## Ampere's Circuital Law

Ampere's circuital law states that the line integral of the magnetic field around a closed path is the net current enclosed by this path. Mathematically,

$$\oint \vec{H}.d\vec{l} = I_{enc}$$ ....................................(3.8)

The total current I enc can be written as,

$$I_{enc} = \int_S \vec{J}.d\vec{s}$$ ....................................(3.9)

By applying Stoke's theorem, we can write

$$\oint \vec{H}.d\vec{l} = \int_S \nabla \times \vec{H}.d\vec{s}$$

$$\therefore \int_S \nabla \times \vec{H}.d\vec{s} = \int_S \vec{J}.d\vec{s}$$

$$\therefore \nabla \times \vec{H} = \vec{J}$$ ....................................(3.10)

which is the Ampere's law in the point form.

**Estimation of Magnetic field intensity for straight and circular conductors:**

Weillustrate the application of Ampere's Law with some examples.

We compute magnetic field due to an infinitely long thin current carrying conductor as shown in Fig. 2.1. Using Ampere's Law, we consider the close path to be a circle of radius

If we consider a small current element $Id\vec{l}(= Idz\hat{a}_z)$ $d\vec{H}$ , is perpendicular to the plane

containing both $d\vec{l}$ and $\vec{R}(= \rho\hat{a}_\rho)$ . Therefore only $\vec{H}$ component of that

will be present $\vec{H}= H_\phi \hat{a}_\phi$

,i.e.,                .

$H_\phi$

By applying Ampere's law we can write,

$$\overrightarrow{H} = \frac{I}{2\pi\rho}\hat{a}_\phi \int_0^{2\pi} H_\phi \rho d\phi = H_\phi \rho 2\pi = I$$

.................(4.11)

Therefore, $\overrightarrow{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$       which is same as equation (3.7)
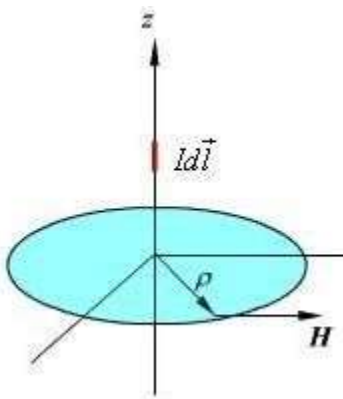


**Fig. 2.1: Magnetic field due to an infinite thin
current carryingconductor**
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

We consider the cross section of an infinitely long coaxial conductor, the inner conductor carrying a current I and outer conductor carrying current - I as shown in figure 2.2. We compute the magnetic field as a function of as follows:

In the region $0 \le \rho \le R_1$

$$I_{enc} = I\frac{\rho^2}{R_1^2}$$
...................................(3.12)
$$H_\phi = \frac{I_{enc}}{2\pi\rho} = \frac{I\rho}{2\pi a^2}$$

$$............................(3.13)$$

In the region $R_1 \le \rho \le R_2$

$$I_{enc} = I$$

$$H_\phi = \frac{I}{2\pi\rho} ...................................(3.14)$$


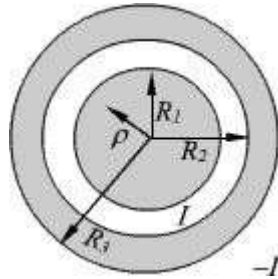
Fig. 2.2: Coaxial conductor carrying equal and opposite

(www.brainkart.com/subject/Electromagnetic-Theory_206/)

$\rho \le R_3$

$$I_{enc} = I$$

$$= I\frac{\rho^2 - R_2^2}{R_3^2 - R_2^2}......................(3.15)$$

$$H_\phi = \frac{I}{2\pi\rho}\frac{R_3^2 - \rho^2}{R_3^2 - R_2^2}......................(3.16)$$

$> R_3$  $\quad I_{enc} = 0 \qquad H_\phi = 0$ $......................(3.17)$

## **Boundary Condition for Magnetic Fields**

Similar to the boundary conditions in the electro static fields, here we will consider $\vec{B}$ $\vec{H}$ behavior at the interface of two different media. In particular, we determine how the tangential and normal components of magnetic fields behave at the boundary of two regions having different permeabilities.

The figure 7.1 shows the interface between two media having

$\hat{a}_n$ permeabities $\mu_1$ and $\mu_2$, being the normal vector from medium 2 to
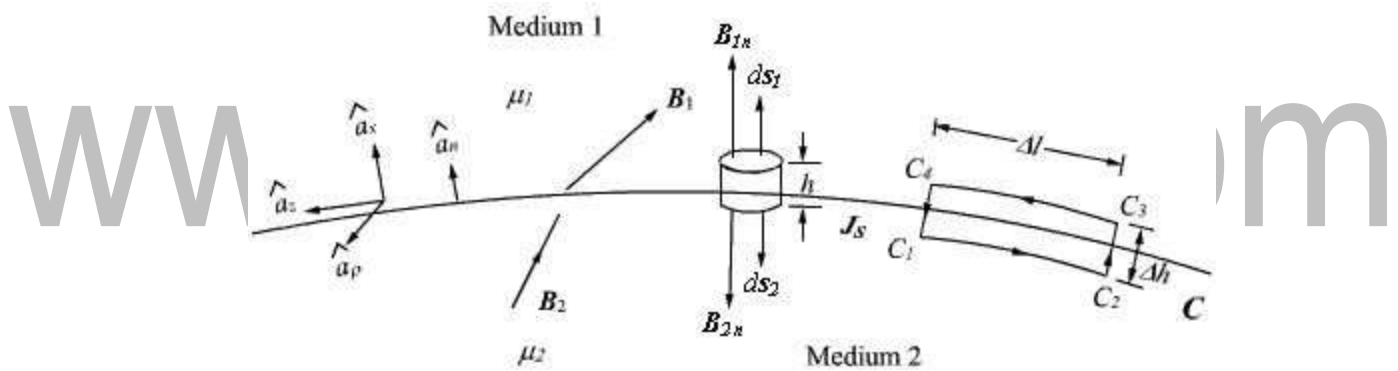
medium 1.



**Figure 7.1: Interface between two magnetic media**
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

To determine the condition for the normal component of the flux $\vec{B}$ densityvector
, we consider a small pill box P with vanishingly small thickness $h$ and $\Delta S$ having an
elementary area for the faces. Over the pill box, we can write

$$\oint_s \vec{B}.d\vec{s} = 0 \quad .................................................(3.36)$$

Since h --> 0, we can neglect the flux through the sidewall of the pill box.

$$\therefore \int_{\Delta S} \vec{B}_1 . d\vec{S}_1 + \int_{\Delta S} \vec{B}_2 . d\vec{S}_2 = 0 \quad \text{..........................(3.37)}$$

$$d\vec{S}_1 = dS\hat{a}_n \text{ and } d\vec{S}_2 = dS\left(-\hat{a}_n\right) \quad \text{...............................(3.38)}$$

where

$$\int_{\Delta S} B_{1n} dS - \int_{\Delta S} B_{2n} dS = 0$$

$$B_{1n} = \vec{B}_1 . \hat{a}_n \text{ and } B_{2n} = \vec{B}_2 . \hat{a}_n \text{ ...............} (3.39)$$

Since $\Delta S$ is small, we can write

$$\left(B_{1n} - B_{2n}\right)\Delta S = 0$$

or,............................ $B_{1n} = B_{2n}$ .............................(3.40)

That is, the normal component of the magnetic flux density vector is continuous acrossthe interface.

