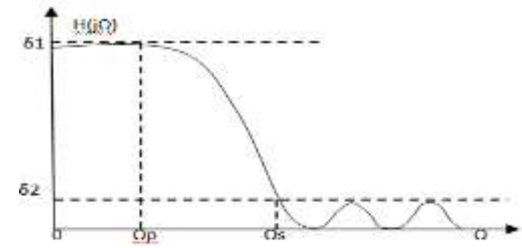
Magnitude Response of LPF:



Design of IIR filters from analog filters:

The different design techniques available for IIR filter are

1) Approximation of derivates
2) Impulse invariant method
3) Bilinear transformation
4) Matched z-transform techniques.

Approximation of derivates:

For analog to digital domain, we get

$$s = \frac{1 - z^{-1}}{T} \quad \text{..................................(3)}$$

$$H(z) = H(s)\Big|_{s=\frac{1-z^{-1}}{T}} \quad \text{-----------------------------------(4)}$$

Mapping of the s-plane to the z-plane using approximation of derivatives.

Convert the analog BPF with system IIR filter $H_a(s) = \frac{1}{(s+0.1)^2 + 9}$ into a digital IIR filter by use of the backward difference for the derivative. [Nov/Dec-2015]

Solution:

Given:

$$H(z) = H_a(s)\Big|_{s=\frac{1-z^{-1}}{T}}$$

$$H(z) = \frac{1}{(s+0.1)^2 + 9}\Bigg|_{s=\frac{1-z^{-1}}{T}}$$

$$= \frac{1}{\left(\frac{1-z^{-1}}{T} + 0.1\right)^2 + 9}$$

$$= \frac{T^2\left(1 + 0.2T + 9.01T^2\right)}{1 - \frac{2(1+0.1T)}{1+0.2T+9.01T^2}z^{-1} + \frac{1}{1+0.2T+9.01T^2}z^{-2}}$$

$T = 0.1\,sec,$

$$= 0.91 \pm j0.27$$

Design of IIR filter using Impulse Invariance Method:

Steps to design a digital filter using Impulse Invariance Method (IIM):

Step 1: For the given specifications, find Ha(s) the Transfer function of an analog filter.

Step 2: Select the sampling rate of the digital filter, T seconds per sample.

Step 3: Express the analog filter transfer function as the sum of single-pole filter.

$$H_a(s) = \sum_{k=1}^{N} \frac{C_k}{S - P_k}$$

Step 4: Compute the z-transform of the digital filter by using formula

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1 - e^{p_k T} z^{-1}}$$

For high sampling rate,

$$\therefore H(z) = \sum_{k=1}^{N} \frac{TC_k}{1 - e^{p_k T} z^{-1}}$$

For the analog transfer function $H(s) = \frac{2}{s^2 + 3s + 2}$ **Determine H (z) using impulse invariant transformation if (a) T=1 second and (b) T=0.1 second. [Nov/Dec-15]**

Solution:

$$Given\ that, H(s) = \frac{2}{s^2 + 3s + 2} = \frac{2}{(s+1)(s+2)}$$

By partial fraction expansion technique we can write,

The roots of quadratic,

$S2 + 3S + 2 = 0$ *are,*

$$s = \frac{-3 \pm \sqrt{3^2 - 4X2}}{2}$$

$$= \frac{-3 \pm 1}{2} = -1, -2$$

$$H(s) = \frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \frac{2}{(s+1)(s+2)} X (s+1) \Big|_{s=-1} = \frac{2}{-1+2} = 2$$

$$B = \frac{2}{(s+1)(s+2)} X (s+2) \Big|_{s=-2} = \frac{2}{-2+1} = -2$$

$$\therefore H(s) = \frac{2}{s+1} + \frac{-2}{s+2}$$

By impulse invariant transformation we know that,

$$\frac{A_i}{s+p_i} \rightarrow \frac{A_i}{1-e^{-p_iT}z^{-1}}$$

$$H(z) = \frac{2}{1-e^{-p_iT}z^{-1}} + \frac{-2}{1-e^{-p_iT}z^{-1}} \quad Where\ p_1 = 1,\ p_2 = -2$$

$$\boxed{H(z) = \frac{2}{1-e^{-T}z^{-1}} + \frac{-2}{1-e^{-2T}z^{-1}}}$$

(a) When T = 1second

$$H(z) = \frac{2}{1-e^{-1}z^{-1}} + \frac{-2}{1-e^{-2}z^{-1}}$$

$$H(z) = \frac{2}{1-0.1353z^{-1}} + \frac{2}{1-0.1353z^{-1}} = \frac{2(1-0.1353z^{-1}) - 2(1-0.3679z^{-1})}{(1-0.367z^{-1})(1-0.1353z^{-1})}$$

$$= \frac{2-0.27606z^{-1} - 2 + 0.7358z^{-1}}{1-0.1353z^{-1} - 0.3679z^{-1} + 0.0498z^{-2}} = \frac{0.4652z^{-1}}{1-0.5032z^{-1} + 0.0498z^{-2}}$$

$$H(z) = \frac{0.4652z^{-1}}{1-0.5032z^{-1} + 0.0498z^{-2}} = \frac{0.4652z^{-1}}{z^{-2}(z^2 - 0.5032z + 0.0498)}$$

$$\boxed{H(z) = \frac{0.4652z}{(z^2 - 0.5032z + 0.0498)}}$$

(b) When $T = 0.1$second

$$H(z) = \frac{2}{1-e^{-0.1}z^{-1}} + \frac{-2}{1-e^{-0.2}z^{-1}}$$

$$= \frac{2}{1-0.9048z^{-1}} + \frac{-2}{1-0.8187z^{-1}} = \frac{2(1-0\ .8187z^{-1}) - 2(1-0.9048z^{-1})}{(1-0.8187z^{-1})(1-0.9048z^{-1})}$$

$$= \frac{2-1.6374z^{-1} - 2 + 1.8096z^{-1}}{1-0.8187z^{-1} - 0.9048z^{-1} + 0.7408z^{-2}} = \frac{0.1722z^{-1}}{1-1.7235z^{-1} + 0.7408z^{-2}}$$

$$H(z) = \frac{0.1722z^{-1}}{1-1.7235z^{-1} + 0.7408z^{-2}} = \frac{0.1722z^{-1}}{z^{-2}(z^2 - 1.7235z + 0.7408)}$$

$$= \frac{0.1722z}{z^2 - 1.7235z + 0.7408}$$

Since, $T < 1$, *we can compute magnitude normalized transfer function*, $H(z)$.

