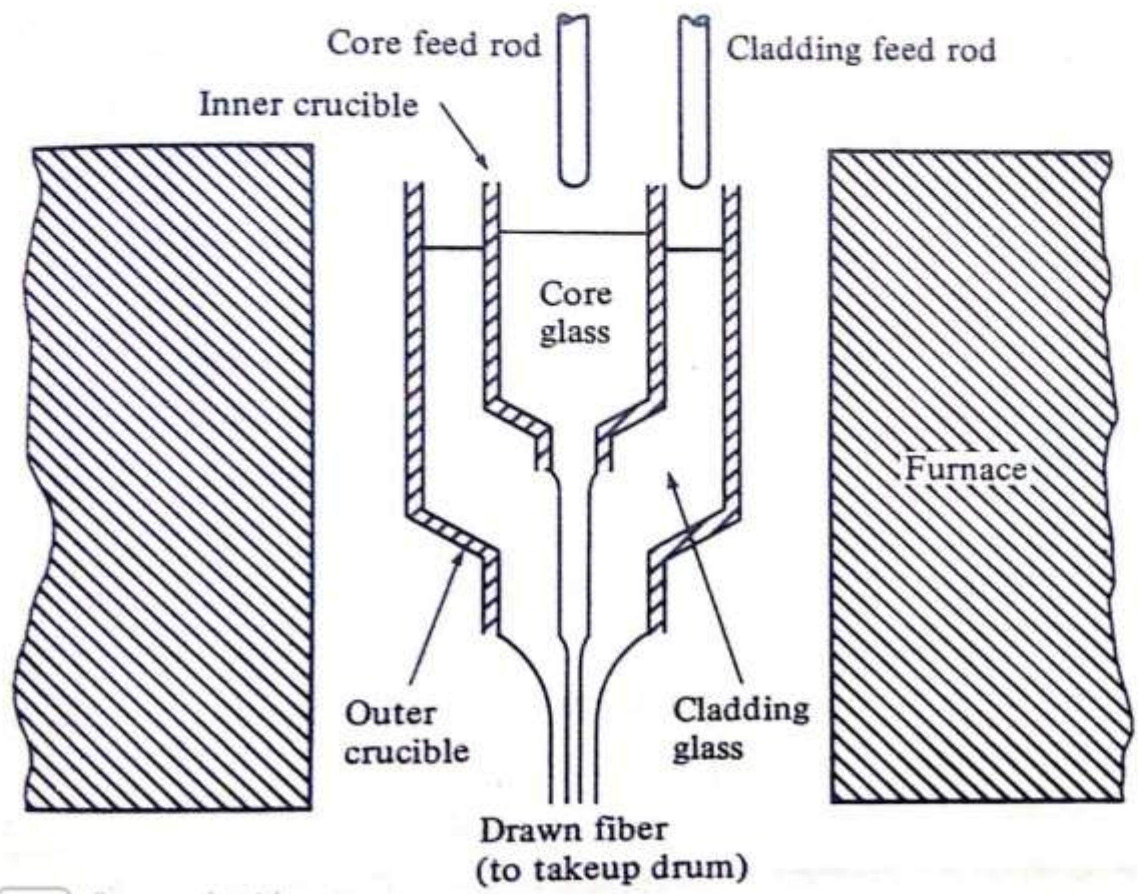

Figure 1.30 DC Method

[Source: Gred Keiser,"Optical Fiber Communication]

INTRODUCTION

Communication may be broadly defined as the transfer of information from one point to another. When the information is to be conveyed over any distance a communication system is usually required. Within a communication system the information transfer is frequently achieved by superimposing or modulating the information onto an electromagnetic wave which acts as a carrier for the information signal. This modulated carrier is then trans- mitted to the required destination where it is received and the original information signal is obtained by demodulation. Sophisticated techniques have been developed for this process using electromagnetic carrier waves operating at radio frequencies as well as microwave and millimeter wave frequencies. However, 'communication' may also be achieved using an electromagnetic carrier which is selected from the optical range of frequencies

Communication is defined as the transfer of information from one point to another. The function of communication system is to convey signal from source to destination over transmission medium.

Communication system consists of

1. Transmitter/ modulator linked to information source.
2. Transmission medium.
3. Receiver/ demodulator at destination point.

Interest in communicating at optical frequencies was created in 1960. Electrical communication was started in 1837 with the invention of telegraph by Samuel F.B. Morse. Advanced telegraph was invented in 1874 with 120 bits per second. In 1876, Alexander Graham Bell invented telephone to transmit voice signal in analog form. In 19‟s LASER was discovered. Around 1980‟s optical fiber technology becomes back bone for long distance communication. Around

1970‟s, optical fiber has low loss window around 800 nm. Optical source is made of GaAs which emit light at 800 nm. This is first window. As glass purification technology improved during 1980‟s, optical source is made of silica. So in1980‟s optical communication is shifted to 1300 nm band. This is second window. In 1990‟s the communication was shifted to 1550 nm due to the invention of erbium dopped fiber amplifier (EDFA). This is third window.

First operating window:

Centered at 850 nm.

Fibers silica multi mode fiber

Sources GaAlAs based sources

Photo detector Silicon Photo detector.

Optical amplifier GaAlAs based amplifiers.

Used in initial telephone system in USA around 1977.

Second Operating window:

Centered at 1310 nm

Fibers Single mode and Multi mode fiber

Sources InGaAsP based sources

Photo detector InGaAs Photo detector.

Optical amplifier PDFA (Praseodymium doped fiber amplifiers). By this repeater is reduced.

Optical fiber exhibit low power loss and less signal dispersion.

Third operating window:

Centered around 1500 nm.

Fibers Dispersion Shifted fiber

Sources InGaAsP based sources

Photo detector InGaAs Photo detector.

Optical amplifier EDFA (Erbium doped fiber amplifiers).

Used in metropolitan networks.

Fiber optics deals with the study of propagation of light through transparent dielectric waveguides. A fiber optic cable is essentially a light pipe that is used to carry a light beam from one place to another. The carrier frequencies used in conventional systems had the limitations in handling the volume and data rate of data transmission. The greater the carrier frequency larger the available bandwidth and information carrying capacity.

The first generation of light wave systems uses GaAs semiconductor laser and operating region was near 0.8μm. Other specifications of this generation are as under:

First generation

Bit rate : 45 Mb/s

Repeater spacing : 10 km

Second generation

Bit rate : 100 Mb/s to 1.7 Gb/s

Repeater spacing : 50 km

Operating wavelength : 1.3 μm

Semiconductor : In GaAsP

Third generation

Bit rate : 10 Gb/s

Repeater spacing : 100 km

Operating wavelength : 1.55 μm

Fourth generation

Fourth generation uses WDM technique.

Bit rate : 10 Tb/s

Repeater spacing : > 10,000 km.

Operating wavelength : 1.45 to 1.62 μm

Fifth generation

Fifth generation uses Roman amplification technique and optical solitons.

Bit rate : 40 Gb/s to 160 Gb/s

Repeater spacing : 24000 km to 35000 km

Operating wavelength : 1.53 to 1.57 μm

S.	Characteristics	Optical fiber	Satellite link
1	Bandwidth	1 to 10GHz	36 to 72 MHz
2	Immunity to interference	Immune to electromagnetic	Subject to interference from various sources
3	Security	Difficult to tap without defection	Signals must be encrypted for security
4	Flexibility	Difficult to reconfigure to meet	Easy to configure
5	Multipoint capability	Point to point mainly	Point to multipoint
6	Connectivity to customer end	Local loop required	Local loop not required but antenna is needed at customer site.

