1.4 BLOCK DIAGRAMS

Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form.

Basic Elements of Block Diagram

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following fig 1.4.1 to identify these elements.
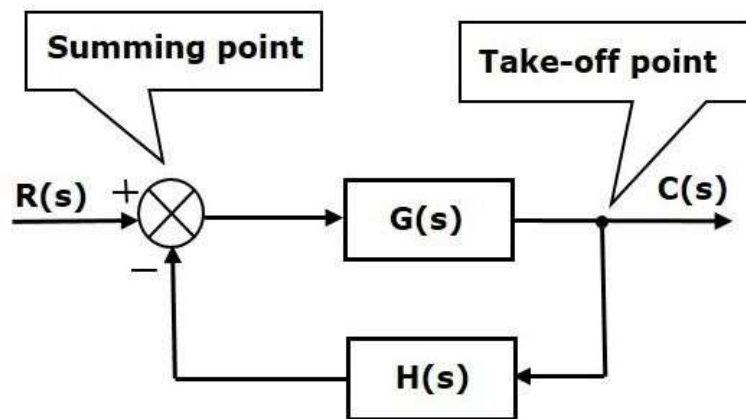

Figure 1.4.1: Basic elements of Block diagram

[Source: "Control System Engineering" by Nagoor Kani, page : 1.50]

The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

Block

The transfer function of a component is represented by a block. Block has single input and single output. The following fig 1.4.2 shows a block having input X(s), output Y(s) and the transfer function G(s).

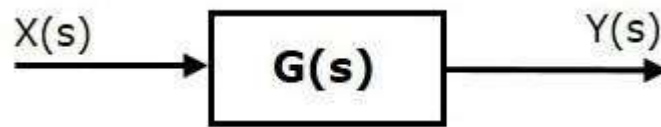**Figure 1.4.2: Open loop transfer function**

[Source: "Control System Engineering" by Nagoor Kani, page : 1.49]

Transfer Function, G(s)=Y(s) / X(s)

$$\Rightarrow Y(s)=G(s)X(s)$$

Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B.**

i.e.,Y = A + B.

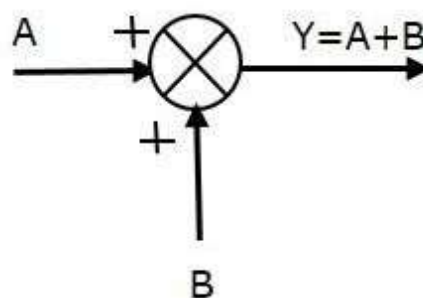The following fig 1.4.3& fig 1.14 shows the summing point with two inputs (A, B) and one output (Y).

Figure 1.4.3: Summing point with two inputs

[Source: "Control System Engineering" by Nagoor Kani, page : 1.50]

Here, the inputs A and B are having opposite signs, i.e., A is having positive sign and B is having negative sign. So, the summing point produces the output **Y** as the **difference of A and B**.
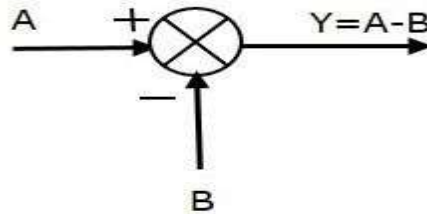
$$Y = A + (-B) = A - B.$$

Figure 1.4.4: Summing point with two inputs (difference of A & B)

[Source: "Control System Engineering" by Nagoor Kani, page : 1.50]

The following fig 1.4.5 shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as
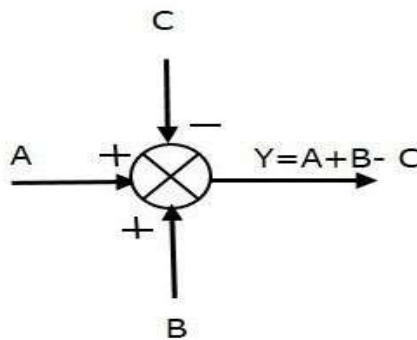
$$Y = A + B + (-C) = A + B - C.$$

Figure 1.4.5: Summing point with three inputs

[Source: "Control System Engineering" by Nagoor Kani, page: 1.50]

Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.

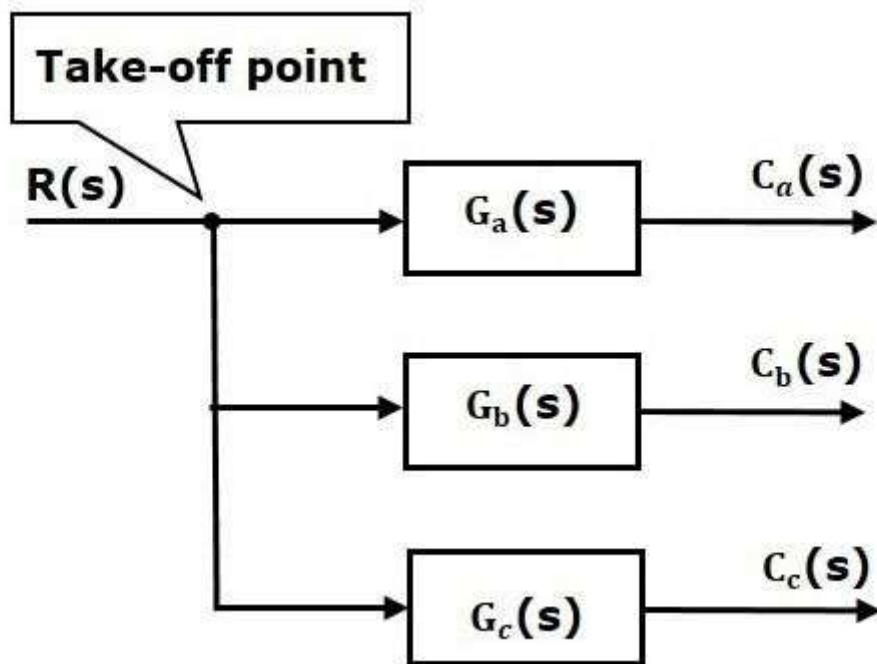In the following fig 1.4.6 shows the take-off point is used to connect the same input, R(s) to two more blocks.

Figure 1.4.6: Take-off point from same input

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

In the following fig 1.4.7 , the take-off point is used to connect the output C(s), as one of the inputs to the summing point.
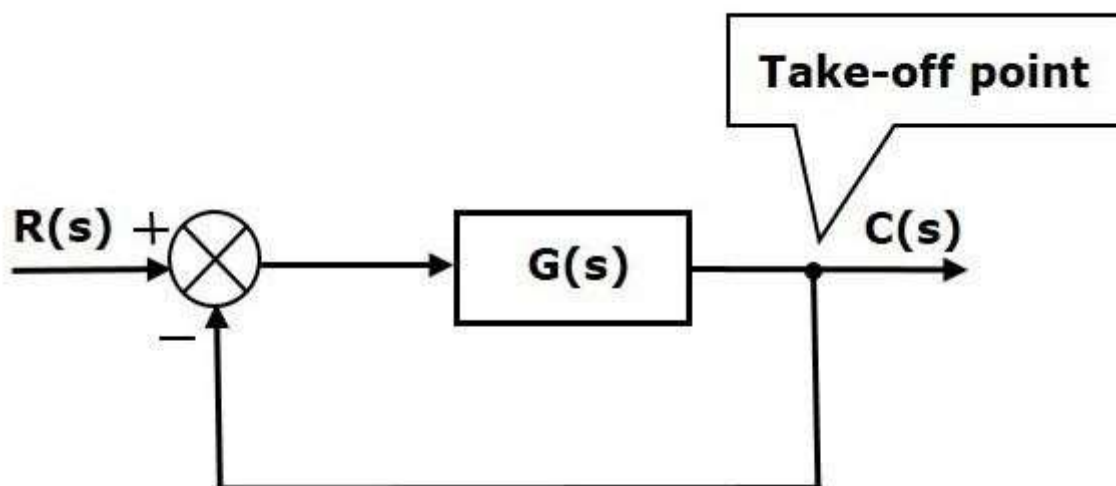

Figure 1.4.7: Take-off point from output

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

Block Diagram Representation of Electrical Systems

In this section, let us represent an electrical system with a block diagram. Electrical systems contain mainly three basic elements — **resistor, inductor and capacitor**.

Consider a series of RLC circuit as shown in the fig 1.4.8. Where, $V_i(t)$ and $V_o(t)$ are the input and output voltages. Let i(t) be the current passing through the circuit. This circuit is in time domain.
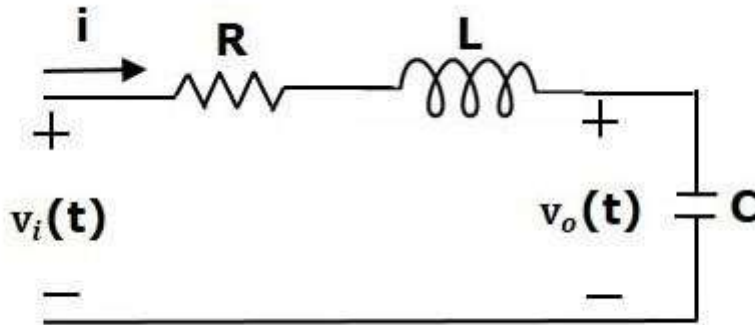

Figure 1.4.8: RLC Series Circuit

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

By applying the Laplace transform to this circuit, will get the circuit in s-domain. The circuit is as shown in the following fig 1.4.9.
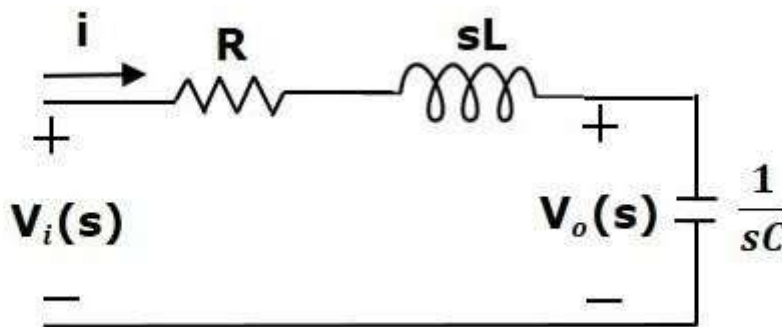

Figure 1.4.9: RLC Series Circuit with s-domain

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

From the above circuit, we can write

$$(\,)\dfrac{(\,)\quad(\,)}{}$$

$$\Rightarrow I(s) = \{1/(R+sL)\}\ \{V_i(s) - V_o(s)\}\ \textbf{(Equation 1)}$$
$$V_o(s) = (1/sC)I(s)\ \textbf{(Equation 2)}$$

Let us now draw the block diagrams for these two equations individually. And then combine those block diagrams properly in order to get the overall block diagram of series of RLC Circuit (s-domain).

Equation 1 can be implemented with a block having the transfer function, 1/R+sL. The input and output of this block are $\{V_i(s) - V_o(s)\}$ and $I(s)$. We require a summing point to get $\{V_i(s) - V_o(s)\}$. The block diagram of Equation 1 is shown in fig 1.4.10.
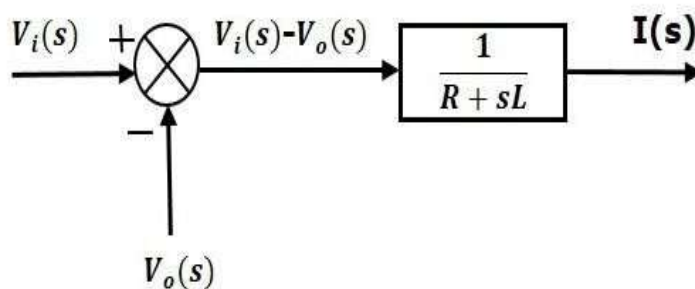

Figure 1.4.10: block diagram of equation1

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

Equation 2 can be implemented with a block having transfer function, 1/sC. The input and output of this block are $I(s)$ and $V_o(s)$. The block diagram of Equation 2 is shown in the fig 1.4.11.
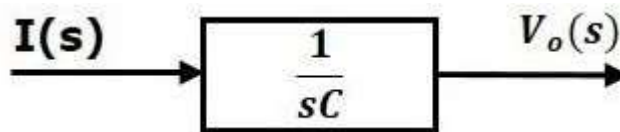

Figure 1.4.11: block diagram of equation2

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

The overall block diagram of the series of RLC Circuit (s-domain) is shown in fig 1.4.12.
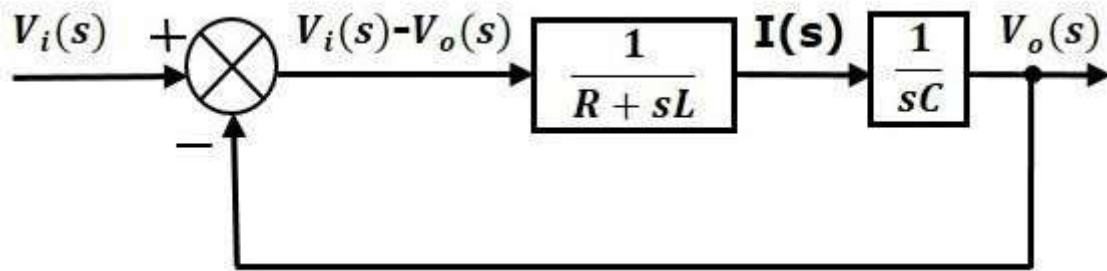
Figure 1.4.12: block diagram of RLC series circuit

[Source: "Control System Engineering" by Nagoor Kani, page: 1.51]

Similarly, you can draw the **block diagram** of any electrical circuit or system just by following this simple procedure.

