

**Figure 4.1.1 A Typical frequency response of an Amplifier**

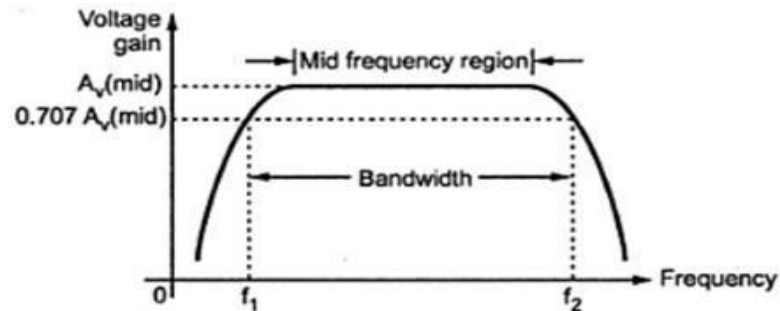
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To plot this curve, input voltage to the amplifier is kept constant and frequency of input signal is continuously varied. The output voltage at each frequency of input signal is noted and the gain of the amplifier is calculated. For an audio frequency amplifier, the frequency range is quite large from 20 Hz to 20 kHz. In this frequency response, the gain of the amplifier remains constant in mid-frequency while the gain varies with frequency in low and high frequency regions

of the curve. Only at low and high frequency ends, gain deviates from ideal characteristics. The decrease in voltage gain with frequency is called roll-off.

### Definition of cut-off frequencies and bandwidth:

The range of frequencies can be specified over which the gain does not deviate more than 70.7% of the maximum gain at some reference mid-frequency.



**Figure 4.1.2 Frequency Response, Half power frequencies and Bandwidth of an RC coupled amplifier**

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From above figure 4.1.2 , the frequencies  $f_1$  &  $f_2$  are called lower cut-off and upper cut-off frequencies. Bandwidth of the amplifier is defined as the difference between  $f_2$  &  $f_1$ .

$$\text{Bandwidth of the amplifier} = f_2 - f_1$$

The frequency  $f_2$  lies in high frequency region while frequency  $f_1$  lies in low frequency region. These two frequencies are also called as half-power frequencies since gain or output voltage drops to 70.7% of maximum value and this represents a power level of one half the power at the reference frequency in mid-frequency region.

## Low frequency analysis of amplifier to obtain lower cut-off frequency:

### 1. Decibel Unit:

The decibel is a logarithmic measurement of the ratio of one power to another or one voltage to another. Voltage gain of the amplifier is represented in decibels (dBs). It is given by,

$$\text{Voltage gain in dB} = 20 \log A_v$$

Power gain in decibels is given by,

$$\text{Power gain in dB} = 10 \log A_p$$

Where  $A_v$  is greater than one, gain is positive and when  $A_v$  is less than one, gain is negative. The positive and negative gain indicates that the amplification and attenuation respectively. Usually the maximum gain is called mid frequency range gain is assigned a 0 db value. Any value of gain below mid frequency range can be referred as 0 db and expressed as a negative db value.

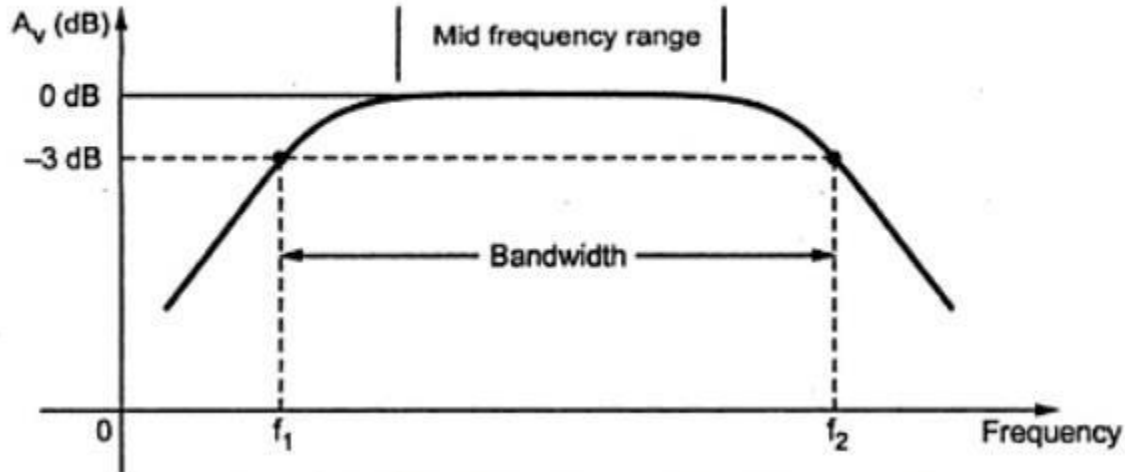
Assume that mid frequency gain of a certain amplifier is 100.

$$\text{Then, Voltage gain} = 20 \log 100 = 40 \text{ db}$$

$$\text{At } f_1 \text{ and } f_2 \text{ } A_v = 100/\sqrt{2} = 70.7$$

$$\text{Voltage gain at } f_1 = \text{Voltage gain at } f_2 = 20 \log 70.7 = 37 \text{ db}$$

From the figure 4.1.3 , it shows that the voltage gain at  $f_1$  and  $f_2$  is less than 3db of the maximum voltage gain. Due to this the frequencies  $f_1$  and  $f_2$  are also called as 3 db frequencies. At  $f_1$  &  $f_2$  power gain drops by 3 db.



**Figure 4.1.3 Normalized voltage Vs frequency**

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For all frequencies within the bandwidth, amplifier power gain is at least half of the maximum power gain. This bandwidth is also referred to as 3 dB bandwidth.

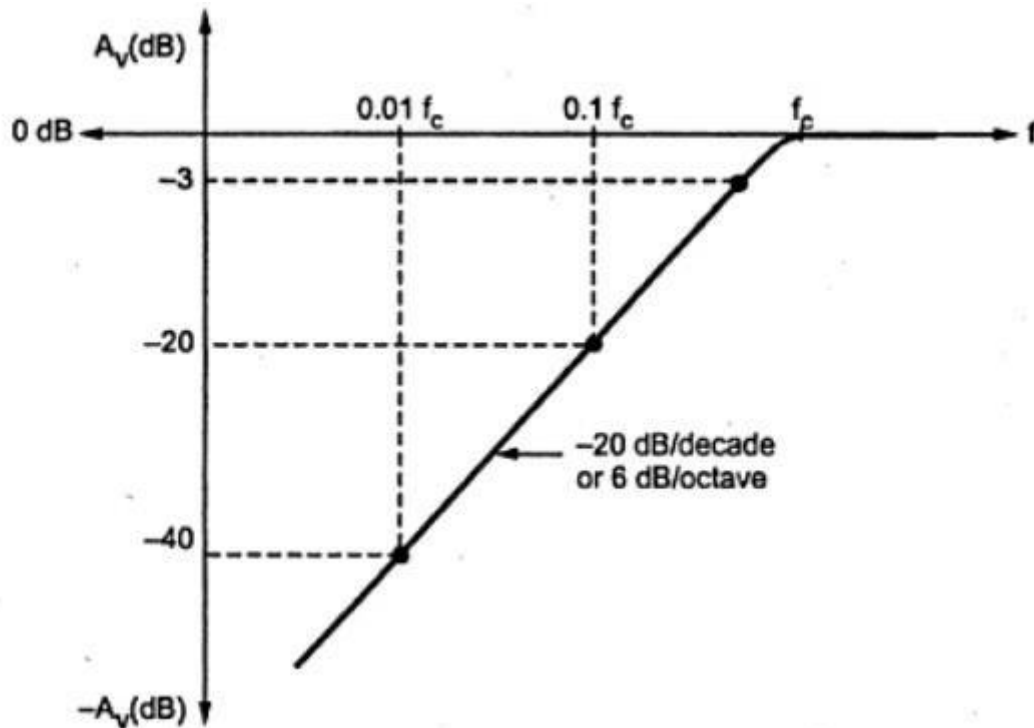
## 2. Significance of octaves and decades:

The octaves and decades are the measures of change in frequency. A ten times change in frequency is called a decade. Otherwise, an octave corresponds to a doubling or halving of the frequency.

Example:

An increase in frequency from 100 Hz to 200 Hz is an octave.