$$H_N(z) = T \ X \ H(z) = 0.1X \frac{0.1722z^{-1}}{1-1.7235z^{-1} + 0.7408z^{-2}} = \frac{0.1722z^{-1}}{1-1.7235z^{-1} + 0.7408z^{-2}}$$

$$H_N(z) = T \ X \ H(z) = 0.1X \ \frac{0.1722z}{z^2 - 1.7235z + 0.7408} = \frac{0.172z}{z^2 - 1.7235z + 0.7408}$$

Design a third order Butterworth digital filter using impulse invariant technique. Assume sampling period *T=1 sec.*

Solution:

Given: For N=3, the transfer function of a normalized Butterworth filter is given by

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$\frac{1}{} \qquad \frac{-1+0.66z^{-1}}{}$$

Ans: $H(z) = \frac{}{1-0.368z^{-1}} + \frac{}{1-0.786z^{-1}+0.368z^{-2}}$

Apply impulse invariant method and find H(z) for $H(s) = \dfrac{s+a}{(s+a)^2 + b^2}$

Solution:

Given: The transfer function $H(s) = \dfrac{s+a}{(s+a)^2 + b^2}$

Sampling the function produces

$$h(nT) = \begin{cases} e^{-anT}\cos(bnT) & for\ n \geq 0 \\ 0 & otherwise \end{cases}$$

$$H(z) = \sum_{n=0}^{\infty} e^{-anT}\cos(bnT) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left[e^{-anT} z^{-n} \left(\frac{e^{jbnT} + e^{-jbnT}}{2} \right) \right]$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \left[\left(e^{-(a-jb)T} z^{-1}\right)^n + \left(e^{-(a+jb)T} z^{-1}\right)^n \right]$$

$$= \frac{1}{2} \left[\frac{1}{1-e^{-(a-jb)T} z^{-1}} + \frac{1}{1-e^{-(a+jb)T} z^{-1}} \right]$$

$$H(z) = \frac{1 - e^{-aT}\cos(bT)z^{-1}}{1 - 2e^{-aT}\cos(bT)z^{-1} + e^{-2aT} z^{-2}}$$

Convert analog filter $H_a(s) = \dfrac{s+0.1}{(s+0.1)^2 + 9}$ **into digital IIR filter using impulse invariant method. [Nov/Dec-2015]**

Solution:

Given: Analog filter $H_a(s) = \dfrac{s+0.1}{(s+0.1)^2 + 9}$

$$\frac{s+a}{(s+a)^2 + b^2} = \frac{1 - e^{-aT}\cos(bT)z^{-1}}{1 - 2e^{-aT}\cos(bT)z^{-1} + e^{-2aT} z^{-2}}$$

$$\frac{s+0.1}{(s+0.1)^2 + 9} = \frac{1 - e^{-aT}\cos(bT)z^{-1}}{1 - 2e^{-aT}\cos(bT)z^{-1} + e^{-2aT} z^{-2}}; T = 1\ sec$$

$$\frac{s+0.1}{(s+0.1)^2 + 9} = \frac{1 - e^{-0.1}\cos(3)z^{-1}}{1 - 2e^{-0.1}\cos(3)z^{-1} + e^{-2*0.1} z^{-2}};$$

$$= \frac{1 - 0.9048 * (-0.9899)z^{-1}}{1 + 1.791z^{-1} + 0.818z^{-2}}$$

$$H(z) = \frac{1 + 0.89566z^{-1}}{1 + 1.7915z^{-1} + 0.818z^{-2}}$$

Convert analog filter $H_a(s) = \frac{6}{(s+0.1)^2 + 36}$ **into digital IIR filter whose system function is given above. The digital filter should have (**$\omega r = 0.2\pi$**). Use impulse invariant mapping T=1sec.**

Solution:

Given: Analog filter $H_a(s) = \dfrac{6}{(s+0.1)^2+36}$

$$\frac{b}{(s+a)^2+b^2} = \frac{e^{-aT}\sin bTz^{-1}}{1-2e^{-aT}\cos bTz^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{6}{(s+0.1)^2+36} = \frac{e^{-0.1}\sin(6)z^{-1}}{1-2e^{-0.1}\cos(6)z^{-1}+e^{-2*0.1}z^{-2}}$$

Assume $T = 1$ *sec.*

$$\frac{6}{(s+0.1)^2+36} = \frac{-0.2528z^{-1}}{1-2*0.8687z^{-1}+0.818z^{-2}}$$

$$= \frac{-0.2528z^{-1}}{1-1.7374z^{-1}+0.818z^{-2}}$$

H.W: Challenge 1: An analog filter has a transfer function $H_a(s) = \dfrac{10}{s^2+7s+10}$**. Design a digital filter equivalent to this using impulse invariant method for T=0.2 sec. [Nov/Dec-15]**

$$\text{Ans : } H(z) = \frac{0.2012z^{-1}}{1-1.0378z^{-1}+0.247z^{-2}}$$

2. An analog filter has a transfer function $H(s) = \dfrac{5}{s^3+6s^2+11s+6}$**. Design a digital equivalent to this using impulse invariant method for T=1 sec.**

3. An analog filter has a transfer function $H(s) = \dfrac{s+3}{s^2+6s+25}$**. Design a digital filter equivalent to this using impulse invariant method T=1 sec.**

<u>Design of IIR filters using Bilinear Transformation:</u>

Steps to design digital filter using bilinear transform technique:

1. From the given specifications, find prewarping analog frequencies using formula $\Omega = \dfrac{2}{T}\tan\dfrac{\omega}{2}$
2. Using the analog frequencies find *H(s)* of the analog filter.
3. Select the sampling rate of the digital filter, call it T seconds per sample.
4. Substitute $s = \dfrac{2}{T}\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)$ into the transfer function found in step2.

Apply bilinear transformation of $H(s) = \dfrac{2}{(s+1)(s+2)}$ **with *T=1 sec* and find *H(z).[Nov/Dec-13]***

Solution:

Given: The system function $H(s) = \frac{2}{(s+1)(s+2)}$

$$\text{Substitute } s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] \text{ in } H(s) \text{ to get } H(z)$$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{2}{(s+1)(s+2)}\Bigg|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

Given *T=1 sec.*

$$H(z) = \frac{2}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right\}\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right\}}$$

$$= \frac{2\left(1+z^{-1}\right)^2}{\left(3-z^{-1}\right)(4)}$$

$$= \frac{\left(1+z^{-1}\right)^2}{6-2z^{-1}}$$

$$H(z) = \frac{0.166(1+z^{-1})^2}{(1-0.33z^{-1})}$$

Using the bilinear transformation, design a high pass filter, monotonic in pass band with cut off frequency of 1000Hz and down 10dB at 350 Hz. The sampling frequency is 5000Hz. [May/June-16]

Solution:

Given: Pass band attenuation $\alpha_P = 3dB$; Stop band attenuation $\alpha_S = 10dB$

Pass band frequency $\omega_P = 2\pi * 1000 = 2000\pi$ *rad/sec.*

Stop band frequency $\omega_S = 2\pi * 350 = 700\pi$ *rad/sec.*

$$T = \frac{1}{f} = \frac{1}{5000} = 2*10^{-4} \text{ sec.}$$

Prewarping the digital frequencies, we have

$$\Omega_P = \frac{2}{T}\tan\frac{\omega_P T}{2} = \frac{2}{2*10^{-4}}\tan\frac{\left(2000\pi*2*10^{-4}\right)}{2} = 10^4\tan(0.2\pi) = 7265\, rad/sec.$$

$$\Omega_S = \frac{2}{T}\tan\frac{\omega_S T}{2} = \frac{2}{2*10^{-4}}\tan\frac{\left(700\pi*2*10^{-4}\right)}{2} = 10^4\tan(0.07\pi) = 2235\, rad/sec.$$

The order of the filter

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_S}-1}{10^{0.1\alpha_P}-1}}}{\log \frac{\Omega_S}{\Omega_P}}$$

$$= \frac{\log \sqrt{\frac{10^{0.1\times 10}-1}{10^{0.1\times 3}-1}}}{\log \frac{7265}{2235}}$$

$$= \frac{\log(3)}{\log(3.25)} = \frac{0.4771}{0.5118} = 0.932$$

$$N = 1$$

The first order Butterworth filter for Ω_C=1 rad/sec is H(s) = 1/S+1

The high pass filter for $\Omega_C=\Omega_P$=7265 rad/sec can be obtained by using the transformation.

$$S \to \frac{\Omega_C}{s}$$

$$S \to \frac{7265}{s}$$

The transfer function of high pass filter

$$H(s) = \left.\frac{1}{s+1}\right|_{s=\frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \left.\frac{s}{s+7265}\right|_{s=\frac{2}{2*10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265}$$

$$= \frac{0.5792\left(1-z^{-1}\right)}{1-0.1584z^{-1}}$$

H.W: 1. Determine H(z) that results when the bilinear transformation is applied to

Ha(s)= $\frac{s^2+4.525}{s^2+0.692s+0.504}$ **[Nov/Dec-15]** Ans: $H(z) = \frac{1.4479+0.1783z^{-1}+1.4479z^{-2}}{1-1.18752z^{-1}+0.5299z^{-2}}$

2. An analog filter has a transfer function $H(s)=\frac{1}{s^2+6s+9}$,design a digital filter using bilinear transformation method.

Additional Examples:

Design a digital Butterworth filter satisfying the constraints

$$0.707 \le |H(e^{j\omega})| \le 1 \quad \text{for } 0 \le \omega \le \frac{\pi}{2}$$

$$|H(e^{jw})| \le 0.2 \quad \text{for } \frac{3\pi}{4} \le \omega \le \pi$$

With *T=1 sec using bilinear transformation.* *[April/May-2015][May/June-14]*

Solution:

Given data:

Pass band attenuation $\alpha_P = 0.707$ *; Pass band frequency* $\omega_P = \frac{\pi}{2}$ *;*

Stop band attenuation $\alpha_S = 0.2$ *; Stops band frequency* $\omega_S = \frac{3\pi}{4}$ *;*

Step 1: Specifying the pass band and stop band attenuation in dB.

Pass band attenuation $\alpha_P = -20\log\delta 1 = -20\log(0.707) = 3.0116\,dB$

Stop band attenuation $\alpha_S = -20\log\delta 2 = -20\log(0.2) = 13.9794\,dB$

Step2. Choose *T* and determine the analog frequencies (i.e) Prewarp band edge frequency

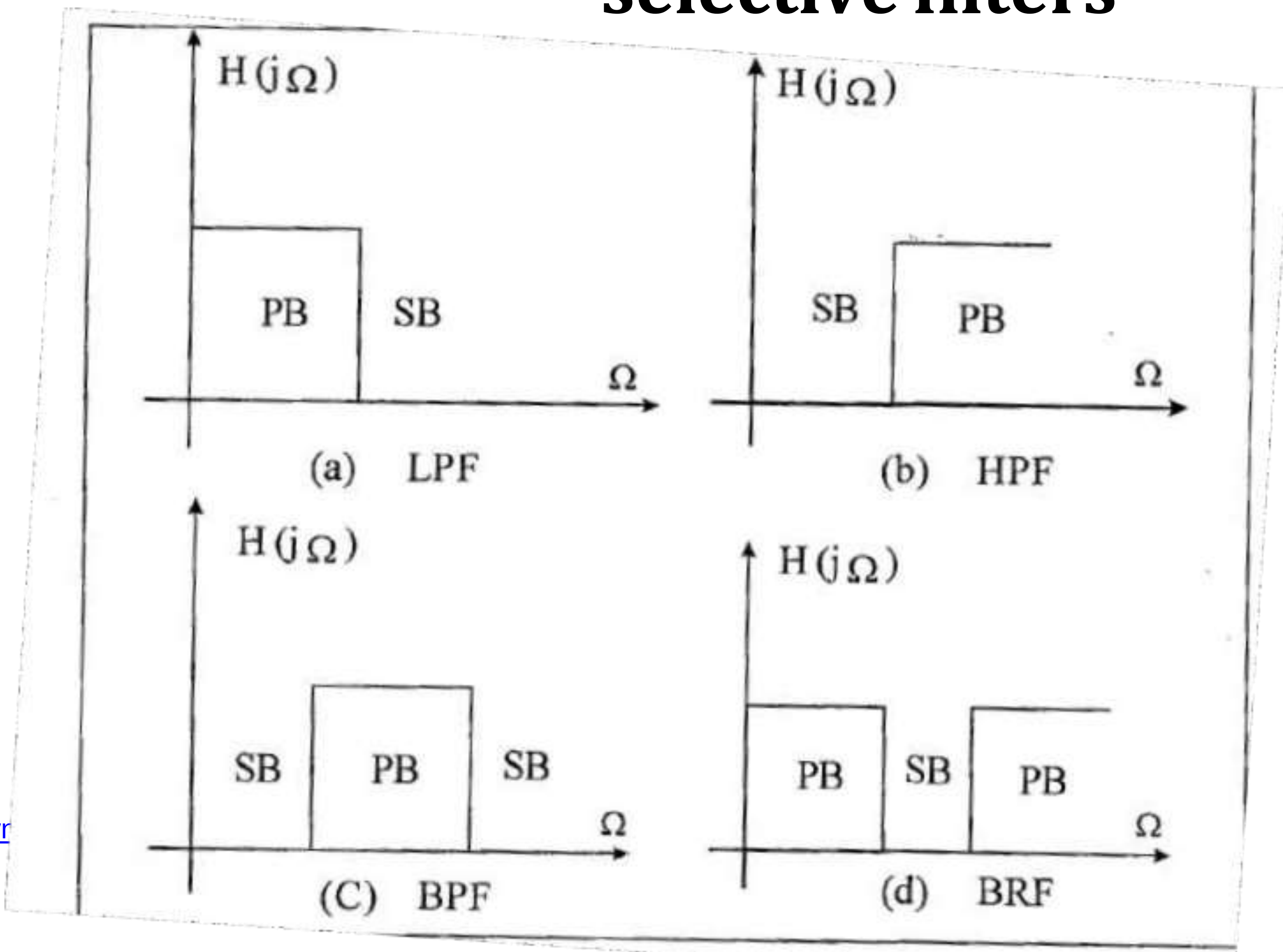
$$\Omega_P = \frac{2}{T}\tan\left(\frac{\omega_P T}{2}\right) = \frac{2}{1}\tan\left(\frac{\frac{\pi}{2}}{2}\right) = 2\tan\left(\frac{\pi}{4}\right) = 2\,Rad/Sec$$