- Convert the time domain electrical circuit into an s-domain electrical circuit by applying Laplace transform.

- Write down the equations for the current passing through all series branch elements and voltage across all shunt branches

- Draw the block diagrams for all the above equations individually.

- Combine all these block diagrams properly in order to get the overall block diagram of the electrical circuit (s-domain).

Block diagram algebra

Block diagram algebra is nothing but the algebra involved with the basic elements of the block diagram. This algebra deals with the pictorial representation of algebraic equations.

Basic Connections for Blocks

There are three basic types of connections between two blocks.

Series Connection

Series connection is also called **cascade connection**. In the following fig 1.4.13, two blocks having transfer functions $G_1(s)$ and $G_2(s)$ are connected in series.
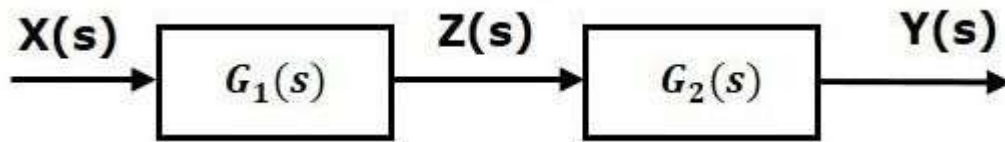
Figure 1.4.13: block diagram of Series connection of two blocks

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

For this combination, we will get the output $Y(s)Y(s)$ as

$$Y(s)=G_2(s)Z(s)$$

Where, $Z(s)=G_1(s)X(s)$

$$\Rightarrow Y(s)=G_2(s)[G_1(s)X(s)]=G_1(s)G_2(s)X(s)$$
$$\Rightarrow Y(s)=\{G_1(s)G_2(s)\}X(s)$$

Compare this equation with the standard form of the output equation, $Y(s)=G(s)X(s)$.

Where, $G(s)=G_1(s)G_2(s)$.

That means we can represent the **series connection** of two blocks with a single block. The transfer function of this single block is the **product of the transfer functions** of those two blocks. The equivalent block diagram is shown in fig 1.4.14.
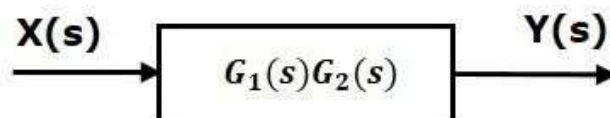

Figure 1.4.14: block diagram of Series connection of two block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

Similarly, you can represent series connection of 'n' blocks with a single block. The transfer function of this single block is the product of the transfer functions of all those 'n' blocks.

Parallel Connection

The blocks which are connected in **parallel** will have the **same input**. In the following fig 1.25, two blocks having transfer functions G1(s) and G2(s) are connected in parallel. The outputs of these two blocks are connected to the summing point.
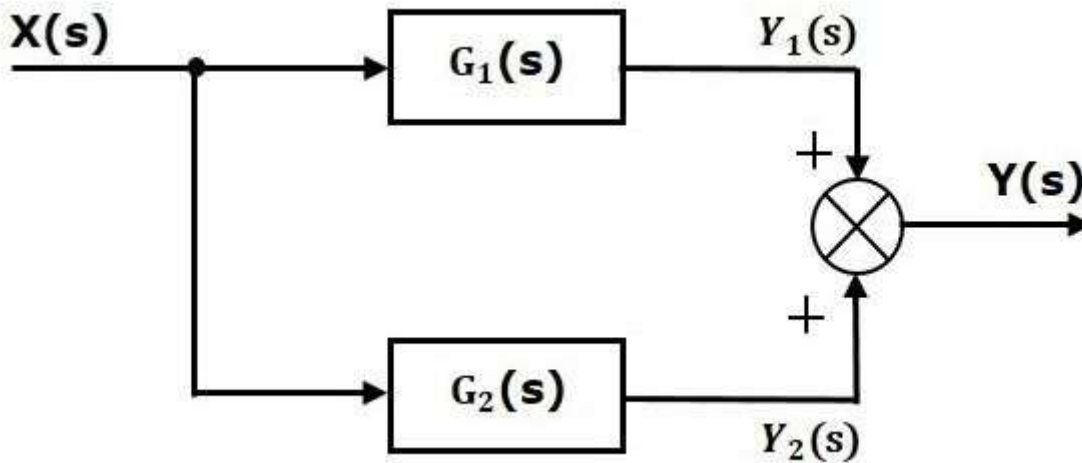

Figure 1.4.15: block diagram of parallel connection of two blocks

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

For this combination, we will get the output Y(s)Y(s) as

$$Y(s)=Y1(s)+Y2(s)$$

Where, Y1(s)=G1(s)X(s) and Y2(s)=G2(s)X(s)

$$\Rightarrow Y(s)=G1(s)X(s)+G2(s)X(s)=\{G1(s)+G2(s)\}$$

Compare this equation with the standard form of the output equation, Y(s)=G(s)X(s).

Where, G(s)=G1(s)+G2(s).

That means we can represent the **parallel connection** of two blocks with a single block. The transfer function of this single block is the **sum of the transfer functions** of those two blocks. The equivalent block diagram is shown below.
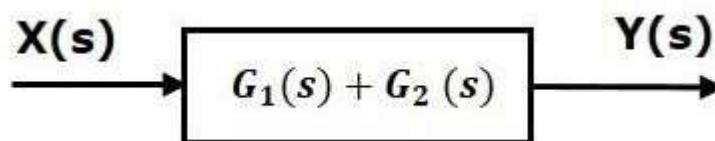

Figure 1.4.16: block diagram of parallel connection of two block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.5]

Similarly, you can represent parallel connection of 'n' blocks with a single block. The transfer function of this single block is the algebraic sum of the transfer functions of all those 'n' blocks.

Feedback Connection

As we discussed in previous chapters, there are two types of **feedback** — positive feedback and negative feedback. The following figure 1.4.17 shows negative feedback control system. Here, two blocks having transfer functions G(s) and H(s) form a closed loop.

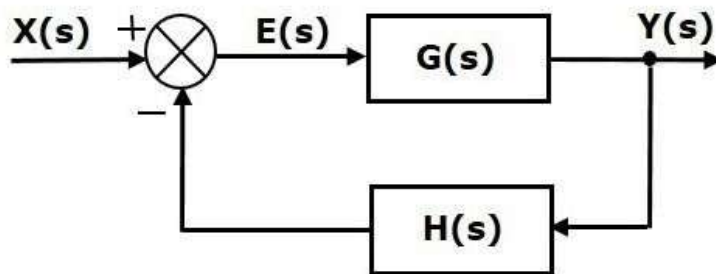**Figure 1.4.17: block diagram of negative feedback system**

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

The output of the summing point is -

$$E(s)=X(s)-H(s)Y(s)$$

The output Y(s)Y(s) is -

$$Y(s)=E(s)G(s)$$

Substitute E(s) value in the above equation.

$$Y(s)=\{X(s)-H(s)Y(s)\}G(s)\}$$

$$Y(s)\{1+G(s)H(s)\}=X(s)G(s)\}$$

$$\Rightarrow Y(s)/X(s) =G(s)/(1+G(s)H(s))$$

Therefore, the negative feedback closed loop transfer function is G(s) /(1+G(s)H(s))

This means we can represent the negative feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the negative feedback. The equivalent block diagram is shown in figure 1.4.18.
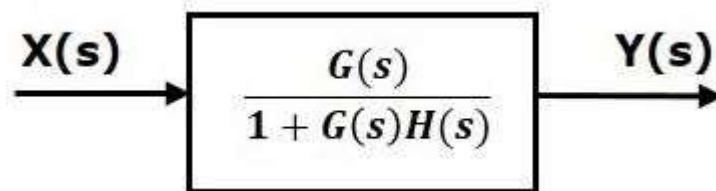

Figure 1.4.18: equivalent block diagram of negative feedback system

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

Similarly, you can represent the positive feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the positive feedback, i.e., $G(s)/(1-G(s)H(s))$

Block Diagram Algebra for Summing Points

There are two possibilities of shifting summing points with respect to blocks −

- Shifting summing point after the block

- Shifting summing point before the block

Let us now see what kind of arrangements need to be done in the above two cases one by one.

Shifting Summing Point After the Block

Consider the block diagram shown in the following figure 1.29. Here, the summing point is present before the block.
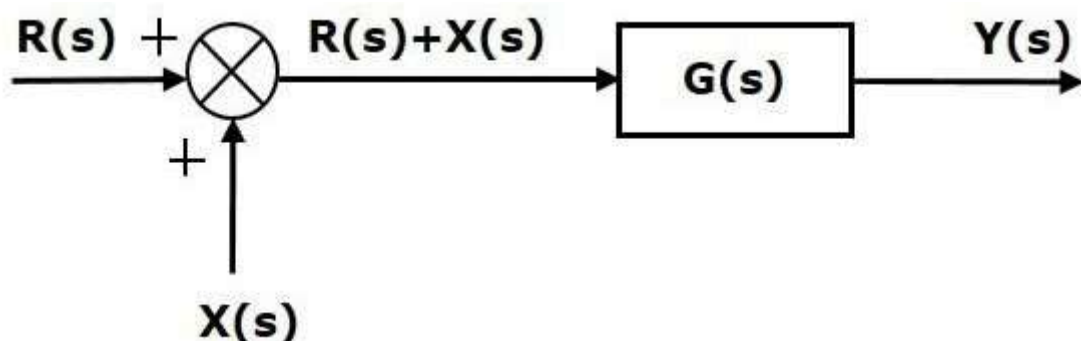
Figure 1.4.19: shifting summing point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

Summing point has two inputs R(s) and X(s). The output of it is {R(s)+X(s)}.

So, the input to the block G(s) is {R(s)+X(s)} and the output of it is –

$$Y(s)=G(s)\{R(s)+X(s)\}$$

$$\Rightarrow Y(s)=G(s)R(s)+G(s)X(s) \text{ (Equation 1)}$$

Now, shift the summing point after the block. This block diagram is shown in the following figure 1.4.20.
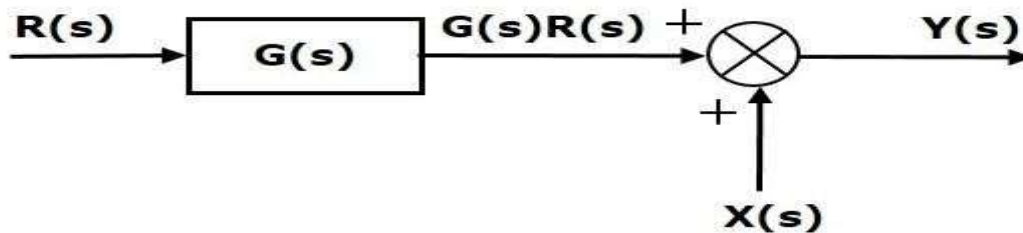

Figure 1.4.20: shifting summing point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.52]

Output of the block G(s) is G(s)R(s).

The output of the summing point is

Y(s)=G(s)R(s)+X(s)**(Equation 2)**

Compare Equation 1 and Equation 2.