In vector form,

$$\hat{a}_n . \left(\vec{B}_1 - \vec{B}_2\right) = 0 \quad \text{.........................(3.41)}$$

To determine the condition for the tangential component for the magnetic field, we considera closed path C as shown in figure 3.8. By applying Ampere's law we can write

$$\oint \vec{H} . d\vec{l} = I \quad \text{..........................(3.42) Since}$$

h --
>0,

$$\int_{c_1 - c_2} \vec{H} . d\vec{l} + \int_{c_3 - c_4} \vec{H} . d\vec{l} = I \quad \text{..................................(3.43)}$$

We have shown in figure 4.8, a set of three unit vectors $\hat{a}_N$, $\hat{a}_t$, $\hat{a}_P$ such that they

$$\hat{a}_t = \hat{a}_P \times \hat{a}_n$$

satisfy        (R.H. rule). Here is tangential to the interface        is the vector perpendicular to the surface enclosed by C at the interface

The above equation can be written as

$$\vec{H}_1 . \Delta l \, \hat{a}_t - \vec{H}_2 . \Delta l \, \hat{a}_t = I = J_{Sn} \Delta l$$

$$H_{1t} - H_{2t} = J_{Sn}$$

or, .............................................................................................................. (3.44)

i.e., tangential component of magnetic field component is discontinuous across the interface where a free surface current exists.

If $J_S = 0$, the tangential magnetic field is also continuous. If one of the medium is a

perfect conductor $J_S$ exists on the surface of

the perfect conductor. In vector form we can write,

$$\left(\vec{H}_1 - \vec{H}_2\right) . \hat{a}_t \, \Delta l$$

$$= \left(\vec{H}_1 - \vec{H}_2\right) . \left(\hat{a}_P \times \hat{a}_n\right) \Delta l$$

$$= J_{Sn\Delta l} = \vec{J}_S . \hat{a}_P \, \Delta l$$

Therefore,

$$\hat{a}_n \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_S$$

...................................(3.45)

$$\hat{a}_n \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}_S \quad \text{...................................(3.46)}$$

## Energy Stored in Magnetic Fields

Since an inductor in a circuit serves to oppose any change in the current through it, work must be done by an external source such as a battery in order to establish a current in the inductor. From the work-energy theorem, we conclude that energy can be stored in an inductor. The role played by an inductor in the magnetic case is analogous to that of a capacitor in the electric case.

The power, or rate at which an external emf $\varepsilon_{ext}$ works to overcome the self-induced emf $\varepsilon_L$ and pass current $I$ in the inductor is

$$P_L = \frac{dW_{ext}}{} = I\varepsilon_{ext}\, dt \qquad (1)$$

If only the external emf and the inductor are present, then $\varepsilon_{ext} = -\varepsilon_L$ which implies

$$P_L = \frac{dW_{ext}}{} = -I\varepsilon_L = +IL[dI/dt] \qquad (2)$$

If the current is increasing with $dI / dt > 0$, then $P > 0$ which means that the external source is doing positive work to transfer energy to the inductor. Thus, the internal energy $U_B$ of the inductor is increased. On the other hand, if the current is decreasing with $dI / dt < 0$, we then have $P < 0$. In this case, the external source takes energy away from the inductor, causing its internal energy to go down. The total work done by the external source to increase the current form zero to $I$ is then

$$W = dW = LI'dI' = \frac{1}{2}LI^2 \qquad (3)$$

This is equal to the magnetic energy stored in the inductor:

$$U_B = LI^2 \qquad (4)$$

The above expression is analogous to the electric energy stored in a capacitor:

$$U_E = \frac{1}{2}\frac{Q^2}{C} \qquad (5)$$

We comment that from the energy perspective there is an important distinction between an inductor and a resistor. Whenever a current $I$ goes through a resistor, energy flows into the resistor and dissipates in the form of heat regardless of whether $I$ is steady or time dependent (recall that power dissipated in a resistor is $P_R = IV_R = IR$). On the other hand, energy flows into an ideal inductor only when the current is varying with $dI/dt > 0$. The energy is not dissipated but stored there; it is released later when the current decreases with $dI/dt < 0$. If the current that passes through the inductor is steady, then there is no change in energy since $P_L = (/dt)$

$$LI\, dI = 0.$$

## Example 11.5 Energy Stored in a Solenoid

A long solenoid with length $l$ and a radius $R$ consists of $N$ turns of wire. A current $I$ passes through the coil. Find the energy stored in the system.

**Solution:**

$$U_B = LI = \mu_0 nI\, \pi\, Rl \qquad (6)$$

The result can be expressed in terms of the magnetic field strength $B = \mu_0 nI$ :

$$U_B = (\mu_0 nI)(\pi\, Rl) = (\pi\, Rl) \qquad (7)$$

Since $P^2 l$ is the volume within the solenoid, and the magnetic field inside is uniform, the term may be identified as the magnetic energy density, or the energy per unit volume of the magnetic field. The above expression holds true even when the magnetic field is nonuniform. The result can be compared with the energy density associated with an electric field:

**Creating and Destroying Magnetic Energy Animation**

Let's consider the process involved in creating magnetic energy. It shows the process by which an external agent(s) creates magnetic energy. Suppose we have five rings that carry a number of free positive charges that are not moving. Since there is no current, there is no magnetic field. Now suppose a set of external agents come along (one for each charge) and simultaneously spin up the charges counterclockwise as seen from above, at the same time and at the same rate, in a manner that has been pre-arranged. Once the charges on the rings start to accelerate, there is a magnetic field in the space between the

rings, mostly parallel to their common axis, which is stronger inside the rings than outside. This is the solenoid configuration.

As the magnetic flux through the rings grows, Faraday's law of induction tells us that there is an electric field induced by the time-changing magnetic field that is circulating clockwise as seen from above. The force on the charges due to this electric field is thus opposite the direction the external agents are trying to spin the rings up (counterclockwise), and thus the agents have to do additional work to spin up the charges because of their charge. This is the source of the energy that is appearing in the magnetic field between the rings — the work done by the agents against the "back emf."