The general system:

An optical fiber communication system is similar in basic concept to any type of communication system. A block schematic of a general communication system is shown in Figure 1.2(a), the function of which is to convey the signal from the information source over the transmission medium to the destination. The communication system therefore consists of a transmitter or modulator linked to the information source, the transmission medium, and a receiver or demodulator at the destination point. In electrical communications the information source provides an electrical signal, usually derived from a message signal which is not electrical (e.g. sound), to a transmitter comprising electrical and electronic components which converts the signal into a suitable form for propagation over the transmission medium. This is often achieved by modulating a carrier, which, as mentioned pre-viously, may be an electromagnetic wave.

For optical fiber communications the system shown in Figure 1.1(a) may be considered in slightly greater detail, as given in Figure 1.1(b). In this case the information source provides an electrical signal to a transmitter comprising an electrical stage which drives an optical source to give modulation of the light wave carrier. The optical source which provides the electrical–optical conversion may be either a semiconductor laser or light-emitting diode (LED).

The transmission medium consists of an optical fiber cable and the receiver consists of an optical detector which drives a further electrical stage and hence provides demodulation of the optical carrier. Photodiodes (*p–n*, *p–i–n* or avalanche) and, in some instances, phototransistors and photoconductors are utilized for the detection of the optical signal and the optical–electrical conversion. Thus there is a requirement for electrical interfacing at either end of the optical link and at present the signal processing is usually performed electrically.

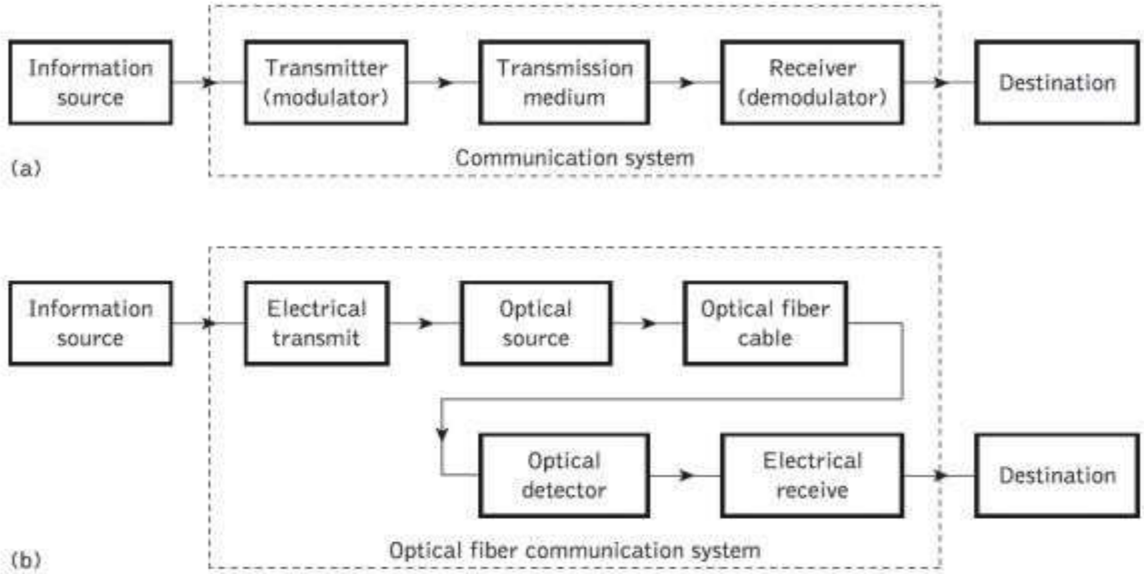

Figure 1.1 (a) The general communication system. (b) The optical fiber communication system

[Source: htpp://img.brainkart.com]

The optical carrier may be modulated using either an analog or digital information signal. In the system shown in Figure 1.1(b) analog modulation involves the variation of the light emitted from the optical source in a continuous manner. With digital modulation, however, discrete changes in the light intensity are obtained (i.e. on–off pulses). Although often simpler to implement, analog modulation with an optical fiber communication sys- tem is less efficient, requiring a far higher signal-to-noise ratio at the receiver than digital modulation. Also, the linearity needed for analog modulation is not always provided by semiconductor optical sources, especially at high modulation frequencies. For these reasons, analog optical fiber communication links are generally limited to shorter distances and lower bandwidth operation than digital links.

The transmission of light via a dielectric waveguide structure was first proposed and investigated at the beginning of the twentieth century. In 1910 Hondros and Debye conducted a theoretical study, and experimental work was reported by Schriever in 1920. However, a transparent dielectric rod, typically of silica glass with a refractive index of around 1.5, surrounded by air, proved to be an impractical waveguide due to its unsupported structure (especially when very thin waveguides were considered in order to limit the number of optical modes propagated) and the excessive losses at any dis- continuities of the glass–air interface. Nevertheless, interest in the application of dielectric optical waveguides in such areas as optical imaging and medical diagnosis (e.g. endoscopes) led to proposals for a clad dielectric rod in the mid-1950s in order to overcome these problems. This structure is illustrated in Figure 2.1, which shows a trans- parent core with a refractive index n_1 surrounded by a transparent cladding of slightly lower refractive index n_2. Theinvention of the clad waveguide structure led to the first serious proposals by Kao and Hockham and

Werts, in 1966, to utilize optical fibers as a communications medium, even though they had losses in excess of 1000 dB km−1.

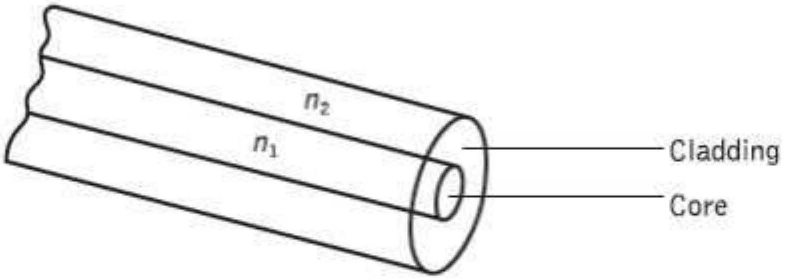

Figure 1.1 (c) Optical fiber waveguide showing the core of refractive index *n*1, surrounded by the cladding of slightly lower refractive index *n*2

[Source: htpp://img.brainkart.com]

These proposals stimulated tremendous efforts to reduce the attenuation by purification of the materials. This has resulted in improved conventional glass refining techniques giving fibers with losses of around 4.2 dB km. Also, progress in glass refining processes such as depositing vapor-phase reagents to form silica allowed fibers with losses below 1 dB km to be fabricated.

Most of this work was focused on the 0.8 to 0.9 μm wavelength band because the first generation of optical sources fabricated from gallium aluminium arsenide alloys operated in this region. However, as silica fibers were studied in further detail it became apparent that transmission at longer wavelengths (1.1 to 1.6 μm) would result in lower losses and reduced signal dispersion. This produced a shift in optical fiber source and detector technology in order to provide operation at these longer wavelengths. Hence at longer wavelengths, especially around 1.55 μm, typical high-performance fibers have losses of 0.2 dB km.

As such losses are very close to the theoretical lower limit for silicate glass fiber, there is interest in glass-forming systems which can provide low-loss transmission in the mid infrared (2 to 5 μm) optical wavelength regions. Although a system based on fluoride glass offers the potential for ultra-low-loss transmission of 0.01 dB kmat a wavelength of 2.55 μm, such fibers still exhibit losses of at least 0.65 dB km properties of silica fibers.

In order to appreciate the transmission mechanism of optical fibers with dimensions approximating to those of a human hair, it is necessary to consider the optical waveguideing of a cylindrical glass fiber. Such a fiber acts as an open optical waveguide, which may be analyzed utilizing simple ray theory. However, the concepts of geometric optics are not sufficient when considering all types of optical fiber, and electromagnetic mode theory must be used to give a complete picture. The following sections will therefore outline the transmission of light in optical fibers prior to a more detailed discussion of the various types of fiber.