A decrease in frequency from 100 kHz to 50 kHz is also an octave



**Figure 4.1.4 Frequency Response showing significance of decade and Octave**

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At lower and higher frequencies the decrease in the gain of amplifiers is often indicated in terms of db/decades or db/octaves shown in figure 4.1.4. If the attenuation in gain is 20 db for each decade, then it is indicated by line having slope of 20 db/decade. A rate of -20 db/decade is approximately equivalent to -6db/octave. A rate of -40 db/decade is approximately equivalent to -12db/octave.

### 3. Midband gain:

It is defined as the band of frequencies between  $10 f_1$  and  $0.1 f_2$ . It is denoted as midband gain or  $A_{\text{mid}}$ . The voltage gain of the amplifier outside the midband is approximately given as,

$$A = \frac{A_{mid}}{\sqrt{1 + (f_1/f)^2} \sqrt{1 + (f/f_2)^2}}$$

In midband,

$$f_1/f \approx 0 \text{ and } f/f_2 \approx 0.$$

**Midband:**

$$A = A_{mid}$$

Below the midband,

$$f/f_2 \approx 0$$

As a result, the equation becomes,

**Below midband:**

$$A = \frac{A_{mid}}{\sqrt{1 + (f_1/f)^2}}$$

Above midband,

$$f_1/f \approx 0.$$

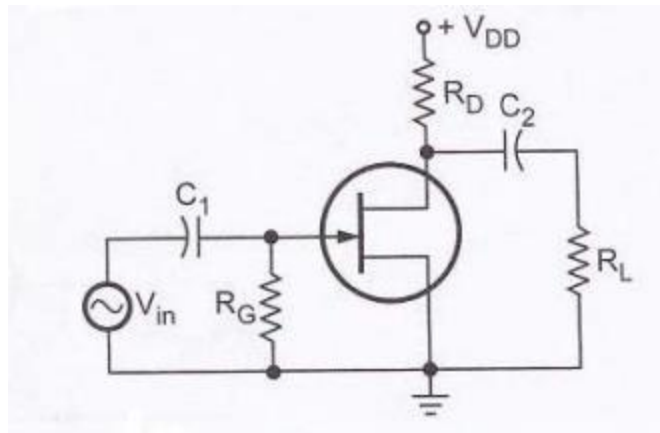
As a result, the equation becomes,

**Above midband:**

$$A = \frac{A_{mid}}{\sqrt{1 + (f/f_2)^2}}$$

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## Low frequency analysis of FET amplifier:



**Figure 4.5.1 Typical RC Coupled common source amplifier**

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From above figure 4.5.1, it has two RC networks that affect its gain as the frequency is reduced below midrange. These are,

1. RC network formed by the input coupling capacitor  $C_1$  and input impedance of the amplifier.
2. RC network formed by the output coupling capacitor and the output impedance looking in at the drain.

### Input RC network:

Lower critical frequency of this network is given as,

$$f_c = \frac{1}{2\pi R_{in} C_1}$$

where  $R_{in} = R_G \parallel R_{in(\text{gate})}$

The value of  $R_{in(\text{gate})}$  can be determined from the data sheet as follows:

$$R_{in(gate)} = \left| \frac{V_{GS}}{I_{GSS}} \right|$$

where  $I_{GSS}$  is the gate reverse current.

The phase shift in low frequency input RC circuit is

$$\theta = \tan^{-1} (X_{C1} / R_{in} )$$

**Output RC network:**

Lower critical frequency of this network is given as,

$$f_c = \frac{1}{2 \pi (R_D + R_L ) C_2}$$

The phase shift in low frequency output RC circuit is  $\theta = \tan^{-1} (X_{C2} / R_D + R_L)$

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## Effect of various capacitors on frequency response:

### 1. Effect of coupling capacitors

The reactance of the capacitor is

$$X_c = 1/2\pi f_c$$

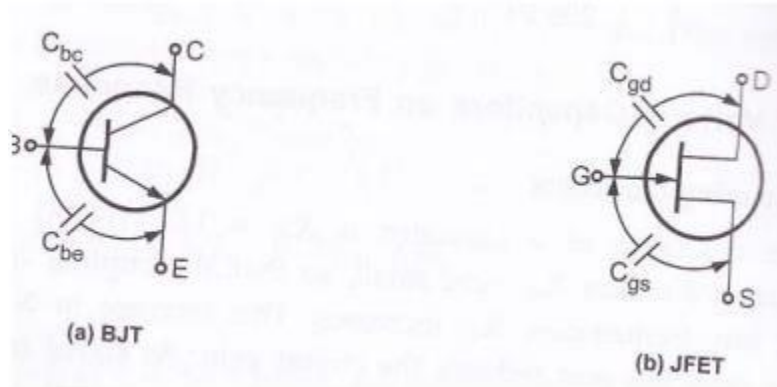
At medium and high frequencies, the factor  $f$  makes  $X_c$  very small, so that all coupling capacitors behave as short circuits. At low frequencies,  $X_c$  increases. This increase in  $X_c$  drops the signal voltage across the capacitor and reduces the circuit gain. As signal frequencies decrease, capacitor reactance increase and gain continues to fall, reducing the output voltage.

### 2. Effect of Bypass capacitors:

At lower frequencies, bypass capacitor  $C_E$  is not a short. So emitter is not at ac ground.  $X_c$  in parallel with  $R_E$  creates an impedance. The signal voltage drops across this impedance reducing the circuit gain.

### 3. Effect of internal transistor capacitances:

At high frequencies, coupling and bypass capacitors act as short circuit and do not affect the amplifier frequency response. At high frequencies, internal capacitances, commonly known as junction capacitances. The following figure shows the junction capacitances for both BJT and FET in figure 4.2.1. In case of BJT,  $C_{be}$  is the base emitter junction capacitance and  $C_{bc}$  is the base collector junction capacitance. In case of FET,  $C_{gs}$  is the internal capacitance between gate and source and  $C_{gd}$  is the internal capacitance between gate and drain.

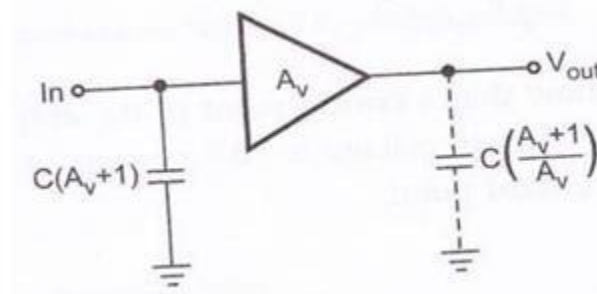


**Figure 4.2.1 Internal transistor capacitances**

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### Miller Theorem:

In transistor amplifiers, it is necessary to split the capacitance between input and output. It can be achieved by using miller's theorem. In the following figure 4.2.2,  $A_v$  represents absolute voltage gain of the amplifier at midrange frequencies and  $C$  represents either  $C_{bc}$  (incase of BJT) or  $C_{gd}$  (incase of FET).



**Figure 4.2.2 Splitting capacitor using Miller's Theorem**

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### Hybrid - $\pi$ equivalent circuits of BJTs:

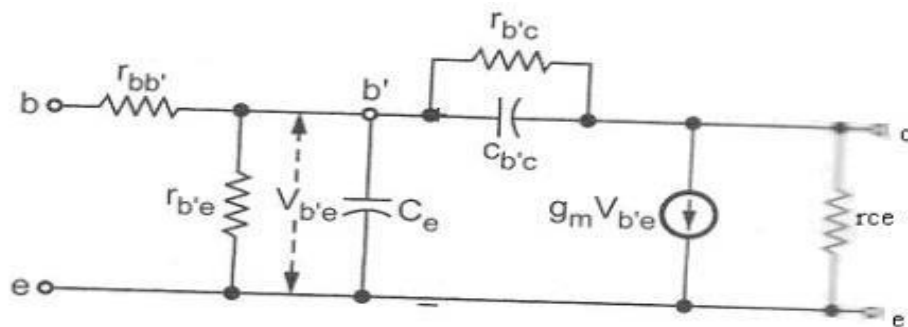
At low frequencies, we can analyze the transistor using h-parameters. But for high frequency, analysis of h-parameter model is not suitable for following reasons.

1. The values of h-parameters are not constant at high frequencies. So it is necessary to analyze transistor at each and every frequency which is impractical.
2. At high frequency h-parameters become complex in nature.

Due to the above reasons, modified T model and hybrid  $\pi$  models are used for high frequency analysis of the transistor. These models give a reasonable compromise between accuracy and simplicity to do high frequency analysis of the transistor.