Over the course of the "create" animation associated with the agents moving the charges to a higher speed against the induced electric field are continually doing work. The electromagnetic energy that they are creating at the place where they are doing work (the path along which the charges move) flows both inward and outward. The direction of the flow of this energy is shown by the animated texture patterns. This is the electromagnetic energy flow that increases the strength of the magnetic field in the space between the rings as each positive charge is accelerated to a higher and higher speed. When the external agents have gotten up the charges to a predetermined speed, they stop the acceleration. The charges then move at a constant speed, with a constant field inside the solenoid, and zero "induced" electric field, in accordance with Faraday's law of induction.

This process proceeds as follows. Our set of external agents now simultaneously start to spin down the moving charges (which are still moving counterclockwise as seen from above), at the same time and at the same rate, in a manner that has been pre-arranged. Once the charges on the rings start to decelerate, the magnetic field in the space between the rings starts to decrease in magnitude. As the magnetic flux through the rings decreases, Faraday's law tells us that there is now an electric field induced by the time-changing magnetic field that is circulating counterclockwise as seen from above. The force on the charges due to this electric field is thus in the same direction as the motion of the charges. In this situation the agents have work done on them as they try to spin the charges down.

Over the course of the "destroy" animation associated with the strength of the magnetic field decreases, and this energy flows from the field back to the path along which the charges move, and is now being provided *to* the agents trying to spin down the moving charges. The energy provided to those agents as they destroy the magnetic field is exactly the amount of energy that they put into creating the magnetic field in the first place, neglecting radiative losses (such losses are small if we move the charges at speeds small

compared to the speed of light). This is a totally reversible process if we neglect such losses. That is, the amount of energy the agents put into creating the magnetic field is exactly returned to the agents as the field is destroyed.

## UNIT III MAGNETOSTATICS

**Introduction**

In previous chapters we have seen that an electrostatic field is produced by static or stationary charges. The relationship of the steady magnetic field to its sources is much more complicated.

**Laws governing magneto static fields**

The source of steady magnetic field may be a permanent magnet, a direct current or an electric field changing with time. In this chapter we shall mainly consider the magnetic field produced by a direct current. The magnetic field produced due to time varying electric field will be discussed later. Historically, the link between the electric and magnetic field was established Oersted in 1820. Ampere and others extended the investigation of magnetic effect of electricity . There are two major laws governing the magnetostatic fields are:

1.Biot-Savart Law

2.Ampere's Law

Usually, the magnetic field intensity is represented by the vector $\vec{H}$ . It is customary to represent the direction of the magnetic field intensity (or current) by a small circle with adot or cross sign depending on whether the field (or current) is out of or into the page asshown in Fig. 1.1.



**Fig. 1.1: Representation of magnetic field (or current) Biot- Savart Law**

(www.brainkart.com/subject/Electromagnetic-Theory_206/)

This law relates the magnetic field intensity $dH$ produced at a point due to a differentialcurrent element $Id\vec{l}$ shown in Fig. 1.2.
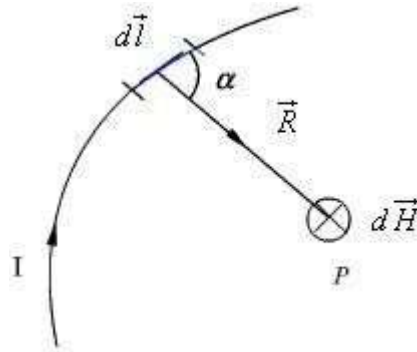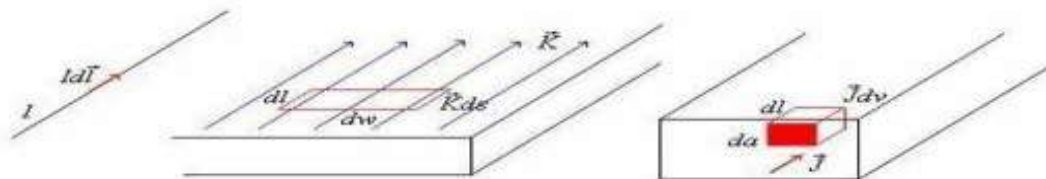


**Fig.1.2: Magnetic field intensity due to a current element**
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

The magnetic field intensity at P can be written as,

$$d\vec{H} = \frac{Id\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3} \ldots\ldots\ldots\ldots(3.1a)$$

$$dH = \frac{IdlSin\alpha}{4\pi R^2}$$

$$\ldots\ldots\ldots\ldots\ldots(3.1b)$$

where $R = |\vec{R}|$ is the distance of the current element from the point P. Similar to different charge distributions, we can have different current distribution such as line current, surface current and volume current. These different types of current densities are shown in Fig. 1.3.

Line Current            Surface Current            Volume Current

**Fig. 1.3: Different types of current distributions**
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

By denoting the surface current density as K (in amp/m) and
volume current density as J(in amp/m$^2$) we can write:

$$Id\vec{l} = \vec{K}ds = \vec{J}dv$$ .....................................(3.2)

( It may be noted that $I = Kdw = Jdq$ )

Employing Biot-Savart Law, we can now express the magnetic
field intensity H. Interms of these current distributions.

$$\vec{H} = \int_l \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$ ... **for line current** ...................(3.3a)

$$\vec{H} = \int_s \frac{Kd\vec{s} \times \vec{R}}{4\pi R^3}$$ **for surface current** ............(3.3b)

$$\vec{H} = \int_v \frac{\vec{J}dv \times \vec{R}}{4\pi R^3}$$ **for volume current** .............(3.3c)

To illustrate the application of Biot - Savart's Law, we consider the
following example.

**Example 3.1:** We consider a finite length of a conductor $\vec{r}$
carrying a current placed along z- axis as shown in the Fig 1.4.
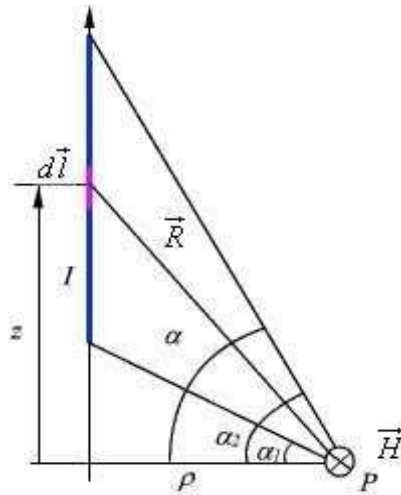We determine the magnetic field at point P due to this current
carrying conductor.