$$\Omega_S = \frac{2}{T}\tan\left(\frac{\omega_S T}{2}\right) = \frac{2}{1}\tan\left(\frac{\frac{3\pi}{4}}{2}\right) = 2\tan\left(\frac{3\pi}{8}\right) = 4.828\,Rad/Sec$$

Step3. To find order of the filter

$$N \ge \left[\frac{\log_{10}\left(\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}\right)}{\log_{10}\left(\frac{\Omega_s}{\Omega_P}\right)}\right]$$

Characteristics of practical frequency selective filters



Digital Vs. Analog filter

	Analog filter	Digital filter
i)	Analog filter processes analog input and generates analog output.	A digital filter processes and generate digital data.
ii)	They are constructed from active (or) passive electronic components.	They consists of elements like adders, multiplier and delay unit.
iii)	Analog filter is described by a differential equation.	Digital filter is represented by a difference equation.
iv)	The frequency response of an analog fitter can be modified by changing the components.	The frequency response can be changed by changing the fitter coefficients.

Advantages of digital filters

i) Unlike analog filter, the digital filter performance is not influenced by component ageing, temperature and power supply variation.

ii) A digital filter is highly immune to noise and possesses considerable parameters stability.

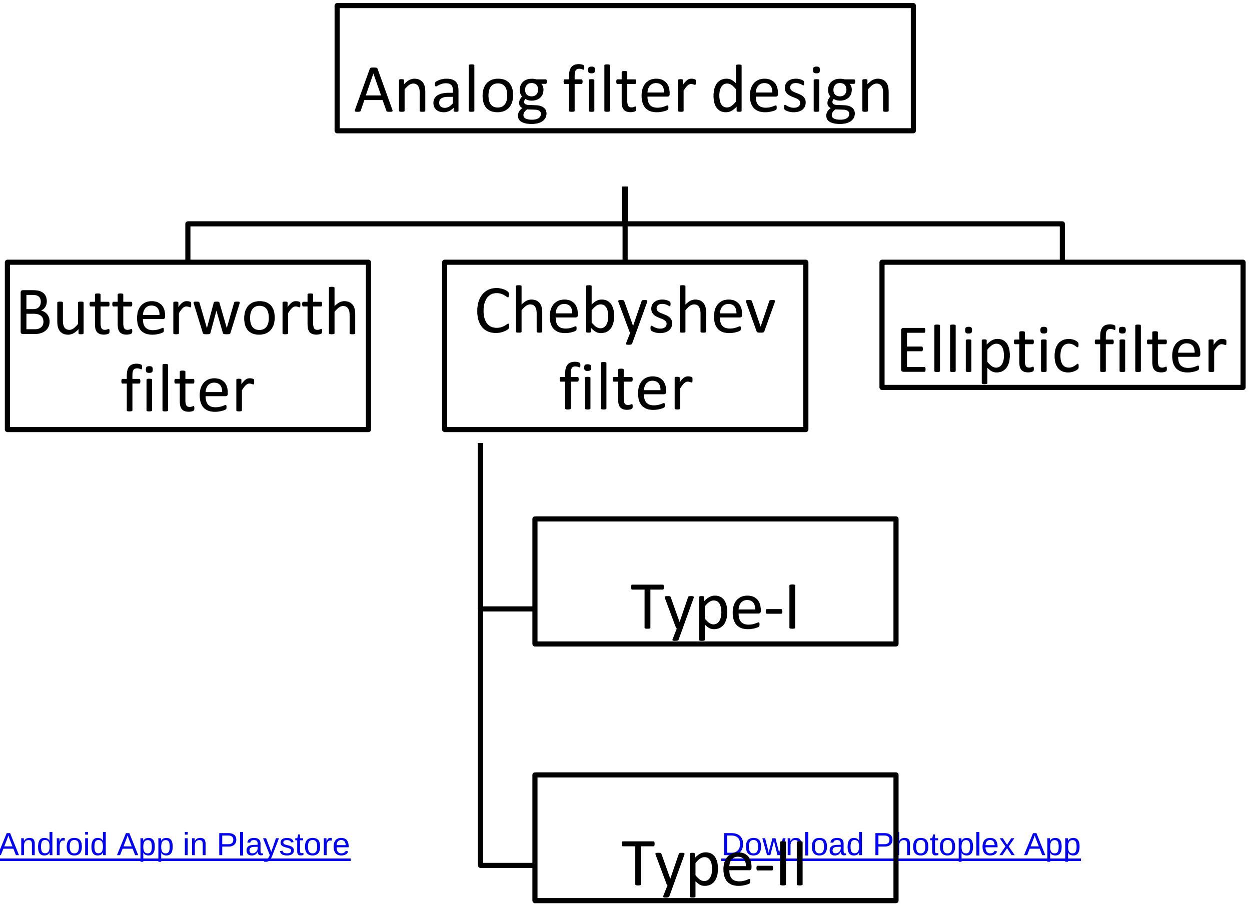
iii) Digital filter afford a wide variety of shapes for the amplitude and phase response. There are no problem of input (or) output impedance matching with digital filter.

iv) Digital filter can be osperated over a wide range of frequencies.

v) The coefficients of digital filter can be programmed and altered any time to obtain the desired characteristics.

vi) Multiple filtering is possible only in digital filter.