### Hybrid - $\pi$ common emitter transistor model:

Common emitter circuit is most important practical configuration and this is useful for the analysis of transistor using hybrid -  $\pi$  model. The following figure shows the hybrid -  $\pi$  model for a transistor in CE configuration. For this model in figure 4.2.3, all parameters are assumed to be independent of frequency. But they may vary with the quiescent operating point.



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**Figure 4.2. 3 Hybrid pi model for a transistor in CE Configuration**

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### Elements in hybrid – $\pi$ model:

**Cb'e and Cb'c** :Forward biased PN junction exhibits a capacitive effect called diffusion capacitance. This capacitive effect of normally forward biased base-emitter junction of the transistor is represented by Cb'e or Ce. The diffusion

capacitance is connected between b' and e represents the excess minority carrier storage in the base.

The reverse bias PN junction exhibits a capacitive effect called transition capacitance. This capacitive effect of normally reverse biased collector base junction of the transistor is represented by  $C_{b'e}$  or  $C_c$ .

**$r_{bb'}$** : The internal node b' is physically not accessible bulk node b represents external base terminal.

**$r_{b'e}$** : It is the portion of the base emitter which may be thought of as being in series with the collector junction. This establishes a virtual base b' for junction capacitances to be connected instead of b.

**$r_{b'c}$** : Due to early effect, varying voltages across collector to emitter junction results in base-width modulation. A change in the effective base-width causes the emitter current to change. This feedback effect between output and input is taken into account by connecting  $r_{b'c}$  or  $r_{b'e}$  between b' and c.

**$g_m$** : Due to small changes in voltage  $V_{b'e}$  across emitter junction, there is excess minority carrier concentration injected into the base which is proportional to  $V_{b'e}$ . So resulting small signal collector current with collector shorted to the emitter is also proportional to  $V_{b'e}$ .

$g_m$  is also called as transconductance and it is given as,

$$g_m = \frac{\Delta I_c}{\Delta V_{b'e}} \text{ at a constant } V_{CE}$$

**$r_{ce}$** : It is the output resistance. It is also the result of early effect.

**Hybrid –  $\pi$  parameter values:**

The following table shows the typical values for hybrid -  $\pi$  parameters at room temperature and for  $I_c = 1.3\text{mA}$ .

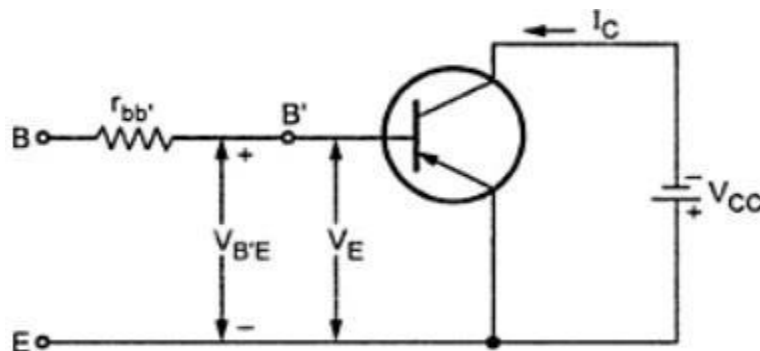
Parameter	Meaning	Value
$g_m$	Mutual conductance	50mA/V
$r_{bb'}$	Base spreading resistance	100 $\Omega$
$r_{b'e}$ OR $g_{b'e}$	Resistance between b' and e	1k $\Omega$
	Conductance between b' and e	1m mho
$r_{b'c}$ OR $g_{b'c}$	Resistance of reverse biased PN junction between base and collector	4M $\Omega$
	Conductance of reverse biased PN junction between base and collector	0.25*10 <sup>-6</sup> mho
$r_{ce}$ OR $g_{ce}$	Output resistance between c and e	80k $\Omega$
	Conductance between c and e	12.5*10 <sup>-6</sup> mho
$C_e$	Junction capacitance between b and e	100pF
$C_c$	Junction capacitance between base and collector	3pF

**Hybrid –  $\pi$  conductances:**

**Transistor Transconductance  $g_m$ :**

Let us consider a p-n-p transistor in CE configuration with  $V_{cc}$  bias in the collector circuit as shown in the above figure 4.2.4.

Transconductance  $g_m$  is given as,



**Figure 4.2. 4** Pertaining to the derivation of  $g_m$

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The collector current in active region is given as,

$$g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{V_{CE}}$$

The collector current in active region is given as,

$$I_C = I_{CO} - \alpha I_E$$

$$\partial I_C = \alpha \partial I_E \quad \because I_{CO} = \text{constant.}$$

Substituting value of  $\partial I_C$

$$g_m = \alpha \frac{\partial I_E}{\partial V_{BE}} = \alpha \frac{\partial I_E}{\partial V_E} \quad \because V_E = V_{BE}$$

The emitter diode resistance,  $r_e$  is given as,

$$r_e = \frac{\partial V_E}{\partial I_E}$$

$$\frac{1}{r_e} = \frac{\partial I_E}{\partial V_E}$$

Substituting  $r_e$  in place of  $\partial I_E / \partial V_E$  we get,

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Substituting  $r_e$  in place of  $\partial I_E / \partial V_E$  we get,

$$g_m = \frac{\alpha}{r_e}$$

The emitter diode is a forward biased diode and its dynamic resistance is given as,

$$r_e = \frac{V_T}{I_E}$$

The emitter diode is a forward biased diode and its dynamic resistance is given as,

$$r_e = \frac{V_T}{I_E}$$

where  $V_T$  is the "volt equivalent of temperature", defined by

$$V_T = \frac{KT}{q}$$

where  $K$  is the Boltzmann constant in joules per degree kelvin ( $1.38 \times 10^{-23} \text{J/}^\circ\text{K}$ ) is the electronic charge ( $1.6 \times 10^{-19} \text{C}$ ).

Substituting value of  $r_e$  in equation (3) we get,

$$g_m = \frac{\alpha I_E}{V_T} = \frac{I_{CO} - I_C}{V_T} \quad \because I_C = I_{CO} - \alpha I_E$$

For p-n-p transistor  $I_C$  is negative. For an n-p-n transistor  $I_C$  is positive, but the foregoing analysis (with  $V_E = +V_{BE}$ ) leads to  $g_m = (I_C - I_{CO}) / V_T$ .

Hence, for either type of transistor,  $g_m$  is positive.

$$g_m = \frac{I_C - I_{CO}}{V_T} \quad \because I_C \gg I_{CO}$$

Input Conductance  $g_b'e$

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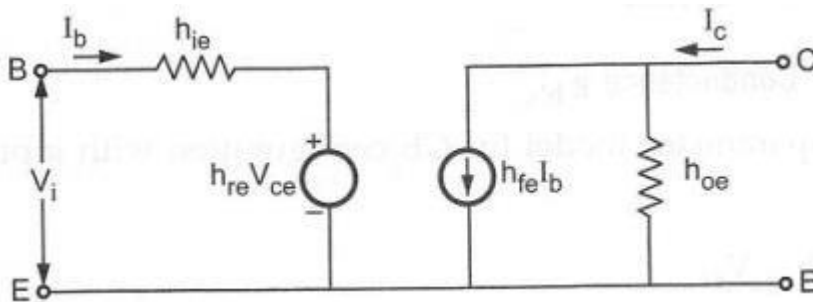
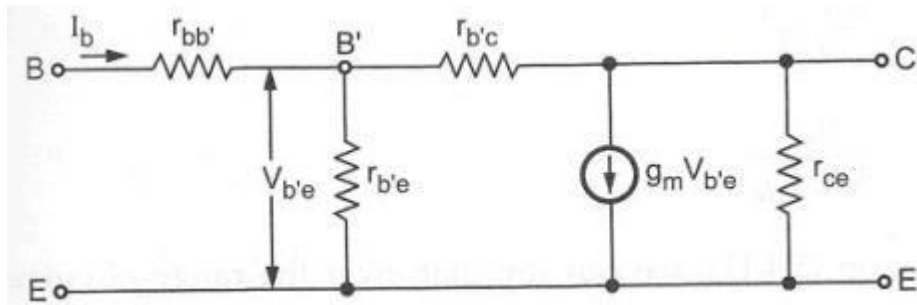


Figure 4.2.5a Hybrid pi model for CE at low frequency

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**Figure 4.2. 5b h parameter model for CE at low frequency**

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First consider h-parameter model for CE configuration in figure 4.2.5 a and b.