**Fig. 1.4: Field at a point P due to a finite length current carrying conductor**

(www.brainkart.com/subject/Electromagnetic-Theory_206/)

With reference to Fig. 1.4, we find that

$$d\vec{l} = dz\,\hat{a}_z \quad and \quad \vec{R} = \rho\hat{a}_\rho - z\hat{a}_z$$

..................................................(3.4)

Applying Biot -Savart's law for the current element $d\vec{l}$
we can write,

$$\overrightarrow{dH} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3} = \frac{\rho dz\hat{a}_\phi}{4\pi[\rho^2 + z^2]^{3/2}} \quad .................(3.5)$$

Substituting $\dfrac{z}{\rho} = \tan\alpha$ we can write,

$$\vec{H} = \int_{\alpha_1}^{\alpha_2} \frac{I}{4\pi} \frac{\rho^2 \sec^2\alpha\, d\alpha}{\rho^3 \sec^3\alpha} \hat{a}_\phi = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\hat{a}_\phi \quad ........................(3.6)$$

## Magnetic Circuits

Magnetic phenomena are described using a fairly large number of terms that are often, at first, somewhat difficult to keep track of. One approach that may help is to describe analogies between electrical circuits, which are usually more familiar, and corresponding magnetic circuits. Consider the electrical circuit shown in Fig. 1.20a and the analogous magnetic circuit shown in Fig 1.20b. The electrical circuit consists of a voltage source, $v$, sending current $i$ through an electrical load with resistance $R$. The electrical load consists of a long wire of length $l$, cross-sectional area $A$, and conductance $\rho$.

The resistance of the electrical load is given by (1):

$$R = \rho l A \qquad\qquad (1)$$

The current flowing in the electrical circuit is given by Ohm's law:

$$i = vR \qquad\qquad (2)$$

In the magnetic circuit of Fig. 9.1, the driving force, analogous to voltage, is called the magnetomotive force (mmf), designated by F. The magnetomotive force is created by wrapping $N$ turns of wire, carrying current $i$, around a toroidal



Cross-sectional area   A
Length            $l$
Conductance      $\rho$

Cross-sectional area   A
Length            $l$
Permeability       $\mu$

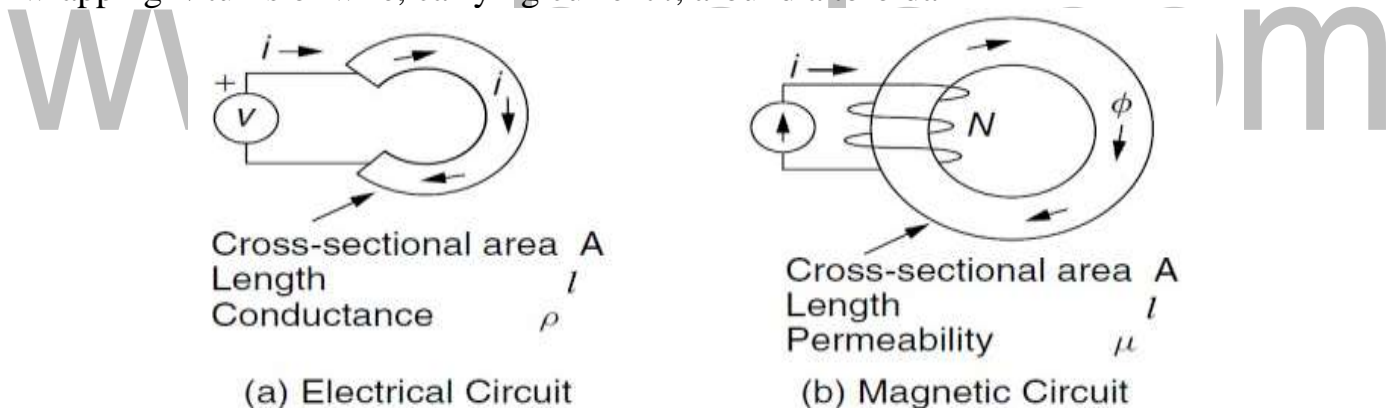(a) Electrical Circuit          (b) Magnetic Circuit

**Figure** 9.1     Analogous electrical and magnetic circuits.

(http://paginas.fisica.uson.mx/horacio.munguia/aula_virtual/Cursos/Instrumentacion%20I/ Documentos/Electricidad%20Basica.pdf)

core. By definition, the magnetomotive force is the product of current $\times$ turns, and has units of ampere-turns.

$$\text{Magnetomotive force (mmf )} F = Ni \text{ (ampere} - \text{turns)} \qquad\qquad (3)$$

The response to that mmf (analogous to current in the electrical circuit) is creation of magnetic flux $\varphi$, which has SI units of webers (Wb). The magnetic flux is proportional to the mmf driving force and inversely proportional to a quantity called reluctance R, which

is analogous to electrical resistance, resulting in the "Ohm's law" of magnetic circuits given by

$$F = R\varphi \qquad (4)$$

From (4), we can ascribe units for reluctance R as amp-turns per weber (A-t/Wb).

Reluctance depends on the dimensions of the core as well as its materials:

$$\text{reluctance} = R = l\mu A \, (A\text{-}t/Wb) \qquad (5)$$

Notice the similarity between (5) and the equation for resistance given in (4). The parameter in (5) that indicates how readily the core material accepts magnetic flux is the material's permeability $\mu$. There are three categories of magnetic materials: diamagnetic, in which the material tends to exclude magnetic fields; paramagnetic, in which the material is slightly magnetized by a magnetic field; and ferromagnetic, which are materials that very easily become magnetized. The vast majority of materials do not respond to magnetic fields, and their permeability is very close to that of free space. The materials that readily accept magnetic flux—that is, ferromagnetic materials—are principally iron, cobalt, and nickel and various alloys that include these elements. The units of permeability are webers per amp-turn-meter (Wb/A-t-m).

The permeability of free space is given by

$$\text{Permeability of free space } \mu 0 = 4\pi \times 10{-}7 \text{ Wb/A-t-m} \qquad (6)$$

Oftentimes, materials are characterized by their relative permeability, $\mu r$, which for ferromagnetic materials may be in the range of hundreds to hundreds of thousands. As will be noted later, however, the relative permeability is not a constant for a given material: It varies with the magnetic field intensity. In this regard, the magnetic analogy deviates from its electrical counterpart and so must be used with some caution.

$$\text{Relative permeability} = \mu r = \mu/\mu 0 \qquad (7)$$

Another important quantity of interest in magnetic circuits is the magnetic flux density, B. As the name suggests, it is simply the "density" of flux given by the following:

$$\text{Magnetic flux density } B = \varphi A \text{ webers/m2 or teslas (T)} \qquad (8)$$

When flux is given in webers (Wb) and area A is given in m2, units for B are teslas (T). The analogous quantity in an electrical circuit would be the current density, given by

$$\text{Electric current density } J = i/A \qquad (9)$$

The final magnetic quantity that we need to introduce is the magnetic field intensity, H. Referring back to the simple magnetic circuit shown in Fig. 9.1, the magnetic field intensity is defined as the magnetomotive force (mmf) per unit of length around the magnetic loop. With N turns of wire carrying current i, the mmf created in the circuit is Ni ampere-turns. With l representing the mean path length for the magnetic flux, the magnetic field intensity is therefore

$$\text{Magnetic field intensity } H = Ni/l \text{ ampere-turns/meter} \tag{10}$$

