3.3 ANALYSIS OF THREE HINGED PARABOLIC ARCHES

Three hinged arch

The three hinged arches are statically determinate structures in which the horizontal movement at the support is prevented to a certain extent or it is wholly prevented. This type of arch has two hinges at the end supports and also an supplementary intermediate hinge at the crown.

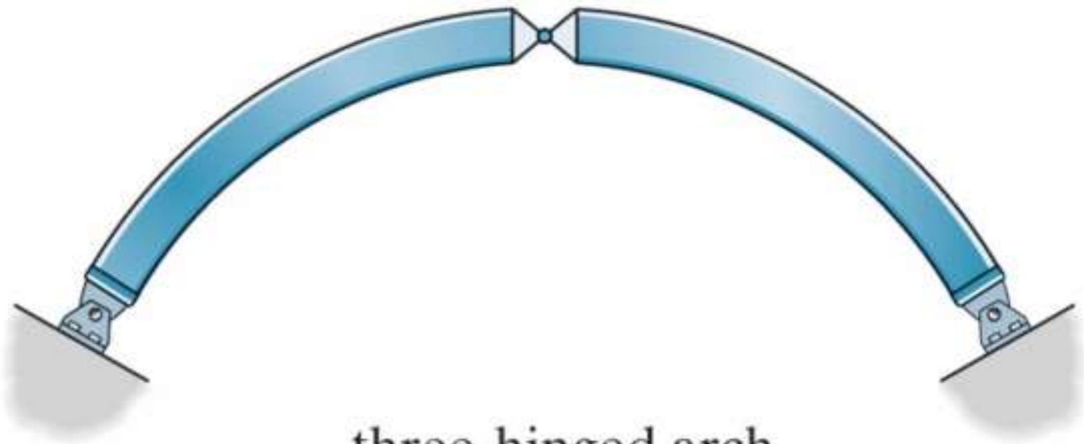

Fig. 3.3.1

Three hinged parabolic arches

An arch which is hinged at three points and whose axis represents a parabolic shape is known as a three hinged parabolic arch.

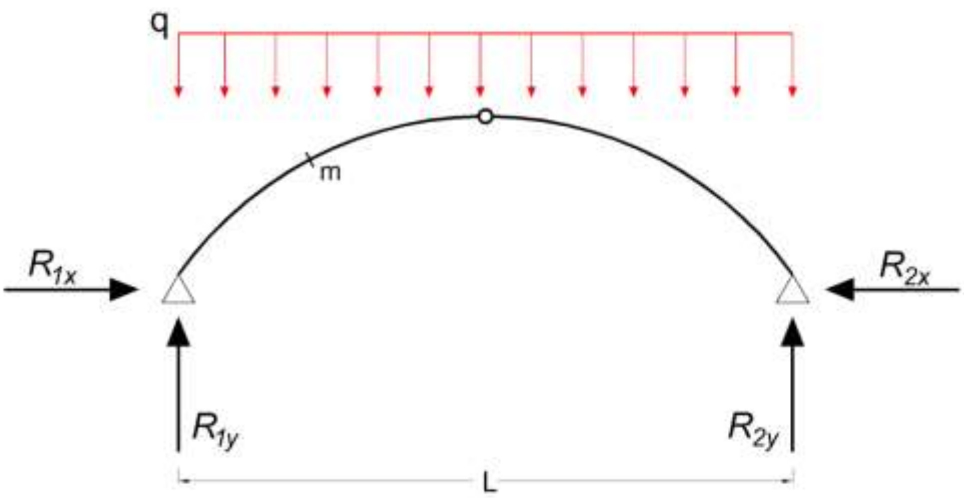

Fig. 3.3.2 Three hinged parabolic arches

Analysis of 3-hinged arches

It is the process of determining external reactions at the support and internal quantities such as normal thrust, shear and bending moment at any section in the arch.

Procedure to find reactions at the supports

Step 1. Sketch the arch with the loads and reactions at the support.

Apply equilibrium conditions namely □ F_x □ 0,□ F_y □ 0and □ M □ 0

Apply the condition that BM about the hinge at the crown is zero (Moment of all the forces either to the left or to the right of the crown).

Example:

A 3hinged arch of span 40m and rise 8m carries concentrated load of 200KN and 150KN at distance of 8m and 16m from the left end and an UDL of 50K/N on the right half of the span.