The first term 'G(s)R(s)′ is same in both the equations. But, there is difference in the second term. In order to get the second term also same, we require one more block G(s). It is having the input X(s) and the output of this block is given as input to summing point instead of X(s). This block diagram is shown in the following figure 1.4.21.
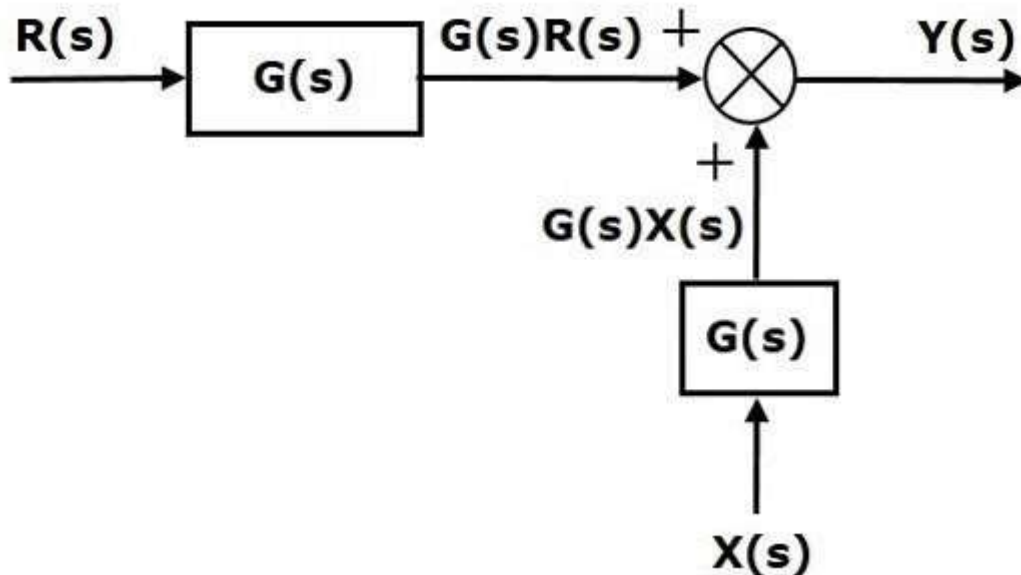
Figure 1.4.21: shifting summing point after the block(equation1 and equation2)

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Shifting Summing Point before the Block

Consider the block diagram shown in the following figure 1.4.22. Here, the summing point is present after the block.
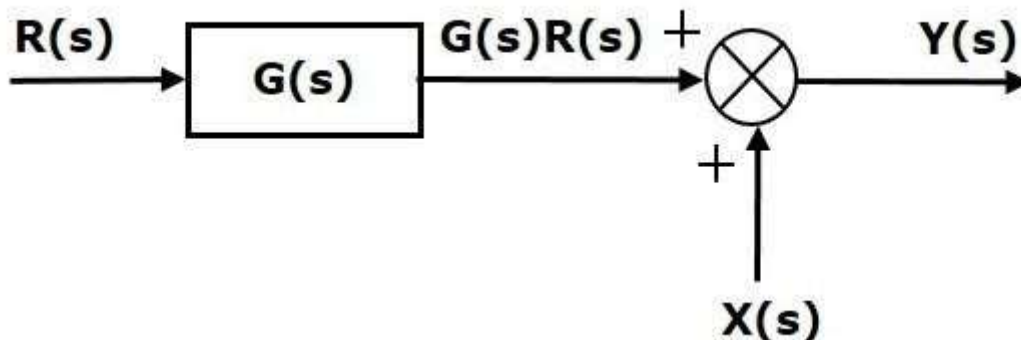

Figure 1.4.22: shifting summing point before the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Output of this block diagram is -

Y(s)=G(s)R(s)+X(s) (**Equation 3)**

Now, shift the summing point before the block. This block diagram is shown in the following figure 1.4.23.
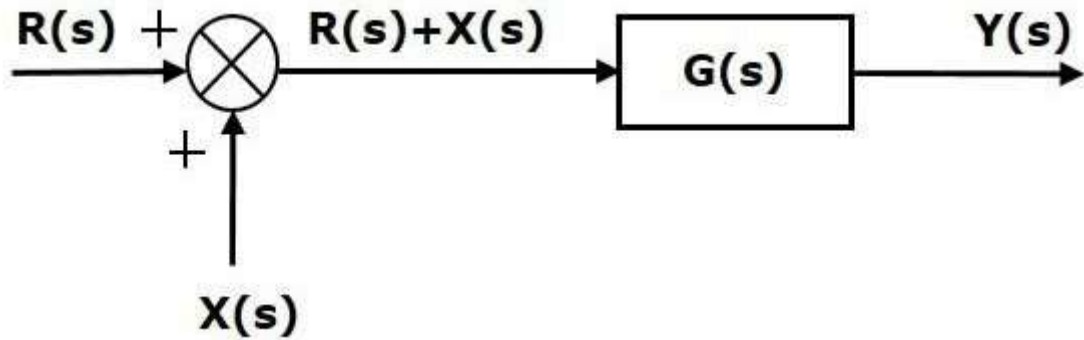
Figure 1.4.23: shifting summing point after the block (equation 3)

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Output of this block diagram is -

$$Y(S) = G(s)R(s) + G(s)X(s) \textbf{ (Equation 4)}$$

Compare Equation 3 and Equation 4,

The first term 'G(s)R(s)' is same in both equations. But, there is difference in the second term. In order to get the second term also same, we require one more block 1/G(s). It is having the input X(s) and the output of this block is given as input to summing point instead of X(s). This block diagram is shown in the following figure 1.4.24.
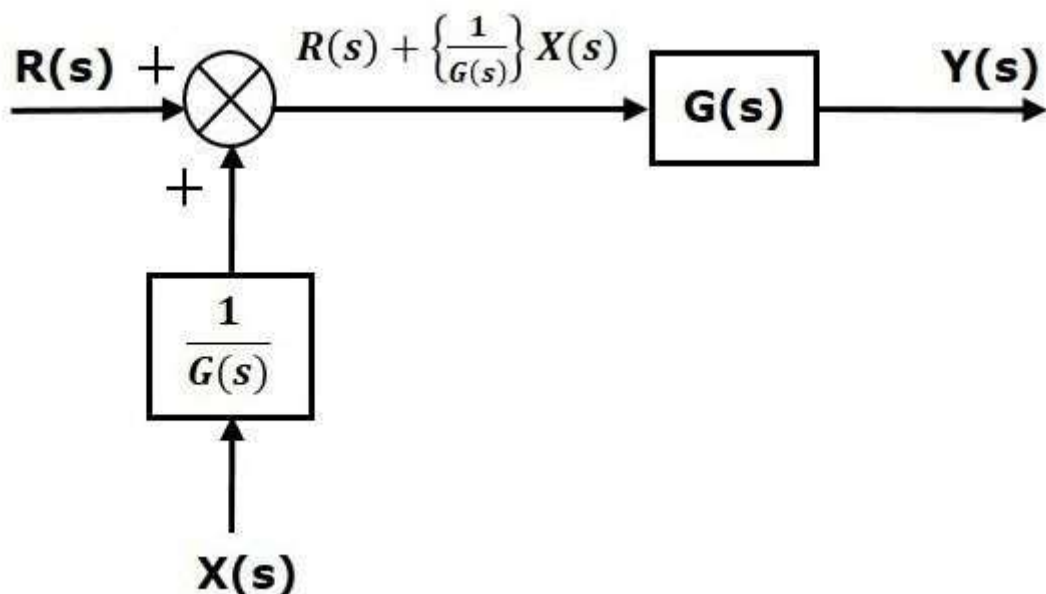

Figure 1.4.24: output of shifting summing point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Block Diagram Algebra for Take-off Points

There are two possibilities of shifting the take-off points with respect to blocks −

- Shifting take-off point after the block

- Shifting take-off point before the block

Let us now see what kind of arrangements is to be done in the above two cases, one by one.

Shifting Take-off Point after the Block

Consider the block diagram shown in the following figure 1.4.25. In this case, the take-off point is present before the block.

Figure 1.4.25: shifting take off point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Here, $X(s)=R(s)$ and $Y(s)=G(s)R(s)$

When you shift the take-off point after the block, the output $Y(s)Y(s)$ will be same. But, there is difference in $X(s)$ value. So, in order to get the same $X(s)$ value, we require one more block $1/G(s)$. It is having the input $Y(s)$ and the output is $X(s)$. This block diagram is shown in the following figure 1.4.26.

Figure 1.4.26: shifting take off point after the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Shifting Take-off Point Before the Block

Consider the block diagram shown in the following figure 1.4.27. Here, the take-off point is present after the block.
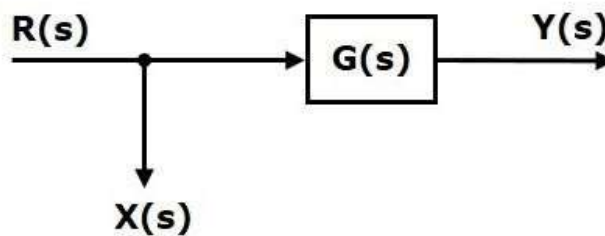

Figure 1.4.27: shifting take off point before the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Here, $X(s) = Y(s) = G(s)R(s)$

When you shift the take-off point before the block, the output $Y(s)$ will be same. But, there is difference in $X(s)X(s)$ value. So, in order to get same $X(s)$ value, we require one more block $G(s)$. It is having the input $R(s)R(s)$ and the output is $X(s)X(s)$. This block diagram is shown in the following figure 1.4.28.
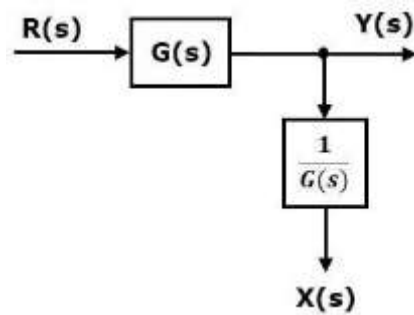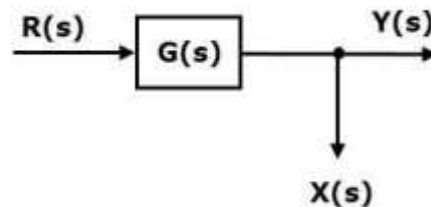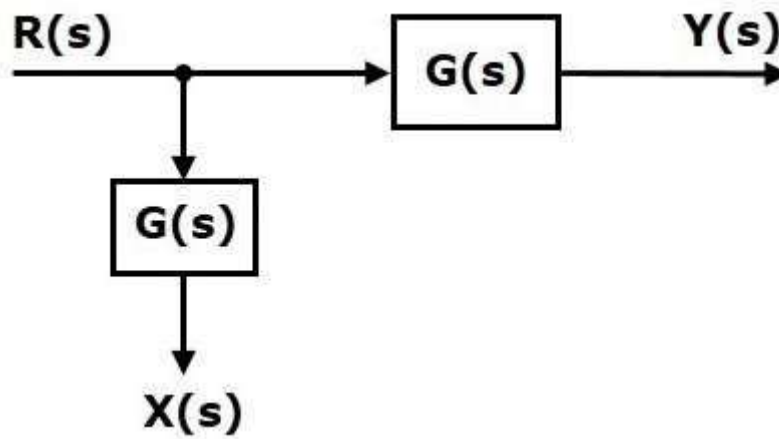
Figure 1.4.28: shifting take off point before the block

[Source: "Control System Engineering" by Nagoor Kani, page: 1.53]

Block Diagram Reduction

Block Diagram Reduction Rules

Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

- **Rule 1** − Check for the blocks connected in series and simplify.

- **Rule 2** − Check for the blocks connected in parallel and simplify.

- **Rule 3** − Check for the blocks connected in feedback loop and simplify.

- **Rule 4** − If there is difficulty with take-off point while simplifying, shift it towards right.

- **Rule 5** − If there is difficulty with summing point while simplifying, shift it towards left.

- **Rule 6** − Repeat the above steps till you get the simplified form, i.e., single block.

Note − the transfer function present in this single block is the transfer function of the overall block diagram.