Advantages of optical fiber communication:

Wider bandwidth and greater information capacity.
Lower losses.
Small size and light weight.
Signal security.
Repeaters spacing.
Environmental immunity.
Low cost and ease of maintenance.
System reliability.
Long distance transmission
Safe and easy installation

Disadvantages of optical fiber communication:

High initial cost
Maintenance and repairing cost high.
Joining and test procedure
Tensile stress
Fiber losses

Applications of optical fiber communication:

Used in telecommunication, instrumentation, cable TV network (CATV) and data transmission and distribution.
Used in telephone system because of small size and large information carrying capacity. Used for transmitting digital data generated by computers between CPU and peripherals between CPU and memory and between CPU‟s.

Ray Theory Transmission

1. Total Internal Reflection

To consider the propagation of light within an optical fiber utilizing the ray theory model it is necessary to take account of the refractive index of the dielectric medium. The refractive index of a medium is defined as the ratio of the velocity of light in a vacuum to the velocity of light in the medium.

A ray of light travels more slowly in an optically dense medium than in one that is less dense, and the refractive index gives a measure of this effect. When a ray is incident on the interface between two dielectrics of differing refractive indices (e.g. glass–air), refraction occurs, as illustrated in Figure 1.2(a). It may be observed that the ray approaching the interface is propagating in a dielectric of refractive index *n* and is at an angle φ to the normal at the surface of the interface. If the dielectric on the other side of the interface has a refractive index *n*which is less than n_1, then the refraction is such that the ray path in this lower index medium is at an angle to the normal, where is greater than . The angles of incidence and refraction are related to each other and to the refractive indices of the dielectrics by Snell's law of refraction, which states that:

$$n_1 \sin \phi_1 = n_2 \sin \phi_2$$

Or

$$\frac{\sin \phi_1}{\sin \phi_2} = \frac{n_2}{n_1} \qquad (1.1)$$

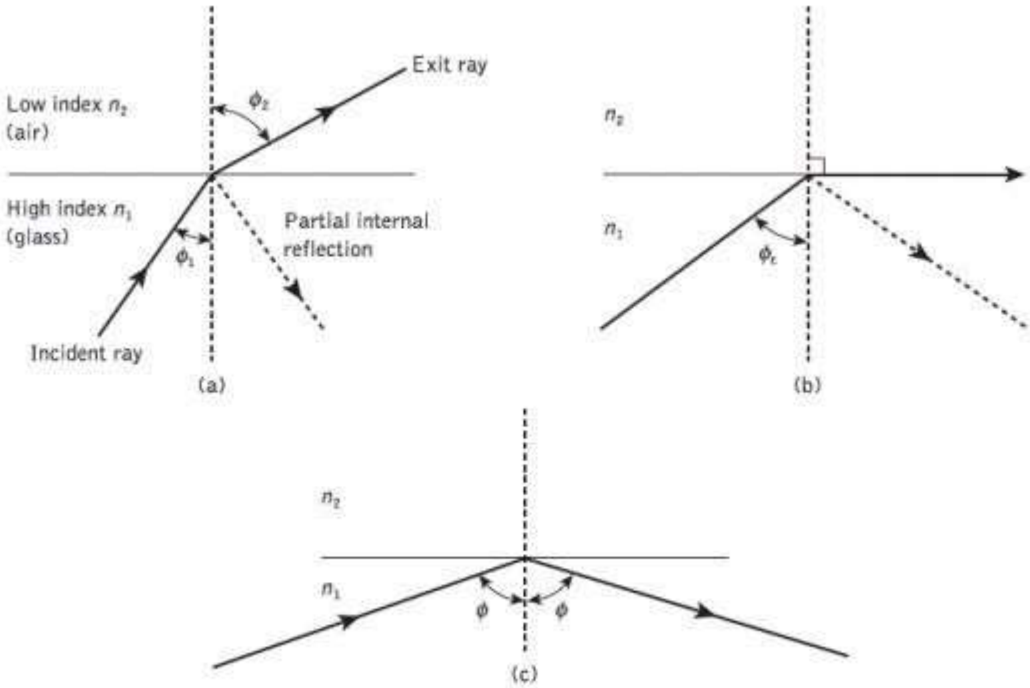

Figure 1.2 Light rays incident on a high to low refractive index interface (e.g. glass air): (a) refraction; (b) the limiting case of refraction showing the critical ray at an angle φ_C (c) total internal reflection where $\varphi > \varphi_C$

[Source: htpp://img.brainkart.com]

It may also be observed in Figure 1.2(a) that a small amount of light is reflected back into the originating dielectric medium (partial internal reflection). As *n*is greater than *n*, the angle of refraction is always greater than the angle of incidence. Thus when the angle of refraction is 90° and the refracted ray emerges parallel to the interface between the dielectrics, the angle of incidence must be less than 90°. This is the limiting case of refraction and the angle of incidence is now known as the critical angle φc, as shown in Figure 1.2(b). From Eq. (1.1) the value of the critical angle is given by

$$\sin \phi_c = \frac{n_2}{n_1} \quad (1.2)$$

At angles of incidence greater than the critical angle the light is reflected back into the originating dielectric medium (total internal reflection) with high efficiency (around 99.9%). Hence, it may be observed in Figure 1.2(c) that total internal reflection occurs at the inter- face between two dielectrics of differing refractive indices when light is incident on the dielectric of lower index from the dielectric of higher index, and the angle of incidence of the ray exceeds the critical value. This is the mechanism by which light at a sufficiently shallow angle (less than 90° − may be considered to propagate down an optical fiber with low loss.

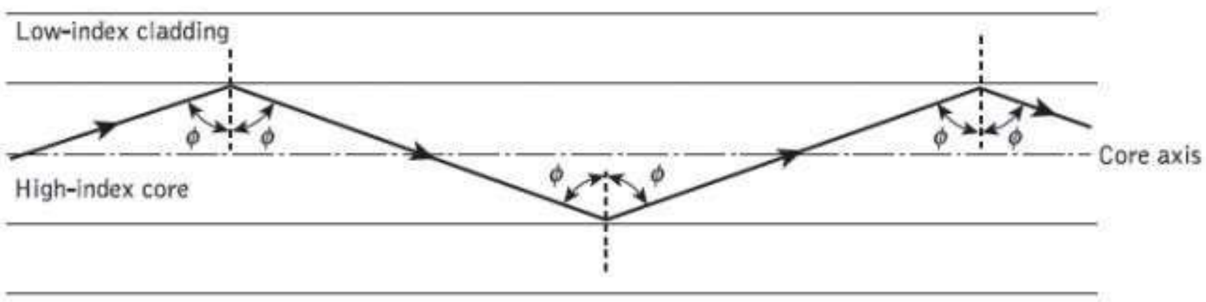

Figure 1.3 The transmission of a light ray in a perfect optical fiber

[Source: htpp://img.brainkart.com]

Figure 1.3 illustrates the transmission of a light ray in an optical fiber via a series of total internal reflections at the interface of the silica core and the slightly lower refractive index silica cladding. The ray has an angle of incidence φ at the interface which is greater than the critical angle and is reflected at the same angle to the normal.

The light ray shown in Figure 1.3 is known as a meridional ray as it passes through the axis of the fiber core. This type of ray is the simplest to describe and is generally used when illustrating the fundamental transmission properties of optical fibers. It must also be noted that the light transmission illustrated in Figure 1.3 assumes a perfect fiber, and that any discontinuities or imperfections at the core–cladding interface would probably result in refraction rather than total internal reflection, with the subsequent loss of the light ray into the cladding.

2. Acceptance Angle

Having considered the propagation of light in an optical fiber through total internal reflection at the core–cladding interface, it is useful to enlarge upon the geometric optics approach with reference to light rays entering the fiber. Since only rays with a sufficiently shallow grazing angle (i.e. with an angle to the normal greater than φ_c) at the core–cladding interface are transmitted by total internal reflection, it is clear that not all rays entering the fiber core will continue to be propagated down its length.