An analogous concept in electric circuits is the electric field strength, which is voltage drop per unit of length. In a capacitor, for example, the intensity of the electric field formed between the plates is equal to the voltage across the plates divided by the spacing between the plates. Finally, if we combine all the above equations, we arrive at the following relationship between magnetic flux density B and magnetic field intensity H:

$$B = \mu H \tag{11}$$

Returning to the analogies between the simple electrical circuit and magnetic circuit shown in Fig. 9.2.
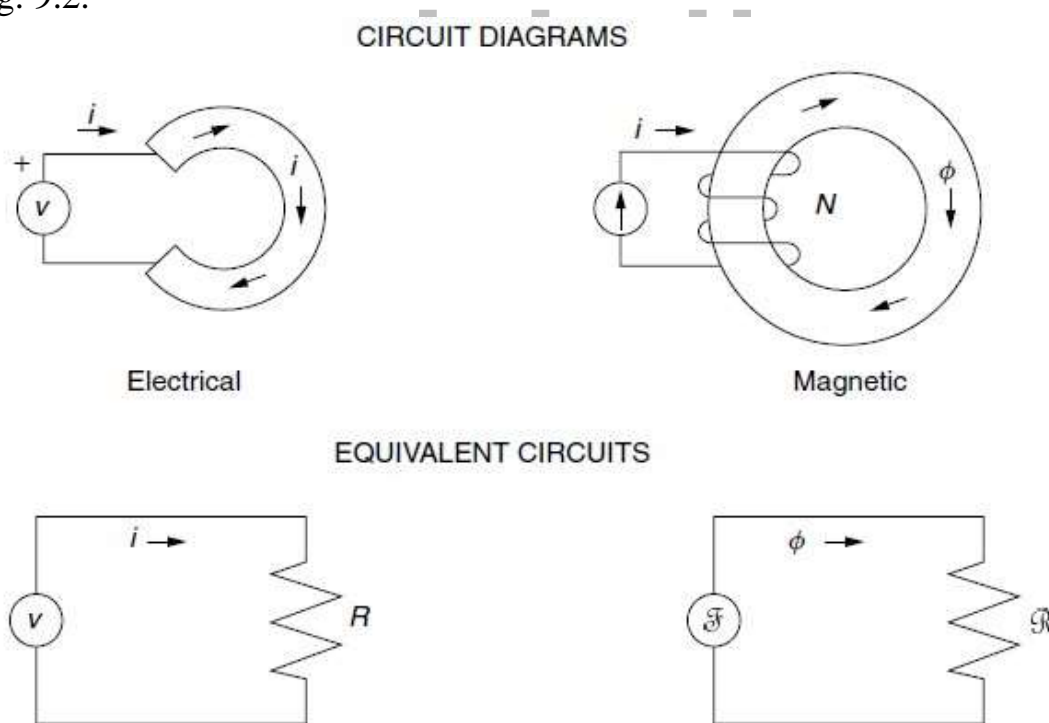


**Figure 9.2**   Equivalent circuits for the electrical and magnetic circuits shown.

(http://paginas.fisica.uson.mx/horacio.munguia/aula_virtual/Cursos/Instrumentacion%20I/ Documentos/Electricidad%20Basica.pdf)

## Magnetic Flux and Density

In simple matter, the magnetic flux density $\vec{B}$, $\vec{H}$, related to the

magnetic $\vec{B} = \mu \vec{H}$

field intensity as where $\mu$ called the permeability. In particular when

we consider the free space

$\vec{B} = \mu_0 \vec{H}$ where $\mu_0$ $4\pi \times 10^{-7}$ H/m is the permeability of the free space. Magnetic flux density is measured in terms of Wb/m$^2$.

The magnetic flux density through a surface is given by:

$$\psi = \int_S \vec{B} . d\vec{s}$$

Wb ..............................(3.18)

In the case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface. In case of magnetic field isolated magnetic charge (i. e. pole) does not exist. Magnetic poles always occur in pair (as N-S). For example, if we desire to have an isolated magnetic pole by dividing the magnetic bar successively into two, we end up with pieces each having north (N) and south (S) pole as shown in Fig. 3.1 (a). This process could be continued until the magnets are of atomic dimensions; still we will have N-S pair occurring together. This means that the magnetic poles cannot be isolated.
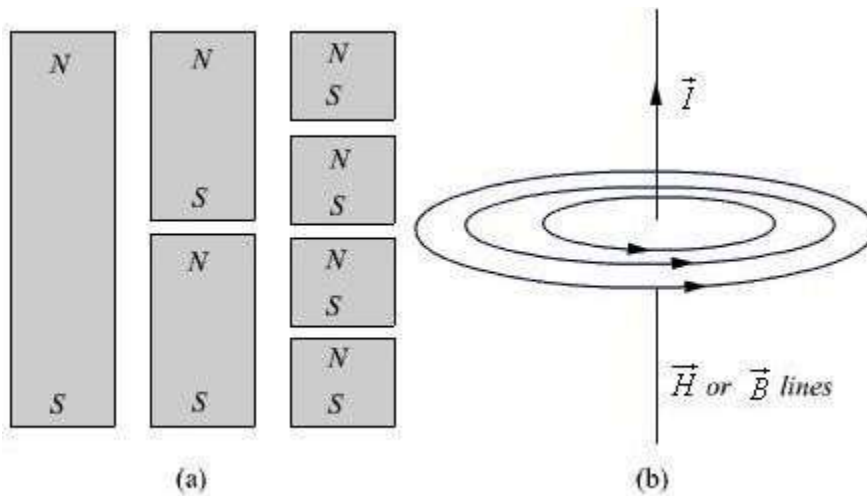
**Fig. 3.1: (a) Subdivision of a magnet (b) Magnetic field/ flux lines of a straightcurrent carrying conductor**
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

Similarly if we consider the field/flux lines of a current carrying conductor as shown in Fig. 3.1 (b), we find that these lines are closed lines, that is, if we consider a closed surface, the number of flux lines that would leave the surface would be same as the number of flux lines that would enter the surface.

From our discussions above, it is evident that for magnetic field,

$$\oint_S \vec{B}.d\vec{s} = 0$$

....................................(3.19)

which is the Gauss's law

for the magnetic field. By

$$\oint_S \vec{B}.d\vec{s} = \int_V \vec{\nabla}.\vec{B}dv = 0$$

applying divergence

theorem, we can write:

Hence,.....................................................$$\nabla.\vec{B} = 0$$.............................................(3.20)

which is the Gauss's law for the magnetic field in point form.

## MAGNETIC FORCES

### Force On A Moving Charge

In electric field, force on a charged

particle is F=QE

Force is in the same direction as the electric field intensity(positive charge)

A charged particle in motion in a magnetic field force magnitude is proportional to the product of magnitudes of the charge Q, its velocity V and the flux density B and to the sineof the angle between the vectors V and B.