Example

Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.
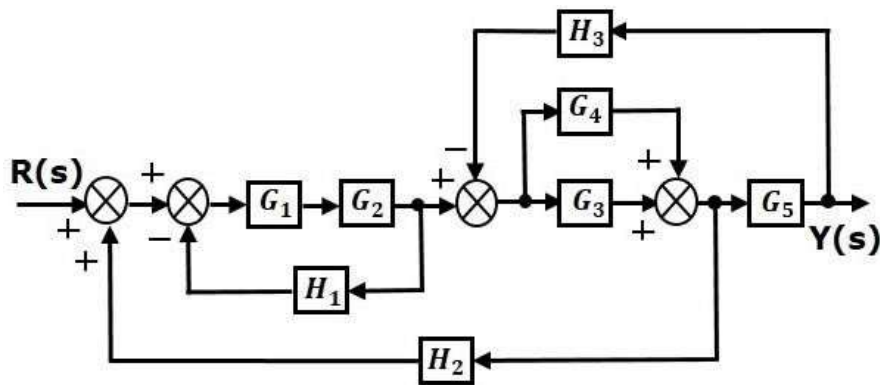
Step 1 − Use Rule 1 for blocks G1 and G2. Use Rule 2 for blocks G3 and G4. The modified block diagram is shown in the following figure.
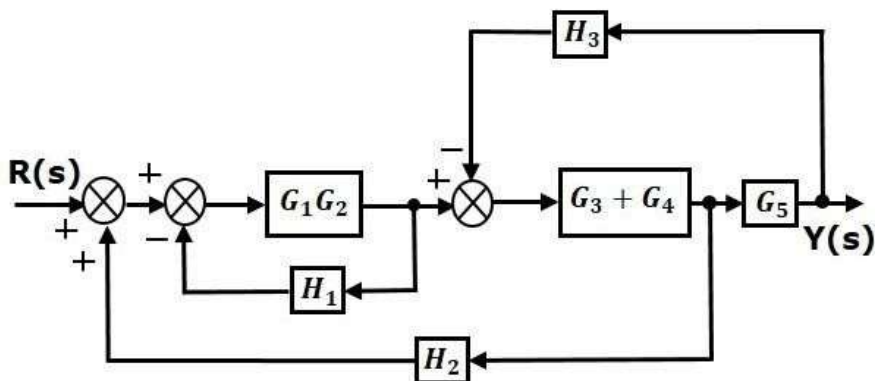
Step 2 − Use Rule 3 for blocks G1G2 and H1. Use Rule 4 for shifting take-off point after the block G5. The modified block diagram is shown in the following figure.
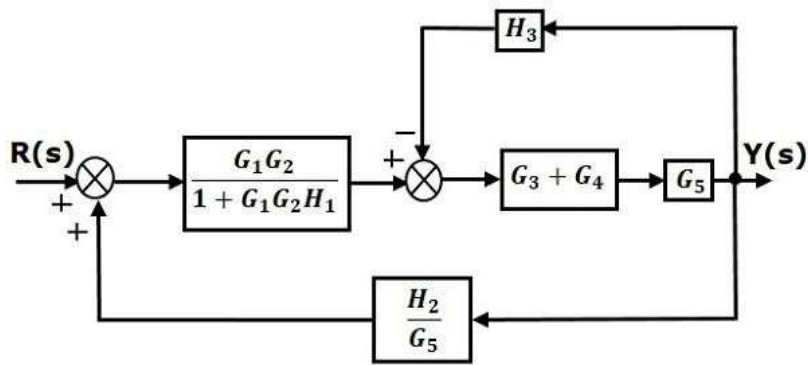
Step 3 − Use Rule 1 for blocks (G3+G4) and G5. The modified block diagram is shown in the following figure.
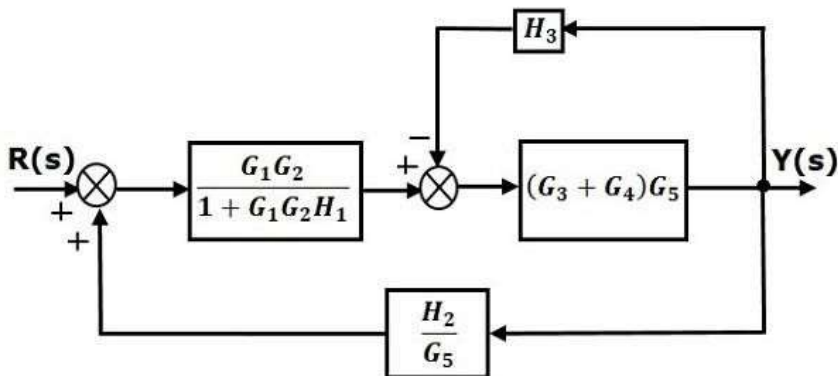
Step 4 − Use Rule 3 for blocks (G3+G4)G5 and H3. The modified block diagram is shown in the following figure.
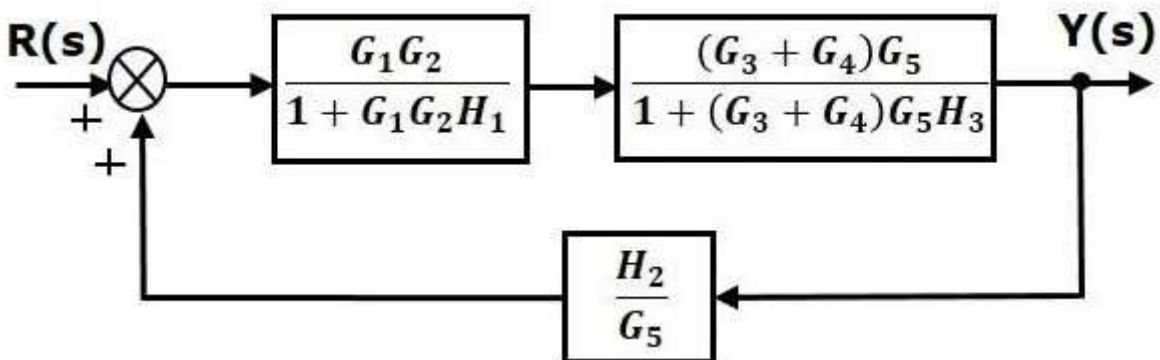
Step 5 − Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.
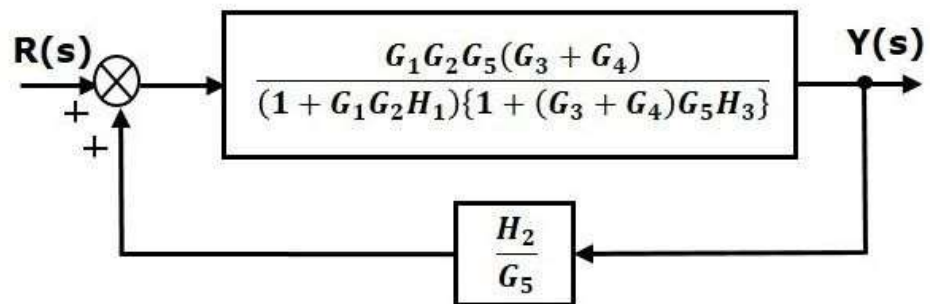
Step 6 − Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.
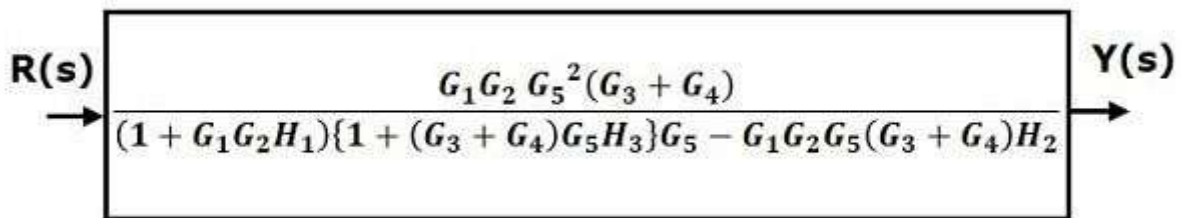
Therefore, the transfer function of the system is

$Y(s) / R(s) = G_1G_2G^2_5(G_3+G_4) / \{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5 - G_1G_2G_5(G_3+G_4)H_2\}$

1.1 CONTROL SYSTEM: TERMINOLOGY AND BASIC STRUCTURE

A control system is a system, which provides the desired response by controlling the output. The following fig 1.1.1 shows the simple block diagram of a control system.

Figure 1.1.1: Blok diagram of Control System
[Source: "Control System Engineering" by Nagoor Kani, page-1.1]

Here, the control system is represented by a single block. Since, the output is controlled by varying input; the control system got this name. We will vary this input with some mechanism. In the next section on open loop and closed loop control systems, we will study in detail about the blocks inside the control system and how to vary this input in order to get the desired response.

System: A combination or arrangement of a number of different physical components to form a whole unit such that that combining unit performs to achieve a certain goal.

Control: The action to command, direct or regulate a system.

Plant or process: The part or component of a system that is required to be controlled.

Input: It is the signal or excitation supplied to a control system.

Output: It is the actual response obtained from the control system.

Controller: The part or component of a system that controls the plant.

Disturbances: The signal that has adverse effect on the performance of a control system.

Control system: A system that can command, direct or regulate itself or another system to achieve a certain goal.

Automation: The control of a process by automatic means

Control System: An interconnection of components forming a system configuration that will provide a desired response.

Actuator: It is the device that causes the process to provide the output. It is the device that provides the motive power to the process.

Design: The process of conceiving or inventing the forms, parts, and details of system to achieve a specified purpose.

Simulation: A model of a system that is used to investigate the behaviour of a system by utilizing actual input signals.

Optimization: The adjustment of the parameters to achieve the most favourable or advantageous design.

Feedback Signal: A measure of the output of the system used for feedback to control the system.

Negative feedback: The output signal is feedback so that it subtracts from the input signal.

Block diagrams: Unidirectional, operational blocks that represent the transfer functions of the elements of the system.

Signal Flow Graph (SFG): A diagram that consists of nodes connected by several directed branches and that is a graphical representation of a set of linear relations.

Specifications: Statements that explicitly state what the device or product is to be and to do. It is also defined as a set of prescribed performance criteria.

Open-loop control system: A system that utilizes a device to control the process without using feedback. Thus the output has no effect upon the signal to the process.

Closed-loop feedback control system: A system that uses a measurement of the output and compares it with the desired output.

Regulator: The control system where the desired values of the controlled outputs are more or less fixed and the main problem is to reject disturbance effects.

Servo system: The control system where the outputs are mechanical quantities like acceleration, velocity or position.

Stability: It is a notion that describes whether the system will be able to follow the input command. In a non-rigorous sense, a system is said to be unstable if its output is out of control or increases without bound.

Multivariable Control System: A system with more than one input variable or more than one output variable.

Trade-off: The result of making a judgment about how much compromise must be made between conflicting criteria.

Examples − Traffic lights control system, washing machine

1.6 DC and AC SERVO SYSTEMS

An AC servo motor is essentially a two phase induction motor with modified constructional features to suit servo applications.

- The schematic of a two phase or servo motor is shown fig 1.6.1
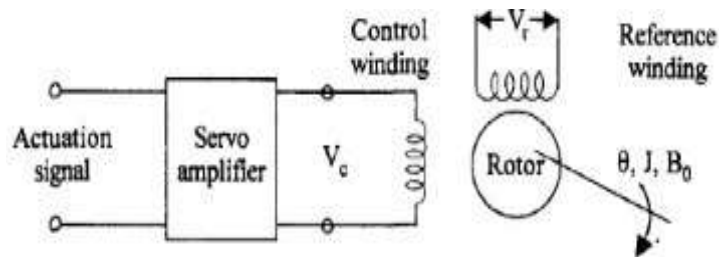
Figure 1.6.1: Two phase servo motor

[Source: "Control System Engineering" by Nagoor Kani, page: 1.22]

- It has two windings displaced by 90° on the stator One winding, called as reference winding, is supplied with a constant sinusoidal voltage.

- The second winding, called control winding, is supplied with a variable control voltage which is displaced by -- 90° out of phase from the reference voltage.

- The major differences between the normal induction motor and an AC servo motor are

- The rotor winding of an ac servo motor has high resistance (R) compared to its inductive reactance (X) so that its X / R ratio is very low.

- For a normal induction motor, X / R ratio is high so that the maximum torque is obtained in normal operating region which is around 5% of slip.

- The torque speed characteristics of a normal induction motor and an ac servo motor are shown in fig 1.6.2
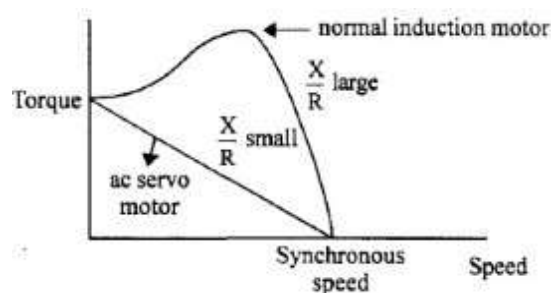
Figure 1.6.2: Torque speed Characteristics of AC servo Motor

[Source: "Control System Engineering" by Nagoor Kani, page: 1.22]

- The Torque speed characteristic of a normal induction motor is highly nonlinear and has a positive slope for some portion of the curve.

- This is not desirable for control applications. As the positive slope makes the systems unstable. The torque speed characteristic of an ac servo motor is fairly linear and has negative slope throughout.