The geometry concerned with launching a light ray into an optical fiber is shown in Figure1.4, which illustrates a meridional ray *A* at the critical angle φ_c within the fiber at the core– cladding interface. It may be observed that this ray enters the fiber core at an angle θ_a to the fiber axis and is refracted at the air–core interface before transmission to the core–cladding interface at the critical angle. Hence, any rays which are incident into the fiber core at an angle greater than θ_a will be transmitted to the core–cladding interface at an angle less than φ_c, and will not be totally internally reflected. This situation is also illustrated in Figure 2.4, where the incident ray *B* at an angle greater than θ_a is refracted into the cladding and eventually lost by radiation. Thus for rays to be transmitted by total internal reflection within the fiber core they must be incident on the fiber core within an acceptance cone defined by the conical half angle θ_a.

Hence θa is the maximum angle to the axis at which light may enter the fiber in order to be propagated, and is often referred to as the acceptance angle for the fiber.

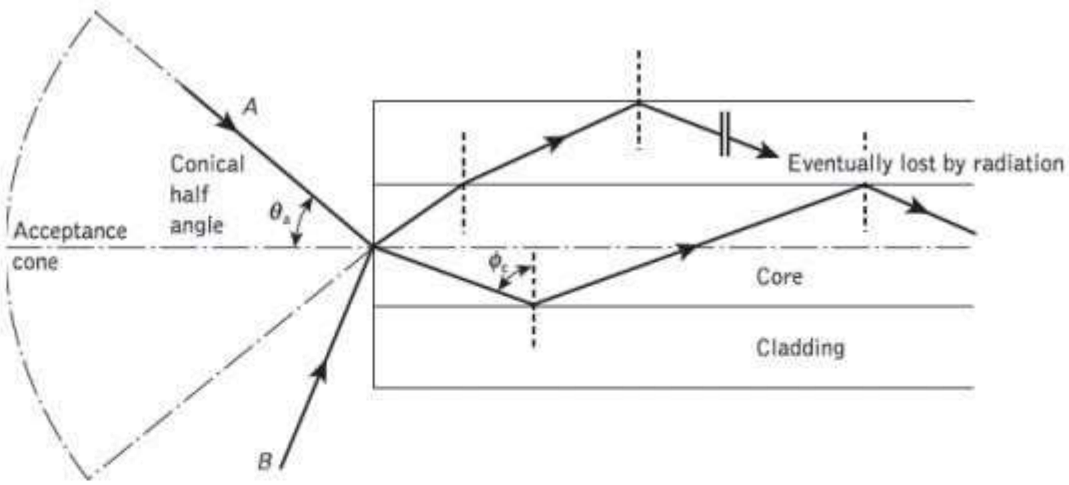

Figure 1.4 The acceptance angle θ_a when launching light into an optical fiber

[Source: htpp://img.brainkart.com]

If the fiber has a regular cross-section (i.e. the core–cladding interfaces are parallel and there are no discontinuities) an incident meridional ray at greater than the critical angle will continue to be reflected and will be transmitted through the fiber. From symmetry considerations it may be noted that the output angle to the axis will be equal to the input angle for the ray, assuming the ray emerges into a medium of the same refractive index from which it was input.

3. Numerical Aperture

The acceptance angle for an optical fiber was defined in the preceding section. However, it is possible to continue the ray theory analysis to obtain a relationship between the acceptance angle and the refractive indices of the three media involved, namely the core, cladding and air. This leads to the definition of a more generally used term, the numerical aperture of the fiber. It must be noted that within this analysis, as with the preceding discussion of acceptance angle, we are concerned with meridional rays within the fiber. Figure 1.5 shows a light ray incident on the fiber core at an angle θ_1 to the fiber axis which is less than the acceptance angle for the fiber θ_a. The ray enters the fiber from a medium (air) of refractive index n_0, and the fiber core has a refractive index n_1, which is slightly greater than the cladding refractive index n_2.

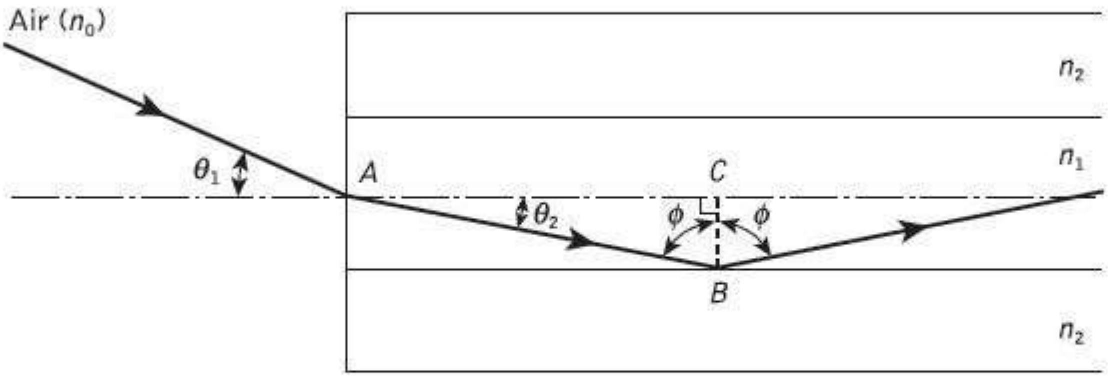

Figure 2.5 The ray path for a meridional ray launched into an optical fiber in air at an input angle less than the acceptance angle for the fiber

[Source: htpp://img.brainkart.com]

Assuming the entrance face at the fiber core to be normal to the axis, then considering the refraction at the air–core interface and using Snell's law given by Eq. (1.1):

$$n_0 \sin \theta_1 = n_1 \sin \theta_2 \tag{1.3}$$

Considering the right-angled triangle *ABC* indicated in Figure 2.5, then:

$$\phi = \frac{\pi}{2} - \theta_2 \tag{1.4}$$

where φ is greater than the critical angle at the core–cladding interface. Hence Eq. (1.3) becomes:

$$n_0 \sin \theta_1 = n_1 \cos \phi \tag{1.5}$$

Using the trigonometrical relationship $\sin^2 \varphi + \cos^2 \varphi = 1$, Eq. (1.5) may be written in the form:

$$n_0 \sin \theta_1 = n_1 (1 - \sin^2 \phi)^{\frac{1}{2}} \tag{1.6}$$

When the limiting case for total internal reflection is considered, φ becomes equal to the critical angle for the core–cladding interface and is given by Eq. (1.2). Also in this limiting case θ_1 becomes the acceptance angle for the fiber θ_a. Combining these limiting cases into Eq. (1.6) gives:

$$n_0 \sin \theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}} \tag{1.7}$$

Equation (1.7), apart from relating the acceptance angle to the refractive indices, serves as the basis for the definition of the important optical fiber parameter, the numerical aperture (*NA*). Hence the *NA* is defined as:

$$NA = n_0 \sin \theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}} \tag{1.8}$$

Since the *NA* is often used with the fiber in air where n_0 is unity, it is simply equal to $\sin \theta_a$. It may also be noted that incident meridional rays over the range $0 \le \theta_1 \le \theta_a$ will be propagated within the fiber. The *NA* may also be given in terms of the relative refractive index difference between the core and the cladding which is defined as:

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$\simeq \frac{n_1 - n_2}{n_1} \quad \text{for } \Delta \ll 1 \tag{1.9}$$

Hence combining Eq. (1.8) with Eq. (1.9) we can write:

$$NA = n_1(2\Delta)^{\frac{1}{2}} \tag{1.10}$$

The relationships given in Eqs (1.8) and (1.10) for the numerical aperture are a very useful measure of the light-collecting ability of a fiber. They are independent of the fiber core diameter and will hold for diameters as small as 8 μm. However, for smaller diameters they break down as the geometric optics approach is invalid. This is because the ray theory model is only a partial description of the character of light. It describes the direction a plane wave component takes in the fiber but does not take into account interference between such components. When interference phenomena are considered it is found that only rays with certain discrete characteristics propagate in the fiber core. Thus the fiber will only support a discrete number of guided modes. This becomes critical in small- core-diameter fibers which only support one or a few modes. Hence electromagnetic mode theory must be applied in these cases.