The direction of force is perpendicular to both V and B and is given by a unit vector in thedirection of V x B.

The force may therefore be

expressed as F=QV x B

Force on a moving particle due to combined electric and magnetic fields is obtained bysuperposition.

$$F=Q (E + V x B)$$

This equation is known as Lorentz force equation.

### Force On A Differential Current Element:

The force on a charged particle moving through a steady magnetic field may be written asthe differential; force exerted on a differential element of charge.

Convection current density in terms of the velocity of the volume charge

density Differential element of charge may also be expressed in terms of

volume charge density.

The magnitude of the force is given by the familiar equation

$$F=BIL\sin\theta$$

## Force on a current-carrying conductor

Charges confined to wires can also experience a force in a magnetic field. A current (I) in a magnetic field ( **B**) experiences a force ( **F**) given by the equation **F** = I **l** × **B** or F = IlB sin θ, where **l** is the length of the wire, represented by a vector pointing in the direction of the current. The direction of the force may be found by a right-hand rule similar. In this case, point your thumb in the direction of the current—the direction of motion of positive charges. The current will experience no force if it is parallel to the magnetic field.

## Force and Torque on a current loop

A loop of current in a magnetic field can experience a torque if it is free to turn. Figure 4.1 (a) depicts a square loop of wire in a magnetic field directed to the right. Imagine in Figure 4.1 (b) thatthe axis of the wire is turned to an angle (θ) with the magnetic field and that the view is lookingdown on the top of the loop. The *x* in a circle depicts the current traveling into the page away from the viewer, and the dot in a circle depicts the current out of the page toward the viewer.
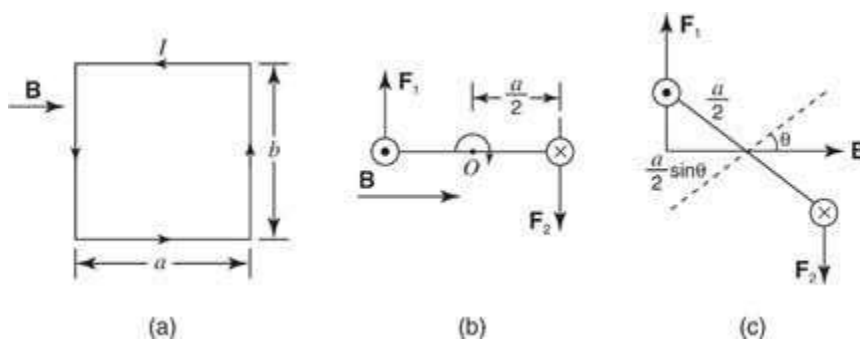


**Figure 4.1**
(a) Square current loop in a magnetic field **B**. (b) View from the top of the current loop. (c) If the loop is tilted with respect to **B**, a torque results.

(www.brainkart.com/subject/Electromagnetic-Theory_206/)

## MAGNETIC INDUCTANCE
### Self inductance and mutual inductance:

Like capacitance, inductance $L$ is a property of a physical arrangement of conduc- tors. It is a measure of magnetic flux which links the circuit when a current $I$ flows in the circuit. It is also a measure of how much energy is stored in the magnetic field of an inductor, such as a coil, solenoid, etc.

The definition of inductance rests on the concept of flux linkage. It is not a very precise concept unless one is willing to introduce a complicated topological description. For our purposes it will be sufficient to define *fluxlinkage* A as the flux that links all

the circuit, multiplied by the number of turns N. For example, in the case of the solenoid shown in Fig.5.1 , flux linkage will be given by
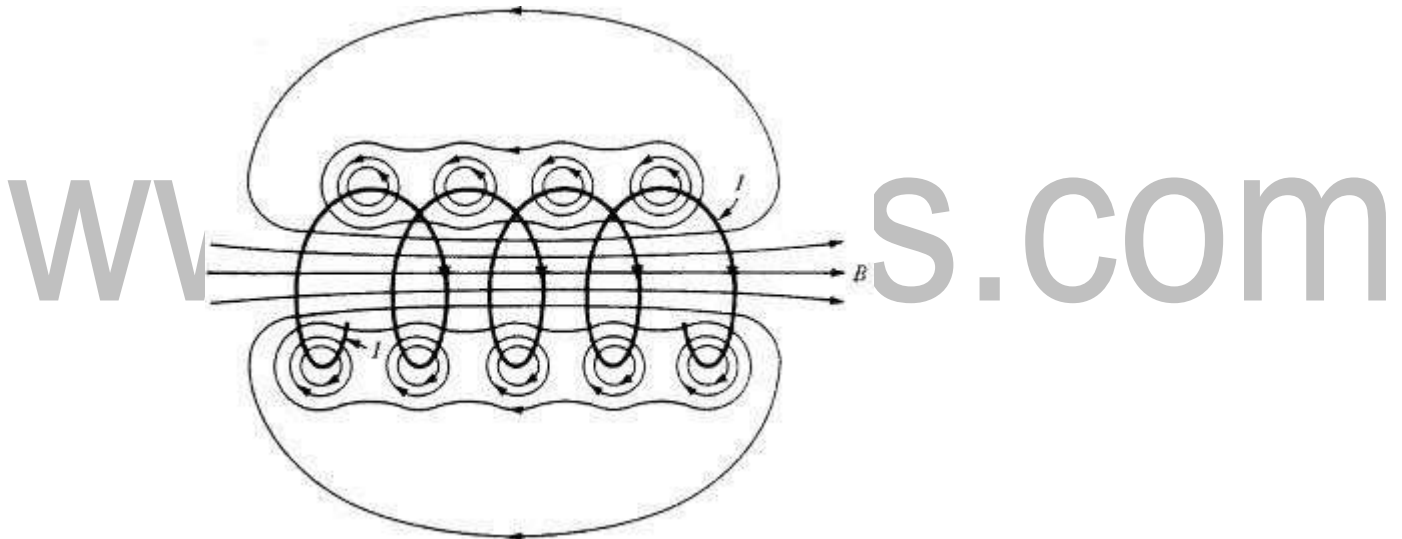


Fig 5.1 solenoid flux linkage
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

A = N¢ = N JJ B· **dA** ∼ *NBA* Wb

that is, only the flux that goes through the inside of the solenoid and therefore links all turns is used. The small flux loops about each turn are ignored in a first-order analysis because they link only one or two turns and flow through a small area. The area $A$ is that area through which the flux that links all turns flows. For the solenoid of Fig. 5.2, a good approximation to $A$ is the cross section of the solenoid

The unit of inductance is the henry (H). Inductors for filter applications in power supplies are usually wire-wound solenoids on an iron core with inductances in the range from 1 to 10 H. Inductors found in high-frequency circuits are air-core solenoids with values in the millihenry (mH) range. The definition for inductance, even though it is derived for steady currents, is valid up to very high frequencies.

