Binary Frequency Shit Keying Modulation

Frequency Shit Keying (FSK) modulation is a popular form of digital modulation used in low-cost application for transmitting data at moderator or low rate over wired as well as wireless channels. In general, an M-ary FSK modulation are becoming efficiency modulation scheme and several forms of M-ary FSK modulation are becoming popular for spread spectrum communications and other wireless applications. In this lesson, our discussion will be limited to binary frequency shift keying(BFSK).

Two carrier frequencies are used for binary frequency shift keying modulation. One frequency is called the „mark" frequency (f2) and the other as the frequency (f1). By convention, the „mark" frequency indicates the higher of the two carriers used. If Tb indicates the duration of one information but, the two time- limited signals can be expressed as:

The binary scheme uses two carriers and for special relationship between the two frequencies one can also define two orthonormal basis functions as shown below.

Generation and Coherent Detection of Binary FSK Signals

The block diagram describes a scheme for generating the binary FSK signal; it consists of two components:

1. *On–off level encoder*, the output of which is a constant amplitude of in response to input symbol 1 and zero in response to input symbol 0.
2. *Pair of oscillators*, whose frequencies *f*1 and *f*2 differ by an integer multiple of the bit rate 1 *T*b in accordance with (7.152). The lower oscillator with frequency *f*2 is preceded by an inverter. When in a signaling interval, the input symbol is 1, the upper oscillator with frequency *f*1 is switched on and signal *s*1(*t*) is transmitted, while the lower oscillator is switched off. On the other hand, when the input symbol is 0, the upper oscillator is switched off, while the lower oscillator is switched on and signal *s*2(*t*) with frequency *f*2 is transmitted. With phase continuity as a requirement, the two oscillators are *synchronized* with each other. Alternatively, we may use a voltage-controlled oscillator, in which case phase continuity is automatically satisfied.

To coherently detect the original binary sequence given the noisy received signal *x*(*t*),

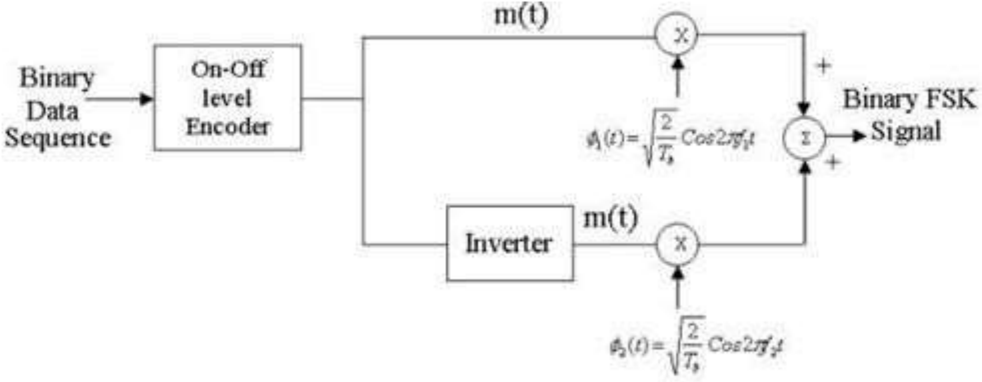


Fig:4.

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page- 397)

we may use the receiver shown in Figure 7.26b. It consists of two correlators with a common input, which are supplied with locally generated coherent reference signals 1(*t*) and 2(*t*). The correlator outputs are then subtracted, one from the other; the resulting difference *y* is then compared with a threshold of zero. If *y* 0, the receiver decides in favor of 1. On the other hand, if *y* 0, it decides in favor of 0. If *y* is exactly zero, the receiver makes a random guess (i.e., flip of a fair coin) in favor of 1 or 0.

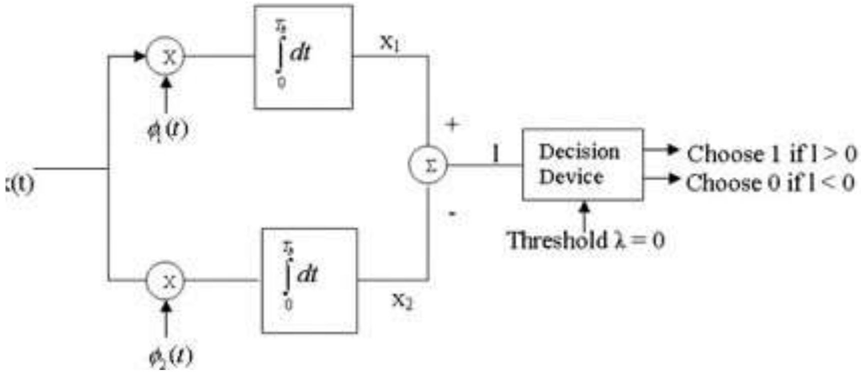


Fig:4.3.2 Coherent Binary FSK receiver

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page- 397)

A binary FSK Transmitter is as shown, the incoming binary data sequence is applied to on-off level encoder. The output of encoder is √ volts for symbol 1 and 0 volts for symbol “0‟. When we have symbol 1 the upper channel is switched on with oscillator frequency f1, for symbol “0“, because of inverter the lower

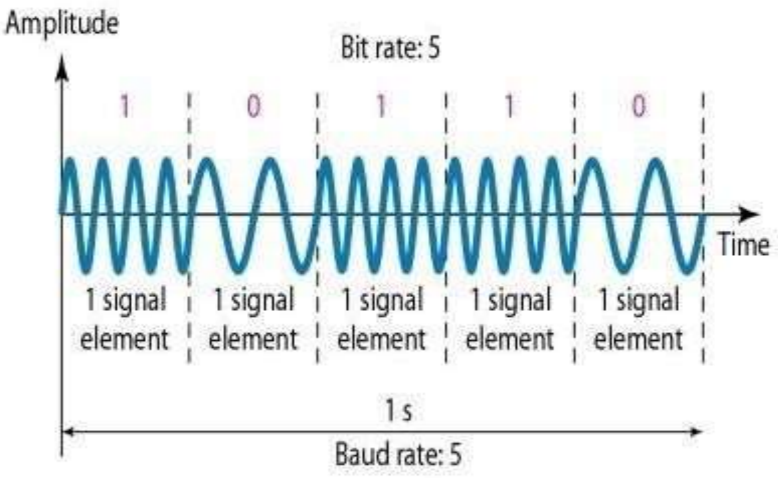
channel is switched on with oscillator frequency f2. These two frequencies are combined using an adder circuit and then transmitted. The transmitted signal is nothing but required BFSK signal. The detector consists of two correlators. The incoming noisy BFSK signal x(t) is common to both correlator. The Coherent reference signal ¦1(t)&¦2(t) are supplied to upper and lower correlators respectively.