4. Skew Rays

In the preceding sections we have considered the propagation of meridional rays in the optical waveguide. However, another category of ray exists which is transmitted without passing through the fiber axis. These rays, which greatly outnumber the meridional rays, follow a helical path through the fiber, as illustrated in Figure 1.6, and are called skew rays.

It is not easy to visualize the skew ray paths in two dimensions, but it may be observed from Figure 1.6(b) that the helical path traced through the fiber gives a change in direction of 2γ at each reflection, where γ is the angle between the projection of the ray in two dimensions and the radius of the fiber core at the point of reflection. Hence, unlike meridional rays, the point of emergence of skew rays from the fiber in air will depend upon the number of reflections they undergo rather than the input conditions to the fiber. When the light input to the fiber is non uniform, skew rays will therefore tend to have a smoothing effect on the distribution of the light as it is transmitted, giving a more uniform output. The amount of smoothing is dependent on the number of reflections encountered by the skew rays. A further possible advantage of the transmission of skew rays becomes apparent when their acceptance conditions are considered. In order to calculate the acceptance angle for a skew ray it is necessary to define the direction of the ray in two perpendicular planes. The geometry of the situation is illustrated in Figure 1.7 where a skew ray is shown incident on the fiber core at the point *A*, at an angle θ_s to the normal at the fiber end face. The ray is refracted at the air–core interface before traveling to the point *B* in the same plane. The angles of incidence and reflection at the point*B* are φ, which is greater than the critical angle for the core–cladding interface.

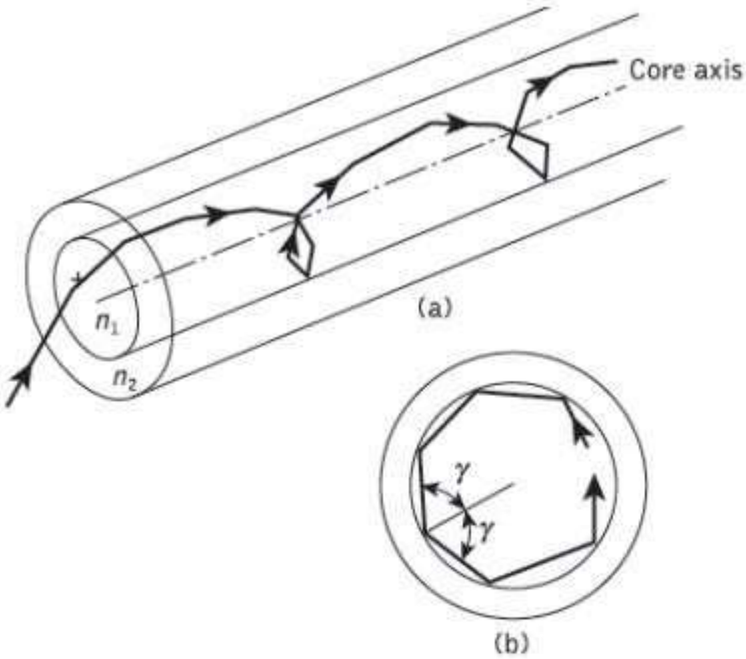

Figure 1.6 The helical path taken by a skew ray in an optical fiber: (a) skew ray path down the fiber; (b) cross-sectional view of the fiber

[Source: htpp://img.brainkart.com]

When considering the ray between *A* and *B* it is necessary to resolve the direction of the ray path *AB* to the core radius at the point *B*. As the incident and reflected rays at the point *B* are in the same plane, this is simply cos φ. However, if the two perpendicular planes through which the ray path *AB*traverses are considered, then γ is the angle between the core radius and the projection of the ray onto a plane *BRS* normal to the core axis, and θ is the angle between the ray and a line *AT* drawn parallel to the core axis. Thus to resolve the ray path *AB* relative to the radius *BR* in these two perpendicular planes requires multiplication by cos γ and sin θ

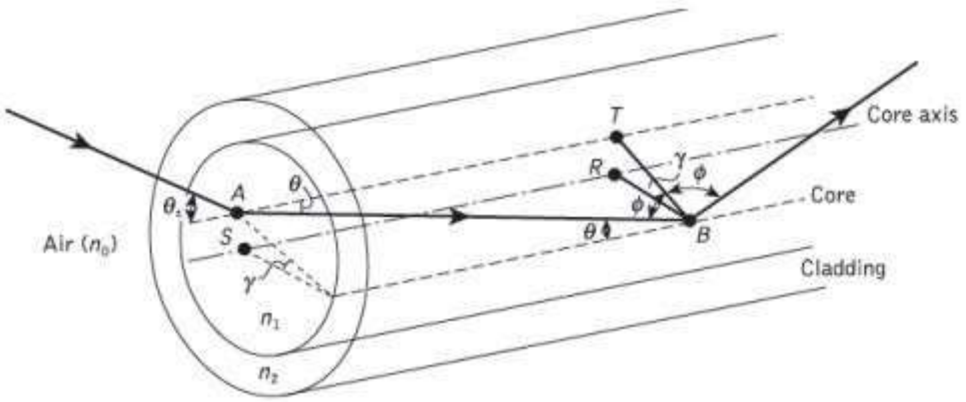

Figure 1.7 The ray path within the fiber core for a skew ray incident at an angle θ_s to the

[Source: htpp://img.brainkart.com]

Hence, the reflection at point *B* at an angle φ may be given by:

$$\cos\gamma \sin\theta = \cos\varphi \tag{1.11}$$

Using the trigonometrical relationship $\sin^2\varphi + \cos^2\varphi = 1$, Eq. (2.11) becomes:

$$\cos\gamma \sin\theta = \cos\phi = (1 - \sin^2\phi)^{\frac{1}{2}} \tag{1.12}$$

If the limiting case for total internal reflection is now considered, then φ becomes equal to the critical angle φ_c for the core–cladding interface and, following Eq. (2.2), is given by $\sin\varphi_c = n_2/n_1$. Hence, Eq. (2.12) may be written as:

$$\cos\gamma \sin\theta \leq \cos\phi_c = \left(1 - \frac{n_2^2}{n_1^2}\right)^{\frac{1}{2}} \tag{1.13}$$

Furthermore, using Snell's law at the point *A*, following Eq. (2.1) we can write:

$$n_0 \sin\theta_a = n_1 \sin\theta \tag{1.14}$$

where θ_a represents the maximum input axial angle for meridional rays, as expressed in Section 1.2.2, and θ is the internal axial angle. Hence substituting for sin θ from Eq.(1.13) into Eq. (1.14) gives:

$$\sin \theta_{as} = \frac{n_1}{n_0}\frac{\cos \phi_c}{\cos \gamma} = \frac{n_1}{n_0 \cos \gamma}\left(1 - \frac{n_2^2}{n_1^2}\right)^{\frac{1}{2}} \tag{1.15}$$

where θ_{as} now represents the maximum input angle or acceptance angle for skew rays. It may be noted that the inequality shown in Eq. (1.13) is no longer necessary as all the terms in Eq. (1.15) are specified for the limiting case. Thus the acceptance conditions for skew rays are:

$$n_0 \sin \theta_{as} \cos \gamma = (n_1^2 - n_2^2)^{\frac{1}{2}} = NA \tag{1.16}$$

and in the case of the fiber in air ($n_0 = 1$):

$$\sin \theta_{as} \cos \gamma = NA \tag{1.17}$$

Therefore by comparison with Eq. (1.8) derived for meridional rays, it may be noted that skew rays are accepted at larger axial angles in a given fiber than meridional rays, depending upon the value of cos γ. In fact, for meridional rays cos γ is equal to unity and θ_{as} becomes equal to θ_a. Thus although θ_a is the maximum conical half angle for the acceptance of meridional rays, it defines the minimum input angle for skew rays. Hence, as may be observed from Figure 1.6, skew rays tend to propagate only in the annular region near the outer surface of the core, and do not fully utilize the core as a transmission medium. However, they are complementary to meridional rays and increase the light-gathering capacity of the fiber. This increased light-gathering ability may be significant for large *NA* fibers, but for most communication design purposes the expressions given in Eqs (1.8) and (1.10) for meridional rays are considered adequate.