Applying KCL to output circuit,

$$I_C = h_{fe} I_b + h_{oe} V_{ce}$$

Making  $V_{ce} = 0$ , the short circuit current gain  $h_{fe}$  is defined as

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$$h_{fe} = \frac{I_C}{I_b}$$

$$\begin{aligned} I_C &= g_m V_{b'e} \\ &= g_m I_b r_{b'e} & \therefore V_{b'e} = I_b r_{b'e} \\ \frac{I_C}{I_b} &= g_m r_{b'e} \end{aligned}$$

Substituting the value of  $I_C / I_b$ ,

$$\begin{aligned} h_{fe} &= g_m r_{b'e} \\ \text{or} \\ r_{b'e} &= \frac{h_{fe}}{g_m} \quad \text{or} \quad g_{b'e} = \frac{g_m}{h_{fe}} \end{aligned}$$

$$g_m = I_C / V_T$$

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$$\begin{aligned} \therefore r_{b'e} &= \frac{h_{fe} V_T}{|I_C|} \\ \text{or} \quad g_{b'e} &= \frac{|I_C|}{V_T h_{fe}} \end{aligned}$$

Let us consider h-parameter model for CE configuration in figure 4.2.6 with input open circuit ( $I_b = 0$ ),  $V_i$  is given as,

$$V_i = h_{re} V_{ce}$$

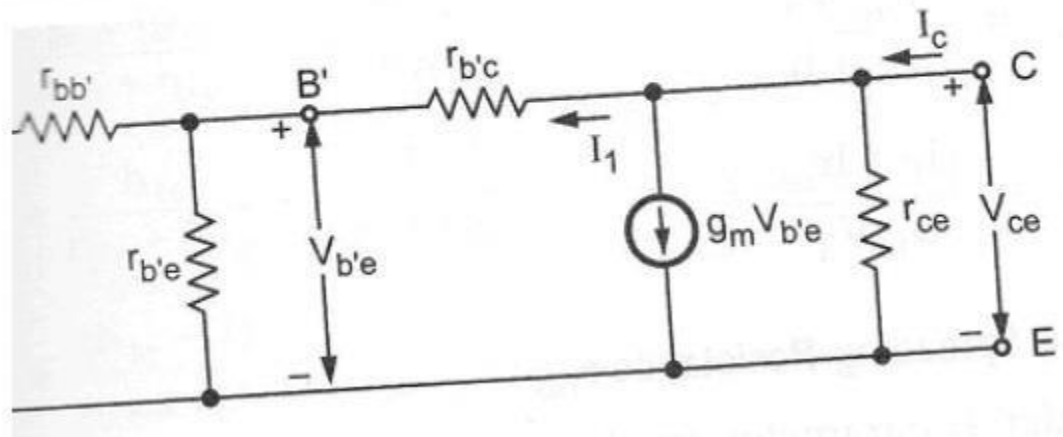


Figure 4.2. 6 h-parameter model for CE configuration with input open circuit ( $I_b = 0$ ),

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Fig. Hybrid  $\pi$  model for CE configuration With

$I_b = 0$ ,  $V_{ce}$  is given as,

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$$V_{ce} = I_1 (r_{b'c} + r_{b'e})$$

$$I_1 = \frac{V_{ce}}{r_{b'c} + r_{b'e}}$$

Voltage between b' and e,  $V_{b'e}$  can be given as,

$$V_{b'e} = I_1 r_{b'e}$$
$$= r_{b'e} \frac{V_{ce}}{r_{b'c} + r_{b'e}}$$

With  $I_b = 0$ ,  
 $V_i = V_{b'e}$

$$= \frac{r_{b'e} V_{ce}}{r_{b'c} + r_{b'e}}$$

Substituting the value of  $r_{b'e}$ ,

Substituting the value of  $V_i$ ,

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$$h_{re} V_{ce} = \frac{r_{b'e} V_{ce}}{r_{b'c} + r_{b'e}}$$
$$= \frac{r_{b'e}}{r_{b'c} + r_{b'e}}$$
$$= h_{re} r_{b'c} + h_{re} r_{b'e}$$
$$\left( \frac{1 - h_{re}}{h_{re}} \right) r_{b'e}$$
$$r_{b'c} = \frac{r_{b'e}}{h_{re}} \quad \because 1 - h_{re} \approx 1$$
$$g_{b'c} = \frac{h_{re}}{r_{b'e}} = h_{re} g_{b'e}$$

$$r_{b'e} = \frac{h_{fe} V_T}{|I_C| h_{re}}$$

$$g_{b'e} = \frac{|I_C| h_{re}}{h_{fe} V_T}$$

**Base Spreading Resistance  $r_{bb'}$ :**

$$h_{ie} = r_{bb'} + r_{b'e}$$

$$r_{bb'} = h_{ie} - r_{b'e}$$

Substituting the value of  $r_{b'e}$ ,

$$r_{bb'} = h_{ie} - \frac{h_{fe} V_T}{I_C}$$

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**Output Resistance  $g_{ce}$ :**

Using h-parameters output conductance is given as,

$$h_{oe} = \frac{I_C}{V_{ce}}$$

Applying KCL to the output circuit,

$$I_C = \frac{V_{ce}}{r_{ce}} + g_m V_{b'e} + I_1$$

$$1/r_{ce} = g_{ce} = h_{oe} - g_{b'c} h_{fe}$$

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**Relation between hybrid- $\pi$  and h-parameters:**

Sr. No.	Parameter relation
1.	$g_m = \frac{I_C}{V_T}$
2.	$r_{b'e} = \frac{h_{fe}}{g_m}$
3.	$r_{bb'} = h_{ie} - r_{b'e}$
4.	$r_{b'c} = \frac{r_{b'e}}{h_{re}}$
5.	$g_{ce} = \frac{1}{r_{ce}} = h_{oe} - g_{b'c} h_{fe}$

**Hybrid –  $\pi$  capacitances:**

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$$C_e = C_{De} + C_{Te} \approx C_{De}$$

$$C_e = \frac{gm}{2\pi f_t}$$

$$C_e = gm/2\pi f_t$$

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## High frequency analysis of FET

### Common source amplifier at high frequencies:

Let  $Y_L$  is admittance corresponds to load Resistor  $R_L$ ,

$Y_{DS}$  admittance corresponds to  $C_{DS}$

$g_d$  Conductance corresponds to  $r_d$

$Y_{GD}$  is admittance corresponds to load Resistor  $C_{GD}$

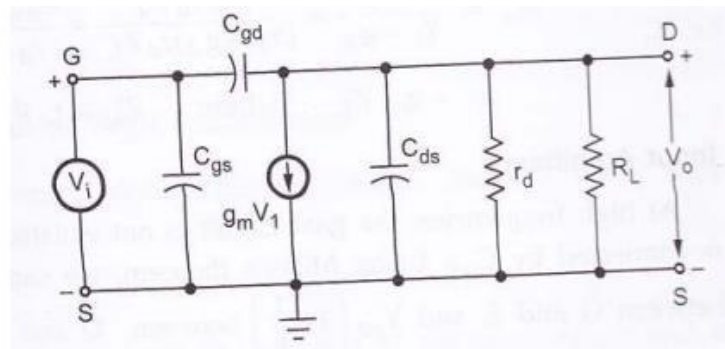


Figure 4.7.1 Small Signal Equivalent Circuit at high Frequencies

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$$Y = \frac{1}{Z} = Y_L + Y_{ds} + g_d + Y_{gd}$$

where

$$Y_L = \frac{1}{R_L}$$

$$Y_{ds} = j\omega C_{ds}$$

$$g_d = \frac{1}{r_d}$$

$$Y_{gd} = j\omega C_{gd}$$

$$I = -g_m V_i + V_i Y_{gd} = V_i (-g_m + Y_{gd})$$

### Voltage gain:

The voltage gain for common source amplifier circuit with the load  $R_L$  is given by,

$$A_v = \frac{V_o}{V_i} = \frac{IZ}{V_i} = \frac{I}{V_i Y}$$

Substituting the values of I and Y from equations (2) and (3) we have,

$$A_v = \frac{-g_m + Y_{gd}}{Y_L + Y_{ds} + g_d + Y_{gd}}$$

At low frequencies,  $Y_{ds}$  and  $Y_{gd} = 0$  and hence equation (4) reduces to

$$\begin{aligned} A_v &= \frac{-g_m}{Y_L + g_d} = \frac{-g_m r_d Z_L}{(Y_L + g_d)r_d Z_L} = \frac{-g_m r_d Z_L}{r_d + Z_L} \\ &= -g_m Z'_i \quad \text{where } Z'_i = r_d \parallel Z_L \end{aligned}$$

### Input Admittance:

$$Y_i = Y_{gs} + (1 - A_v) Y_{gd}$$

### Input capacitance (Miller Effect):

$$A_v = -g_m R'_d \quad \text{where } R'_d = r_d R_d$$

Substituting the value of  $A_v$

$$\frac{Y_i}{j\omega} \equiv C_i = C_{gs} + (1 + g_m R'_d) C_{gd}$$

This increase in input capacitance  $C_i$  over the capacitance from gate to source is called Miller effect.