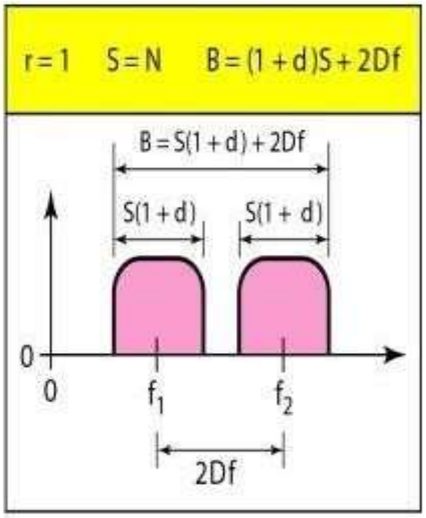
The correlator outputs are then subtracted one from the other and resulting a random vector “l” (l=x1 - x2). The output “l” is compared with threshold of zero volts.

If l > 0, the receiver decides in favour of symbol 1. l< 0, the receiver decides in favour of symbol 0.

FSK Bandwidth:

- Limiting factor: Physical capabilities of the carrier
- Not susceptible to noise as much as ASK





Error Probability of Binary FSK

The observation vector $\mathbf{x}$ has two elements $x1$ and $x2$ that are defined by, respectively,

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$x_2 = \int_0^{T_b} x(t)\,\phi_2(t)\,dt$$

where $x(t)$ is the received signal, whose form depends on which symbol was transmitted. Given that symbol 1 was transmitted, $x(t)$ equals $s1(t) + w(t)$, where $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N0$ 2. If, on the other hand, symbol 0 was transmitted, $x(t)$ equals $s2(t) + w(t)$.

Now, applying the decision rule of (7.57) assuming the use of coherent detection at the receiver, we find that the observation space is partitioned into two decision regions, labeled $Z1$ and $Z2$ in Figure 7.25. The decision boundary, separating region $Z1$ from region $Z2$, is the perpendicular bisector of the line joining the two message points. The receiver decides in favor of symbol 1 if the received signal point represented by the observation vector $\mathbf{x}$ falls inside region $Z1$. This occurs when $x1$ $x2$. If, on the other hand, we have $x1$ $x2$, the received signal point falls inside region $Z2$ and the receiver decides in favor of symbol 0. On the decision boundary, we have $x1 = x2$, in which case the receiver makes a random guess in favor of symbol 1 or 0. To proceed further, we define a new Gaussian random variable Y whose sample value y is equal to the difference between $x1$ and $x2$; that is, $y = x1 – x2$. The mean value of the random variable Y depends on which binary symbol was transmitted. Given that symbol 1 was sent, the Gaussian random variables $X1$ and $X2$, whose sample values are denoted by $x1$ and $x2$, have mean values equal to and zero, respectively. Correspondingly, the conditional mean of the random variable Y given that symbol 1 was sent is

Applications

– On voice-grade lines, used up to 1200bps

– Used for high-frequency (3 to 30 MHz) radio transmission

– used at higher frequencies on LANs that use coaxial cable.

Therefore Binary FSK system has 2 dimensional signal space with two messages S1(t) and S2(t), [N=2 , m=2] they are represented.

QUADRATURE PHASE - SHIFT KEYING (QPSK)

The provision of reliable performance, exemplified by a very low probability of error, is one important goal in the design of a digital communication system. Another important goal is the efficient utilization of channel bandwidth. In this subsection we study a *bandwidth-conserving modulation scheme* known as *quadriphase-shift keying* (QPSK), using coherent detection. In a sense, QPSK is an expanded version from binary PSK where in a symbol consists of two bits and two orthonormal basis functions are used. A group of two bits is often called a “dibit”. So, four dibits are possible. Each symbol carries same energy. Let, E: Energy per Symbol and T: Symbol duration = 2.* Tb, where Tb: duration of 1 bit.

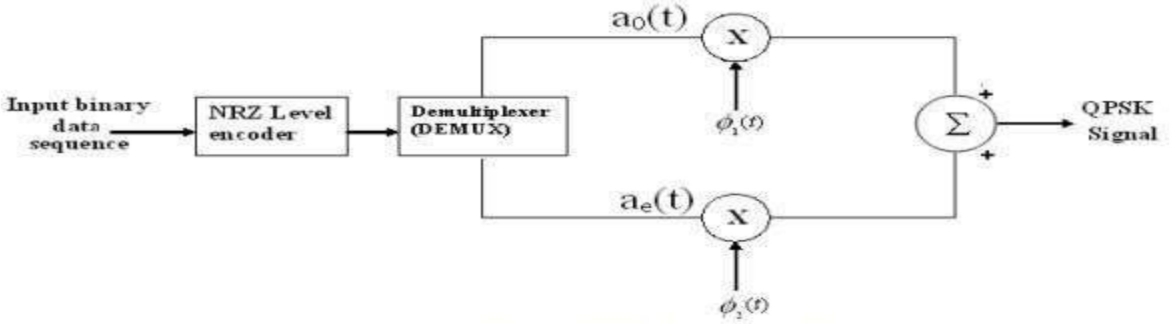


Fig. (a) QPSK Transmitter

Fig 4.3.4 QPSK

(Source:Ece4uplp)

In QPSK system the information carried by the transmitted signal is contained in the phase. Expanding on the binary PSK transmitter of Figure 7.14a, we may build on (7.113) to construct the QPSK transmitter shown in Figure 7.18a. A distinguishing feature

of the QPSK transmitter is the block labeled *demultiplexer*. The function of the demultiplexer is to divide the binary wave produced by the polar NRZ-level encoder into

two separate binary waves, one of which represents the odd-numbered dibits in the incoming binary sequence and the other represents the even-numbered dibits.

Accordingly, we may make the following statement:

The QPSK transmitter may be viewed as two binary PSK generators that work in parallel, each at a bit rate equal to one-half the bit rate of the original binary sequence at the QPSK transmitter input.

QPSK Receiver:-

The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals ϕ1(t) &ϕ2(t)as shown in fig(b).The correlator outputs x1 and x2 produced in response to the received signal x(t) are each compared with a threshold value of zero.

Expanding on the binary PSK receiver of Figure 7.14b, we find that the QPSK receiver is

structured in the form of an *in-phase path* and a *quadrature path*, working in parallel as

depicted in Figure 7.18b. The functional composition of the QPSK receiver is as follows:

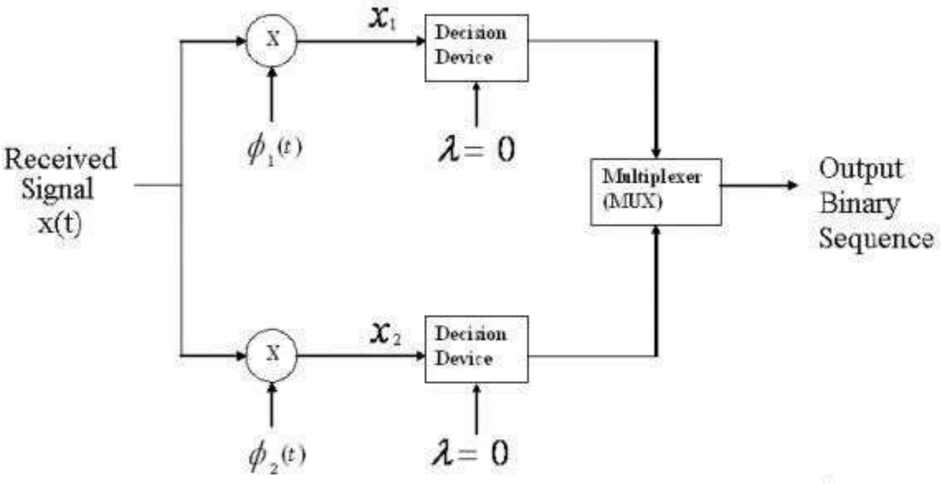


Fig. (b) QPSK Receiver

Fig 4..3.5 QPSK Receiver

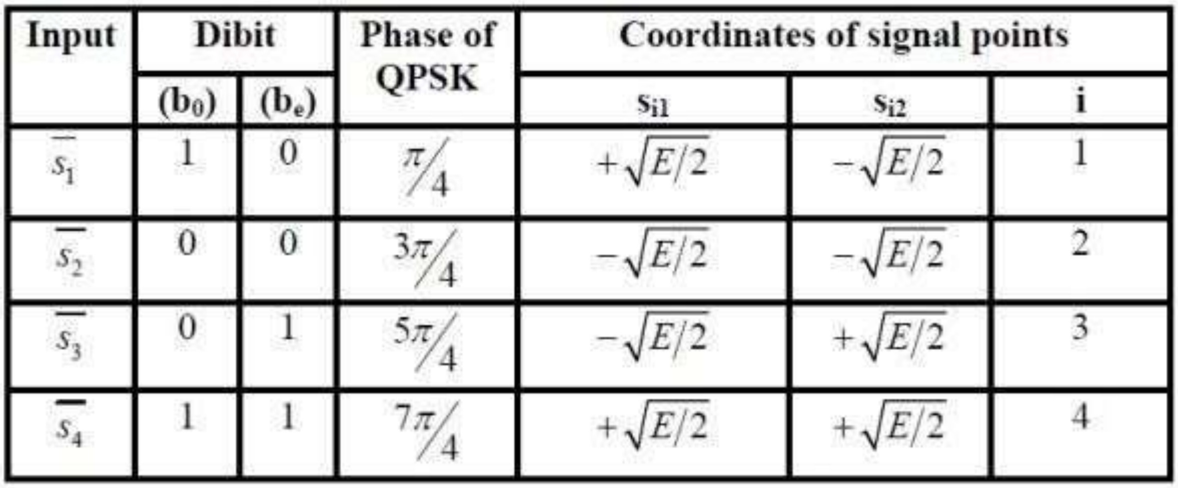
(Source:Ece4uplp)

1. *Pair of correlators*, which have a common input $x(t)$. The two correlators are supplied with a pair of *locally generated orthonormal basis functions* $1(t)$ and $2(t)$, which means that the receiver is synchronized with the transmitter. The correlator outputs, produced in response to the received signal $x(t)$, are denoted by $x1$ and $x2$, respectively.
2. *Pair of decision devices*, which act on the correlator outputs $x1$ and $x2$ by comparing each one with a zero-threshold; here, it is assumed that the symbols 1 and 0 in the original binary stream at the transmitter input are equally likely. If $x1 > 0$, a decision is made in favor of symbol 1 for the in-phase channel output; on the other hand, if $x1$ 0, then a decision is made in favor of symbol 0. Similar binary decisions are made for the quadrature channel.
3. *Multiplexer*, the function of which is to combine the two binary sequences produced by the pair of decision devices. The resulting binary sequence so produced provides an *estimate* of the original binary stream at the transmitter input.

The in-phase channel output:

If $x1> 0$ a decision is made in favour of symbol 1 $x1< 0$ a decision is made in favour of symbol 0.

Similarly quadrature channel output:
If $x2>0$ a decision is made in favour of symbol 1 and $x2<0$ a decision is made in favour of symbol 0 Finally these two binary sequences at the in phase and quadrature channel outputs are combined in a multiplexer (Parallel to Serial) to reproduce the original binary sequence.



Input	Dibit		Phase of QPSK	Coordinates of signal points		
	(b_0)	(b_e)		s_{i1}	s_{i2}	i
$\overline{s_1}$	1	0	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	1
$\overline{s_2}$	0	0	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	2
$\overline{s_3}$	0	1	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	3
$\overline{s_4}$	1	1	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	4

Signal-Space Diagram of QPSK Signal

Using a well-known trigonometric identity, we may expand (7.112) to redefine the transmitted signal in the canonical form:

where i = 1, 2, 3, 4. Based on this representation, we make two observations:

1. There are two orthonormal basis functions, defined by a pair of quadrature carriers:

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad 0 \le t \le T$$

2

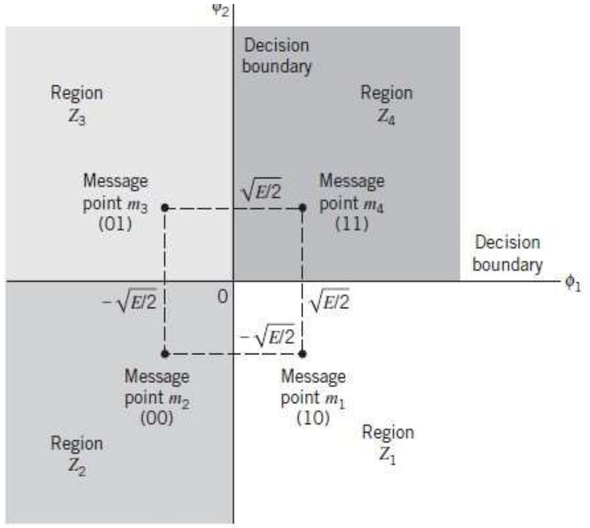


Fig:4.3.6 Signal-space diagram of QPSK system.

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page-358)

Accordingly, a QPSK signal has a two-dimensional signal constellation (i.e., $N = 2$) and four message points (i.e., $M = 4$) whose phase angles increase in a counterclockwise direction, as illustrated ,As with binary PSK, the QPSK signal has *minimum average energy*.

Probability of error:-

A QPSK system is in fact equivalent to two coherent binary PSK systems working arallel and using carriers that are in-phase and quadrature. The in- phase channel output x1 and the Q-channel output x2 may be viewed as the individual outputs of the two coherent binary PSK systems.

Thus the two binary PSK systems may be characterized as follows.

- The signal energy per bit √E/2**Offset QPSK** embodies all the possible phase transitions that can arise in the generation of a QPSK
signal. More specifically, examining the QPSK waveform illustrated
we may make three observations:

1. The carrier phase changes by 180° whenever both the in-phase and quadrature components of the QPSK signal change sign. An example of this situation is illustrated in Figure 7.17 when the input binary sequence switches from dibit 01 to

- dibit 10.

- The carrier phase changes by 90° whenever the in-phase or quadrature component changes sign. An example of this second situation is illustrated in Figure 7.17 when the input binary sequence switches from dibit 10 to dibit 00, during which the inphase component changes sign, whereas the quadrature component is unchanged. 3. The carrier phase is unchanged when neither the in-phase component nor the quadrature component changes sign. This last situation is illustrated in Figure 7.17 when dibit 10 is transmitted in two successive symbol intervals.