- The rotor construction is usually squirrel cage or drag cup type for an ac servo motor.

- The diameter is small compared to the length of the rotor which reduces inertia of the moving parts.

- Thus it has good accelerating characteristic and good dynamic response

- The supplies to the two windings of ac servo motor are not balanced as in the case of a normal induction motor.

- The control voltage varies both in magnitude and phase with respect to the constant reference vulture applied to the reference winding.

- The direction of rotation of the motor depends on the phase ($\pm$ 90°) of the control voltage with respect to the reference voltage.

- For different RMS values of control voltage the torque speed characteristics are shown in Fig.1.6.3
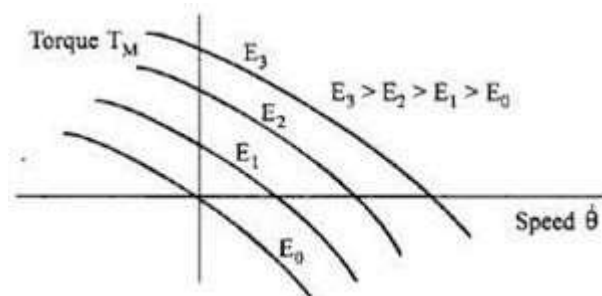

Figure 1.6.3: different RMS control of AC servo Motor

[Source: "Control System Engineering" by Nagoor Kani, page: 1.22]

- The torque varies approximately linearly with respect to speed and also controls voltage. The torque speed characteristics can be linearized at the operating

point and the transfer function of the motor can be obtained

DC Servo Motor

- A DC servo motor is used as an actuator to drive a load. It is usually a DC motor of low power rating.

- DC servo motors have a high ratio of starting torque to inertia and therefore they have a faster dynamic response.

- DC motors are constructed using rare earth permanent magnets which have high residual flux density and high coercively.

- As no field winding is used, the field copper losses am zero and hence, the overall efficiency of the motor is high.

- The speed torque characteristic of this motor is flat over a wide range, as the armature reaction is negligible.

- Moreover speed in directly proportional to the armature voltage for a given torque.

- Armature of a DC servo motor is specially designed to have low inertia.

- In some application DC servo motors are used with magnetic flux produced by field windings.

- The speed of PMDC motors can be controlled by applying variable armature voltage. These are called armature voltage controlled DC servo motors.

- Wound field DC motors can be controlled by either controlling the armature voltage or controlling rho field current. Let us now consider modelling of these two types or DC servo motors.

(a) Armature controlled DC servo motor

- The physical model of an armature controlled DC servo motor is given in fig 1.6.4
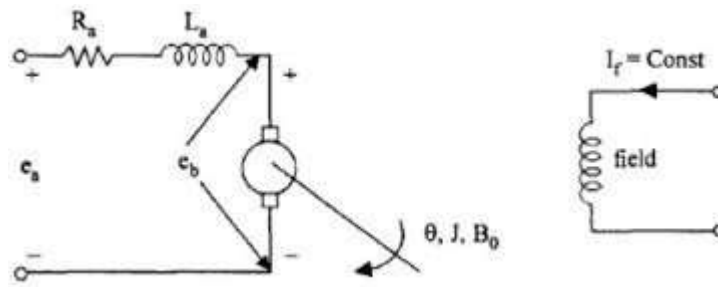
Figure 1.6.4: Armature Controlled DC Servo motor

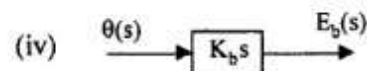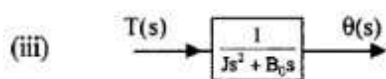*[Source: "Control System Engineering" by Nagoor Kani, page: 1.23]*

- The armature winding has a resistance R_a and inductance L_a.

- The field is produced either by a permanent magnet or the field winding is separately excited and supplied with constant voltage so that the field current If is a constant.

- When the armature is supplied with a DC voltage of e a volts, the armature rotates and produces a back e.m.f e_b.

- The armature current ia depends on the difference of eb and en. The armature has a permanent of inertia J, frictional coefficient B0. The angular displacement of the motor is 8.

- The torque produced by the motor is given by

  Where K_T is the motor torque constant.

- The back emf is proportional to the speed of the motor and hence

- we get

$$\theta(s) = \frac{K_T\, E_a(s)}{s[(R_a + sL_a)(Js + B_0) + K_T\, K_b]}$$

- The block diagram representation of the armature controlled DC servo motor is developed in Steps
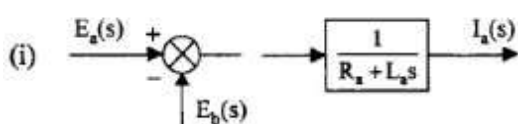
Combining these blocks in fig 1.6.5
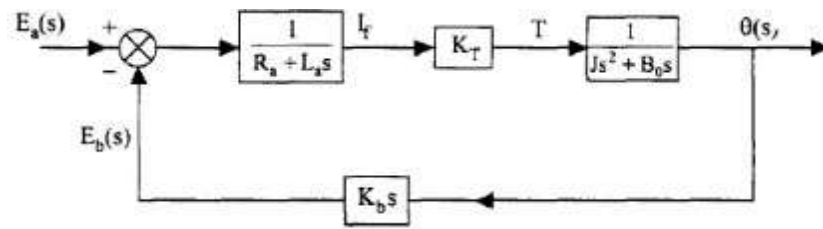
Figure 1.6.5: Block diagram of Armature Controlled DC Servo motor

[Source: "Control System Engineering" by Nagoor Kani, page: 1.23]

Field Controlled Dc Servo Motor

- The speed of a DC motor is directly proportional to armature voltage and inversely proportional to flux.

- In the field controlled DC motor the armature voltage is kept Constant and the speed is varied by varying the flux of machine.

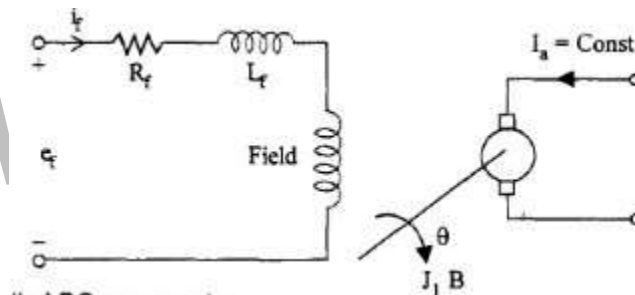- The field controlled DC motor speed control System is shown in fig 1.6.6

Figure 1.6.6: Field Controlled DC motor

[Source: "Control System Engineering" by Nagoor Kani, page: 1.24]

- Let, R_f = field resistance

  L_f = Field inductance

  J = moment of inertia of rotor and load; B = frictional coefficient of rotor and load

1.3 ELECTRICAL AND MECHANICAL TRANSFER FUNCTION MODELS

The control systems can be represented with a set of mathematical equations known as mathematical model. These models are useful for analysis and design of control systems. Analysis of control system means finding the output when we know the input and mathematical model. Design of control system means finding the mathematical model when we know the input and the output.

The following mathematical models are mostly used.

- Differential equation model

- Transfer function model

- State space model

Let us discuss the first two models in this chapter.

Differential Equation Model

Differential equation model is a time domain mathematical model of control systems.

Follow these steps for differential equation model.

- Apply basic laws to the given control system.

- Get the differential equation in terms of input and output by eliminating the intermediate variable(s).

Example

Consider the following electrical system as shown in the following figure 1.3.1. This circuit consists of resistor, inductor and capacitor. All these electrical elements are connected in **series**. The input voltage applied to this circuit is v_i and the voltage across the capacitor is the output voltage v_o.
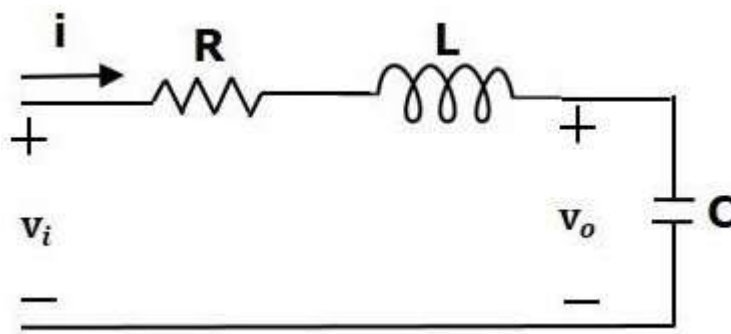

Figure 1.3.1: RLC series electrical system

[Source: "Control System Engineering" by Nagoor Kani, page : 1.20]

Mesh equation for this circuit is

—

Substitute, the current passing through capacitor in the above equation.

⇒   —   —

The above equation is a second order **differential equation**.

Transfer Function Model

Transfer function model is an s-domain mathematical model of control systems.
The **Transfer function** of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.

If x(t) and y(t) are the input and output of an LTI system, then the corresponding Laplace transforms are X(s) and Y(s).

Therefore, the transfer function of LTI system is equal to the ratio of Y(s) and X(s).

i.e.,   —

The transfer function model of an LTI system is shown in the following figure1.3.2.
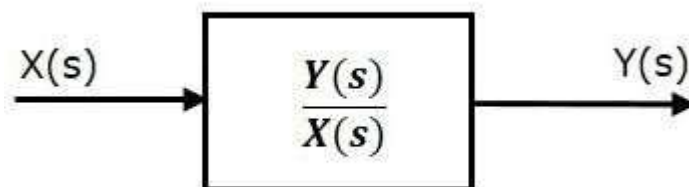

Figure 1.3.2: transfer function model

[Source: "Control System Engineering" by Nagoor Kani, page : 1.5]

Here, we represented an LTI system with a block having transfer function inside it. And this block has an input X(s) & output Y(s).
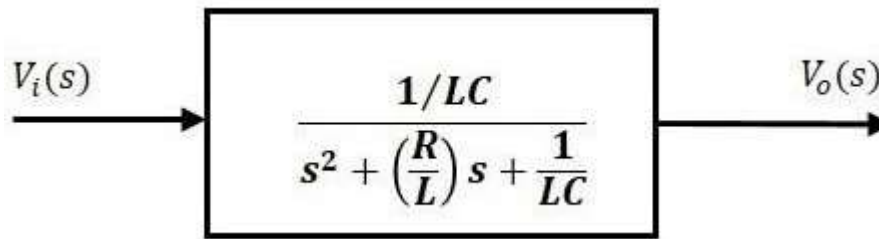
Transfer function of previous example electrical system

$$\frac{1/LC}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$

$V_i(s)$ $V_o(s)$

Figure 1.3.3: transfer function of RLC series electrical system

[Source: "Control System Engineering" by Nagoor Kani, page : 1.20]

Here, we show a second order electrical system with a block having the transfer function inside it. And this block has an input Vi(s) & an output Vo(s).

The differential equation modeling of mechanical systems. There are two types of mechanical systems based on the type of motion.

- Translational mechanical systems
- Rotational mechanical systems

Modeling of Translational Mechanical Systems

Translational mechanical systems move along a **straight line**. These systems mainly consist of three basic elements. Those are mass, spring and dashpot or damper.

If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system. Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero. Let us now see the force opposed by these three elements individually.

Mass

Mass is the property of a body, which stores **kinetic energy**. If a force is applied on a body having mass **M**, then it is opposed by an opposing force due to mass. This opposing force is proportional to the acceleration of the body. Assume elasticity and frictions are negligible.