Single-Mode Fiber

The advantage of the propagation of a single mode within an optical fiber is that the signal dispersion caused by the delay differences between different modes in a multimode fiber may be avoided. Multimode step index fibers do not lend themselves to the propagation of a single mode due to the difficulties of maintaining single-mode operation within the fiber when mode conversion (i.e. coupling) to other guided modes takes place at both input mismatches and fiber imperfections.

Hence, for the transmission of a single mode the fiber must be designed to allow propagation of only one mode, while all other modes are attenuated by leakage or absorption. Following the preceding discussion of multimode fibers, this may be achieved through choice of a suitable normalized frequency for the fiber. For single-mode operation, only the fundamental LP_{01} mode can exist. Hence the limit of single-mode operation depends on the lower limit of guided propagation for the LP_{11} mode. The cutoff normalized frequency for the LP_{11} mode in step index fibers occurs at V_c = 2.405. Thus single-mode propagation of the LP_{01} mode in step index fibers is possible over the range:

$$0 \leq V < 2.405 \quad (1.51)$$

as there is no cutoff for the fundamental mode. It must be noted that there are in fact two modes with orthogonal polarization over this range, and the term single-mode applies to propagation of light of a particular polarization. Also, it is apparent that the normalized frequency for the fiber may be adjusted to within the range given in Eq. (1.51) by reduction of the core radius.

1. Cutoff Wavelength

It may be noted that single-mode operation only occurs above a theoretical cutoff wavelength λ_c given by:

$$\lambda_c = \frac{2\pi a n_1}{V_c}(2\Delta)^{\frac{1}{2}} \qquad (1.52)$$

where V_c is the cutoff normalized frequency. Hence λ_c is the wavelength above which a particular fiber becomes single-moded.

Dividing Eq. (1.52) by $V = \frac{2\pi}{\lambda} a n_1 (2\Delta)^{\frac{1}{2}}$

$$\frac{\lambda_c}{\lambda} = \frac{V}{V_c} \qquad (1.53)$$

Thus for step index fiber where $V_c = 2.405$, the cutoff wavelength is given by:

$$\lambda_c = \frac{V\lambda}{2.405} \qquad (1.54)$$

An effective cutoff wavelength has been defined by the ITU-T which is obtained from a 2 m length of fiber containing a single 14 cm radius loop. This definition was produced because the first higher order LP_{11} mode is strongly affected by fiber length and curvature near cutoff. Recommended cutoff wavelength values for primary coated fiber range from 1.1 to 1.28 μm for single-mode fiber designed for operation in the 1.3μm wavelength region in order to avoid modal noise and dispersion problems. Moreover, practical transmission systems are generally operated close to the effective cutoff wave-length in order to enhance the fundamental mode confinement, but sufficiently distant from cutoff so that no power is transmitted in the second-order LP_{11} mode.

2. Mode-Field Diameter and Spot Size

Many properties of the fundamental mode are determined by the radial extent of its electromagnetic field including losses at launching and jointing, micro bend losses, waveguide dispersion and the width of the radiation pattern. Therefore, the MFD is an important parameter for characterizing single-mode fiber properties which takes into account the wavelength-dependent field penetration into the fiber cladding. In this context it is a better measure of the functional properties of single-mode fiber than the core diameter. For step index and graded (near parabolic profile) single-mode fibers operating near the cutoff wavelength λ_c, the field is well approximated by a Gaussian distribution. In this case the MFD is generally taken as the distance between the opposite 1/e = 0.37 field amplitude points and the power $1/e^2$ = 0.135 points in relation to the corresponding values on the fiber axis.Another parameter which is directly related to the MFD of a single-mode fiber is the spot size (or mode-field radius) ω_0. Hence MFD = $2\omega_0$, where $\omega_0$is the nominal half width of the input excitation.

The MFD can therefore be regarded as the single- mode analog of the fiber core diameter in multimode fibers. However, for many refractive index profiles and at typical operating wavelengths the MFD is slightly larger than the single-mode fiber core diameter. Often, for real fibers and those with arbitrary refractive index profiles, the radial field distribution is not strictly Gaussian and hence alternative techniques have been proposed. However, the problem of defining the MFD and spot size for non-Gaussian field distributions is a difficult one and at least eight definitions exist.

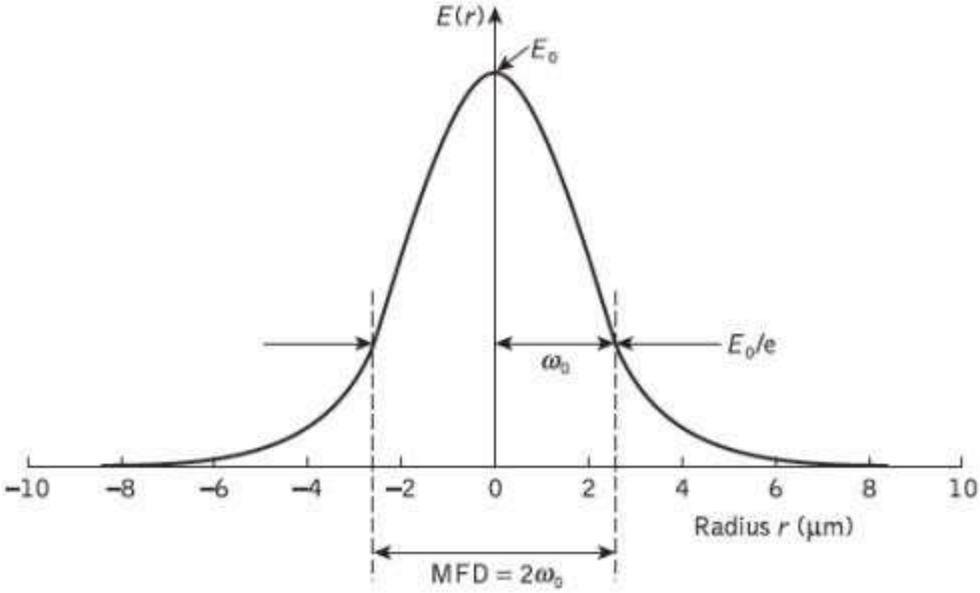

Figure 1.19 Field amplitude distribution $E(r)$ of the fundamental mode in a single-mode fiber illustrating the mode-field diameter (MFD) and spot size (ω_0)

[Source: htpp://img.brainkart.com]

3. Effective Refractive Index

The rate of change of phase of the fundamental LP_{01} mode propagating along a straight fiber is determined by the phase propagation constant . It is directly related to the wavelength of the LP01 mode λ01 by the factor 2π, since β gives the increase in phase angle per unit length. Hence:

$$\beta\lambda_{01} = 2\pi \quad \text{or} \quad \lambda_{01} = \frac{2\pi}{\beta} \tag{1.55}$$

Moreover, it is convenient to define an effective refractive index for single-mode fiber, sometimes referred to as a phase index or normalized phase change coefficient n_{eff}, by the ratio of the propagation constant of the fundamental mode to that of the vacuum propagation constant:

$$n_{eff} = \frac{\beta}{k} \tag{1.56}$$

Hence, the wavelength of the fundamental mode λ_{01} is smaller than the vacuum wave-length λ by the factor $1/n_{eff}$ where:

$$\lambda_{01} = \frac{\lambda}{n_{eff}} \tag{1.57}$$

It should be noted that the fundamental mode propagates in a medium with a refractive index $n(r)$ which is dependent on the distance r from the fiber axis. The effective refractive index can therefore be considered as an average over the refractive index of this medium. Within a normally clad fiber, not depressed-cladded fibers, at long wavelengths (i.e. small V values) the MFD is large compared to the core diameter and hence the electric field extends farinto the cladding region. In this case the propagation constant β will be approximately equal to n_2k (i.e. the cladding wave number) and the effective index will be similar to the refractive index of the cladding n_2. Physically, most of the power is transmitted in the cladding material.

