1.2 CLASSIFICATION OF SIGNALS

CONTINUOUS TIME AND DISCRETE TIME SIGNAL

Continuous time signal:

A signal that is defined for every instants of time is known as continuous time signal. Continuous time signals are continuous in amplitude and continuous in time. It is denoted by x(t) and shown in Figure 1.2.1

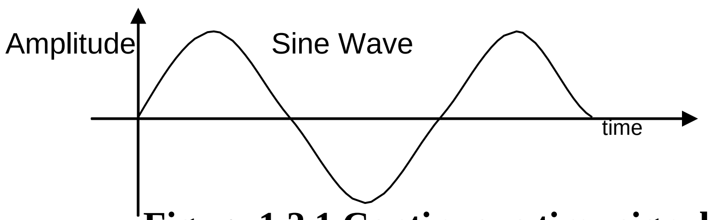

Figure 1.2.1 Continuous time signal

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

Discrete time signal:

A signal that is defined for discrete instants of time is known as discrete time signal. Discrete time signals are continuous in amplitude and discrete in time. It is also obtained by sampling a continuous time signal. It is denoted by x(n) and shown in Figure 1.2.2

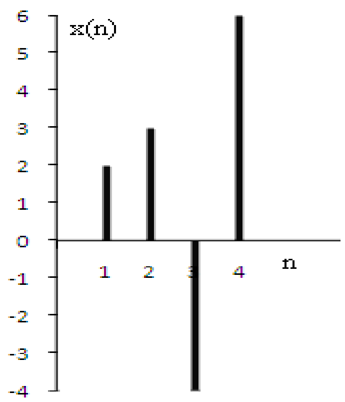

Figure 1.2.2 Discrete time signal

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EVEN (SYMMETRIC) AND ODD (ANTI-SYMMETRIC) SIGNAL

Continuous domain:

Even signal:

A signal that exhibits symmetry with respect to t=0 is called even signal

Even signal satisfies the condition $x(t) = x(-t)$

Odd signal:

A signal that exhibits anti-symmetry with respect to t=0 is called odd signal

Odd signal satisfies the condition $x(t) = -x(-t)$

Even part $\boldsymbol{x_e(t)}$ and Odd part $\boldsymbol{x_0(t)}$ of continuous time signal $\boldsymbol{x(t)}$:

Even component of $x(t)$ is $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

Odd component of $x(t)$ is $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

Discrete domain:

Even signal:

A signal that exhibits symmetry with respect to n=0 is called even signal

Even signal satisfies the condition $x(n) = x(-n)$.

Odd signal:

A signal that exhibits anti-symmetry with respect to n=0 is called odd signal

Odd signalsatisfies the condition $x(n) = -x(-n)$.

Even part $\boldsymbol{x_e(n)}$ and Odd part $\boldsymbol{x_0(n)}$ of discrete time signal $\boldsymbol{x(n)}$:

Even component of $x(n)$ is $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

Odd component of $x(n)$ is $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

PERIODIC AND APERIODIC SIGNAL

Periodic signal:

A signal is said to periodic if it repeats again and again over a certain period of time.

Aperiodic signal:

A signal that does not repeat at a definite interval of time is called aperiodic signal.

Continuous domain:

A Continuous time signal is said to periodic if it satisfies the condition

$$x(t)=x(t+T)\, where\, T\, is\, fundamental\, time\, period$$

If the above condition is not satisfied, then the signal is said to be aperiodic

Fundamental time period

$$T = 2\pi/\Omega$$

where Ω is fundamental angular frequency in rad/sec

Discrete domain:

A Discrete time signal is said to periodic if it satisfies the condition

$$x(n) = x(n + N)\ where\ N\ is\ fundamental\ time\ period$$

If the above condition is not satisfied, then the signal is said to be aperiodic

Fundamental time period

$$N = 2\pi m/\omega$$

where ω is fundamental angular frequency in rad/sec,

m is smallest positive integer that makes N as positive integer.

ENERGY AND POWER SIGNAL

Energy signal:

The signal which has finite energy and zero average power is called energy signal. The non-periodic signals like exponential signals will have constant energy and so non periodic signals are energy signals.

i.e., For energy signal, $0 < E < \infty\ and\ P = 0$

For Continuous time signals,

$$Energy\ E = \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2 dt$$

For Discrete time signals,

$$Energy\ E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

Power signal:

The signal which has finite average power and infinite energy is called power signal. The periodic signals like sinusoidal complex exponential signals will have constant power and so periodic signals are power signals.

i.e., For power signal, $0 < P < \infty\ and\ \ E = \infty$

For Continuous time signals,

$$Average\ power\ P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

For Discrete time signals,

$$Average\ power\ P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

DETERMINISTIC AND RANDOM SIGNALS

Deterministic signal:

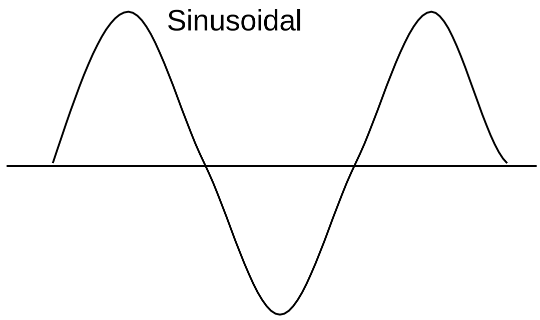
A signal is said to be deterministic if there is no uncertainty over the signal at any instant of time i.e., its instantaneous value can be predicted. It can be represented by mathematical equation.