This input capacitance affects the gain at high frequencies in the operation of cascaded amplifiers. In cascaded amplifiers, the output from one stage is used as the input to a second amplifier. The input impedance of a second stage acts as a shunt across output of the first stage and  $R_d$  is shunted by the capacitance  $C_i$ .

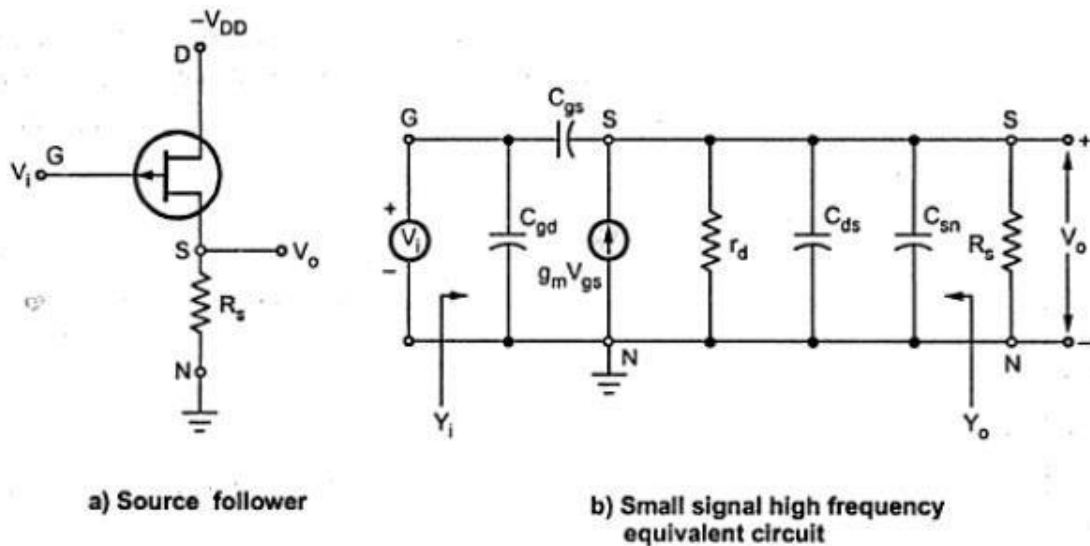
**Output Admittance:**

From above figure, the output impedance is obtained by looking into the drain with the input voltage set equal to zero. If  $V_i = 0$  in figure,  $r_d$ ,  $C_{ds}$  and  $C_{gd}$  in parallel. Hence the output admittance with  $R_L$  considered external to the amplifier is given by

$$Y_o = g_d + Y_{ds} + Y_{gd}$$

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**Common Drain Amplifier at High Frequencies:**



**Figure 4.7.2 a Common Drain Amplifier Circuit (Source Follower) & 4.7.2 b, Small signal equivalent circuit at high frequencies**

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Figure 4.7.2 a Common Drain Amplifier Circuit (Source Follower) & 4.7.2 b, Small signal equivalent circuit at high frequencies.

### Voltage gain:

The output voltage  $V_o$  can be found from the product of the short circuit and the impedance between terminals S and N. Voltage gain is given by,

$$\frac{V_o}{V_i} = \frac{g_m + j\omega C_{gs}}{R_s + (g_m + g_d + j\omega C_T)}$$

where  $C_T = C_{gs} + C_{ds} + C_{sn}$

$$A_v = \frac{(g_m + j\omega C_{gs})R_s}{1 + (g_m + g_d + j\omega C_T)R_s}$$

At low frequencies the gain reduces to

$$A_v = \frac{g_m R_s}{1 + (g_m + g_d)R_s}$$

### Input Admittance:

Input Admittance  $Y_i$  can be obtained by applying Miller's theorem to  $C_{gs}$ .

It is given by,

because  $Y_i = j\omega C_{gd} + j\omega C_{gs}(1 - A_v) \approx j\omega C_{gd}$   
 $A_v \approx 1.$

### Output Admittance:

Output Admittance  $Y_o$  with  $R_s$  considered external to the amplifier, it is given by

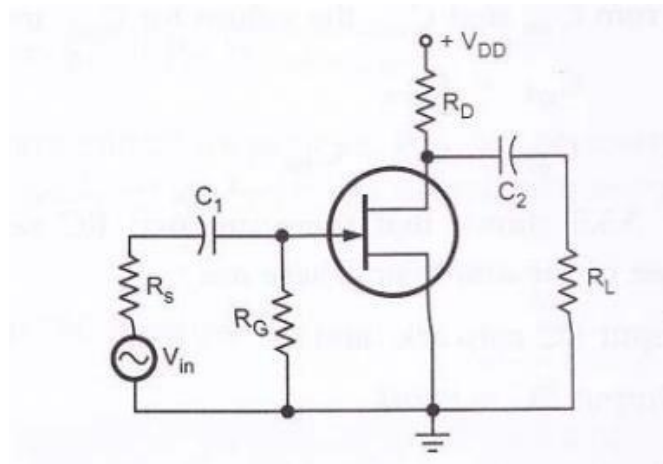
$$Y_o = g_m + g_d + j\omega C_T$$

At low frequencies, output resistance  $R_o$  is given by,

$$R_o = \frac{1}{g_m + g_d} \approx \frac{1}{g_m}$$

since  $g_m \gg g_d$

### Frequency Response of Common Source Amplifier:



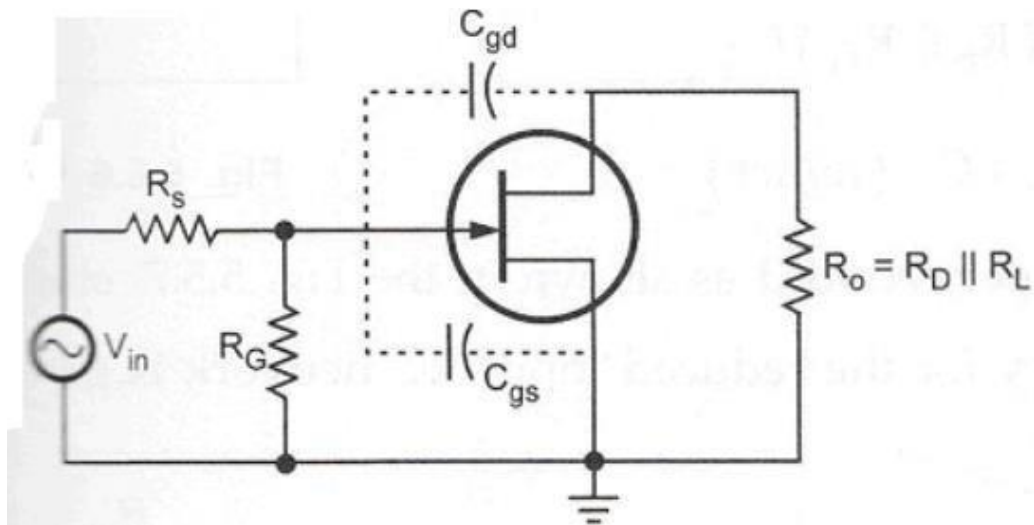
**Figure 4.7.3** A typical RC Coupled common source amplifier

*Diagram Source Brain Kart*

Let us consider a typical common source amplifier as shown in the above figure 4.7.3.

**Figure 4.7.4** High Frequency Equivalent Circuit

*Diagram Source Brain Kart*



From above figure 4.7.4, it shows the high frequency equivalent circuit for the given amplifier circuit. It shows that at high frequencies coupling and bypass capacitors act as short circuits and do not affect the amplifier high frequency response. The equivalent circuit shows internal capacitances which affect the high frequency response.