At short wavelengths, however, the field is concentrated in the core region and the propagation constant β approximates to the maximum wave number n_1k. Following this discussion, and as indicated previously, then the propagation constant in single-mode fiber varies over the interval $n_2k < \beta < n_1k$. Hence, the effective refractive index will vary over the range $n_2 < n_{eff} < n_1$. In addition, a relationship between the effective refractive index and the normalized propagation constant b defined as:

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} = \frac{\beta^2 - n_2^2k^2}{n_1k^2 - n_2^2k^2} \tag{1.58}$$

may be obtained. Making use of the mathematical relation $A^2 - B^2 = (A + B)(A - B)$, Eq. (1.58) can be written in the form:

$$b = \frac{(\beta + n_2 k)(\beta - n_2 k)}{(n_1 k + n_2 k)(n_1 k - n_2 k)} \tag{1.59}$$

However, taking regard of the fact that $\beta \simeq n_1 k$, then

$$b \simeq \frac{\beta - n_2 k}{n_1 k - n_2 k} = \frac{\beta/k - n_2}{n_1 - n_2}$$

Finally, in Eq. (1.56) n_{eff} is equal to β/k, therefore:

$$b \simeq \frac{n_{eff} - n_2}{n_1 - n_2} \tag{1.60}$$

The dimensionless parameter b which varies between 0 and 1 is particularly useful in the theory of single-mode fibers because the relative refractive index difference is very small, giving only a small range for β. Moreover, it allows a simple graphical representation of results to be presented as illustrated by the characteristic shown in Figure 2.32 of the normalized phase constant of β as a function of normalized frequency V in a step index fiber.* It should also be noted that $b(V)$ is a universal function which does not depend explicitly on other fiber parameters.

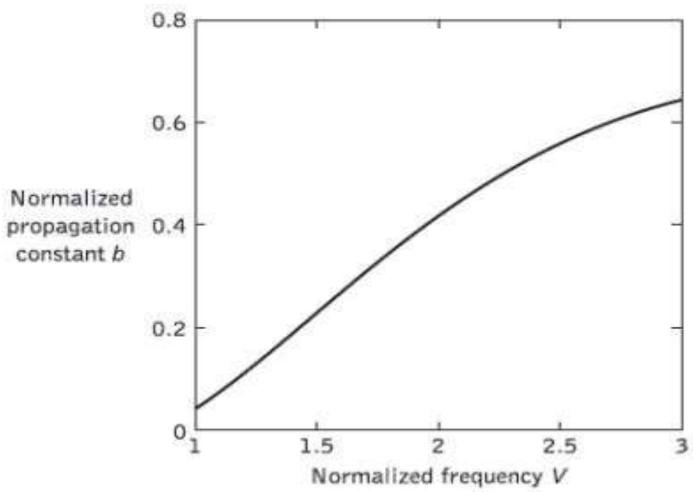

Figure 1.20 The normalized propagation constant (b) of the fundamental mode in a step index fiber shown as a function of the normalized frequency (V)

[Source: htpp://img.brainkart.com]

4. Group Delay and Mode Delay Factor

The transit time or group delay τ_g for a light pulse propagating along a unit length of fiber is the inverse of the group velocity υ_g . Hence:

$$\tau_g = \frac{1}{\upsilon_g} = \frac{d\beta}{d\omega} = \frac{1}{c}\frac{d\beta}{dk} \quad (1.61)$$

The group index of a uniform plane wave propagating in a homogeneous medium has been determined as:

$$N_g = \frac{c}{\upsilon_g}$$

However, for a single-mode fiber, it is usual to define an effective group index* N_{ge} by:

$$N_{ge} = \frac{c}{\upsilon_g} \quad (1.62)$$

Where υ_g is considered to be the group velocity of the fundamental fiber mode. Hence, the specific group delay of the fundamental fiber mode becomes:

$$\tau_g = \frac{N_{ge}}{c} \quad (1.63)$$

Moreover, the effective group index may be written in terms of the effective refractive index n_{eff} defined in Eq. (1.56) as:

$$N_{ge} = n_{eff} - \lambda\frac{dn_{eff}}{d\lambda} \quad (1.64)$$

$$\beta = k[(n_1^2 - n_2^2)b + n_2^2] \simeq kn_2[1 + b\Delta] \quad (1.65)$$

Furthermore, approximating the relative refractive index difference as $(n1 - n2)/n2$, for a weakly guiding fiber where Δ<<1, we can use the approximation :

$$\frac{n_1 - n_2}{n_2} \simeq \frac{N_{g1} - N_{g2}}{N_{g2}} \qquad (1.66)$$

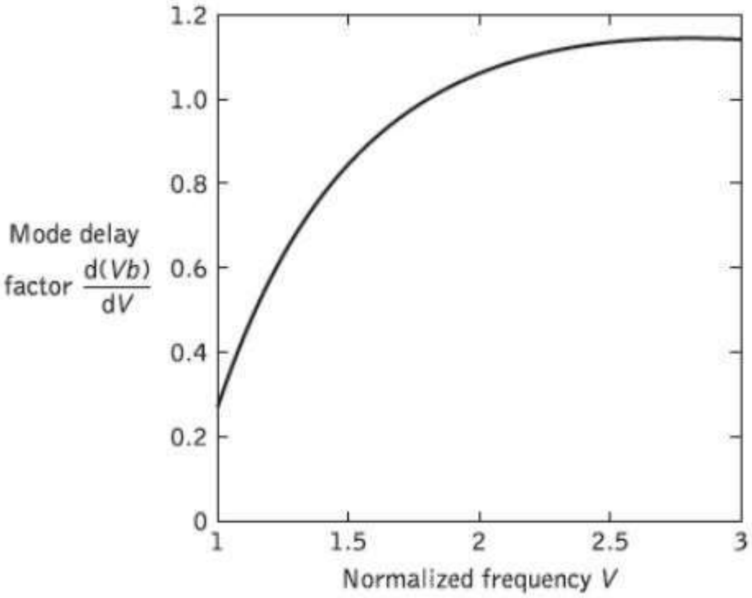

Figure 1.21 The mode delay factor (d(*Vb*)/d*V*) for the fundamental mode in a step index fiber shown as a function of normalized frequency (*V*)

[Source: htpp://img.brainkart.com]

Where N_{g1} and N_{g2} are the group indices for the fiber core and cladding regions respectively. Substituting Eq. (1.65) for β into Eq. (1.67) and using the approximate expression given in Eq. (1.66), we obtain the group delay per unit distance as:

$$\tau_g = \frac{1}{c}\left[N_{g2} + (N_{g1} - N_{g2})\frac{d(Vb)}{dV}\right] \qquad (1.67)$$

The dispersive properties of the fiber core and the cladding are often about the same and therefore the wavelength dependence of can be ignored. Hence the group delay can be written as:

$$\tau_g = \frac{1}{c}\left[N_{g2} + n_2\Delta\frac{d(Vb)}{dV}\right] \tag{1.68}$$

The initial term in Eq. (1.68) gives the dependence of the group delay on wavelength caused when a uniform plane wave is propagating in an infinitely extended medium with a refractive index which is equivalent to that of the fiber cladding. However, the second term results from the waveguiding properties of the fiber only and is determined by the mode delay factor d(*Vb*)/d*V*, which describes the change in group delay caused by the changes in power distribution between the fiber core and cladding. The mode delay factor [Ref. 50] is a further universal parameter which plays a major part in the theory of single-mode fibers. Its variation with normalized frequency for the fundamental mode in a step index fiber is shown in Figure 1.21.