Example: sinusoidal signal

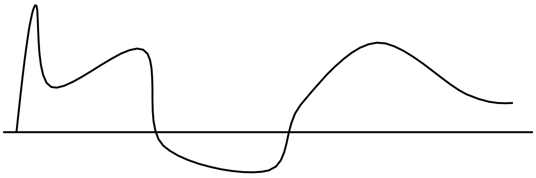
Random signal (Non-Deterministic signal):

A signal is said to be random if there is uncertainty over the signal at any instant of time i.e., its instantaneous value cannot be predicted. It cannot be represented by mathematical equation.

Example: noise signal



Deterministic signal



Random signal

CAUSAL AND NON-CAUSAL SIGNAL

Continuous domain:

Causal signal:

A signal is said to be causal if it is defined for t≥0.

$$i.e., \quad x(t) = 0 \quad for\ t < 0$$

Non-causal signal:

A signal is said to be non-causal, if it is defined for t< 0 or for both $t < 0$ and $t \geq 0$

$$i.e., \quad x(t) \neq 0 \quad for\ t < 0$$

When a non-causal signal is defined only for t<0, it is called as anti-causal signal

Discrete domain:

Causal signal:

A signal is said to be causal, if it is defined for n≥0.

$$i.e., \quad x(n) = 0 \quad for\ n < 0$$

Non-causal signal:

A signal is said to be non-causal, if it is defined, for n< 0 or for both n < 0 $and\ n \geq 0$

$$i.e., \quad x(n) \neq 0 \quad for\ n < 0$$

When a non-causal signal is defined only for n<0, it is called as anti-causal signal.

1.4 CLASSIFICATION OF SYSTEM

- Continuous time and Discrete time system
- Linear and Non-Linear system
- Static and Dynamic system
- Time invariant and Time variant system
- Causal and Non-Causal system
- Stable and Unstable system

CONTINUOUS TIME AND DISCRETE TIME SYSTEM

Continuous time system:

Continuous time system operates on a continuous time signal (input or excitation) and produces another continuous time signal (output or response) which is shown in Figure 1.4.1. The signal $x(t)$ is transformed by the system into signal $y(t)$, this transformation can be expressed as,

Response $y(t) = T\ x(t)$

where $x(t)$ is input signal, $y(t)$ is output signal, and T denotes transformation

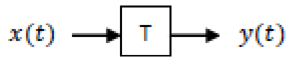


Figure 1.4.1 Representation of continuous time system

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

Discrete time system:

Discrete time system operates on a discrete time signal (input or excitation) andproduces another discrete time signal (output or response) which is shown in Figure 1.4.2.

The signal $x(n)$ is transformed by the system into signal $y(n)$, this transformation can beexpressed as,

Response $y(n) = T\{x(n)\}$

where x(n) is input signal, y(n) is output signal, and T denotes transformation

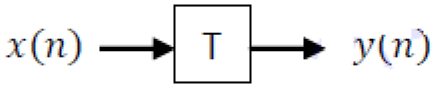


Figure 1.4.2 Representation of discrete time system

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

Linear system and Non Linear system

Linear system:

A system is said to be linear if it obeys superposition theorem. Superposition theorem states that the response of a system to a weighted sum of the signals is equal to the corresponding weighted sum of responses to each of the individual input signals.

Condition for Linearity in continuous time systems:

$$T[ax_1(t) + bx_2(t)] = ay_1(t) + by_2(t)$$

where $y_1(t)$ and $y_2(t)$ are the responses of $x_1(t)$ and $x_2(t)$ respectively

Condition for Linearity in Discrete time systems:

$$T[ax_1(n) + bx_2(n)] = ay_1(n) + by_2(n)$$

where $y_1(n)$ and $y_2(n)$ are the responses of $x_1(n)$ and $x_2(n)$ respectively

Non Linear system:

A system is said to be Nonlinear if it does not obeys superposition theorem.

Condition for Non Linearity in continuous time systems:

$$i.e.,\ T[ax_1(t) + bx_2(t)] \neq ay_1(t) + by_2(t)$$

where $y_1(t)$ and $y_2(t)$ are the responses of $x_1(t)$ and $x_2(t)$ respectively

Condition for Non Linearity in Discrete time systems:

$$i.e.,\ T[ax_1(n) + bx_2(n)] \neq ay_1(n) + by_2(n)$$

where $y_1(n)$ and $y_2(n)$ are the responses of $x_1(n)$ and $x_2(n)$ respectively

Static (Memoryless) and Dynamic (Memory) system

Static system:

A system is said to be memoryless or static if the response of the system is due topresent input alone.

Example in Continuous time domain: $y(t) = 2x(t)$

$$y(t) = x^2(t) + x(t)$$

Example in Discrete time domain: $y(n) = x(n)$

$$y(n) = x^2(n) + 3x(n)$$

Dynamic system:

A system is said to be memory or dynamic if the response of the system depends on factors other than present input also.

Example in Continuous time domain: $y(t) = 2x(t) + x(-t)$

$$y(t) = x^2(t) + x(2t)$$

Example in Discrete time domain: $y(n) = 2x(n) + x(-n)$

$$y(n) = x^2(1-n) + x(2n)$$

Time invariant (Shift invariant) and Time variant (Shift variant) system

Time invariant system:

A system is said to time invariant if the relationship between the input and output doesnot change with time.