Figure 1.3.4: block diagram of mass

[Source: "Control System Engineering" by Nagoor Kani, page: 1.7]

$$F_m \propto a$$

$$\Rightarrow \quad \underline{\qquad}$$

$$F = \quad \underline{\qquad}$$

Where,

- **F** is the applied force
- **F$_m$** is the opposing force due to mass
- **M** is mass
- **a** is acceleration
- **x** is displacement

Spring

Spring is an element, which stores **potential energy**. If a force is applied on spring **K**, then it is opposed by an opposing force due to elasticity of spring. This opposing force is proportional to the displacement of the spring. Assume mass and friction are negligible.
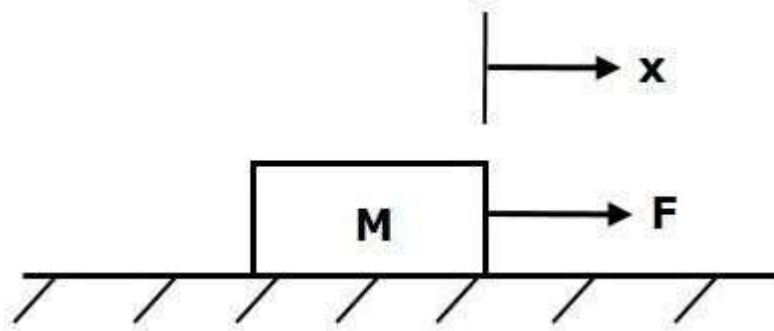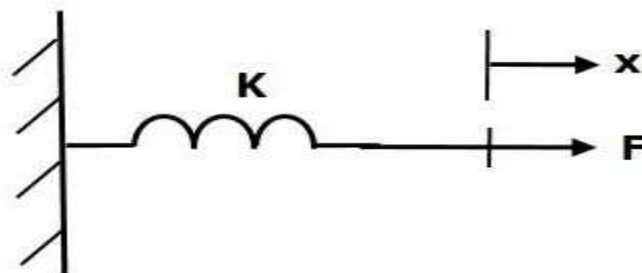
Figure 1.3.5: block diagram of spring

[Source: "Control System Engineering" by Nagoor Kani, page: 1.7]

$$F \propto x$$
$$\Rightarrow F_k = Kx$$
$$F = F_k = Kx$$

Where,

- **F** is the applied force

- **F_k** is the opposing force due to elasticity of spring

- **K** is spring constant

- **x** is displacement

Dashpot

If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot. This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible.
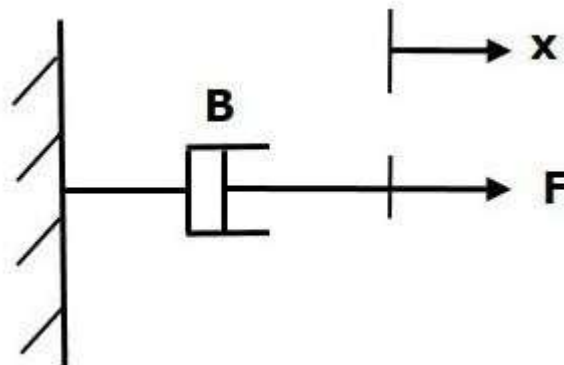

Figure 1.3.6: block diagram of friction

[Source: "Control System Engineering" by Nagoor Kani, page: 1.7]

$$F_b \propto v$$
$$\Rightarrow F_b = Bv = B \, dx/dt$$
$$F = F_b = B \, dx/dt$$

Where,

- **F_b** is the opposing force due to friction of dashpot

- **B** is the frictional coefficient

- **v** is velocity
- **x** is displacement

Modeling of Rotational Mechanical Systems

Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. Those are **moment of inertia, torsional spring** and **dashpot**. If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system. Since the applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero. Let us now see the torque opposed by these three elements individually.

Moment of Inertia

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.

If a torque is applied on a body having moment of inertia **J**, then it is opposed by an opposing torque due to the moment of inertia. This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.

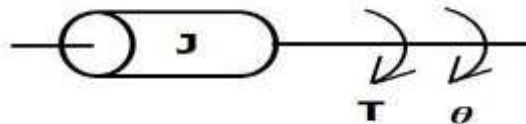**Figure 1.3.7: block diagram of moment of inertia**

[Source: "Control System Engineering" by Nagoor Kani, page: 1.15]

$$T_j \propto \alpha$$
$$\Rightarrow T_j = J\alpha = J\ d^2\theta/dt^2$$
$$T = T_j = J\ d^2\theta\ /\ dt^2$$

Where,

- **T** is the applied torque
- **T$_j$** is the opposing torque due to moment of inertia
- **J** is moment of inertia
- **α** is angular acceleration

- **θ** is angular displacement

Torsional spring

In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores **potential energy**.

If a torque is applied on torsional spring **K**, then it is opposed by an opposing torque due to the elasticity of torsional spring. This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.
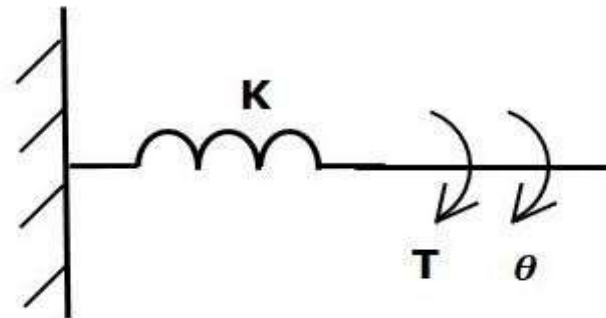

Figure 1.3.8: block diagram of torsional spring

[Source: "Control System Engineering" by Nagoor Kani, page: 1.15]

$$T_k \propto \theta$$
$$\Rightarrow T_k = K\theta$$
$$T = T_k = K\theta$$

Where,

- **T** is the applied torque
- **T_k** is the opposing torque due to elasticity of torsional spring
- **K** is the torsional spring constant
- **θ** is angular displacement

Dashpot

If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot. This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.
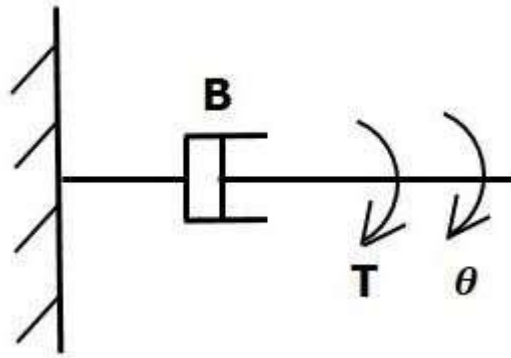
Figure 1.3.9: block diagram of dashpot

[Source: "Control System Engineering" by Nagoor Kani, page: 1.15]

$$T_b \propto \omega$$
$$\Rightarrow T_b = B\omega = B \, d\theta/dt$$
$$T = T_b = B \, d\theta/dt$$

Where,

- T_b is the opposing torque due to the rotational friction of the dashpot
- **B** is the rotational friction coefficient
- ω is the angular velocity
- θ is the angular displacement

Electrical Analogies of Mechanical Systems

Two systems are said to be **analogous** to each other if the following two conditions are satisfied.

- The two systems are physically different
- Differential equation modeling of these two systems are same

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force Voltage Analogy

In force voltage analogy, the mathematical equations of **translational mechanical system** are compared with mesh equations of the electrical system.

Consider the following translational mechanical system as shown in the following figure 1.3.10.
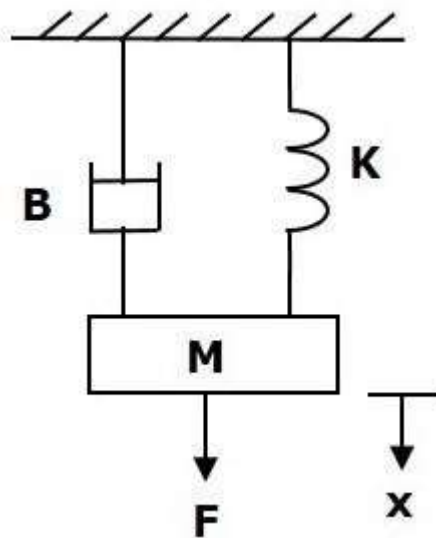
Figure 1.3.10: block diagram mechanical system

[Source: "Control System Engineering" by Nagoor Kani, page: 1.8]

The **force balanced equation** for this system is

$$F = F_m + F_b + F_k$$

$$\Rightarrow F = M\, d^2x/dt^2 + B\, dx/dt + Kx \text{ (\textbf{Equation 1})}$$

Consider the following electrical system as shown in the following figure 1.3.11. This circuit consists of a resistor, an inductor and a capacitor. All these electrical elements are connected in a series. The input voltage applied to this circuit is V volts and the current flowing through the circuit is I Amps.
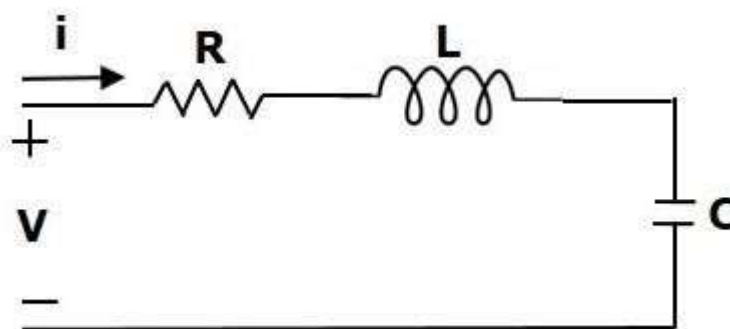

Figure 1.3.11: block diagram of electrical analogy of mechanical system

[Source: "Control System Engineering" by Nagoor Kani, page: 1.8]

Mesh equation for this circuit is

$$V = R_i + L\, di/dt + 1/c \int I\, dt \text{ (\textbf{Equation 2})}$$

Substitute, i=dq/dt in Equation 2.

$$V=R \, dq/dt+L \, d^2q/dt^2+q/C$$
$$\Rightarrow V=L \, d^2q/dt^2 +R \, dq/dt +(1/c)q \text{ (Equation 3)}$$

By comparing Equation 1 and Equation 3, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance (1/c)
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

Similarly, there is torque voltage analogy for rotational mechanical systems. Let us now discuss about this analogy.

Torque Voltage Analogy

In this analogy, the mathematical equations of **rotational mechanical system** are compared with mesh equations of the electrical system.

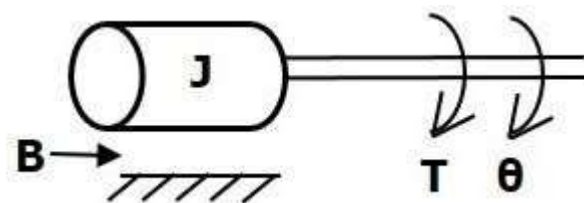Rotational mechanical system is shown in the following figure 1.3.12.

Figure 1.3.12: block diagram of mechanical rotational system

[Source: "Control System Engineering" by Nagoor Kani, page: 1.16]

The torque balanced equation is

$$T = T_j + T_b + T_k$$
$$\Rightarrow T = J\, d^2\theta/dt^2 + B\, d\theta/dt + k\theta \quad \textbf{(Equation 4)}$$

By comparing Equation 4 and Equation 3, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance (1/c)
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Force Current Analogy

In force current analogy, the mathematical equations of the **translational mechanical system** are compared with the nodal equations of the electrical system.

Consider the following electrical system as shown in the following figure 1.3.13. This circuit consists of current source, resistor, inductor and capacitor. All these electrical elements are connected in parallel.
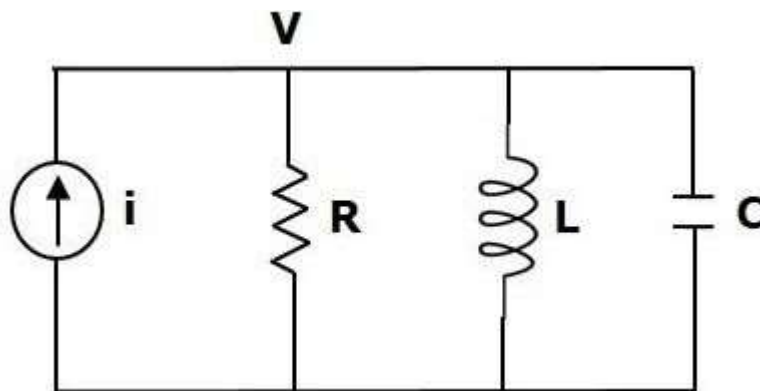

Figure 1.3.13: electrical analogy of mechanical rotational system

The nodal equation is

$$i = V/R + 1/L \int V dt + C \, dV/dt \textbf{(Equation 5)}$$

Substitute, $V = d\Psi/dt$ in Equation 5.

$$i = 1/R \, d\Psi/dt + (1/L)\Psi + C \, d^2\Psi/dt^2$$

$$\Rightarrow i = C \, d^2\Psi/dt^2 + (1/R) \, d\Psi/dt + (1/L)\Psi \textbf{ (Equation 6)}$$

By comparing Equation 1 and Equation 6, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance(1/R)
Spring constant(K)	Reciprocal of Inductance(1/L)
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

Similarly, there is a torque current analogy for rotational mechanical systems. Let us now discuss this analogy.

Torque Current Analogy

In this analogy, the mathematical equations of the **rotational mechanical system** are compared with the nodal mesh equations of the electrical system.

By comparing Equation 4 and Equation 6, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System

Torque(T)	Current(i)
Moment of inertia(J)	Capacitance(C)
Rotational friction coefficient(B)	Reciprocal of Resistance(1/R)
Torsional spring constant(K)	Reciprocal of Inductance(1/L)
Angular displacement(θ)	Magnetic flux(ψ)
Angular velocity(ω)	Voltage(V)

In this chapter, we discussed the electrical analogies of the mechanical systems. These analogies are helpful to study and analyze the non-electrical system like mechanical system from analogous electrical system.