Using Miller theorem, this high frequency equivalent circuit can be further simplified as follows:

The internal capacitance  $C_{gd}$  can be splitted into  $C_{in(miller)}$  and  $C_{out(miller)}$  as shown in the following figure.

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#### Simplified high frequency equivalent circuit

$$\begin{aligned} C_{in(miller)} &= C_{gd} (A_v + 1) \\ C_{out(miller)} &= C_{gd} \frac{(A_v + 1)}{(A_v)} \end{aligned}$$

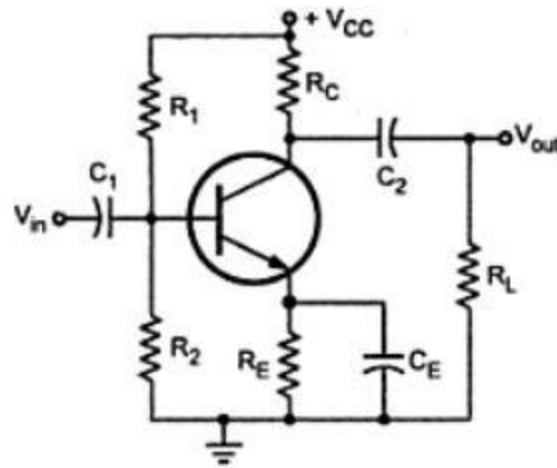
Where

$$C_{gd} = C_{rss}$$

$$C_{gs} = C_{iss} - C_{rss}$$

From simplified high frequency equivalent circuit, it has two RC networks which affect the high frequency response of the amplifier. These are Input RC network and Output RC network

## Low frequency analysis of BJT:



**Figure 4.3.1 Typical RC coupled Common Emitter Amplifier**

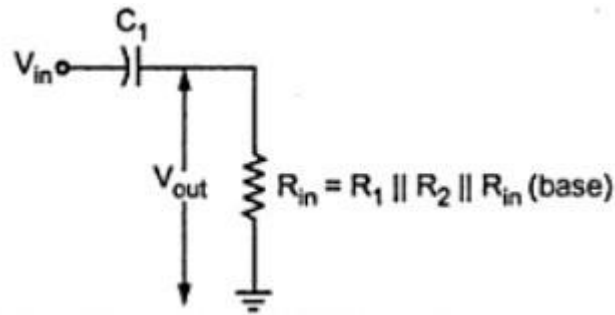
*Diagram Source Brain Kart*

From above figure 4.3.1, it has three RC networks that affect its gain as the frequency is reduces below midrange. These are RC network formed by the input coupling capacitor  $C_1$  and input impedance of the amplifier. RC network formed by the output coupling capacitor  $C_2$ , resistance looking in at the collector and load resistance. RC network formed by the emitter bypass capacitor  $C_E$  and resistance looking in at the emitter.

### Input RC network:

The following figure 4.3.2 shows the input RC network formed by  $C_1$  and the input impedance of the amplifier.

The resistance value is  $R_{in} = R_1 \parallel R_2 \parallel R_{in}(\text{base})$



**Figure 4.3.2 Voltage Divider Rule**

*Diagram Source Brain Kart*

Applying voltage divider rule,

$$V_{out} = \left( \frac{R_{in}}{\sqrt{R_{in}^2 + X_{C1}^2}} \right) V_{in}$$

A critical point in the amplifier response is generally accepted to occur when the output voltage is 70.7 % of the input. At critical point,

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$$\frac{R_{in}}{\sqrt{R_{in}^2 + X_{C1}^2}} = 0.707 = \frac{1}{\sqrt{2}}$$

At this condition,  $R_{in} = X_{C1}$ . Overall gain is reduced due to attenuation provided by the input RC network. The reduction in overall gain is given by,

$$A_v = 20 \log \left( \frac{V_{out}}{V_{in}} \right) = 20 \log ( 0.707 ) = -3 \text{ dB}$$

The frequency  $f_c$  at this condition is called lower critical frequency and it is given by,

$$f_c = \frac{1}{2\pi R_{in} C_1}$$

where

$$R_{in} = R_1 \parallel R_2 \parallel h_{ie}$$

$$\therefore f_c = \frac{1}{2\pi (R_1 \parallel R_2 \parallel h_{ie}) C_1}$$

If the resistance of input source is taken into account the above equation becomes,

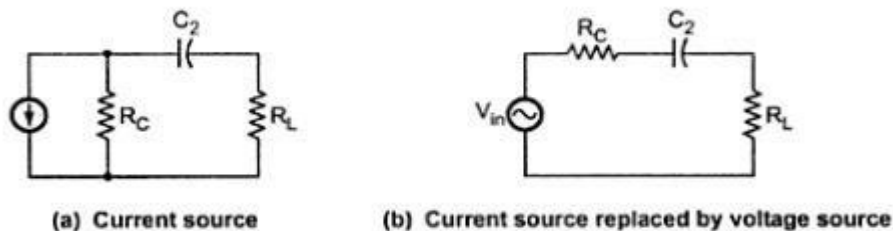
$$f_c = \frac{1}{2\pi (R_s + R_{in}) C_1}$$

The phase angle in an input RC circuit is expressed as

$$\theta = \tan^{-1} \left( \frac{X_{C1}}{R_{in}} \right)$$

**Output RC network:**

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**Figure 4.3.3 a Current Source & 4.3.3 b Current source replaced by Voltage Source**

*Diagram Source Brain Kart*

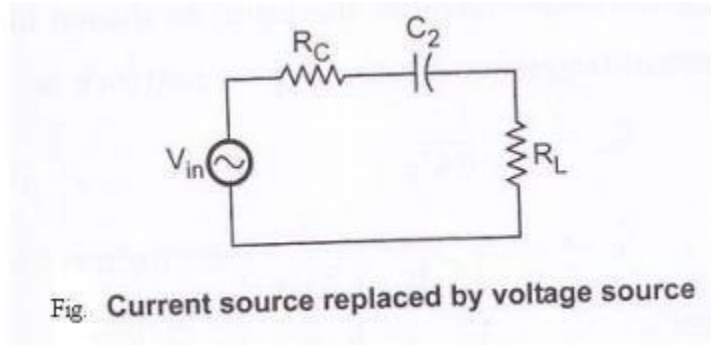
The above figure 4.3.3 a and b shows Current Source & 4.3.3 b Current source replaced by Voltage Source the output RC network formed by  $C_2$ , resistance looking in at the collector and load resistance. The critical frequency for this RC network is given by,

$$f_c = \frac{1}{2\pi (R_C + R_L) C_2}$$

The phase angle in output RC network is given as,

$$\theta = \tan^{-1} \left( \frac{X_{C2}}{R_C + R_L} \right)$$

**Bypass network:**



**Figure 4.3.4 Current Source replaced by Voltage Source**

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From above figure 4.3.4

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$$\left[ \frac{h_{ie} + R_{TH}}{\beta} \right]$$

is the resistance looking in at the emitter. It is derived as follows,  $R = (V_b / \beta I_b) + h_{ie} / \beta$

$$= \frac{I_b R_{TH}}{\beta I_b} + \frac{h_{ie}}{\beta} = \frac{R_{TH} + h_{ie}}{\beta}$$

Where  $R_{TH} = R_1 \parallel R_2 \parallel R_s$ . It is the thevenin's equivalent resistance looking from the base of the transistor towards the input.

The critical frequency for the bypass network is



$$f_c = \frac{1}{2\pi R C_E}$$

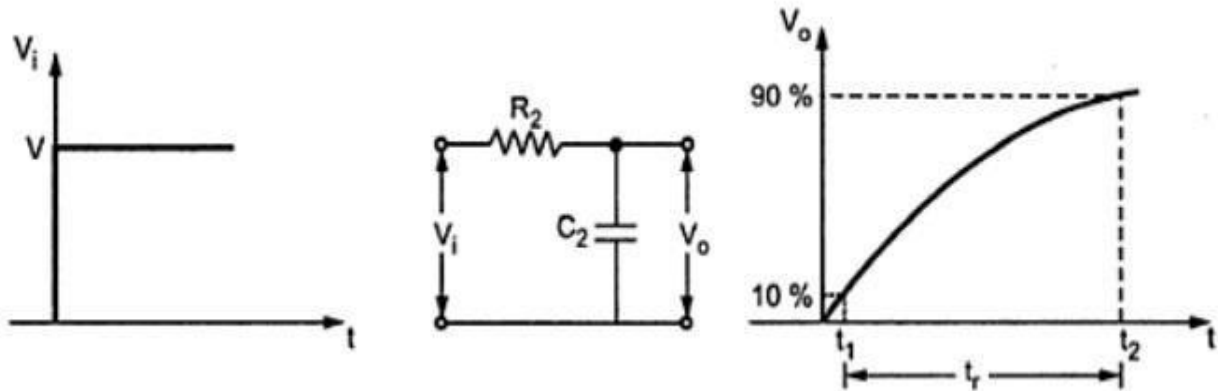
or

$$f_c = \frac{1}{2\pi \left[ \left( \frac{h_{ie} + R_{TH}}{\beta} \right) \parallel R_E \right] C_E}$$

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### Rise time and its Relation to Upper Cut-off Frequency:

#### Upper 3 dB Frequency:



**Figure 4.8.1 Calculation of Rise Time and Fall Time**

*Diagram Source Brain Kart*

When a step input is applied, amplifier high frequency RC network prevent the output from responding immediately to the step input. The output voltage starts from zero and rises towards the steady state value  $V$ , with a time constant  $R_2C_2$  as shown in the above figure 4.8.1.