In Continuous time domain: If $y(t) = T[x(t)]$

Then $T[x(t - t_0)] = y(t - t_0)$ should be satisfied for the system to be time invariant In

Discrete time domain: If $y(n) = T[x(n)]$

Then $T[x(n - n_0)] = y(n - n_0)$ should be satisfied for the system to be time invariant

Time variant system:

A system is said to time variant if the relationship between the input and output changeswith time.

Continuous time domain: If $y(t) = T[x(t)]$
Then $T[x(t - t_0)] \neq y(t - t_0)$ should be satisfied for the system to be time variant

In Discrete time domain: If $y(n) = T[x(n)]$

Then $T[x(n - n_0)] \neq y(n - n_0)$ should be satisfied for the system to be time variant

Causal and Non-Causal system

Causal system:

A system is said to be causal if the response of a system at any instant of time depends only on the present input, past input and past output but does not depends upon the future input and future output.

For continuous time systems,

Example: $y(t) = 3x(t) + x(t - 1)$

A system is said to be causal if impulse response $h(t)$ is zero for negative values of t i.e., $h(t) = 0 \; for \; t < 0$

For discrete time systems,

Example: $y(n) = 3x(n) + x(n - 1)$

A system is said to be causal if impulse response h(n) is zero for negative values of ni.e., h$(n) = 0 \; for \; n < 0$

Non-Causal system:

A system is said to be Non-causal if the response of a system at any instant of timedepends on the future input and also on the present input, past input, past output.

For continuous time systems,

Example: $y(t) = x(t + 2) + x(t - 1)$

$y(t) = x(-t) + x(t + 4)$

A system is said to be non-causal if impulse response $h(t)$ is non-zero for negative values of t

i.e., $h(t) \neq 0 \; for \; t < 0$ For discrete time systems,

Example: $y(n) = x(n + 2) + x(n - 1)$

$$y(n) = x(-n) + x(n + 4)$$

A system is said to be non-causal if impulse response h(n) is non-zero for negative values of ni.e., h(n) $\neq 0$ for $n < 0$

Stable and Unstable system

A system is said to be stable if and only if it satisfies the BIBO stability criterion.

BIBO stable condition for continuous time systems:

- Every bounded input yields bounded output.

$i.e.,$ if $0 < x(t) < \infty$ $then$ $0 < y(t) < \infty$ should be satisfied for the system to be stable

- Impulse response should be absolutely integrable

$$i.e., 0 < \int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

If the BIBO stable condition is not satisfied, then the system is said to be unstable system

BIBO stable condition for Discrete time sytems:

- Every bounded input yields bounded output.
- Impulse response should be absolutely summable

$$i.e., 0 < \sum_{k=-\infty}^{\infty} | h(k) | < \infty$$

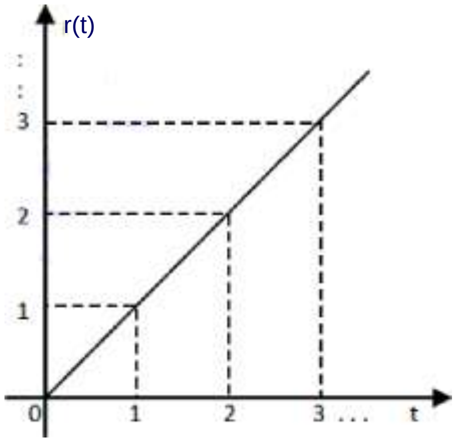
If the BIBO stable condition is not satisfied, then the system is said to be unstable system.

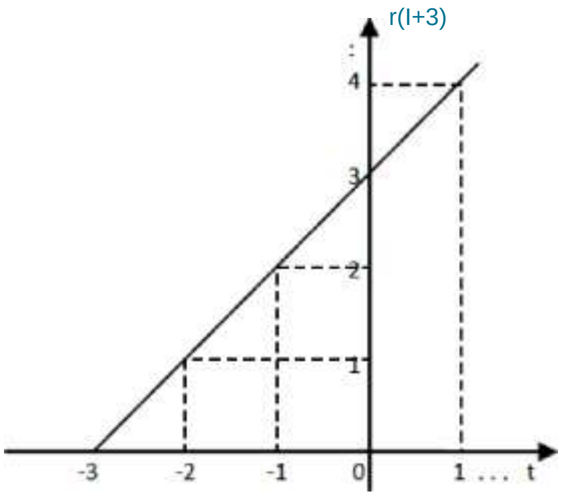
1.3 SOLVED PROBLEMS

1. Draw $r(t+3)$, where r(t) is ramp signal.

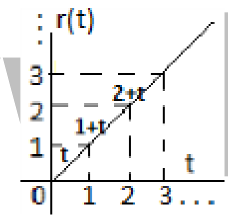
   Solution:

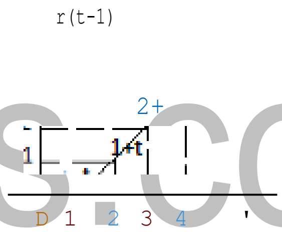
   $$\mathrm{r}(t) = t;\ t \geq 0$$

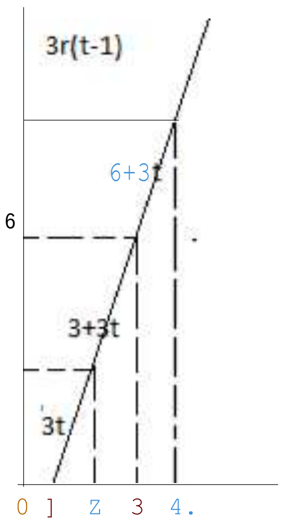


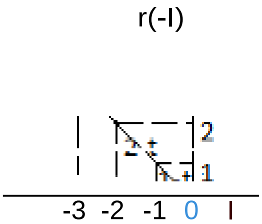


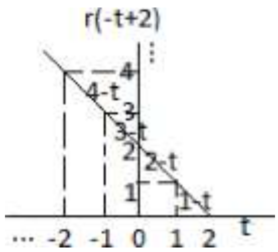
2. Sketch $\boldsymbol{x(t) = 3r(t-1) + r(-t+2)}$.