- Situation 1 and, to a much lesser extent, situation 2 can be of a particular concern when the QPSK signal is filtered during the course of transmission, prior to detection. Specifically, the 180° and 90° shifts in carrier phase can result in changes in the carrier amplitude (i.e.,
- envelope of the QPSK signal) during the course of transmission over the channel, thereby causing additional symbol errors on detection at the receiver.
- To mitigate this shortcoming of QPSK, we need to reduce the extent of its amplitude fluctuations. To this end, we may use *offset QPSK*.[4] In this variant of QPSK, the bit stream responsible for generating the quadrature component is delayed (i.e., offset) by half a symbol interval with respect to the bit stream responsible for generating the in-phase component. Specifically, the two basis functions of offset QPSK are defined by

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad \frac{T}{2} \le t \le \frac{3T}{2}$$

Accordingly, unlike QPSK, the phase transitions likely to occur in offset QPSK are confined to 90°, as indicated in the signalspace diagram of Figure 7.20b. However, 90° phase transitions in offset QPSK occur twice as frequently but with half the intensity encountered in QPSK. Since, in addition to 90° phase transitions, 180° phase transitions also occur in QPSK, we find that amplitude fluctuations in offset QPSK due to filtering have a smaller amplitude than in the case of QPSK.

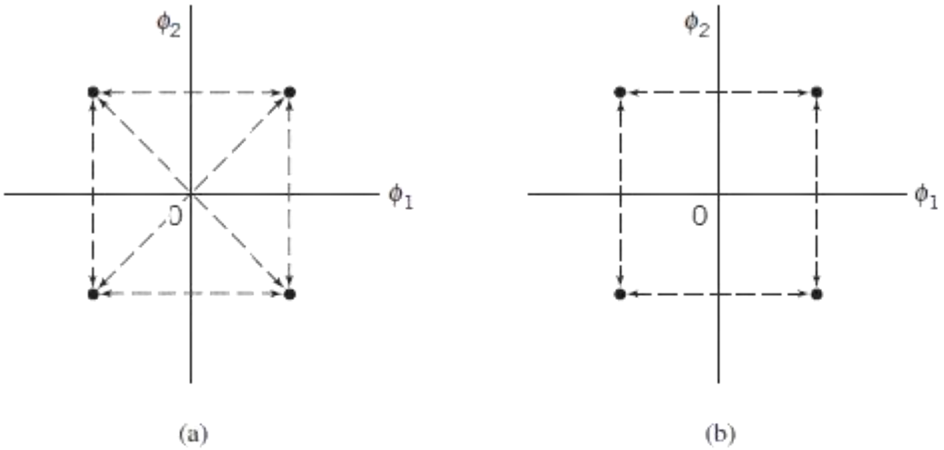


Fig 4.3.7: Possible paths for switching between the message points in (a) QPSK and (b) offset QPSK.

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page-358)

the offset QPSK has exactly the same probability of symbol error in an AW GN channel as QPSK. The equivalence in noise performance between these PSK schemes assumes the use of coherent detection at the receiver. The reason for the equivalence is that the statistical independence of the in-phase and quadrature components applies to both QPSK and offset QPSK. We may, therefore, say that Equation (7.123) for the average probability of symbol error applies equally well to the offset QPSK.

GENERATION AND COHERENT DETECTION OF BPSK SIGNALS

This is also called as **2-phase PSK** (or) **Phase Reversal Keying**. In this technique, the sine wavecarrier takes two phase reversals such as 0° and 180°.

BPSK is basically a DSB-SC (Double Sideband Suppressed Carrier) modulation scheme, formessage being the digital information.

Following is the image of BPSK Modulated output wave along with its input.

The simplest form of PSK is *binary phase-shift keying* (BPSK), where N = 1 and *M* = *2*.Therefore, with BPSK, two phases ($2^1 = 2$) are possible for the carrier.One phase represents a logic 1, and the other phase represents a logic 0. As the input digital signal changes state (i.e., froma 1 to a 0 or from a 0 to a 1), the phase of the output carrier shifts between two angles that are separated by 180°.

Hence, other names for BPSK are *phase reversal keying* (PRK) and *biphase modulation*. BPSK is a form of square-wave modulation of a *continuous wave (CW)* signal.

(i) Generation

To generate the BPSK signal, we build on the fact that the BPSK signal is a special case of DSB-SC modulation. Specifically, we use a product modulator consisting of two components.

(i) Non-return-to-zero level encoder, whereby the input binary data sequence is encoded inpolar form with symbols 1 and 0 represented by the constant- amplitude.
Product modulator, which multiplies the level encoded binary wave by the sinusoidal carrier of amplitude to produce the BPSK signal. The timing pulses used to generate the level encoded binary wave and the sinusoidal carrier wave are usually, but not necessarily, extracted from a common master clock.

The signal at (B) is:

$$r_1 = \sqrt{\frac{2}{T_b}}\int_0^{T_b}\left[d(t).\sqrt{\frac{2E_b}{T_b}}.\cos(w_c t+\theta)+w(t)\right]\cos(w_c t+\theta)dt$$

$$=\sqrt{E_b}.d(t)+\sqrt{\frac{2}{T_b}}\int_0^{T_b} w(t).\cos(w_c t+\theta)dt$$

(ii) Detection

To detect the original binary sequence of 1s and 0s, the BPSK signal at the channel output is applied to a receiver that consists of four sections

(a) *Product modulator*, which is also supplied with a locally generated reference signal thatis a replica of the carrier wave

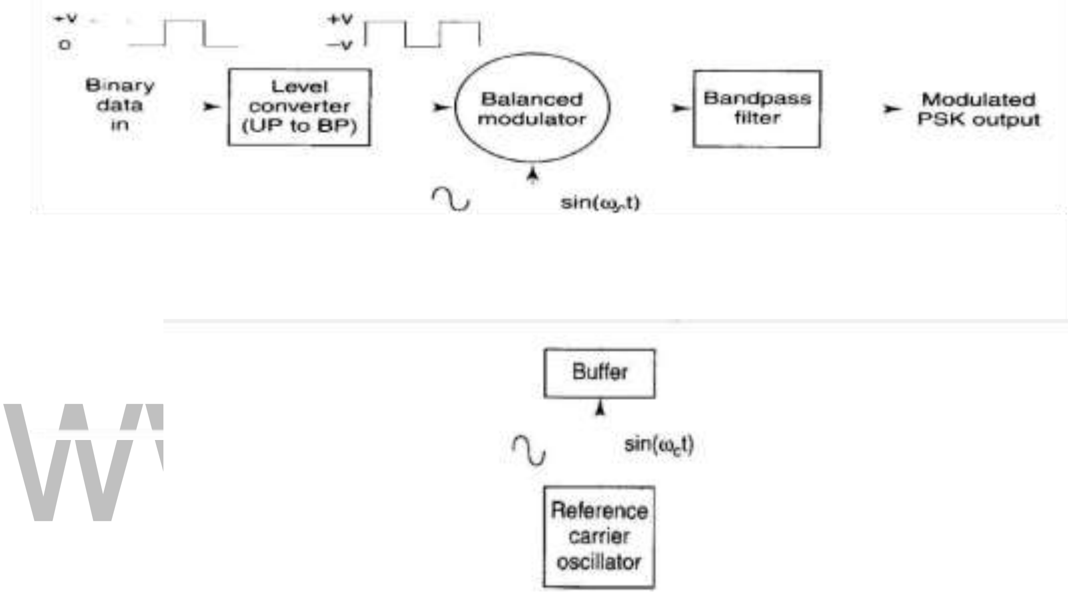
(b) *Low-pass filter*, designed to remove the double-frequency components of the product modulator output (i.e., the components centered on) and pass the zero-frequency components.