The output voltage is given by,

$$V_o = V(1 - e^{-t/R_2 C_2})$$

The time required for  $V_o$  to reach one-tenth of its final value is calculated as,

The time required for  $V_o$  to reach one-tenth of its final value is calculated as,

$$0.1 V = V (1 - e^{-t_1/R_2 C_2})$$

$$\therefore 0.1 = 1 - e^{-t_1/R_2 C_2}$$

$$\therefore 0.9 = e^{-t_1/R_2 C_2}$$

$$\therefore \frac{t_1}{R_2 C_2} = 0.1$$

$$\therefore t_1 = 0.1 R_2 C_2$$

Similarly, the time required for  $V_o$  to reach nine-tenths of its final value is calculated as,

$$0.9 V = V (1 - e^{-t_2/R_2 C_2})$$

$$\therefore 0.9 = 1 - e^{-t_2/R_2 C_2}$$

$$\therefore 0.1 = e^{-t_2/R_2 C_2}$$

$$\therefore \frac{t_2}{R_2 C_2} = 2.3$$

The difference between these two values is called as rise time  $t_r$  of the circuit. The rise time is given as

$$\begin{aligned} t_r &= t_2 - t_1 = 2.3 R_2 C_2 - 0.1 R_2 C_2 \\ &= 2.2 R_2 C_2 \end{aligned}$$

The Upper 3dB frequency is given as,

$$f_H = \frac{1}{2\pi R_2 C_2}$$

Upper 3dB frequency in terms of rise time is given as,

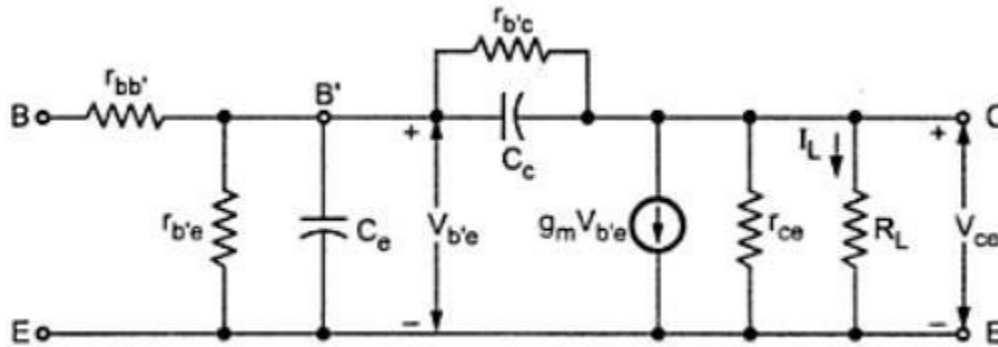
$$f_H = \frac{2.2}{2\pi t_r} = \frac{0.35}{t_r}$$

From above equation, it shows that upper 3dB frequency is inversely proportional to the rise time  $t_r$ .

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**CE short circuit current gain using hybrid-  $\pi$  model:**

Figure 4.6.1 shows the Hybrid-  $\pi$  model for a single transistor with a resistive load  $R_L$ .



**Figure 4.6.1 Hybrid-  $\pi$  model for a single transistor with a resistive load  $R_L$**

*Diagram Source Brain Kart*

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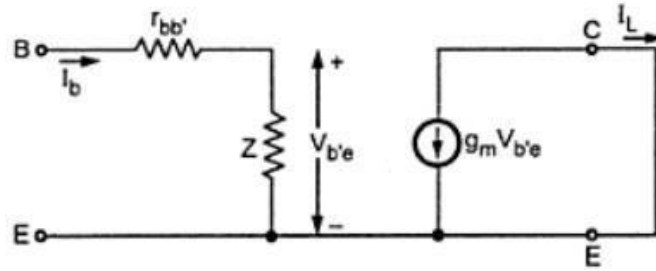
Miller capacitance is  $C_M = C_{b'c} (1 + g_m R_L)$

Here,  $R_L = 0$

$$\therefore C_M = C_{b'c} (C_c)$$

Parallel combination of  $r_{b'e}$ , and  $(C_e + C_c)$  is given as

$$\begin{aligned} Z &= \frac{r_{b'e} \times \frac{1}{j\omega(C_e + C_c)}}{r_{b'e} + \frac{1}{j\omega(C_e + C_c)}} \\ &= \frac{r_{b'e}}{1 + j\omega r_{b'e} (C_e + C_c)} \end{aligned}$$



**Figure 4.6.2 Simplified Hybrid pi model**

*Diagram Source Brain Kart*

$$V_{b'e} = I_b Z$$

$$Z = V_{b'e} / I_b$$

The current gain for the circuit figure 4.6.2 is,

$$A_i = \frac{I_L}{I_b} = \frac{-g_m V_{b'e}}{I_b} \quad \because I_L = -g_m V_{b'e}$$

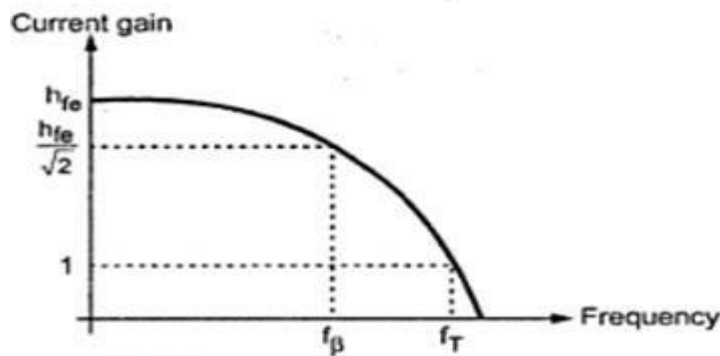
$$A_i = -g_m Z$$

$$= \frac{-g_m r_{b'e}}{1 + j\omega r_{b'e} (C_e + C_c)}$$

$$A_i = \frac{-h_{fe}}{1 + j\omega r_{b'e} (C_e + C_c)}$$

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Figure 4.6.3 shows the Frequency Vs Current Gain



**Figure 4.6.3 Frequency Vs Current Gain**

*Diagram Source Brain Kart*

$$f_{\beta} = \frac{1}{2\pi r_{b'e} (C_e + C_c)}$$
$$A_i = \frac{-h_{fe}}{1 + j\frac{f}{f_{\beta}}}$$
$$|A_i| = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f}{f_{\beta}}\right)^2}}$$

**$f_{\beta}$  (Cutoff frequency):**

It is the frequency at which the transistor short circuit CE current gain drops by 3dB or  $1/\sqrt{2}$  times from its value at low frequency. It is given as,

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$$f_{\beta} = \frac{1}{2\pi r_{b'e} (C_e + C_c)}$$

or

$$= \frac{g_{b'e}}{2\pi (C_e + C_c)}$$

or

$$= \frac{1}{h_{fe}} \frac{g_m}{2\pi (C_e + C_c)} \quad \because g_{b'e} = \frac{1}{r_{b'e}} = \frac{g_m}{h_{fe}}$$

**$f_{\alpha}$  (Cut-off frequency):**

It is the frequency at which the transistor short circuit CB current gain drops by 3dB or  $1/\sqrt{2}$  times from its value at low frequency.