Let us calculate *L* for some useful geometries.

### Solenoid

A good approximation of the *B* field in a solenoid that links all turns is the *B* field at the center of the solenoid; that is, $B = \mu_0 Ni/l$. There is some leakage at the ends of the solenoid (recall that the value of the *B* field drops to one-half at the ends), which we will ignore because it occurs mainly at the ends. The inductance *L* of a solenoid is therefore

$$L = \lambda/i = NBA/i = \mu_0 N^2 A/l$$

where *l* is the length and *A* is the cross section of the solenoid.

If we have a short solenoid of N turns, that is, one where the length *l* is smaller

### TOROID

For example, the inductance of a 2000-turn toroid having a cross-sectional area of 1 cm² and mean radius of 5 cm is

$$L = (4\pi \times 10^{-7} \text{ H/m})(2000)^2 (10^{-4} \text{ m}^2)/2\pi(0.05 \text{ m}) = 1.6 \text{ mH}$$

If the toroid were filled with iron instead of air, the inductance could be increased many thousand fold.

Note that we have neglected the variation of *B* across the cross section of the toroid. By using an average *r*, as, for example, $r = (a + b)/2$, we have in effect used an average value of *B* in the calculation of inductance. If this is not sufficiently accurate, the variation of *B* should be considered by integrating (7.25) between *a* and *b*.

### Coaxial Transmission Line

The student usually does not have any difficulty in grasping the concept ofinduc- tance as long as the geometries involve windings (such as in coils and toroids). In the following examples flux linkage is used in a broader sense and should clarify that concept further. Figure 5.2 shows a longitudinal and transverse cross section of a coaxial line (already considered when the capacitance' per unit length was calculated). The current *I* flows in the center conductor
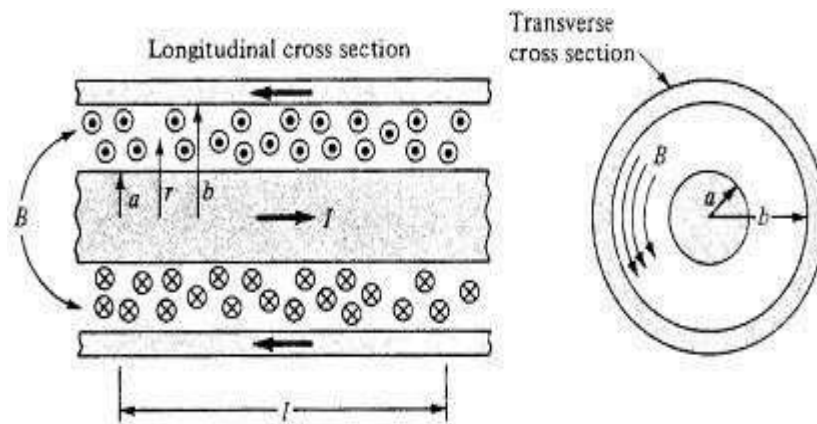


**Fig 5.2 coxial transmission-cross section (www.brainkart.com/subject/Electromagnetic-Theory_206/)**

We have approximated the upper limit *d - a* by *d* because for practical transmis- sion lines *d ~ a*. This approximation also accounts for the flux from the lower conductor which partly links the current inside the upper wire. As a matter of fact it can be shown that the replacement of *d - a* by *a* gives an exact result for the flux linkages. t The inductance per unit length *LII.*which is the desired result and gives the total stored magnetic energy in an inductance *L* carrying current *I.*

# MAGNETIC MATERIALS

All material shows some magnetic effects. In many substances the effects are so weak thatthe materials are often considered to be non magnetic.

A vacuum is the truly nonmagnetic medium.

## DIAMAGNETIC:

In diamagnetic materials magnetic effects are weak. Atoms in which the small magneticfields produced by the motion of the electrons in their orbit and those produced by the electron spin combine to produce a net field of zero.The fields produced by the electron motion itself in the absence of any external magnetic field.

This material as one in which the permanent magnetic moment m0 of each atom is zero.Such a material is termed diamagnetic.

## PARAMAGNETIC:

In paramagnetic materials the magnetic moments of adjacent atoms align in opposite directions so that the net magnetic moment of a specimen is nil even in the presence ofapplied field.

## FERROMAGNETIC:

In ferromagnetic substance the magnetic moments of adjacent atoms are also aligned opposite, but the moments are not equal, so there is a net magnetic moment.

It is less than in ferromagnetic materials.The ferrites have a low electrical conductivity, which makes them useful in the cores of acinductors and transformers.

Since induced currents are less and ohmic losses are reduced. There are a number of elements that are attracted by magnets, commonly referred to as 'magnetic'. These are the same materials that can be magnetised to create permanent magnets, Here you can see a list of magnetic elements and minerals:

Magnetic materials are categorised as magnetically hard, or magnetically soft materials. Magnetically soft materials are easily magnetised but the induced magnetism is usually temporary. For example, if you rub a permanent magnet along a nail, or a screwdriver, the nail or screwdriver will become temporarily magnetised and will emit their own weak magnetic field. This is because a large number of their iron atoms are temporarily aligned in the same direction by the external magnetic field.

Like magnetically soft materials, magnetically hard materials can be magnetised by a strong external magnetic field, such as those generated by an electromagnet. The difference being that magnetically hard materials will remain magnetised indefinitely, unless they are demagnetised by an opposing magnetic field, raised above their curie temperature or allowed to corrode. Magnetically hard materials are used to create permanent magnets made from alloys generally consisting of varying amounts of iron, aluminium, nickel, cobalt and rare earth elements samarium, dysprosium and neodymium. The strongest permanent magnets are known as neodymium magnets and are made from an alloy of neodymium, iron and boron. Permanent magnets are difficult to magnetise as unlike magnetically soft materials their atomic magnetic domains aren't easily aligned but once they are aligned, they remain so indefinitely.

Because all materials have a different atomic structure different materials react differently when placed in a magnetic field. In its simplest form, the magnetic behaviour of a material is determined by its number of unpaired electrons in each atom. In the atoms of most elements electrons exist in pairs with each electron spinning in a different direction causing them to cancel out each other's magnetic field, therefore no net magnetic field exists. However, some materials have unpaired electrons which will generate a net magnetic field and therefore have a greater reaction to an external magnetic field. Most materials are classified either as ferromagnetic, diamagnetic or paramagnetic.

Ferromagnetic materials have some unpaired electrons in their atoms and therefore generate a net magnetic field, albeit a very weak one. This is because the individual atoms or groups of atoms, known as magnetic domains, are randomly aligned cancelling each other out. When an external magnetic field is applied to the ferromagnetic material the individual domains are forced into alignment which they maintain once the external field is removed therefore maintaining their magnetism, known as remanence. Iron, nickel and cobalt are all ferromagnetic materials.