(c) *Sampler*, which uniformly samples the output of the low-pass filter at where; the local clock governing the operation of the sampler is *synchronized* with the clock responsible for bit-timing in the transmitter.

(d) *Decision-making device*, which compares the sampled value of the low- pass filters output to an externally supplied *threshold*, every seconds. If the threshold is exceeded, the device decides in favor of symbol 1; otherwise, it decides in favor of symbol 0. levels.

BPSK TRANSMITTER:

Figure 2-12 shows a simplified block diagram of a BPSK transmitter. The balanced modulator acts as a phase reversing switch. Depending on the logic condition of the digital input, the carrier is transferred to the output either in phase or 180° out of phase with the reference carrier oscillator.



(Figure 4.2.1- shows the schematic diagram of a balanced ring modulator.

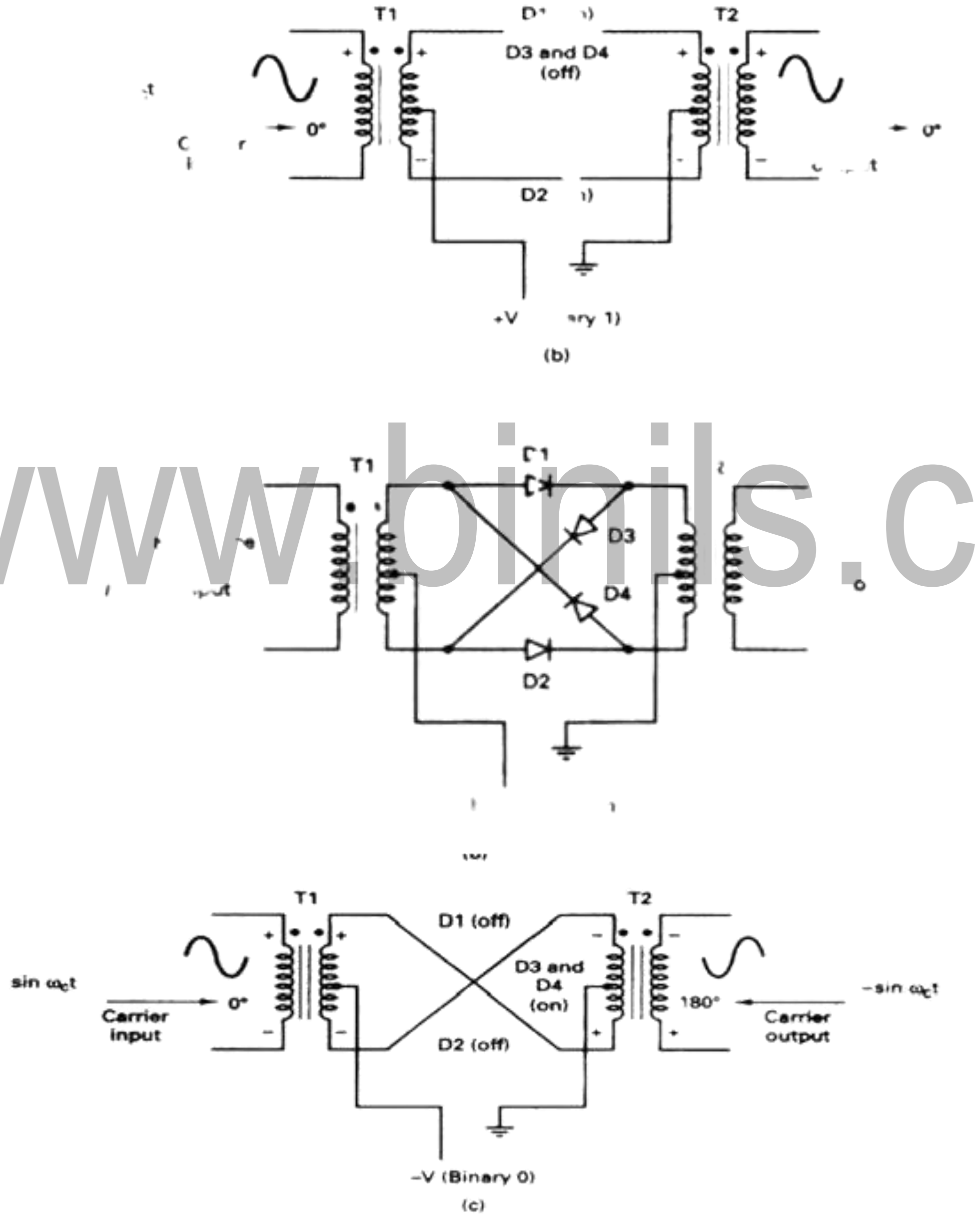
Source: Tutorial point)

The balanced modulator has two inputs: a carrier that is in phase with the reference oscillator and the binary digital data. For the balanced modulator to operate properly, the digital input voltage must be much greater than the peak carrier voltage.

This ensures that the digital input controls the on/off state of diodes D1 to D4. If the binary input is a logic 1(positive voltage), diodes D 1 and D2 are forward biased and on, while diodes D3 and D4 are reverse biased and off (Figure 2-13b). With the polarities shown, the carrier voltage is developed across transformer T2 in phase with the carrier voltage across T

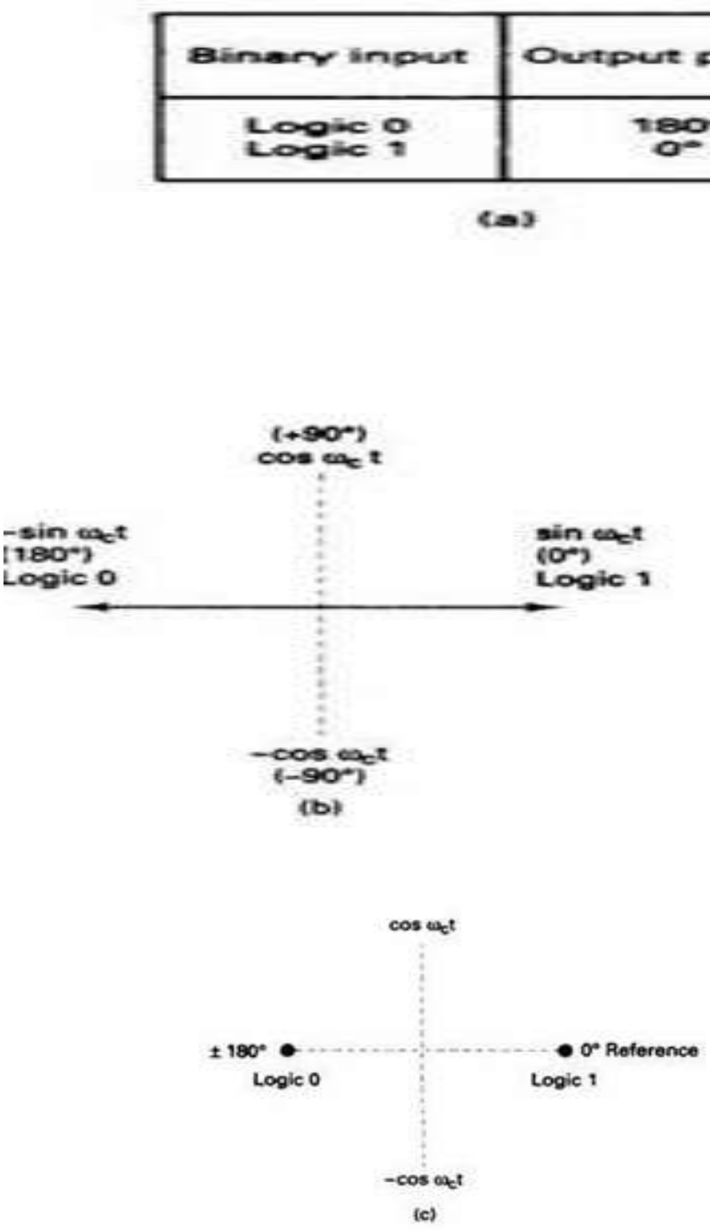
1. Consequently, the output signal is in phase with the reference oscillator.