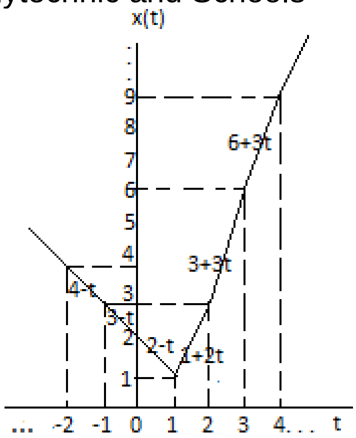








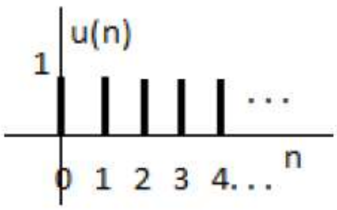
$x(t) = 3r(t-1) + r(-t+2)$
$= 0 + 4 - t \; for \; -2 \le t \le -1$
$= 0 + 3 - t \; for \; -1 \le t \le 0$
$= 0 + 2 - t \; for \; 0 \le t \le 1$
$= 3t + 1 - t \; for \; 1 \le t \le 2$
$= 3 + 3t + 0 \; for \; 2 \le t \le 3$
$= 6 + 3t + 0 \; for \; 3 \le t \le 4$ and so on

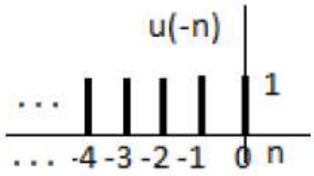


3. Draw time reversal signal of unit step signal

Solution:

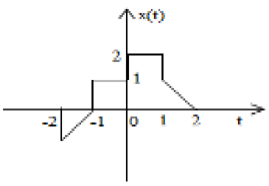
$$u(n) = 1; n \ge 0$$





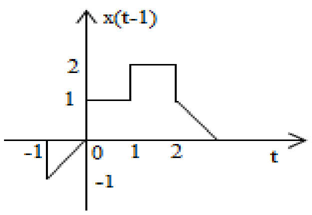
4. A continuous time signal x(t) is shown in figure below. sketch and label carefully each ofthe following signal: 1) x(t-1) 2) x(2-t) 3)x(t)[δ(t+3/2) - δ(t-3/2)] 4) x(2t+1)

Fig:
x(t)

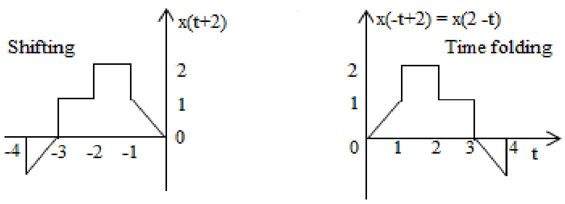


Solutions:

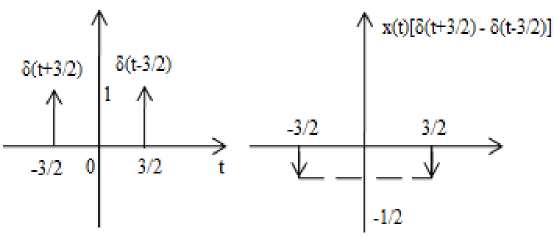
1) x(t-1)



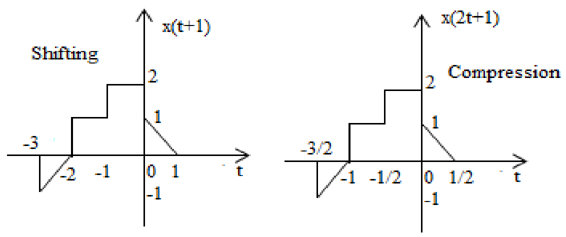
2) x(2-t)



3) x(t)[δ(t+3/2)- δ(t-3/2)]



4) x(2t+1)



5. Determine whether the signal is energy or power signal. $x(t) = e^{-3t}u(t)$.

$$\textbf{Energy } E_\infty = \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2dt = \lim_{T\to\infty}\int_{0}^{T}|e^{-3t}|^2dt = \lim_{T\to\infty}\int_{0}^{T}e^{-6t}\,dt = \lim_{T\to\infty}\left[\frac{e^{-6t}}{-6}\right]_0^T$$

$$= \lim_{T\to\infty}\left[\frac{e^{-6T}}{-6} - \left[\frac{e^{-0}}{-6}\right]\right] = \frac{\mathbf{1}}{\mathbf{6}} < \infty \qquad \because e^{-\infty} = 0, e^{-0} = 1$$

$$\textbf{Power } P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2dt = \lim_{T\to\infty}\frac{1}{2T}\int_{0}^{T}|e^{-3t}|^2dt = \lim_{T\to\infty}\frac{1}{2T}\int_{0}^{T}e^{-6t}\,dt$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[\frac{e^{-6t}}{-6}\right]_0^T = \lim_{T\to\infty}\frac{1}{2T}\left[\frac{e^{-6T}}{-6} - \frac{e^{-0}}{-6}\right] = \lim_{T\to\infty}\frac{1}{2T}\left[\frac{1}{6}\right] = \mathbf{0} \qquad \because e^{-\infty} = 0, e^{-0} = 1, \frac{1}{\infty} = 0$$

Since energy is finite and power is zero .It is a energy signal.