IIR Filter design



Analog Butterworth filter design

- Step-1: from the given specification, find the order of filter ‘N’

$$N \geq \frac{\log\left[\sqrt{\frac{10^{0.1\alpha_s -1}}{10^{0.1\alpha_p -1}}}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} \geq \frac{\log(\lambda/\varepsilon)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

Where $\varepsilon_1 = \sqrt{10^{0.1\alpha_p} - 1}$

$$\lambda = \sqrt{10^{0.1\alpha_s} - 1}$$

Analog Butterworth filter design

- Step-2: Round off the above found ‘N’ to its next highest integer
- Step-3: Find the normalized transfer function H(s) for Ωc=1 rad/s, for the value of N

$$H(s) = \frac{N_r \text{ polynomial}}{D_r \text{ polynomial}}$$

- For Butterworth filter Nr polynomial is alway1

- Step-4: calculate the value of cut-off frequency Ωc using,

$$\Omega_C = \frac{\Omega_P}{\left(10^{0.1\alpha_P} - 1\right)^{1/2N}} = \frac{\Omega_P}{\varepsilon^{1/N}}$$

Step-6: To find Ha(s)

Step 5:

Find the transfer function

$H_a(s)$ for the above values of Ω_c by substituting $s \rightarrow s/\Omega_c$ in H (s)

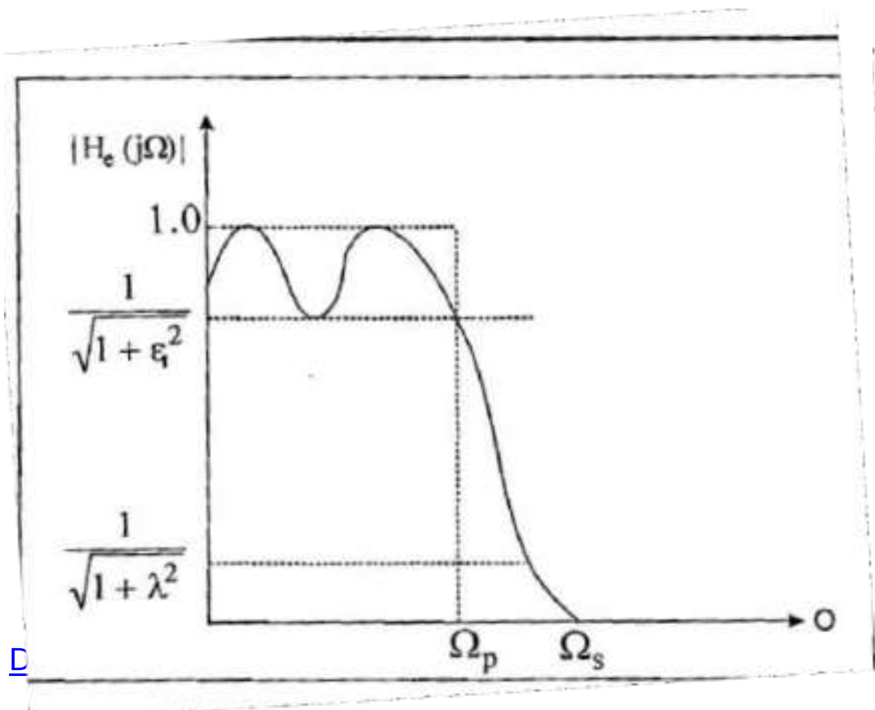
For HPF Ha(s) = H (s) | $s \rightarrow \Omega c/s$

For LPF Ha(s) = H (s) | $s \rightarrow s/\Omega c$

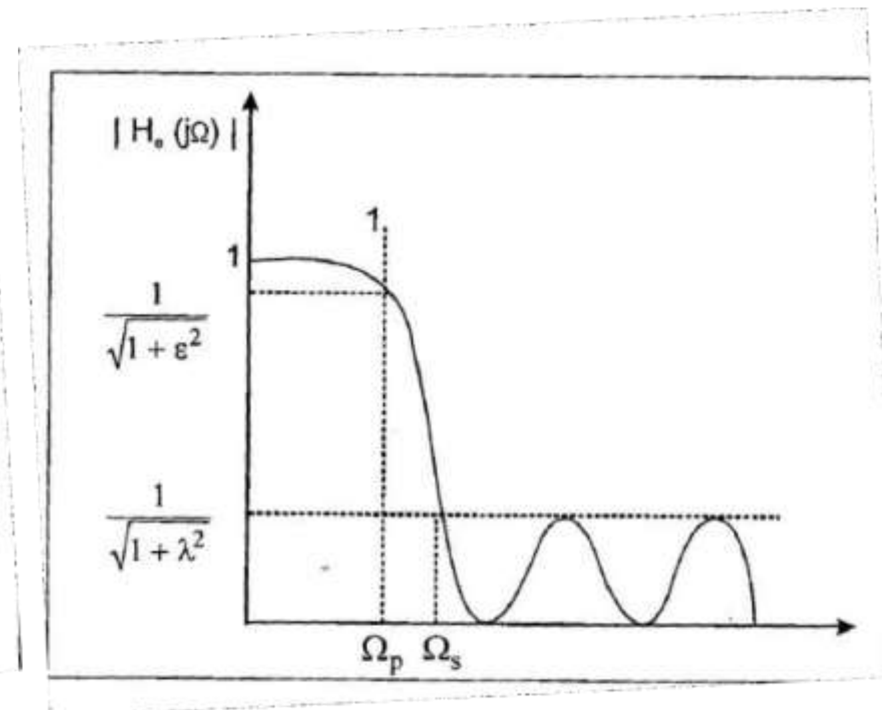
D

Analog Chebyshev filter

Type-I Chebyshev filter



Type-II Chebyshev filter



Steps to design of analog Chebyshev filter

- Step-1: Find N

$$N \geq \frac{\cosh^{-1} \lambda / \xi}{\cosh^{-1} \frac{\Omega_s}{\Omega_p}} \geq \frac{\cosh^{-1} \sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\cosh^{-1} \frac{\Omega_s}{\Omega_p}}$$

- Step-2: Round off the above found 'N' to its next highest integer

Steps to design of analog Chebyshev filter

- step-3: Using following formula find the values of a & b which are minor and major axis of ellipse respectively

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2} \quad b = \Omega_p \frac{[\mu^{1/N} + \mu^{-1/N}]}{2}$$

Where $\mu = \xi^{-1} + \sqrt{\xi^{-2} + 1}$

$$\varepsilon = \sqrt{10^{0-1\alpha_p} - 1}$$

Steps to design of analog Chebyshev filter

- Step-4:

Calculate the poles of chebyshev filter which lie on the ellipse by using the formula

$$S_k = a\cos\varphi_k + jb\sin\varphi_k \quad k = 1, 2 \ldots N$$

$$\text{Where } \varphi_k = \pi/2 + \left(\frac{2k-1}{2N}\right)\pi, \; k = 1, 2 \ldots N$$

- Step-5: Find the Denominator polynomial of the transfer function using the above poles

Steps to design of analog Chebyshev filter

- Step-6:

The Numerator of the transfer function depends on the value of N.