If the binary input is a logic 0 (negative voltage), diodes Dl and D2 are reverse biased and off,while diodes D3 and D4 are forward biased and on (Figure 9-13c). As a result, the carrier voltage isdeveloped across transformer T2 180° out of phase with the carrier voltage across T 1.



Fig;4.2.2 (a) Balanced ring modulator; (b) logic 1 input; (c) logic 0 input

(Source: Tutorial point)



Binary input	Output phase
Logic 0	180°
Logic 1	0°

FIGURE 4.2.3 BPSK modulator: (a) truth table; (b) phasor diagram; (c) constellationdiagram

(Source: Tutorial point)

BPSK receiver:.

The input signal maybe+ sin ct or - sin ct .The coherent carrier recovery circuit detects and regenerates a carrier signal that is both frequency and phase coherent with the original transmitcarrier.

The balanced modulator is a product detector; the output is the product d the two inputs (the BPSKsignal and the recovered carrier).

The low-pass filter (LPF) operates the recovered binary data from the complex demodulated signal.

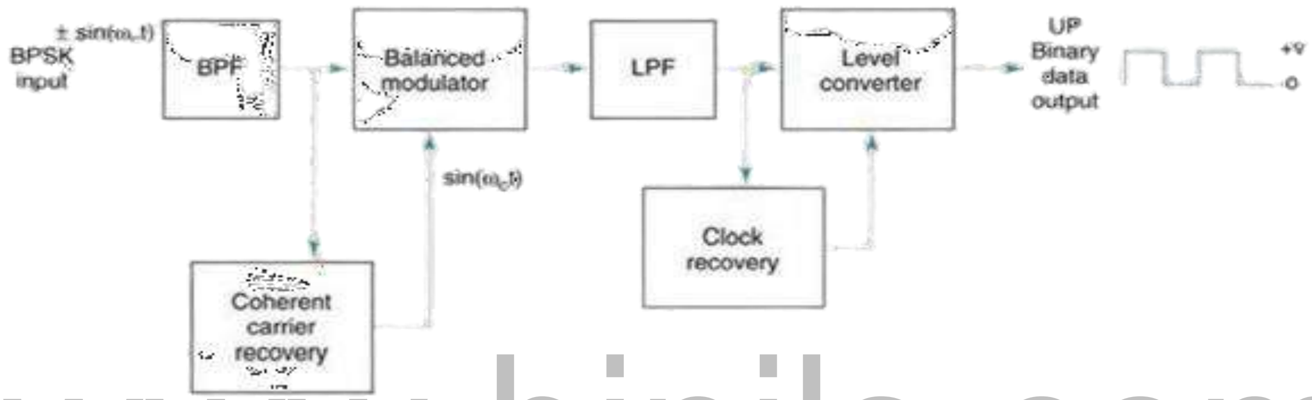


FIGURE 2-16 Block diagram of a BPSK receiver

(Source: Brainskart)

The output of the balanced modulator contains a negative voltage (-[l/2]V) and a cosine wave at twice the carrier frequency (2 ct).

Again, the LPF blocks the second harmonic of the carrier and passes only the negative constant component. A negative voltage represents a demodulated logic 0.

Power Spectrum or BPSK Modulated Signal

Continuing with our simplifying assumption of zero initial phase of the carrier and with no pulse shaping filtering, we can express a BPSK modulated signal.

DIFFERENTIAL PHASE SHIFT KEYING (DPSK):

The distinguishing feature of DPSK is that it eliminates the need for synchronizing the receiver to the transmitter by combining two basic operations at the transmitter:

- *differential encoding* of the input binary sequence and
- *PSK* of the encoded sequence, from which the name of this new binary signaling scheme

high or low state of the previous element. This DPSK technique doesn't need a reference oscillator.

In DPSK (Differential Phase Shift Keying) the phase of the modulated signal is shifted relative tothe previous signal element. No reference signal is considered here. The signal phase follows the

The following figure represents the model waveform of DPSK.

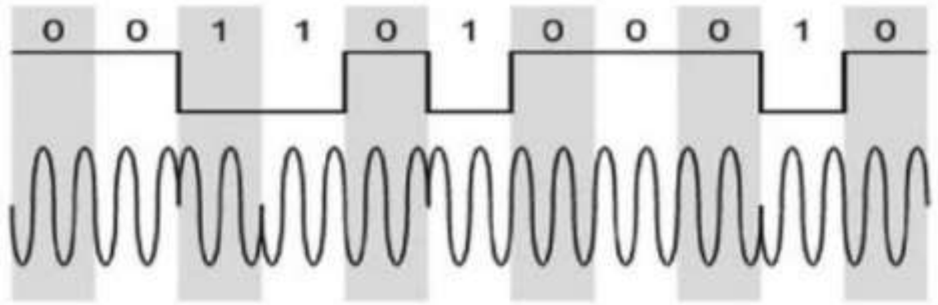


Fig: Represents the model waveform of DPSK.

Source: Brainskart

It is seen from the above figure that, if the data bit is LOW i.e., 0, then the phase of the signal is not reversed, but is continued as it was. If the data is HIGH i.e., 1, then the phase of the signal is reversed, as with NRZI, invert on 1 (a form of differential encoding).

If we observe the above waveform, we can say that the HIGH state represents an **M** in the modulating signal and the LOW state represents a **W** in the modulating signal.

The word binary represents two-bits. **M** simply represents a digit that corresponds to the number of conditions, levels, or combinations possible for a given number of binary variables.

This is the type of digital modulation technique used for data transmission in which instead of one- bit, two or **more bits are transmitted at a time**. As a single signal is used for multiple bit transmission, the channel bandwidth is reduced.

DBPSK TRANSMITTER.:

Figure 2-37a shows a simplified block diagram of a *differential binary phase-shift keying* (DBPSK) transmitter. An incoming information bit is XNORed with the preceding bit prior toentering the BPSK modulator (balanced modulator).