Diamagnetic materials repel any externally applied magnetic field. This occurs because their magnetic domains realign to oppose an externally applied magnetic field when influenced by a magnetic field. All materials show some diamagnetic properties, however, in most materials the effect is extremely weak and unnoticed. All the electrons within the atoms of diamagnetic materials are paired, therefore they do not generate their own net magnetic field. Most elements in the periodic table are diamagnetic.

Paramagnetic materials have a small susceptibility to magnetic fields meaning that they are slightly attracted by a magnetic field. However, unlike ferromagnetic materials they do not maintain their magnetic properties once the external magnetic field is removed. Most elements are paramagnetic, however, because their attractive

force is many thousands of times weaker than ferromagnetic material they are also generally considered as 'non-magnetic.

Neodymium magnets are the strongest magnets available and therefore have a high power to volume and weight ratio. Because they are so strong it also means they have a relatively low cost per unit of strength (Maximum Energy Product, MGOe). They have an incredibly high resistance to being demagnetised but generally have low maximum operating temperatures compared to other materials and are susceptible to corrosion if their coating is damaged.

Other grades of neodymium magnet, including special high maximum operating temperature grades are available. For more information regarding the properties of grades not mentioned above, please refer to our neodymium magnet grades.

www.binils.com

## Magnetic Scalar and Vector Potentials

In studying electric field problems, we introduced the concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a **scalar magnetic potential** and write:

$$\vec{H} = -\nabla V_m \quad ................................(3.21)$$

From Ampere's law , we know that

$$\nabla \times \vec{H} = \vec{J} \quad ................................(3.22)$$

Therefore,

$$\nabla \times (-\nabla V_m) = \vec{J}$$

$$\vec{H} = -\nabla V_m$$

$$................................................................................(3.23)$$

$$\nabla \times (\nabla V) = 0$$

But using vector identity, we find that is valid only where $\vec{J} = 0$. Thus the scalar magnetic potential is defined only in the region where $\vec{J} = 0$.

Moreover, $V_m$ in general is not a single valued function of position.

This point can be illustrated as follows. Let us consider the cross section of a coaxial lineas shown in fig 6.1.

In the region $a < \rho < b$ , $\vec{J} = 0$ and $\qquad \vec{H} = \dfrac{I}{2\pi\rho}\hat{a}_\phi$
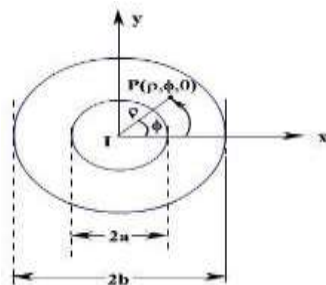


**Fig. 6.1: Cross Section of a Coaxial Line**
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

If $V_m$ is the magnetic potential then,

$$-\nabla V_m = -\frac{1}{\rho}\frac{\partial V_m}{\partial \phi}$$

$$= \frac{I}{2\pi\rho}$$

$$\therefore V_m = -\frac{I}{2\pi}\phi + c$$

$$\phi = 0 \qquad V_m = -\frac{I}{2\pi}\phi$$

$$\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi}\phi_0$$

If we set $V_m = 0$ at    then c=0 and
We observe that as we make a complete lap around the current

carrying conductor , wereach $\phi_0$ again but $V_m$ this time becomes

$$V_m = -\frac{I}{2\pi}(\phi_0 + 2\pi)$$

We now introduce the **vector magnetic potential** which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.

$\nabla . \vec{B} = 0$ and we have the vector identity that      $\vec{A}$     $, \nabla . \left( \nabla \times \vec{A} \right) = 0$ ,
$\vec{B} = \nabla \times \vec{A}$

Here, the vector field $\vec{A}$ is called the vector magnetic potential. Its SI unit is Wb/m. Thus if can find $\vec{A}$ of a given current distribution, $\vec{B}$ can be found from   $\vec{A}$ through a curl operation.

We have introduced the vector function $\vec{A}$ on                          $\vec{B}$ and related its curl to    .
A vector function is                                            $\nabla . \vec{A}$

defined fully in terms of its curl as well as divergence. The choice ofis made asfollows.

$$\nabla \times \nabla \times \vec{A} = \mu \nabla \times \vec{H} = \mu \vec{J}$$ ..........................................(3.24)

By using vector identity,

$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla.\vec{A}) - \nabla^2\vec{A}$$ ....................................................(3.25)

$$\nabla(\nabla.\vec{A}) - \nabla^2\vec{A} = \mu\vec{J}$$ ..........................................(3.26)

Great deal of simplification can be achieved if choose $\nabla.\vec{A} = 0$ .
Putting $\nabla.\vec{A} = 0$, we get $\nabla^2\vec{A} = -\mu\vec{J}$ which is vector poissonequation.
In Cartesian coordinates, the above equation can be written in terms of the componentsas

$$\nabla^2 A_x = -\mu J_x$$ ....................................(3.27a)

$$\nabla^2 A_y = -\mu J_y$$

....................................(3.27b)

$$\nabla^2 A_z = -\mu J_z$$ ....................................(3.27c)

The form of all the above equation is
same as thatof

$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$ ..........................................(3.28)

for which the solution is

$$V = \frac{1}{4\pi\varepsilon}\int_v \frac{\rho}{R}dv', \qquad R = |\vec{r} - \vec{r}'|$$
..................(3.29)

In case of time varying fields we shall see t`hat $\nabla.\vec{A} = \mu\varepsilon\frac{\partial V}{\partial t}$, which is known asLorentz condition, $V$ being the electric potential. Here we are dealing with static $\nabla.\vec{A} = 0$

magnetic field, so .

By comparison, we can write the solution for Axas

$$A_x = \frac{\mu}{4\pi}\int_{v'}\frac{J_x}{R}dv'$$

.....................................(3.30)

Computing similar solutions for other two components of the vector potential, thevector potential can be written as

$$\vec{A} = \frac{\mu}{4\pi}\int_{v'}\frac{\vec{J}}{R}dv'$$

.......................................(3.31)

This equation enables us to find the vector potential at a given point because of a volume current density $\vec{J}$. Similarly for line or surface current density we can write

$$\vec{A} = \frac{\mu}{4\pi}\int_{l'}\frac{I}{R}d\vec{l'}$$

.........................................(3.32)

$$\vec{A} = \frac{\mu}{4\pi}\int_{s'}\frac{\vec{K}}{R}ds'$$

respectively ...................... (3.33)

The magnetic fluxthrough a given area S is given by

$$\psi = \int_{s}\vec{B}.d\vec{s}$$

.............................................(3.34)

$$\vec{B} = \nabla \times \vec{A}$$
$$\psi = \int_{s}\nabla \times \vec{A}.d\vec{s} = \oint_{c}\vec{A}.d\vec{l}$$

.......................................(3.35)

Vector potential thus have the physical significance that its integral around any closed path is equal to the magnetic flux passing through that path.