1.5 SOLVED PROBLEMS

1. **Determine whether the following systems are linear or not $y(n) = x(n-2) + x(n^2)$.**

   Solution: Output due to weighted sum of inputs:

$$y_3(n) = ax_1(n-2) + bx_2(n-2) + ax_1(n^2) + bx_2(n^2) \text{———} (1)$$

Weighted sum of outputs:

For input $x_1(n)$:

$$y_1(n) = x_1(n-2) + x_1(n^2)$$

For input $x_2(n)$:

$$y_2(n) = x_2(n-2) + x_2(n^2)$$

$$ay_1(n) + by_2(n) = ax_1(n-2) + ax_1(n^2) + bx_2(n-2) + bx_2(n^2) \text{-------------} (2)$$

$$\because y_3(n) = ay_1(n) + by_2(n)$$

(1)=(2)

∴∴∵∵∴The system is Linear.

2. **Determine whether the following systems are linear or not $\frac{dy(t)}{dt} + 3ty(t) = t^2x(t)$.**

   Solution:

   Condition for Linearity:

$$T[ax_1(t) + bx_2(t)] = ay_1(t) + by_2(t)$$

$T[ax1(t) + bx2(t)] \rightarrow$

$$\frac{d}{dt}[ay_1(t) + by_2(t)] + 3t[ay_1(t) + by_2(t)]$$

$$= t^2[ax_1(t) + bx_2(t)] ----(1)$$

$$ay_1(t) \rightarrow \frac{ady1(t)}{dt} + 3tay_1(t) = t^2ax_1(t) ----(2)$$

$$ay_2(t) \rightarrow \frac{ady2(t)}{dt} + 3tay_2(t) = t^2ax_2(t) ----(3)$$

Adding equ (2) and (3) we get,

$$\frac{d}{dt}[ay_1(t)+by_2\ (t)]+3t[ay_1\ \ (t)+by_2\ \ (t)]$$

$$= t^2[ax_1(t) + bx_2(t)] - - - -(4)$$

(1)=(4)

∴ This is a Linear system.

3. **Determine whether the following systems are static or dynamic $y(t) = x(2t) + 2x(t)$.**

Solution:

For t=0, $y(0) = x(0) + 2x(0) \Rightarrow$ present inputs

For t=-1, $y(-1) = x(-2) + 2x(-1) \Rightarrow$ past and present inputs

For t=1, $y(1) = x(2) + 2x(1) \Rightarrow$ future and present inputs

Since output depends on past and future inputs the given system is dynamic system.

4. **Determine whether the following systems are static or dynamic $y(n) = sinx(n)$.**

Solution:

For n=0, $y(0) = sinx(0) \Rightarrow$ present input

For n=-1, $y(-1) = sinx(-1) \Rightarrow$ present input

For n=1, $y(1) = sinx(1) \Rightarrow$ present input

Since output depends on present input the given system is Static system

5. **Determine whether the following systems are time invariant or not $y(t) = x(t)sinwt$.**

Solution:

Output due to input delayed by T seconds

$$y(t, T) = x(t - T)sinwt$$

Output delayed by T seconds

$$y(t - T) = x(t - T)sinw(t - T)$$

$$\because y(t, T) \neq y(t - T)$$

The given system is time variant

6. **Determine whether the following systems are time invariant or not**

$$\boldsymbol{y(n) = x(-n + 2)}.$$

Solution:

Output due to input delayed by k seconds

$$y(n, k) = x(-n + 2 - k)$$

Output delayed by k seconds

$$y(n - k) = x(-(n - k) + 2) = x(-n + k + 2)$$

$$\because \boldsymbol{y(n, k) \neq y(n - k)}$$

The given system is time variant

7. **Determine whether the following systems are causal or not**

$$\boldsymbol{y(t) = \frac{dx(t)}{dt} + 2x(t)}$$

Solution:

The given equation is differential equation and the output depends on past input. Hence thegiven system is **Causal**.

8. **Determine whether the following systems are causal or not**

$$\boldsymbol{y(n) = sinx(n)}$$

Solution:

For n=0, $y(0) = sinx(0) \Rightarrow$ present input For

n= -1, $y(-1) = sinx(-1)$ $\Rightarrow$ presentinput

For n=1, $y(1) = sinx(1) \Rightarrow$ present input

Since output depends on present input the given system is Causal system

9. Determine whether the following system is stable or not.$\boldsymbol{y(n) = 3x(n)}$.

Solution:

$$\text{Let } x(n) = \delta(n), y(n) = h(n)$$
$$\Rightarrow h(n) = 3\delta(n)$$
$$\text{Condition for stability } \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$
$$\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=0}^{\infty} |3\delta(k)| = \sum_{k=0}^{\infty} 3\delta(k) = 3$$
$$\because \delta(k) = 0 \; for \; k \neq 0 \quad and \quad \delta(k) = 1 \; for \; k = 0$$
$$\because \sum_{k=-\infty}^{\infty} |h(k)| < \infty \text{ the given system is } \textbf{stable}$$

10.Determine whether the following system is stable or not

$\boldsymbol{h(t) = e^{3t}u(t-2)}$

Solution:

$$\text{Condition for stability} \int_{-\infty}^{\infty} |\, h(t)\,|\, dt < \infty$$

$$\int_{-\infty}^{\infty} |\, h(t)\,|\, dt = \int_{2}^{\infty} e^{3t} dt$$

$$= \left[\frac{e^{3t}}{3}\right]_2^{\infty} = \infty$$

∴ The system is unstable.