For the first data bit, there is no preceding bit with which to compare it. Therefore, an initial reference bit is assumed. Figure 2-37b shows the relationship between the input data, the XNOR output data, and the phase at the output of the balanced modulator. If the initial reference bit is assumed a logic 1, the output from the XNOR circuit is simply the complement of that shown.

In Figure 2-37b, the first data bit is XNORed with the reference bit. If they are the same, the XNORoutput is a logic 1; if they are different, the XNOR output is a logic 0. The balanced modulator operates the same as a conventional BPSK modulator; a logic I produces +sin ct at the output, and A logic 0 produces sin ct at the output.

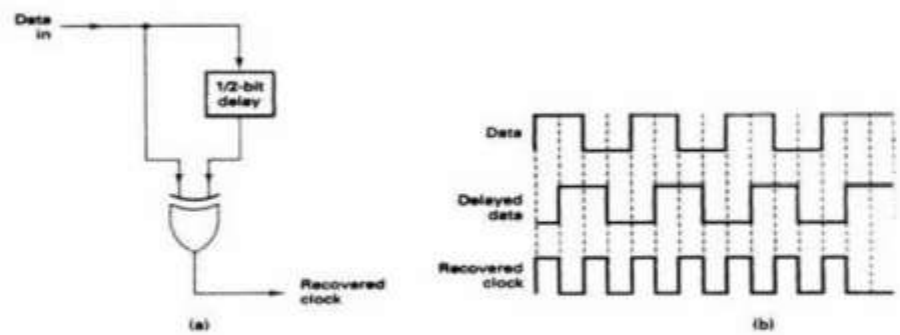


FIGURE 9-40 (a) Clock recovery circuit; (b) timing diagram

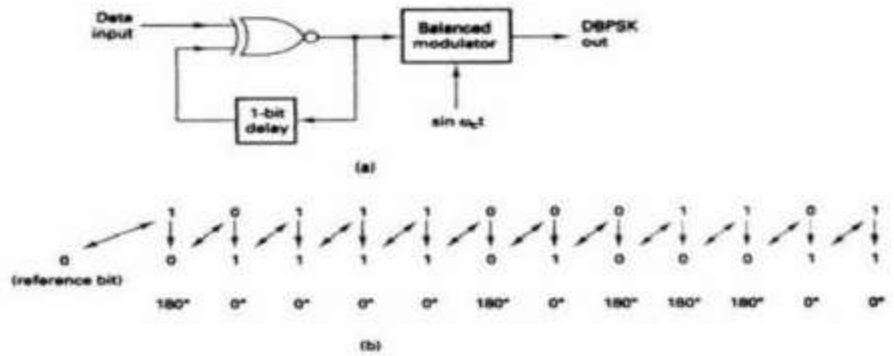


FIGURE 2-37 DBPSK modulator (a) block diagram (b) timing diagram

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page-414)

DBPSK RECEIVER:

Figure 9-38 shows the block diagram and timing sequence for a DBPSK receiver. The received signal is delayed by one bit time, then compared with the next signaling element in the balanced modulator. If they are the same. J logic 1(+ voltage) is generated. If they are different, a logic 0 (- voltage) is generated. [f the reference phase is incorrectly assumed, only the first demodulated bit is in error. Differential encoding can be implemented with higher-than-binary digital modulation schemes, although the differential algorithms are much more complicated than for DBPSK.

The primary advantage of DBPSK is the simplicity with which it can be implemented. With DBPSK, no carrier recovery circuit is needed. A disadvantage of DBPSK is, that it requires between 1 dB and 3 dB more signal-to-noise ratio to achieve the same bit error rate as that of absolute PS

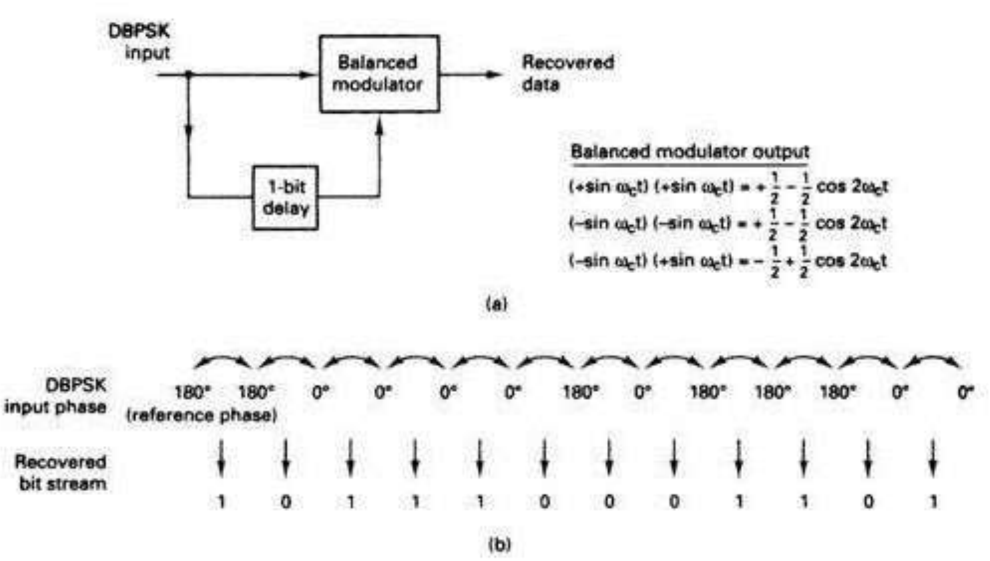


FIGURE 2-38 DBPSK demodulator: (a) block diagram; (b) timing sequence

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page-415)

Error Probability of DPSK

Basically, the DPSK is also an example of noncoherent orthogonal modulation when its behavior is considered over successive two-bit intervals; that is, 0 *t* 2*T*b. To elaborate, let the transmitted DPSK signal be for the first-bit interval 0 *t* *T*b, which corresponds to symbol 1. Suppose, then, the input symbol for the second-bit interval *T*b *t* 2*T*b is also symbol 1. According to part 1 of the DPSK encoding rule, the carrier phase remains unchanged, thereby yielding the DPSK signal

$$s_1(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t), & \text{symbol 1 for } \quad 0 \le t \le T_b \\ \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t), & \text{symbol 0 for } \quad T_b \le t \le 2T_b \end{cases}$$

Suppose, next, the signaling over the two-bit interval changes such that the symbol at the transmitter input for the second-bit interval Tb t 2Tb is 0. Then, according to part 2 of the DPSK encoding rule, the carrier phase is shifted by radians (i.e., 180°), thereby yielding the new DPSK signal

$$s_2(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t), & \text{symbol 1 for } \quad 0 \le t \le T_b \\ \sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t + \pi), & \text{symbol 1 for } \; T_b \le t \le 2T_b \end{cases}$$

We now readily see from (7.246) and (7.247) that s1(t) and s2(t) are indeed orthogonal over the two-bit interval 0 t 2Tb, which confirms that DPSK is indeed a special form of

noncoherent orthogonal modulation with one difference compared with the case of binary FSK: for DPSK, we have T = 2Tb and E = 2Eb. Hence, using (7.227), we find that thE