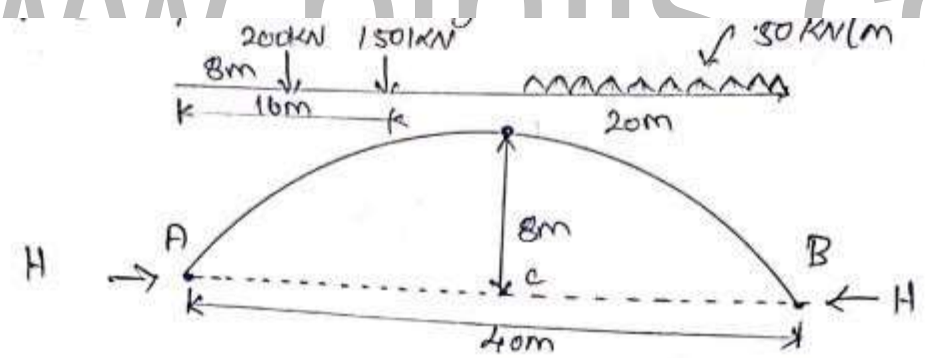

Fig. 3.3.3

SOLUTION

a) vertical reactions VA and VB

Taking moment about A

(200X8)+(150X16)+(50X20X(20+20/2)]-VB X40 = 0

1600+2400+30000-VBX40 =0

VB =850KN

Total load =VA+VB

200+150+(50X20) =850+VA

VA =500KN

b)Horizontal thrust (H)

Taking moment about C

HX8-VA(20)+(200X12)+(150X4) =0

8H-4000+2400+600 =0

H =325KN

Example:

A parabolic 3 hinged arch carries loads as shown in fig. Determine the resultant reactions at supports. Find the bending moment normal thrust and radial shear at D,5m from A. what is the max bending.

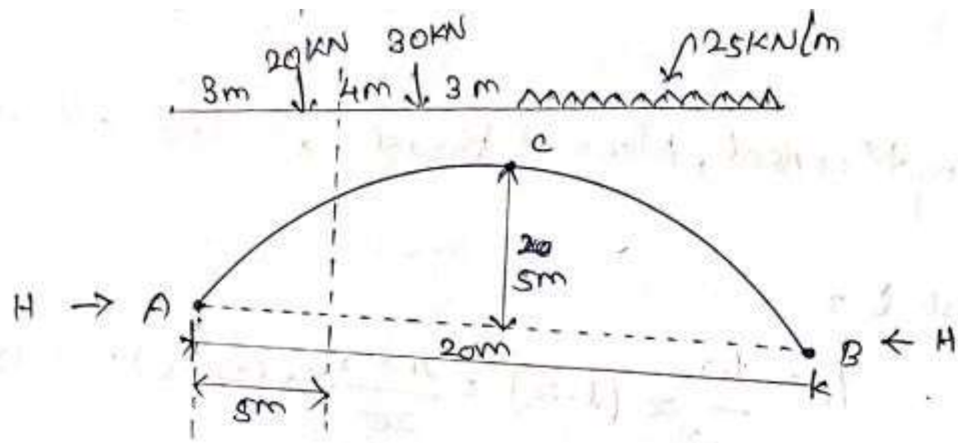

Fig. 3.3.4

SOLUTION

Taking moment about A

$(20x3)+(30x7)+[25x10x(10+10/2)]-VBx20 = 0$

$VB = 201$ KN

$VA = 99$ KN

Horizontal pull

$(H\times5)+(20\times7)+(30\times3)-VA\times10 = 0$

$5H-140+90-990 = 0$

$H = 152$ KN

Resultant Reaction (RA and RB)

$RA = \sqrt{H^2+VA^2}$

$= \sqrt{152^2+99^2}$

$= 181.39$KN

$RB = \sqrt{H^2+VB^2}$

$= \sqrt{152^2+201^2}$

$= 252$ KN

$= \tan^{-1} VA/H$

$= \tan^{-1} 99/152$

$= 33°4'36''.6$

$= \tan^{-1} VB/H$

$= \tan^{-1} 201/152$

$= 52°54'9''.86$

Bending moment, Normal thrust, radial SF at D

BM at D

Y_D =4r/l^2 x(l- x)