For N = odd, substitute s = 0 in Denominator polynomial & find the value. This value is equal to the Numerator of transfer function.

For N = even, substitute s = 0 in Denominator polynomial & divide the result by $\sqrt{1+\varepsilon^2}$, This value is equal to the numerator.

7	$(s+1)(s^2+1.80194s+1)(s^2+1.247s+1)(s^2+0.445s+1)$

Design an analog Butterworth filter that has a -2dB pass band attenuation at a frequency of 20 rad/sec and atleast -10 dB stop band attenuation at 30 rad/sec.

Solution:

Given data:

Pass band attenuation α_P= 2 dB;

Stop band attenuation α_S= 10 dB;

Pass band frequency Ω_P= 20 rad/sec.

Stop band frequency Ω_S=30 rad/sec.

The order of the filter

$$N \geq \left[\frac{\log_{10} \sqrt{\left(\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1} \right)}}{\log_{10} \left(\frac{\Omega_s}{\Omega_P} \right)} \right]$$

$$N \geq \left[\frac{\log_{10} \sqrt{\left(\frac{10^{0.1*10} - 1}{10^{0.1*2} - 1} \right)}}{\log_{10} \left(\frac{30}{20} \right)} \right]$$

$$\geq \frac{\log \sqrt{\left(\frac{10 - 1}{10^{0.2} - 1} \right)}}{\log \frac{30}{20}}$$

$$\geq 3.37$$

Rounding off ‘N’ to the next higher integer, we get

$$N=4$$

The normalized transfer function for $N=4$.

$$H_a(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.8477s+1)}$$

To find cut off frequency

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha p} - 1)^{1/2N}}$$

20

$$\Omega_c = \frac{}{\left(10^{0.1*20} - 1\right)^{\frac{1}{2*4}}} = 21.3868$$

The transfer function for Ωc=21.3868,

$$H(s) = H_a(s)\Big|_{S \to \frac{s}{21.3868}}$$

$$H(s) = \frac{1}{\left(s^2 + 0.76537s + 1\right)\left(s^2 + 1.8477s + 1\right)}\Bigg|s -> \frac{s}{21.3868}$$

$$H(s) = \frac{1}{\left(\frac{s}{21.868}\right)^2 + 0.76535\left(\frac{s}{21.3868}\right) + 1} * \frac{1}{\left(\frac{s}{21.868}\right)^2 + 1.8477\left(\frac{s}{21.3868}\right) + 1}$$

$$H(s) = \frac{0.20921 \times 10^6}{\left(s^2 + 16.3686s + 457.394\right)\left(s^2 + 39.5176s + 457.394\right)}$$

****

****

H.W: Challenge 1: For the given specification design an analog Butterworth filter

$$0.9 \le |H(j\Omega)| \le 1 \quad for\ 0 \le \Omega \le 0.2\pi$$

$$|H(j\Omega)| \le 0.2 \quad for\ 0.4\pi \le \Omega \le \pi.$$

Ans: $H(s) = \dfrac{0.323}{\left(s^2 + 0.577s + 0.0576\pi^2\right)\left(s^2 + 1.393s + 0.0576\pi^2\right)}$

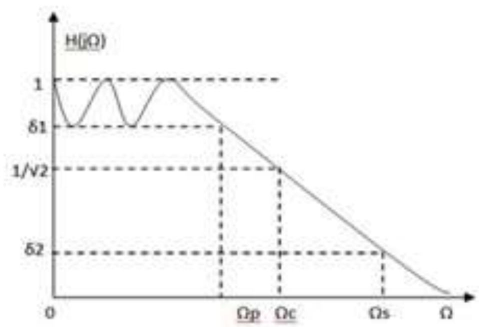
Challenge 2: Determine the order and the poles of low pass Butterworth filter that has 3 dB attenuation at 500 Hz and an attenuation of 40dB at 1000Hz.

Ans: $H(s) = (s+1)\left(s^2 + 1.80194s + 1\right)\left(s^2 + 1.247s + 1\right)\left(s^2 + 0.445s + 1\right)$

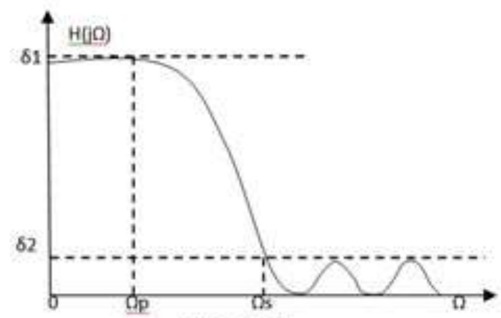
Given the specification $\alpha_p = 1dB; \alpha_s = 30dB; \Omega_p = 200\ rad/\sec; \Omega_s = 600\ rad/sec.$ **determine the order of the filter. Ans:** ***N=4***

Analog Low pass Chebyshev Filter:

There are two types of Chebyshev filters.



Odd - Type I



Even - Type II

Given specifications αp= 3dB,αs=16 dB, f_P=1KHz and f_S=2KHz. Determine the order of the filter using Chebyshev approximation. Find H(s).

Solution:

Given:

Step 1:

Pass band attenuation αp= 3dB,

Stop band attenuation αs=16 dB,

Pass band frequency f_P=1 KHz=2π*1000=2000π rad/sec

Stop band frequency f_S=2 KHz=2π*2*1000=4000π rad/sec

Step 2: Order of the filter

$$N \geq \frac{\cosh^{-1}\sqrt{\left(\frac{10^{0.1\alpha_S}-1}{10^{0.1\alpha_P}-1}\right)}}{\cosh^{-1}\left(\frac{\Omega_S}{\Omega_P}\right)}$$

$$N \geq \frac{\cosh^{-1}\sqrt{\left(\frac{10^{0.1*16}-1}{10^{0.1*3}-1}\right)}}{\cosh^{-1}\left(\frac{4000\pi}{2000\pi}\right)}$$

$$\geq 1.91$$

Rounding the next higher integer value *N=2.*

Step 3: The value of minor axis and major axis can be found as below

$$\varepsilon = \sqrt{\left(10^{0.1*\alpha_P}-1\right)} = \sqrt{\left(10^{0.1*3}-1\right)} = 1$$