BERfor DPSK is given by

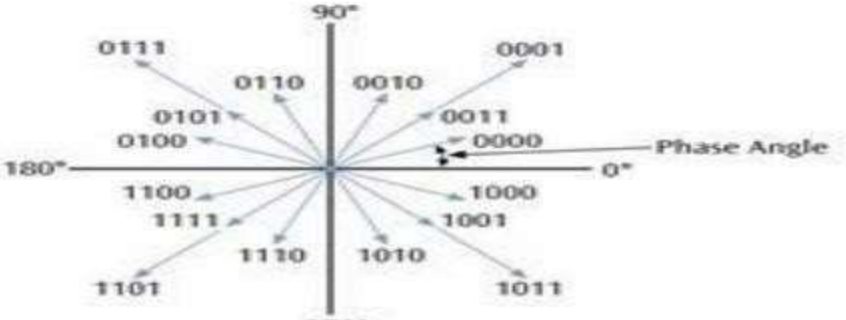
$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

QAM(Quadrature Amplitude Modulation)

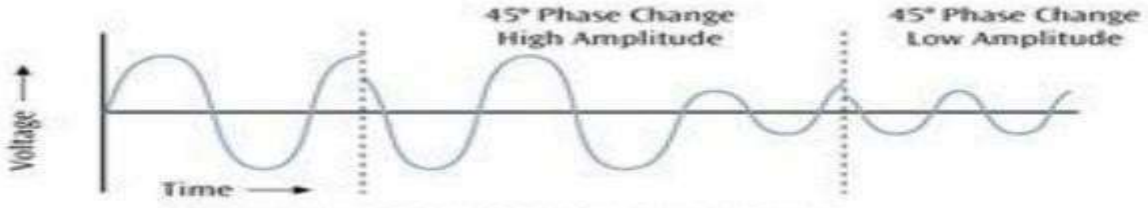
QAM is a combination of ASK and PSK

Two different signals sent simultaneously on the same carrier frequency ie, M=4, 16, 32, 64, 128, 256.

As an example of QAM, 12 different phases are combined with two different modulated signal are interrelated in such a way that the *envelope is constrained to remain constant*. This constraint manifests itself in a circular constellation for the message points, as illustrated in Figure 7.21a. However, if this constraint is removed so as to permit the in-phase and quadrature components to be independent, we get a new modulation scheme called *M-ary* QAM. The QAM is a *hybrid* form of modulation, in that the carrier experiences amplitude as well as amplitudes. Since only 4 phase angles have 2 different amplitudes, there are a total of 16 combinations. With 16 signal combinations, each baud equals 4 bits of information (2 ^ 4 = 16). Combine ASK and PSK such that each signal corresponds to multiple bits. More phases than amplitudes. Minimum bandwidth requirement of QAM is same as ASK or PSK.



(a) Twelve Phase Angles



(b) A Phase Change with Two Amplitudes

Fig: Phase change with two amplitudes (Source:Brainkart)

In an *M*-ary PSK system, the in-phase and quadrature components of the phase-modulation. In *M*-ary PAM, the signal-space diagram is one-dimensional. *M*-ary QAM is a twodimensional generalization of *M*-ary PAM, in that its formulation involves two orthogonal passband basis functions:

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad 0 \le t \le T$$

Let *d*min denote the minimum distance between any two message points in the QAM constellation. Then, the projections of the *i*th message point on the 1- and _2- axes are respectively defined by *ai d*min 2 and *bi d*min 2, where *i* = 1, 2, , *M*. With the separation between two message points in the signal-space diagram being proportional to the square root of energy, we may therefore set

$$\frac{d_{\min}}{2} = \sqrt{E_0}$$

where *E*0 is the *energy of the message signal with the lowest amplitude*. The transmitted *M*-ary QAM signal for symbol *k* can now be defined in terms of *E*0

$$s_k(t) = \sqrt{\frac{2E_0}{T}}a_k\cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}b_k\sin(2\pi f_c t), \qquad \begin{cases} 0 \le t \le T \\ k = 0, \pm 1, \pm 2, \dots \end{cases}$$

The signal *sk*(*t*) involves two phase-quadrature carriers, each one of which is modulated by a set of discrete amplitudes; hence the terminology “quadrature amplitude modulation.” In *M*-ary QAM, the constellation of message points depends on the number of possible symbols, *M*. In what follows, we consider the case of *square constellations*, for which the number of bits per symbol is even.

QAM Square Constellations

With an *even* number of bits per symbol, Under this condition, an *M*-ary QAM square constellation can always be viewed as the *Cartesian product of a one-dimensional L-ary PAM constellation with itself*. By definition, the Cartesian product of two sets of coordinates (representing a pair of one-dimensional constellations) is made up of the set of all possible ordered pairs of coordinates with the first coordinate in each such pair being taken from the first set involved in the product and the second coordinate taken from the second set in the product.

Thus, the ordered pairs of coordinates naturally form a square matrix, as shown by

$$\{a_i, b_i\} = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \cdots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \cdots & (L-1, L-3) \\ \vdots & \vdots & & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, -L+1) \end{bmatrix}$$

To calculate the probability of symbol error for this *M*-ary QAM, we exploit the following property:

A QAM square constellation can be factored into the product of the corresponding *L*-ary PAM constellation with itself.

To exploit this statement, we may proceed in one of two ways:

Approach 1: We start with a signal constellation of the *M*-ary PAM for a prescribed *M*, and then build on it to construct the corresponding signal constellation of the *M*-ary QAM.

Approach 2: We start with a signal constellation of the *M*-ary QAM, and then use it to construct the corresponding orthogonal *M*-ary PAMS.

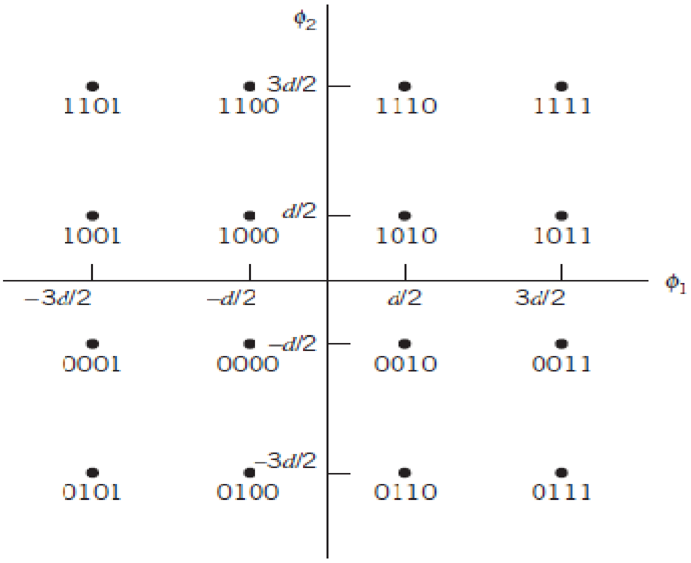


Fig: Signal-space diagram of *M*-ary QAM fo *M* = 16; the message points in each quadrant are identified with Gray-encoded quadbits.

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page- 373)