The current gain for CB configuration is given as,

$$A_i = \frac{-h_{fb}}{1 + j \frac{f}{f_\alpha}}$$

where

$$f_\alpha = \frac{1}{2\pi r_{b'e} (1 + h_{fb}) C_e}$$
$$= \frac{1 + h_{fe}}{2\pi r_{b'e} C_e} \approx \frac{h_{fe}}{2\pi r_{b'e} C_e}$$
$$|A_i| = \frac{h_{fb}}{\sqrt{1 + \left(\frac{f}{f_\alpha}\right)^2}}$$

At  $f = f_\alpha$

$$|A_i| = \frac{h_{fb}}{\sqrt{2}}$$

The ratio of  $f_T / f_\beta$  is quite large compared to 1.

$$f_T = g_m / 2\pi C_e$$

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$$1 = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f_T}{f_\beta}\right)^2}}$$



**Current gain with resistive load:**

$$C_{eq} = C_e + C_c (1 + g_m R_L)$$

For further simplification in figure 4.6.4,

At output circuit value of  $C_c$  can be calculated as,

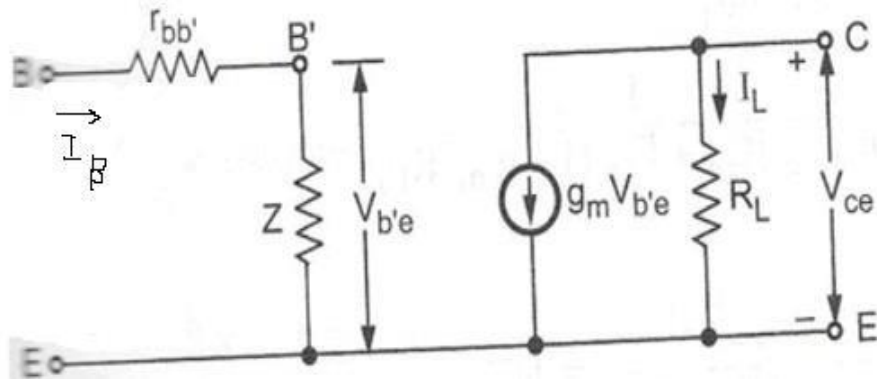


Figure 4.6.4. Simplified hybrid –  $\pi$  model for CE with  $R_L$

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$$\frac{1}{\frac{j\omega C_c}{k-1}} \approx \frac{1}{j\omega C_c}$$

$$C_c \left( \frac{k}{k-1} \right) \approx C_c$$

$$Z = \frac{V_{b'e}}{I_b}$$

$$A_i = \frac{-h_{fe}}{1 + j\left(\frac{f}{f_H}\right)}$$

$$|A_i| = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}$$

At  $f = f_H$   
 $A_i = \frac{h_{fe}}{\sqrt{2}}$

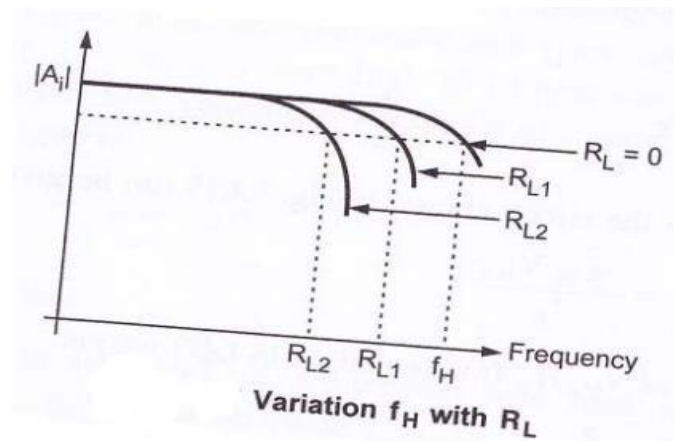
$f_H$  is the frequency at which the transistor gain drops by 3dB or  $1/\sqrt{2}$  times from its value at low frequency in figure 4.6.5. It is given as

$$f_H = \frac{1}{2\pi r_{b'e} C_{eq}}$$

$$= \frac{1}{2\pi r_{b'e} [C_e + C_c (1 + g_m R_L)]}$$

At  $R_L = 0$   
 $f_H = \frac{1}{2\pi r_{b'e} [C_e + C_c]} = f_{\beta}$

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**Figure 4.6.5. Variation  $f_H$  with  $R_L$**

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**Current gain including source resistance:**

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$$\frac{I_L}{I_s} = \frac{-g_m r_{b'e} R_s}{R_s + r_{bb'} + r_{b'e}}$$

$A_{is}$  at low frequency =

$$= \frac{-h_{fe} R_s}{R_s + h_{ie}}$$

**Voltage gain including source resistance:**

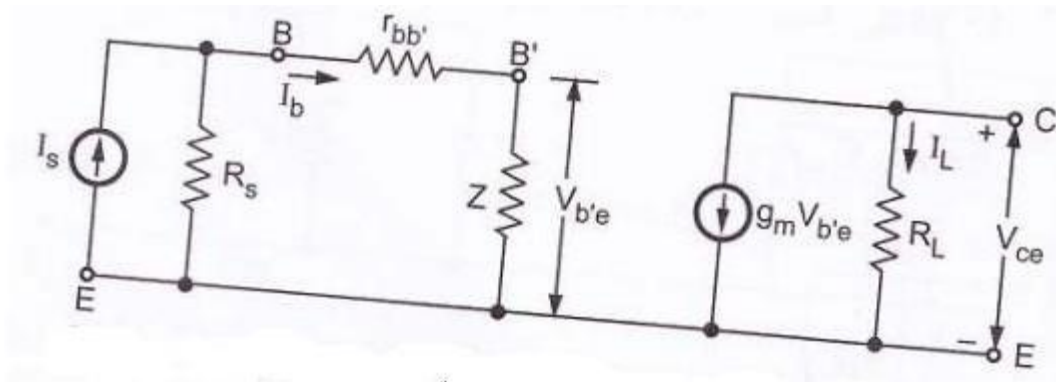
$$A_{vs} = \frac{V_o}{V_s} = \frac{I_L}{I_s} \frac{R_L}{R_s} = \frac{-g_m Z R_s}{R_s + r_{bb'} + Z} \times \frac{R_L}{R_s}$$

$$= \frac{-g_m Z R_L}{R_s + r_{bb'} + Z}$$

$$A_{vs \text{ low}} = \frac{I_L}{I_s} \frac{R_L}{R_s} = \frac{-h_{fe} R_s}{R_s + h_{ie}} \times \frac{R_L}{R_s}$$

$$= \frac{-h_{fe} R_L}{R_s + h_{ie}}$$

Figure 4.6.6. shows hybrid pi Equivalent circuit with current source



**Figure 4.6.6. Equivalent circuit with current source**

**Cutoff frequency including source resistance:**

$$A_{is \text{ high}} = \frac{A_{is}}{1 + j \left( \frac{f}{f_H} \right)}$$

$$A_{vs \text{ high}} = \frac{A_{vs}}{1 + j \left( \frac{f}{f_H} \right)}$$

where,  $f_H = \frac{1}{2\pi R_{eq} C_{eq}}$

where,  $R_{eq} = r_{b'e} \parallel (r_{bb'} + R_s)$

and  $C_{eq} = C_e + C_c [1 + g_m R_L]$

**For  $R_L = 0$ ,**

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$$f_H = \frac{1}{2\pi R(C_e + C_c)}$$

$$= \frac{f_T}{g_m R} \quad \because f_T = \frac{g_m}{2\pi(C_e + C_c)}$$

$$= \frac{h_{fe} f_\beta}{g_m R} \quad \because f_T = h_{fe} f_\beta$$

$$= \frac{f_\beta}{g_{b'e} R} \quad \because g_{b'e} = \frac{g_m}{h_{fe}}$$

## Gain Bandwidth Product:

### i. Gain Bandwidth Product for Voltage:

$$\begin{aligned} |A_{vs \text{ low } f_H}| &= |A_{vso} f_H| = \frac{-h_{fe}R_L}{R_s + h_{ie}} \times \frac{1}{2\pi R_{eq} C_{eq}} \\ &= \frac{R_L}{R_s + r_{bb'}} * \frac{f_T}{1 + 2\pi f_T C_C R_L} \end{aligned}$$

### ii Gain Bandwidth Product for

$$\begin{aligned} |A_{iso} \times f_H| &= \frac{g_m R_s}{2\pi C(R_s + r_{bb'})} \\ &= \frac{f_T}{1 + 2\pi f_T C_C R_L} \cdot \frac{R_s}{R_s + r_{bb'}} \end{aligned}$$

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