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 1^{-1} + \sqrt{1+1^{-2}} = 2.414$$

$$a = \Omega_P \frac{\left[\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}\right]}{2} = 2000\pi \frac{\left[(2.414)^{\frac{1}{2}} - (2.414)^{-\frac{1}{2}}\right]}{2} = 910\pi$$

$$b = \Omega_S \frac{\left[\mu^{\frac{1}{N}} + \mu^{-\frac{1}{N}}\right]}{2} = 4000\pi \frac{\left[(2.414)^{\frac{1}{2}} + (2.414)^{-\frac{1}{2}}\right]}{2} = 2197\pi$$

Step 4: The poles are given by

$S_K = a\cos\phi_k + jb\sin\phi_k;$ $\quad k = 1,2$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}; \quad k = 1,2$$

$For\ k = 1$

$$\phi_1 = \frac{\pi}{2} + \frac{(2-1)\pi}{2*2} = \frac{\pi}{2} + \frac{\pi}{4} = 135^o$$

$For k = 2$

$$\phi_2 = \frac{\pi}{2} + \frac{(2*2-1)\pi}{2*2} = \frac{\pi}{2} + \frac{3\pi}{4} = 225^o$$

$$s_1 = a\cos\phi_1 + jb\sin\phi_1 = (910\pi * \cos 135) + j(2197 * \sin 135)$$

$$s_1 = -643.46\pi + j1553\pi$$

$$s_2 = a\cos\phi_2 + jb\sin\phi_2 = (910\pi * \cos 225) + j(2197 * \sin 225)$$

$$s_2 = -643.46\pi - j1553\pi$$

Step 5: The denominator of *H(s):*

$$H(s) = (s + 643.46\pi)^2 + (1554)^2$$

Step 6: The numerator of *H(s):*

$substitute, s = 0$

$$H(s) = \frac{(643.46\pi)^2 + (1554\pi)^2}{\sqrt{1+\varepsilon^2}}$$

$$= \frac{(643.46\pi)^2 + (1554\pi)^2}{\sqrt{1+1^2}} = (1414.38)^2\pi^2$$

The transfer function $H(s) = \dfrac{(1414.38)^2\pi^2}{s^2 + 1287\pi s + (1682)^2\pi^2}$

HW: Challenge 1: Obtain an analog Chebyshev filter transfer function that satisfies the constraints

$$\frac{1}{\sqrt{2}} \le |H(j\Omega)| \le 1; \ for\ 0 \le \Omega \le 2$$

$$|H(j\Omega)| \le 0.1 \ for\ \Omega \ge 4$$

Ans: $H(s) = \dfrac{2}{(s+0.596)(s^2 + 0.596s + 3.354)}$

2. Design a Chebyshev filter with a maximum pass band attenuation of 2.5dB at Ω_P=20rad/sec and stop band attenuation of 30 dB at Ω_S=50rad/sec.

Ans: *N*=3. $H(s) = \dfrac{2265.27}{(s+6.6)(s^2 + 6.6s + 343.2)}$

---

3. For the given specifications find the order of the Chebyshev-I filter

$\alpha_P = 1.5dB; \alpha_S = 10dB; \Omega_P = 2rad/Sec; \ \Omega_S = 30rad/\text{sec}.$

---

4. For the given specifications find the order of the Chebyshev-I filter

$\alpha_P = 1dB; \alpha_S = 25dB; \Omega_P = 1rad/Sec; \ \Omega_S = 20rad/\text{sec}.$

Discrete time IIR filter from analog filter:

STRUCTURES FOR IIR SYSTEMS:

IIR Systems are represented in four different ways

1. Direct Form Structures Form I and Form II
2. Cascade Form Structure
3. Parallel Form Structure
4. Lattice and Lattice-Ladder structure.

DIRECT FORM-I :

Challenge: Obtain the direct form-I, direct form-II,Cascade and parallel form realization of the system y(n)=-0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2) ***[April/May-2015]***

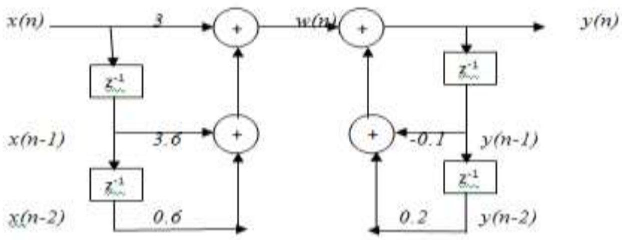
Solution:

Direct Form I:

$Let\ 3x(n)+3.6x(n-1)+0.6x(n-2)=w(n)$

$y(n)=-0.1y(n-1)+0.2y(n-2)+w(n)$

Thedirect form I realizationis

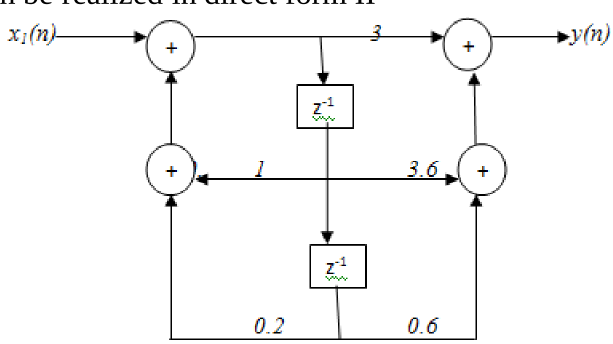


Direct form II:

From the given difference equation we have

$$H(z)=\frac{Y(z)}{X(z)}=\frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

The above system function can be realized in direct form II



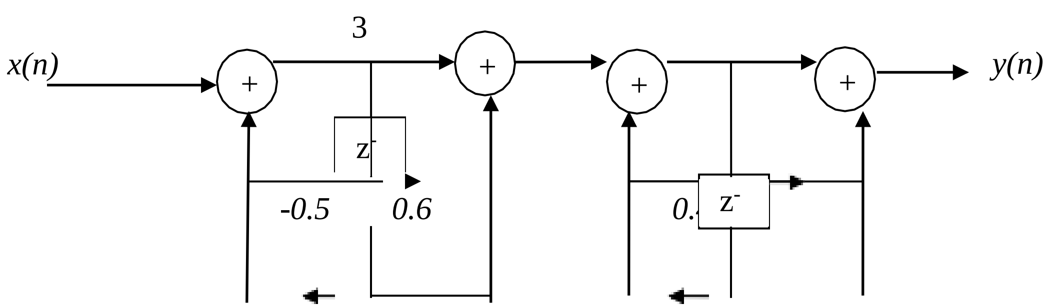
$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-} + 0.6z^{-}}{1+0.1z^{-} - 0.2z^{-}}$$

$$= \frac{(3+0.6z^{-})(1+z^{-})}{(1+0.5z^{-})(1-0.4z^{-})}$$

$$H(z) = \frac{3+0.6z^{-}}{1+0.5z^{-}}$$

$$H(z) = \frac{1+z^{-}}{1-0.4z^{-1}}$$

Now we realize $H_1(z)$ *and* $H_2(z)$ and cascade both to get realization of H(z)