=4×5/20^2 X 5(20-5)

=3.75 m

BMD =+VA×5+$^-$ H_{YD} +$^-$ 20×2

=495+$^-$ 570 +$^-$ 40

= - 11.5 KNm

Slope of the arch at D

Slope =tan^{-1} [4r/l^2 (l- 2x)]

=tan^{-1} [4×5/20^2(20-2×5)]

=26°33'55''.18

Normal thrust

P =Vxsin + H cos

Vx =net beam shear force

Vx = VA -20

=99-20

= 79 KN

P =79 sin26°33'55''.18+152 cos26°33'55''.18

= 171.28KN

Radial shear force

F =vx cos - H sin

= 79 cos - 152 sin

=2.683 KN

Max BM in CB

BMx =VBx - Wx^2/2-Hyx

Yx =4r/l^2 x(l- x)

=4×5/20^2 X x(20-x)

=0.05x(20-x)

Mx = 201x-25×x^2/2 -152(0.05x(20-x))

=201x-12.5x^2 7.6x(20-x)

=201x -12.5x^2 -152x +7.6x^2

Mx =49x - 4.9x^2

Diff W.r to x

dM/dX =49-9.8 x

BM to be max

9.8x-49 =0

x =5m

Mx =49(x5)-4.9(5^2)

=122.5 KN

Example:

A symmetrical three hinged parabolic arch of span 40m and rise 8m carries a UDL of 30KN/m, over the left half of the span. The hinges are provide at the support and at the centre of the arch. Calculation the reaction at the support.als calculate the bending moment, radial shear and normal thrust a distance of 10m from the left support.

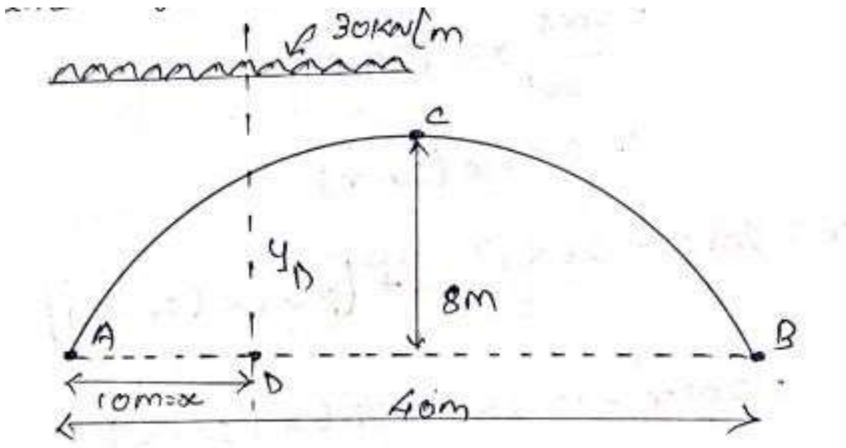

Fig. 3.3.5

Solution

Vertical Components VA and VB

Taking moment about A

$$VB \times 40 - 30 \times 20^2/2 = 0$$

$$VB = 150KN$$

$$VA = \text{total load} - VB$$

$$= 30 \times 20 - 150$$

$$= 450KN$$

Horizontal components

$$VA \times 20 - H \times 8 - 30 \times 20 \times 20/2 = 0$$

H =375KN

Resultant Reaction RA,RB

RA $=\sqrt{H^2+VA^2}$

$=\sqrt{450^2+375^2}$

=585.71KN

RB $=\sqrt{H^2+VB^2}$

$=\sqrt{150^2+375^2}$

=403.89KN

Bending moment at 10m from A

Y $=4r/l^2\ x(l-x)$

$=4\times8/40^2\times10(40-10)$

=6m

Bending moment at 10m

$=VA(10)-HA(Y)-30\times10\times10/2$

=450(10)-(375y)-30(50)

=3000-375Y

=3000-375(6)

=750KNm

Radial shear force at x=10m

R $=V x\cos\theta-H\sin\theta$

Vx =VA-30×10

=450-300

=150KN

slope at D

θ $=\tan^{-1}[4r/l^2 (l-2x)]$

θ $=\tan^{-1}[4\times8/40^2(40-2\times10)]$

=21°48'

R =150 cos 21°48'-375 sin 21°48'

= 0

Normal thrust

P = Vx sinθ+H cosθ

=150sin21°48'+375 cos 21°48'

=403.89KN

Example:

A three hinged parabolic arch of 40m span has abutments at unequal levels. The highest point of the arch is 4m above the left support and 9m above the right support abutments. The arch is subjected to an UDL of 15KNm over its entire horizontal span. Find the horizontal thrust and bending moment at the point 8m from the left support.