1.1 INTRODUCTION

SIGNAL

Signal is one that carries information and is defined as a physical quantity that varies with one or more independent variable.

Example: Music, speech

The signal may depend on one or more independent variables. If a signal depends on only one variable, then it is known as one dimensional signal. Example: AC power signal, speech signal, ECG etc.

When a signal is represented as a function of two or more variable, it is said to be multidimensional signal Example: An image represented as F(x,y). Here x & y represents the horizontal and vertical co-ordinates. The intensity of the image varies at each co-ordinate.

SIGNAL MODELING

The representation of a signal by mathematical expression is known as signal modeling.

ANALOG SIGNAL

A signal that is defined for every instants of time is known as analog signal. Analog signals are continuous in amplitude and continuous in time. It is denoted by x(t). It is also called as Continuous time signal. Example for Continuous time signal is shown in Figure 1.1.1

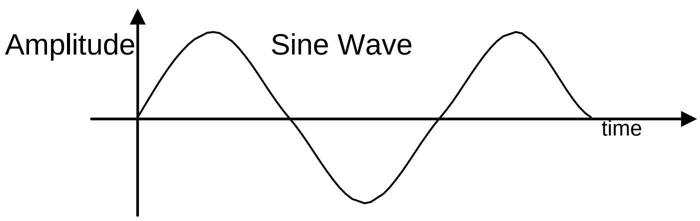


Figure 1.1.1 Continuous time signal

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

DIGITAL SIGNAL

The signals that are discrete in time and quantized in amplitude is called digital signal(Figure 1.1.2)

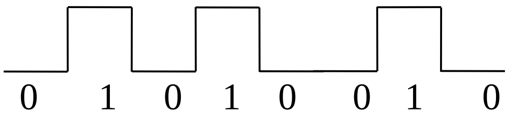


Figure 1.1.2 Digital Signal

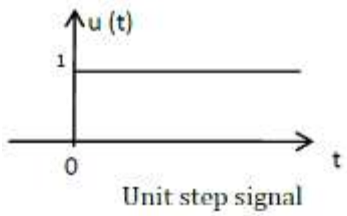
[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

BASIC (ELEMENTARY OR STANDARD) CONTINUOUS TIME SIGNALS

Step signal

Unit Step signal is defined as

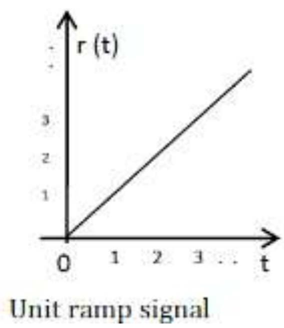
$$u(t) = 1 \ \ for\ t \geq 0$$
$$= 0\ for\ t < 0$$



Ramp signal

Unit ramp signal is defined as

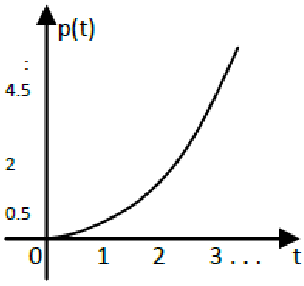
$$r(t) = t \ \ for\ t \geq 0$$
$$= 0\ for\ t < 0$$



Parabolic signal

Unit Parabolic signal is defined as

$$x(t) = \frac{t^2}{2} \; for\ t \geq 0$$
$$= 0\ for\ t < 0$$



Unit Parabolic signal

Relation between Unit Step signal, Unit ramp signal and Unit Parabolic signal:

- Unit ramp signal is obtained by integrating unit step signal

$$i.e., \int u(t)dt = \int 1dt = t = r(t)$$

- Unit Parabolic signal is obtained by integrating unit ramp signal

$$i.e., \int r(t)dt = \int t\,dt = \frac{t^2}{2} = p(t)$$

- Unit step signal is obtained by differentiating unit ramp signal

$$i.e., \frac{d}{dt}(r(t)) = \frac{d}{dt}(t) = 1 = u(t)$$

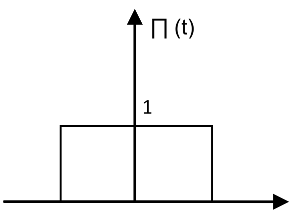
- Unit ramp signal is obtained by differentiating unit Parabolic signal

$$i.e., \frac{d}{dt}(p(t)) = \frac{d}{dt}\left(\frac{t^2}{2}\right) = \frac{1}{2}(2t) = t = r(t)$$

Unit Pulse Signal

Unit Pulse signal is defined as

$$\Pi(t) = 1 \text{ for } |t| \leq \frac{1}{2}$$
$$= 0 \text{ elsewhere}$$

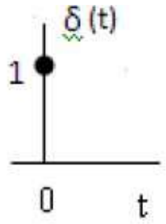


Unit pulse signal

Impulse signal

Unit Impulse signal is defined as

$$\delta(t) = 0 \; for \; t \neq 0$$
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$



Unit Impulse signal

Properties of Impulse signal:

Property 1:

$$\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0)$$

$$\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0)\delta(0) = x(0) \qquad [\because \delta(t) \text{ exists only at } t = 0 \text{ and } \delta(0) = 1]$$

Hence proved

Property 2:

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_o)dt = x(t_o)$$

proof:

$$\int_{-\infty}^{\infty} x(t)\delta(t-t_o)dt = x(t_o)\delta(t_o - t_o) = x(t_o)\delta(0) = x(t_o)$$

$$\therefore [\delta(t-t_o) \text{ exists only at } t = t_o \text{ and } \delta(0) = 1]$$

Hence proved

Sinusoidal signal

Cosinusoidal signal is defined as

$$x(t) = A\cos(\Omega t + \Phi)$$

Sinusoidal signal is defined as

$$x(t) = A\sin(\Omega t + \Phi)$$

Where $\Omega = 2\pi f = \frac{2\pi}{T}$ *and* Ω *is the angular frequency in rad/sec* f is frequency in cycles/sec or Hertz and

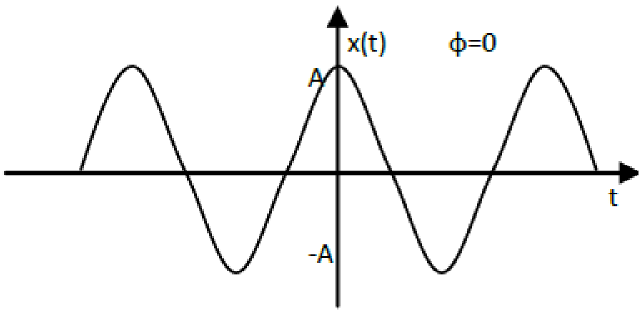
A is amplitude

T is time period in seconds

Φ is phase angle in radians

Cosinusoidal signal

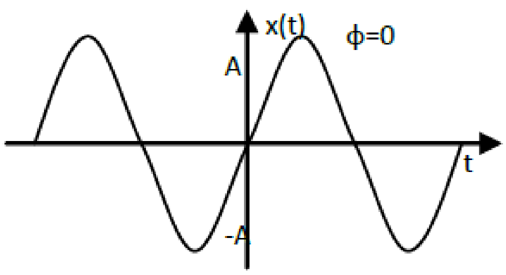
$$when\ \phi = 0,\ x(t) = A\cos(\Omega t)$$



Cosinusoidal signal

Sinusoidal signal

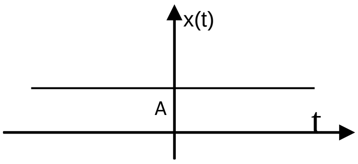
$$when\ \phi = 0,\ x(t) = A\sin(\Omega t)$$



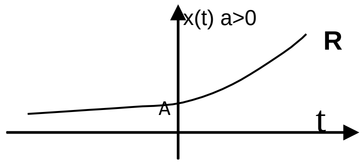
Sinusoidal signal

Exponential signal

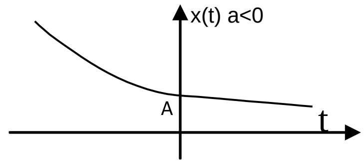
Real Exponential signal is defined as $x(t) = Ae^{at}$,where A is amplitude Depending on the value of 'a' we get dc signal or growing exponential signal or decayingexponential signal



DC signal



Exponentially growing signal



Exponentially decaying signal

Complex exponential signal is defined as

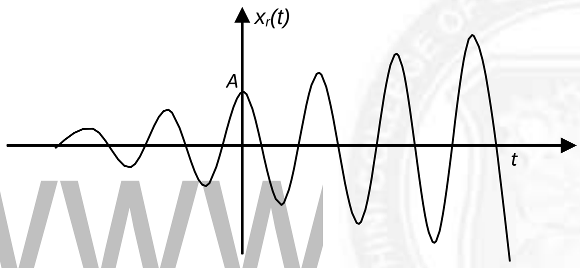
$$x(t) = Ae^{st}$$

where A is amplitude, s is complex variable and $s = \sigma + j\Omega$

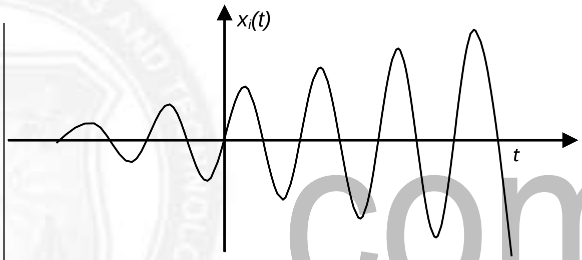
$$x(t) = Ae^{st} = Ae^{(\sigma+j\Omega)t} = Ae^{\sigma t}\, e^{j\Omega t} = Ae^{\sigma t}\,(cos\Omega t + jsin\Omega t)$$

$when\ \sigma = +ve,\ tten\ x(t) = Ae^{\sigma t}\,(cos\Omega t + jsin\Omega t),$

$where\ x_r(t) = Ae^{\sigma t}\, cos\Omega t\ and\ x_i(t) = Ae^{\sigma t}\, sin\Omega t$



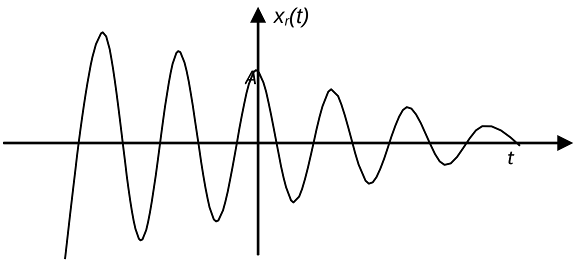
Exponentially growing Cosinusoidal signal