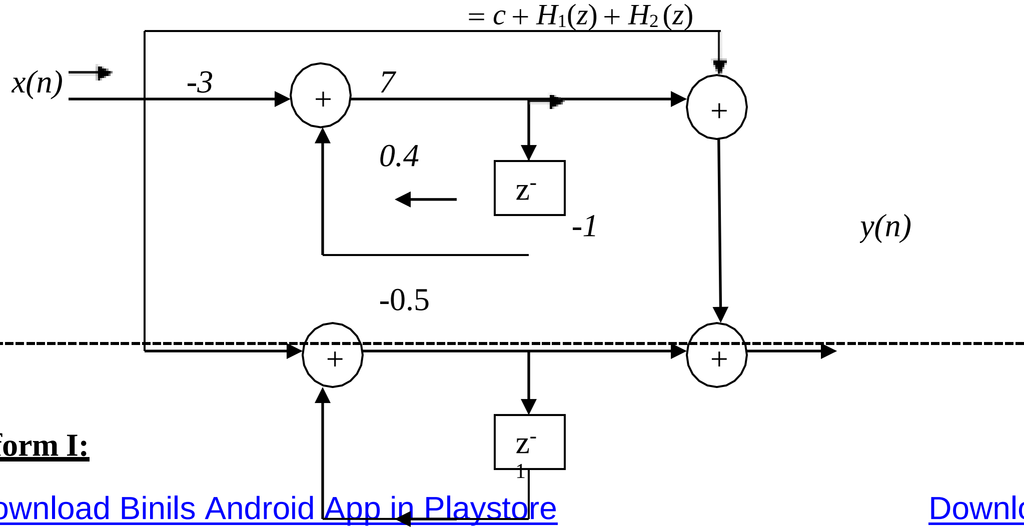


Parallel form:

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}$$

$$= c + H_1(z) + H_2(z)$$



Direct form I:

H.W: Obtain the direct form-I realization for the system described by the following differenceequations.

(i) $y(n) = 2y(n-1) + 3y(n-2) + x(n) + 2x(n-1) + 3x(n-2)$

(ii) $y(n) = 0.5y(n-1) + 0.06y(n-2) + 0.3x(n) + 0.5x(n-1)$

Obtain the direct form-I realization for the system described by difference equation

$y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

Direct form II

H.W: Determine the direct form II realization for the following system

(i) $y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-2)$

(ii) $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{2}x(n-1)$ **[May/June-14]**

Determine the direct form II realization for the following system

$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$

CASCADE FORM:

H.W: For the system function $H(z) = \dfrac{1 + 2z^{-1} + z^{-2}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$ **obtain cascade structure.**

Realize the system with difference equation $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ **in cascade form.**

****

Parallel form:

H.W: Realize the system given by difference equation

$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ **in parallel form.**

****

Analog filter design:

There are two types of analog filter design are,

- Butterworth Filter
- Chebyshev Filter.

Analog Low pass Butterworth Filter:

N	Denominator of H(s)
1	$S+1$
2	$s^2 + \sqrt{2}s + 1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76537s+1)(s^2+1.8477s+1)$
5	$(s+1)(s^2+0.61803s+1)(s^2+1.61803s+1)$
6	$(s^2+1.931855s+1)(s^2+\sqrt{2}s+1)(s^2+0.51764s+1)$

Problem

Obtain the parallel form realisation of the system government by the equation $y(n) = -3/8\, y(n-1) + 3/32\, y(n-2) + 1/64\, y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$

- Soln.: Taking z-transform on both sides

$$Y(z) = -\frac{3}{8}Y(z)z^{-1} + \frac{3}{32}Y(z)z^{-2} + \frac{1}{64}Y(z)z^{-3} + X(z) + 3X(z)z^{-1} + 2X(z)z^{-2}$$

$$Y(z)\left[1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}\right] = X(z)\left[1 + 3z^{-1} + 2z^{-2}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{\left[1+3z^{-1}+2z^{-2}\right]}{\left[1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}\right]}$$

Problem Cont..

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z[z+2][z+1]}{[z-0.25][z+0.5][z+0.125]}$$

$$\frac{H(z)}{z} = \frac{[z+2][z+1]}{[z-0.25][z+0.5][z+0.125]}$$

$$\frac{H(z)}{z} = \frac{A}{[z-0.25]} + \frac{B}{[z+0.5]} + \frac{C}{[z+0.125]}$$

Problem Cont..

$$A = [z-0.25]\frac{H(z)}{z}\bigg|_{z=0.25} = [z-0.25]\frac{[z+2][z+1]}{[z-0.25][z+0.5][z+0.125]}\bigg|_{z=0.25}$$

- A=10

$$B = [z+0.5]\frac{H(z)}{z}\bigg|_{z=-0.5} = [z+0.5]\frac{[z+2][z+1]}{[z-0.25][z+0.5][z+0.125]}\bigg|_{z=-0.5}$$

- B=2.666

$$C = [z+0.125]\frac{H(z)}{z}\bigg|_{z=-0.125} = [z+0.125]\frac{[z+2][z+1]}{[z-0.25][z+0.5][z+0.125]}\bigg|_{z=-0.125}$$

- C=-11.67

Problem Cont..

$$\frac{H(z)}{z} = \frac{10}{[z-0.25]} + \frac{2.666}{[z+0.5]} - \frac{11.67}{[z+0.125]}$$

$$H(z) = 10\left[\frac{z}{z-0.25}\right] + 2.666\left[\frac{z}{z+0.5}\right] - 11.67\left[\frac{z}{z+0.125}\right]$$

$$H(z) = 10\left[\frac{1}{1-0.25z^{-1}}\right] + 2.666\left[\frac{1}{1+0.5z^{-1}}\right] - 11.67\left[\frac{1}{1+0.125z^{-1}}\right]$$

$$H(z) = H_1(z) + H_2(z) + H_3(z)$$

Problem Cont..

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{10}{1-0.25\,z^{-1}}$$

$$Y_1(z) = 10\,X(z) + 0.25\,z^{-1}\,Y_1(z)$$

$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{2.66}{1+0.5\,z^{-1}}$$

$$Y_2(z) = 2.66\,X(z) - 0.5z^{-1}\,Y_2(z)$$

$$H_3(z) = \frac{Y_3(z)}{X(z)} = \frac{-11.67}{1+0.125\,z^{-1}}$$

$$Y_3(z) = -11.67\,X(z) - 0.125z^{-1}\,Y_3(z)$$

Problem Cont..