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1.2 FEED FORWARD AND FEEDBACK CONTROL THEORY

Classification of Control Systems

Continuous time and Discrete-time Control Systems

- Control Systems can be classified as continuous time control systems and discrete time control systems based on the type of the signal used.

- In continuous time control systems, all the signals are continuous in time. But, in discrete time control systems, there exists one or more discrete time signals.

SISO and MIMO Control Systems

- Control Systems can be classified as SISO control systems and MIMO control systems based on the number of inputs and outputs present.

- SISO (Single Input and Single Output) control systems have one input and one output. Whereas, MIMO (Multiple Inputs and Multiple Outputs) control systems have more than one input and more than one output.

Open Loop and Closed Loop Control Systems

Control Systems can be classified as open loop control systems and closed loop control systems based on the feedback path.

In open loop control systems, output is not fed-back to the input. So, the control action is independent of the desired output.

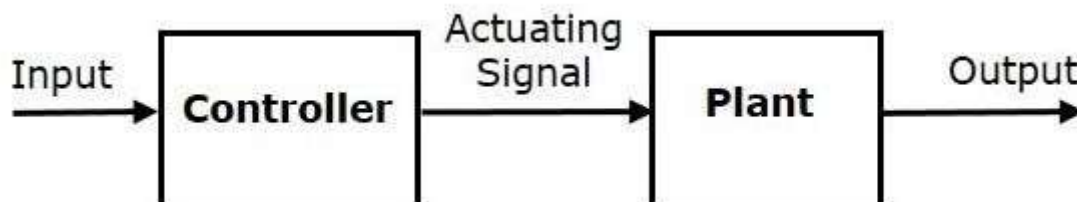The following figure 1.2.1 shows the block diagram of the open loop control system.

Figure 1.2.1: block diagram of open loop system

[Source: "Control System Engineering" by Nagoor Kani, page: 1.1]

Here, an input is applied to a controller and it produces an actuating signal or controlling signal. This signal is given as an input to a plant or process which is to be

controlled. So, the plant produces an output, which is controlled. The traffic lights control system which we discussed earlier is an example of an open loop control system.

In closed loop control systems, output is fed back to the input. So, the control action is dependent on the desired output.

The following figure 1.2.2 shows the block diagram of negative feedback closed loop control system.
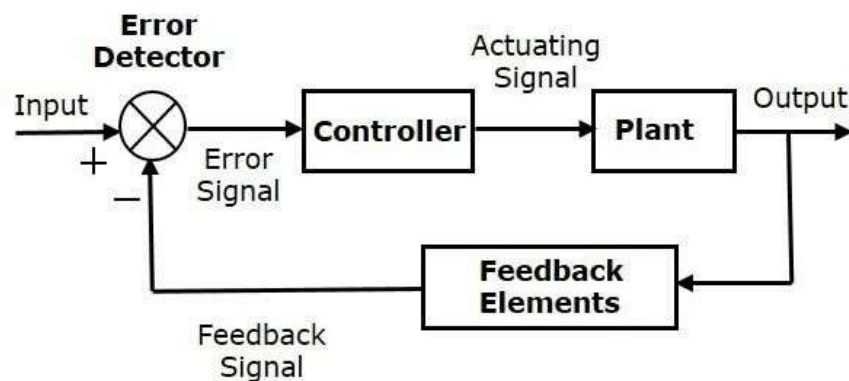

Figure 1.2.2: block diagram of closed loop system

[Source: "Control System Engineering" by Nagoor Kani, page: 1.1]

The error detector produces an error signal, which is the difference between the input and the feedback signal. This feedback signal is obtained from the block (feedback elements) by considering the output of the overall system as an input to this block. Instead of the direct input, the error signal is applied as an input to a controller.

So, the controller produces an actuating signal which controls the plant. In this combination, the output of the control system is adjusted automatically till we get the desired response. Hence, the closed loop control systems are also called the automatic control systems. Traffic lights control system having sensor at the input is an example of a closed loop control system.

The differences between the open loop and the closed loop control systems are mentioned in the following table.

Open Loop Control Systems	Closed Loop Control Systems
Control action is independent of the desired output.	Control action is dependent of the desired output.
Feedback path is not present.	Feedback path is present.
These are also called as non-feedback control systems.	These are also called as feedback control systems.
Easy to design.	Difficult to design.
These are economical.	These are costlier.
Inaccurate.	Accurate.

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1.5 SIGNAL FLOW GRAPHS

Signal flow graph is a graphical representation of algebraic equations. In this chapter, let us discuss the basic concepts related signal flow graph and also learn how to draw signal flow graphs.

Basic Elements of Signal Flow Graph

Nodes and branches are the basic elements of signal flow graph.

Node

Node is a point which represents either a variable or a signal. There are three types of nodes — input node, output node and mixed node.

- **Input Node** − It is a node, which has only outgoing branches.

- **Output Node** − It is a node, which has only incoming branches.

- **Mixed Node** − It is a node, which has both incoming and outgoing branches.

Example

Let us consider the following signal flow graph to identify these nodes.
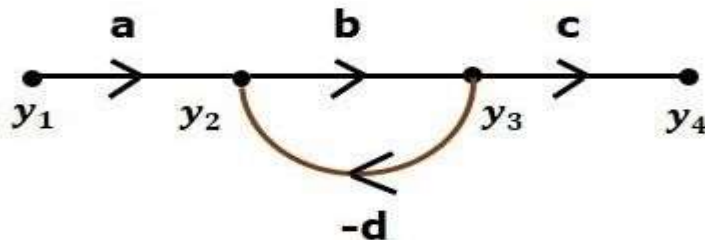
- The **nodes** present in this signal flow graph are y_1, y_2, y_3 and y_4.

- y_1 and y_4 are the **input node** and **output node** respectively.

- y_2 and y_3 are **mixed nodes**.

Branch

Branch is a line segment which joins two nodes. It has both **gain** and **direction**. For example, there are four branches in the above signal flow graph. These branches have **gains** of **a, b, c** and **-d**.

Construction of Signal Flow Graph

Let us construct a signal flow graph by considering the following algebraic equations −

$$y_2 = a_{12}y_1 + a_{42}y_4$$

$$y_3 = a_{23}y_2 + a_{53}y_5$$
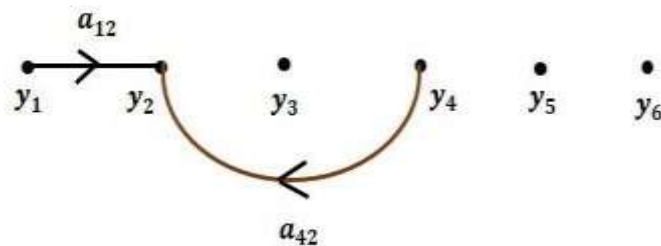
$$y_4 = a_{34}y_3$$
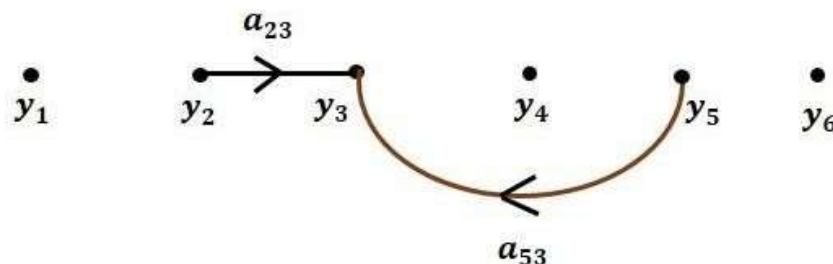
$$y_5 = a_{45}y_4 + a_{35}y_3$$

$$y_6 = a_{56}y_5$$

There will be six **nodes** (y_1, y_2, y_3, y_4, y_5 and y_6) and eight **branches** in this signal flow graph. The gains of the branches are a_{12}, a_{23}, a_{34}, a_{45}, a_{56}, a_{42}, a_{53} and a_{35}.

To get the overall signal flow graph, draw the signal flow graph for each equation, then combine all these signal flow graphs and then follow the steps given below −
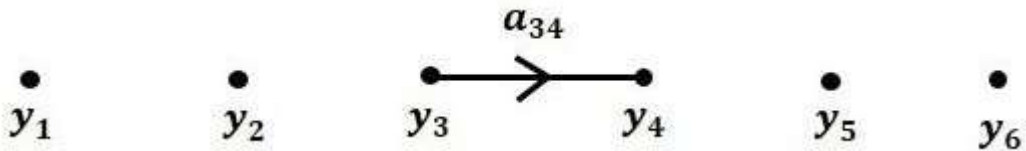
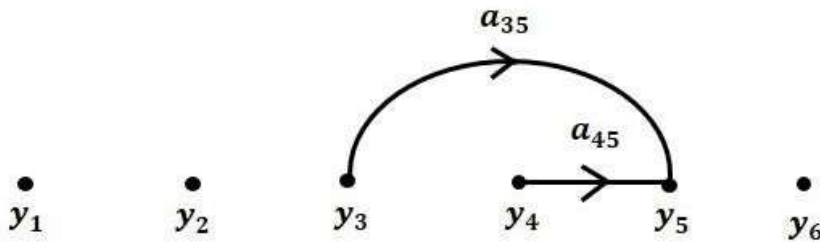**Step 1** − Signal flow graph for $y_2 = a_{13}y_1 + a_{42}y_4$ is shown in the following figure.

Step 2 − Signal flow graph for $y_3 = a_{23}y_2 + a_{53}y_5$ is shown in the following figure.

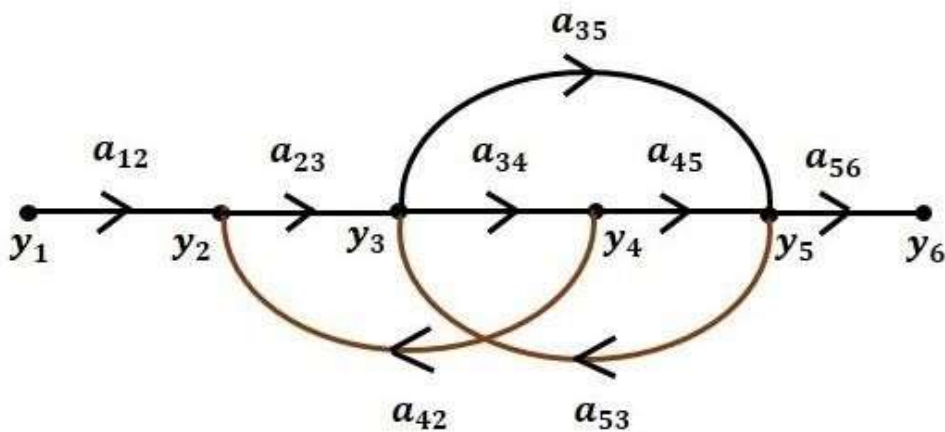**Step 3** − Signal flow graph for $y_4 = a_{34}y_3$ is shown in the following figure.

Step 4 − Signal flow graph for $y_5 = a_{45}y_4 + a_{35}y_3$ is shown in the following figure.

Step 5 − Signal flow graph for $y_6 = a_{56}y_5$ is shown in the following figure.

**Step 6** − Signal flow graph of overall system is shown in the following figure.

Conversion of Block Diagrams into Signal Flow Graphs

Follow these steps for converting a block diagram into its equivalent signal flow graph.

- Represent all the signals, variables, summing points and take-off points of block diagram as **nodes** in signal flow graph.

- Represent the blocks of block diagram as **branches** in signal flow graph.

- Represent the transfer functions inside the blocks of block diagram as **gains** of the branches in signal flow graph.

- Connect the nodes as per the block diagram. If there is connection between two nodes (but there is no block in between), then represent the gain of the branch as one. **For example**, between summing points, between summing point and takeoff point, between input and summing point, between take-off point and output.