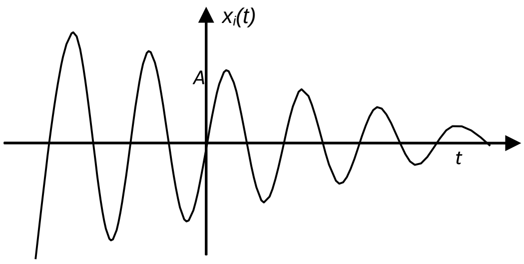
Exponentially growing sinusoidal signal

$when\ \sigma = -ve,\ then\ x(t) = Ae^{-\sigma t}\,(cos\Omega t + jsin\Omega t),$

$where\ x_r(t) = Ae^{-\sigma t}\, cos\Omega t\ and\ x_i(t) = Ae^{-\sigma t}\, sin\Omega t$



Exponentially decaying Cosinusoidal signal



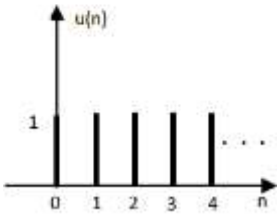
Exponentially decaying sinusoidal signal

BASIC (ELEMENTARY OR STANDARD) DISCRETE TIME SIGNALS

Step signal

Unit Step signal is defined as

$$u(n) = 1 \; for \; n \geq 0$$
$$= 0 \; for \; n < 0$$



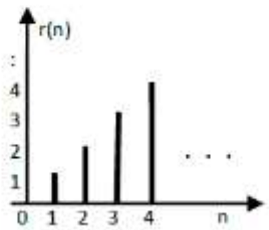
Unit step signal

Unit Ramp signal

Unit Ramp signal is defined as

$$r(n) = n \;\; for \; n \geq 0$$
$$= 0 \quad for \; n < 0$$

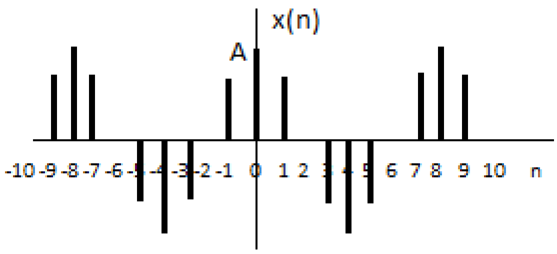
Sinusoidal signal



Unit Ramp signal

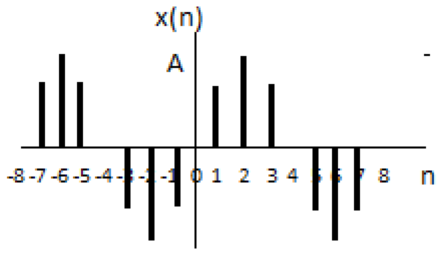
Cosinusoidal signal is defined as

$$x(n) = A cos(\omega n)$$



Sinusoidal signal is defined as

$$x(n) = A sin(\omega n)$$



Sinusoidal signal

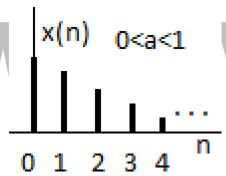
where $\omega = 2\pi f = \frac{2\pi}{N} m$ and ω is frequency in radians/sample

m is the smallest integer

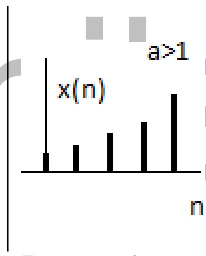
f is frequency in cycles/sample, A is amplitude

Exponential signal

Real Exponential signal is defined as $x(n) = a^n$ for $n \geq 0$



Decreasing exponential signal

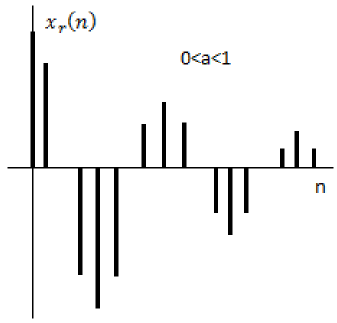


Increasing exponential signal

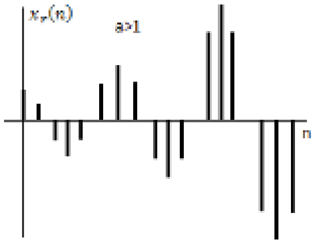
Complex Exponential signal is defined as

$$x(n) = a^n e^{j(\omega_0 n)} = a^n [\cos\omega_0 n + j\sin\omega_0 n]$$

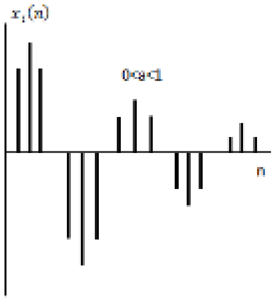
$$\text{where } x_r(n) = a^n \cos\omega_0 n \text{ and } x_i(n) = a^n \sin\omega_0 n$$



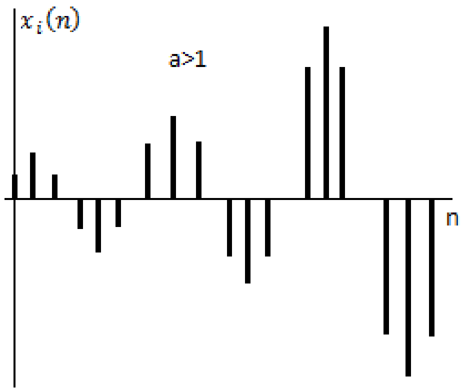
Exponentially decreasing Cosinusoidal signal



Exponentially growing Cosinusoidal signal



Exponentially decreasing sinusoidal signal



Exponentially growing sinusoidal signal