5. The Gaussian Approximation

The field shape of the fundamental guided mode within a single-mode step index fiber for two values of normalized frequency is displayed in Figure 1.22.

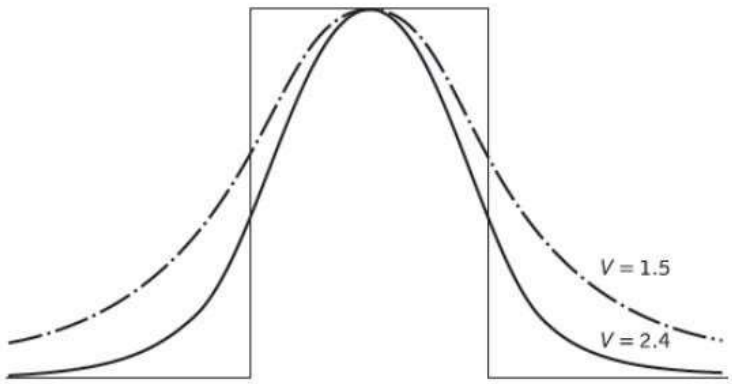

Figure 1.22 Field shape of the fundamental mode for normalized frequencies, V =1.5and V =2.4

[Source: htpp://img.brainkart.com]

As may be expected, considering the discussion in Section 2.4.1, it has the form of a Bessel function ($J_0(r)$) in the core region matched to a modified Bessel function ($K_0(r)$) in the cladding. Depending on the value of the normalized frequency, a significant proportion of the modal power is propagated in the cladding region, as mentioned earlier. Hence, even at the cutoff value (i.e. V_c) only about 80% of the power propagates within the fiber core.

It may be observed from Figure 1.22 that the shape of the fundamental LP_{01} mode is similar to a Gaussian shape, which allows an approximation of the exact field distribution by a Gaussian function.* The approximation may be investigated by writing the scalar wave equation in the form:

$$\nabla^2\psi + n^2k^2\psi = 0 \tag{1.69}$$

where k is the propagation vector defined in Eq. (1.33) and $n(x, y)$ is the refractive index of the fiber, which does not generally depend on z, the coordinate along the fiber axis. It should be noted that the time dependence exp(jωt) has been omitted from the scalar wave equation to give the reduced wave equation† in Eq. (1.69). This representation is valid since the guided modes of a fiber with a small refractive index difference have one predominant transverse field component, for example E_y. By contrast E_x and the longitudinal component are very much smaller.The field of the fundamental guided mode may therefore be considered as a scalar quantity and need not be described by the full set of Maxwell's equations. Hence Eq. (1.69) may be written as:

$$\nabla^2\phi + n^2k^2\phi = 0 \tag{1.70}$$

where φ represents the dominant transverse electric field component. The near-Gaussian shape of the predominant transverse field component of the fundamental mode has been demonstrated for fibers with a wide range of refractive index distributions. This proves to be the case not only for the LP_{01} mode of the step index fiber, but also for the modes with fibers displaying arbitrary graded refractive index distributions.

6. Equivalent Step Index Method

Another strategy to obtain approximate values for the cutoff wavelength and spot size in graded index single-mode fibers (or arbitrary refractive index profile fibers) is to define an equivalent step index (ESI) fiber on which to model the fiber to be investigated. Various methods have been proposed in the literature which commence from the observation that the fields in the core regions of graded index fibers often appear similar to the fields within step index fibers. Hence, as step index fiber characteristics are well known, it is convenient to replace the exact methods for graded index single-mode fibers by approximate techniques based on step index fibers. In addition, such ESI methods allow the propagation characteristics of single-mode fibers to be represented by a few parameters. Several different suggestions have been advanced for the choice of the core radius a_{ESI}, and the relative index difference $_{ESI}$, of the ESI fiber which lead to good approximations for the spot size (and hence joint and bend losses) for the actual graded index fiber. They are all conceptually related to the Gaussian approximation in that they utilize the close resemblance of the field distribution of the LP_{01} mode to the Gaussian distribution within single-mode fiber.

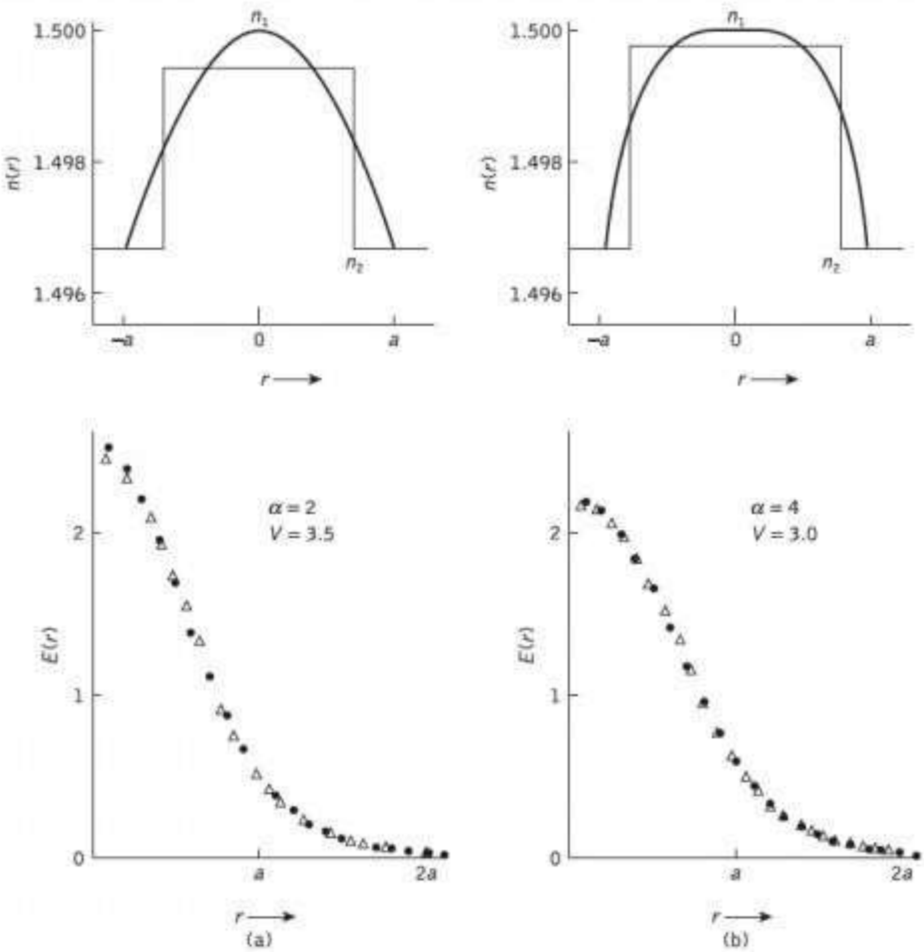

Figure 1.23 Refractive index distributions $n(r)$ and electric field distributions $E(r)$ for graded index fibers and their ESI fibers for: (a) α=2, V=3.5; (b) α=4, V=3.0.

[Source: htpp://img.brainkart.com]

An early proposal for the ESI method involved transformation of the basic fiber parameters following/:

$$a_s = Xa \qquad V_s = YV \qquad NA_s = (Y/X)NA$$

where the subscript s is for the ESI fiber and X, Y are constants which must be determined. However, these ESI fiber representations are only valid for a particular value of normalized frequency V and hence there is a different X, Y pair for each wavelength differences. Figure 1.23 compares the refractive index profiles and the electric field distributions for two graded index fibres (α = 2, 4) and their ESI fibers. It may be observed that their fields differ slightly only near the axis.