Example

Let us convert the following block diagram into its equivalent signal flow graph.
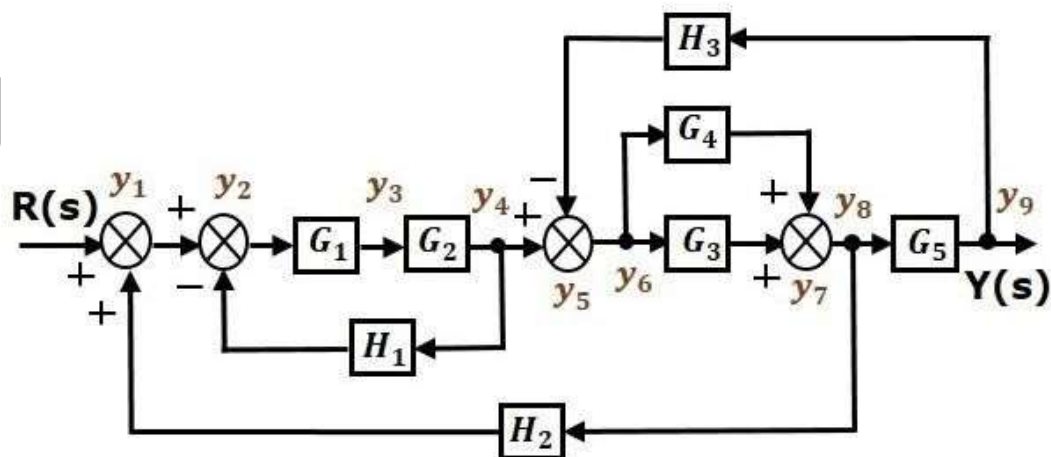
Represent the input signal R(s) and output signal C(s) of block diagram as input node R(s) and output node C(s) of signal flow graph.

Just for reference, the remaining nodes (y_1 to y_9) are labeled in the block diagram. There are nine nodes other than input and output nodes. That is four nodes for four summing points, four nodes for four take-off points and one node for the variable between blocks G1 and G2.

The following figure shows the equivalent signal flow graph.
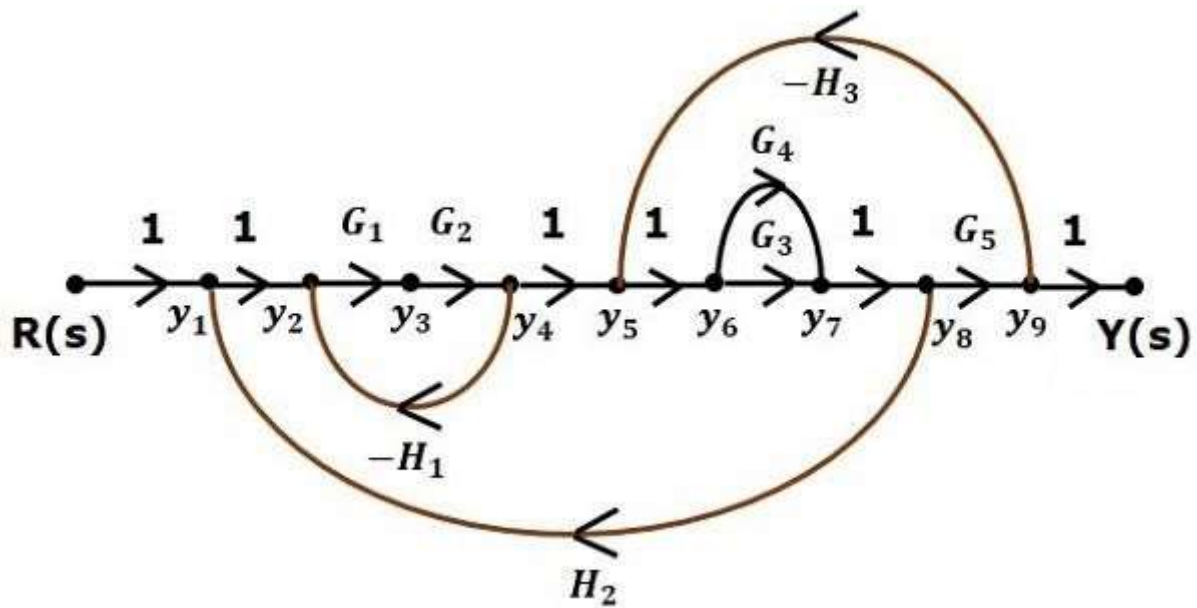
With the help of Mason's gain formula, you can calculate the transfer function of this signal flow graph. This is the advantage of signal flow graphs. Here, we no need to simplify (reduce) the signal flow graphs for calculating the transfer function

There are 'N' forward paths in a signal flow graph. The gain between the input and the output nodes of a signal flow graph is nothing but the **transfer function** of the system. It can be calculated by using Mason's gain formula.

Mason's gain formula is

Where,

- **C(s)** is the output node

- **R(s)** is the input node

- **T** is the transfer function or gain between R(s) and C(s)

- P_i is the ith forward path gain

$\Delta=1-$(sum of all individual loop gains)+(sum of gain products of all possible two nontouching loops)−(sum of gain products of all possible three nontouching loops)+......

Δ_i *is obtained from Δ by removing the loops which are touching the ith forward path*

Consider the following signal flow graph in order to understand the basic terminology involved here.
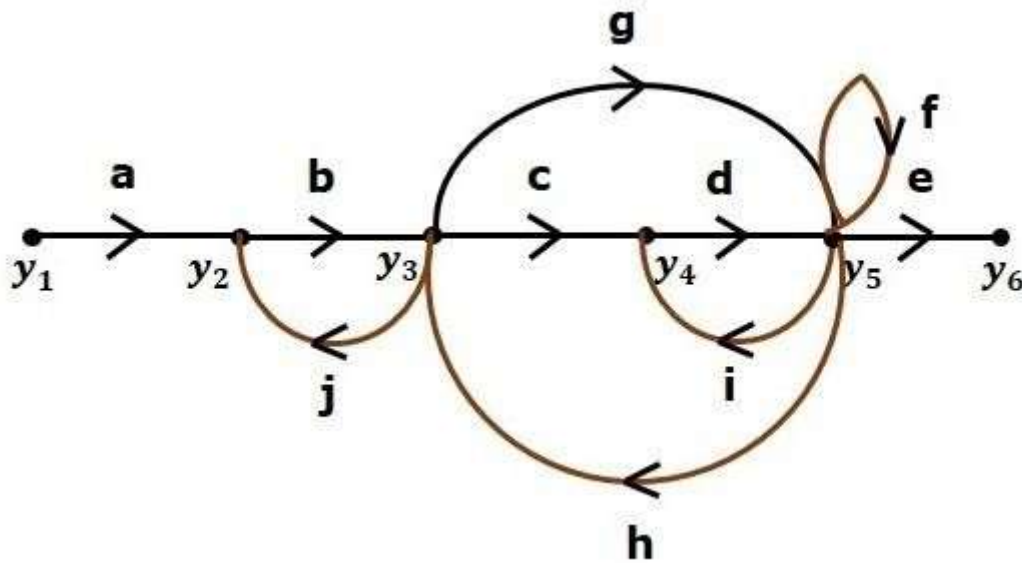
Path

It is a traversal of branches from one node to any other node in the direction of branch arrows. It should not traverse any node more than once.

Examples − y2→y3→y4→y5y2→y3→y4→y5 and y5→y3→y2y5→y3→y2

Forward Path

The path that exists from the input node to the output node is known as **forward path**.

Examples − y1→y2→y3→y4→y5→y6y1→y2→y3→y4→y5→y6 and y1→y2→y3→y5→y6y1→y2→y3→y5→y6.

Forward Path Gain

It is obtained by calculating the product of all branch gains of the forward path.

Examples − abcde is the forward path gain of y1→y2→y3→y4→y5→y6y1→y2→y3→y4→y5→y6 and abge is the forward path gain of y1→y2→y3→y5→y6y1→y2→y3→y5→y6.

Loop

The path that starts from one node and ends at the same node is known as **loop**. Hence, it is a closed path.

Examples − y2→y3→y2y2→y3→y2 and y3→y5→y3y3→y5→y3.

Loop Gain

It is obtained by calculating the product of all branch gains of a loop.

Examples − bjbj is the loop gain of y2→y3→y2y2→y3→y2 and gh is the loop gain of y3→y5→y3y3→y5→y3.

Non-touching Loops

These are the loops, which should not have any common node.

Examples − the loops, y2→y3→y2y2→y3→y2 and y4→y5→y4y4→y5→y4 are non-touching.

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1.7 SYNCHROS

A commonly used error detector of mechanical positions of rotating shafts in AC control systems is the Synchro.

- It consists of two electro mechanical devices.
    - Synchro transmitter
    - Synchro receiver or control transformer.
- The principle of operation of these two devices is same but they differ slightly in their construction.
- The construction of a Synchro transmitter is similar to a phase alternator.
- The stator consists of a balanced three phase winding and is star connected.
- The rotor is of dumbbell type construction and is wound with a coil to produce a magnetic field.
- When a no voltage is applied to the winding of the rotor, a magnetic field is produced.
- The coils in the stator link with this sinusoidal distributed magnetic flux and voltages are induced in the three coils due to transformer action.
- Than the three voltages are in time phase with each other and the rotor voltage.
- The magnitudes of the voltages are proportional to the cosine of the angle between the rotor position and the respective coil axis.
- The position of the rotor and the coils are shown in Fig 1.7.1.
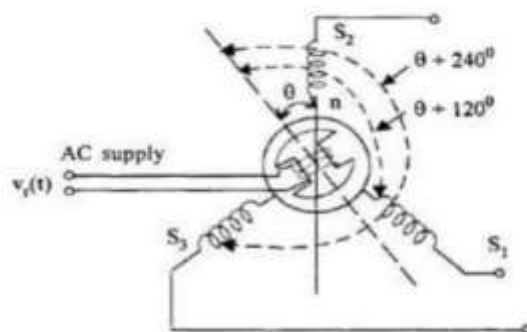

Figure 1.7.1: synchros rotor

- When $q=90^0$ the axis of the magnetic field coincides with the axis of coil S2 and maximum voltage is induced in it as seen.

- For this position of the rotor, the voltage c, is zero, this position of the rotor is known as the 'Electrical Zero' of die transmitter and is taken as reference for specifying the rotor position.

- In summary, it can be seen that the input to the transmitter is the angular position of the rotor and the set of three single phase voltages is the output.

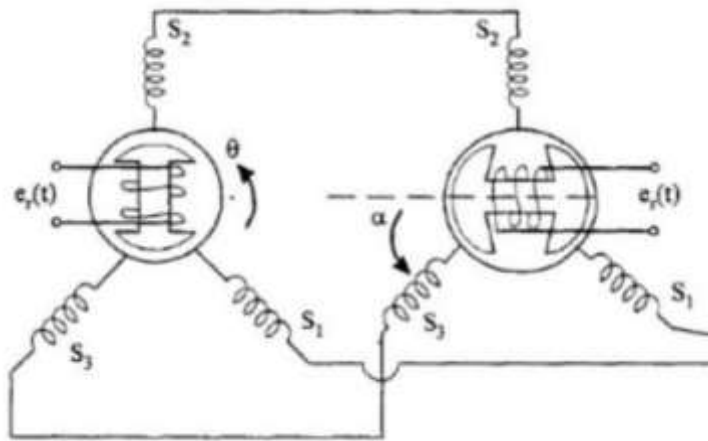- The magnitudes of these voltages depend on the angular position of the rotor as given in fig 1.7.2

Figure 1.7.2: syncros rotor position

- Now consider these three voltages to he applied to the stator of a similar device called control transformer or synchro receiver.

- The construction of a control transformer is similar to that of the transmitter except that the rotor is made cylindrical in shape whereas the rotor of transmitter is dumbbell in shape.

- Since the rotor is cylindrical, the air gap is uniform and the reluctance of the magnetic path is constant.

- This makes the output impedance of rotor to be a constant.

- Usually the rotor winding of control transformer is connected teas amplifier which requires signal with constant impedance for better performance.

- A synchro transmitter is usually required to supply several control transformers and hence the stator winding of control transformer is wound with higher impedance per phase.

- Since the some currents flow through the stators of the synchro transmitter and receiver, the same pattern of flux distribution will be produced in the air gap of the control transformer.

- The control transformer flux axis is in the same position as that of the synchro transmitter.

- Thus the voltage induced in the rotor coil of control transformer is proportional to the cosine of the angle between the two rotors.

Multivariable Control System

- Multivariable control system define a system in which the variable the variable interacts strongly. This kind of system must have more than one input and more than one output. A disturbance in any input causes a change of response from at least one output.

- This kind of system have as many inputs and outputs as needed to control the process. A system with an equal number of inputs and outputs is said to be **square.**

- A disturbance in any variable can cause a change in response in any output in its signal path. Depending on the system design,these paths can be direct or indirect.

- In most control system,a particular input disturbance cause one output to respond by a larger percentage than the other outputs. The response of the other output is called interaction. Interaction is often a result of system design and cannot be avoided. However the system must either correct or compensate for interaction