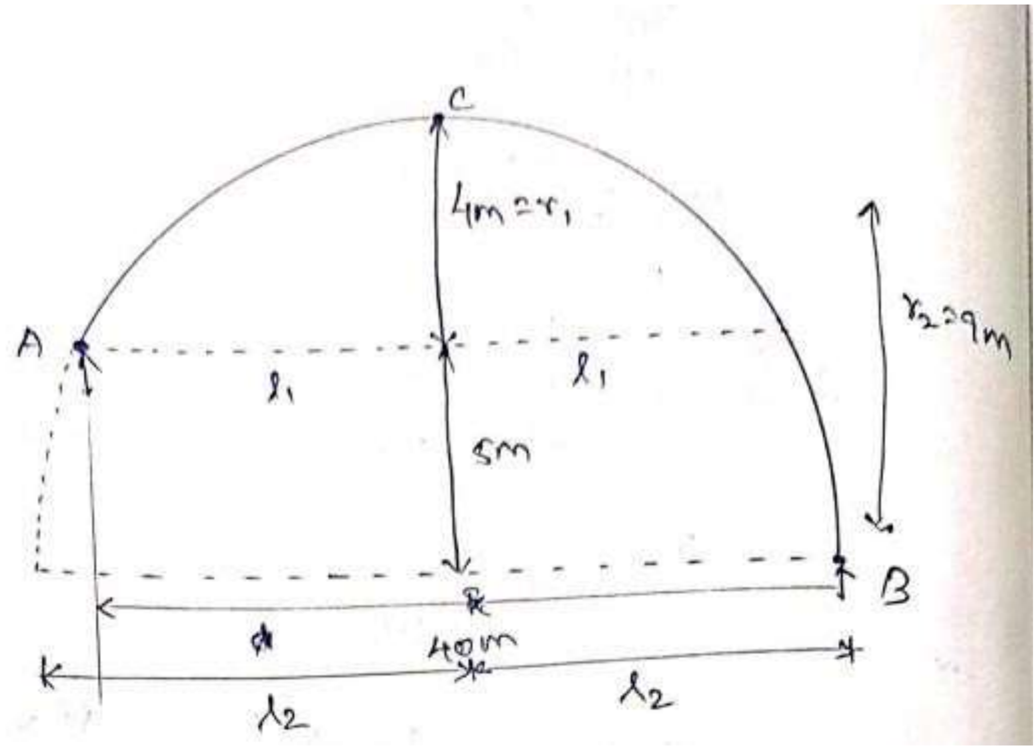

Fig. 3.3.6

Solution

Reaction A,B and H

Find l1, l2

$$l1/l2 = \sqrt{r1/r2}$$

$$l1/40\text{-}l1 = \sqrt{4/9}$$

$$l1 = (40\text{-}l1)\times 2/3$$

$$l1 = 16m$$

$$l2 = 40\text{-}l1$$

$$= 40\text{-}16$$

$$= 24m$$

considering left slab of C

$$VA(16)-4H-15\times16\times16/2 = 0$$
$$16VA-4H-1920 = 0$$
$$4VA-H-480 = 0 \quad \_\_\_\_\_(1)$$

Considering the right slab of C

$$\_VB(24)+H(9)+15\times24\times24/2 = 0$$
$$-24VB+9H+4320 = 0$$
$$-8VB+3H+1440 = 0\_\_\_\_\_(2)$$

$$VA+VB = 600$$
$$VB = 600-VA\_\_\_\_\_(3)$$

Sub 3 in 2

$$-8(600-VA)+3H+1440 = 0$$
$$8VA-4800+3H+1440 = 0$$
$$8VA+3H-3360 = 0\_\_\_\_\_\_(4)$$

1&4 solve

$$4VA-H-480 = 0 \qquad 8VA-2H-960 = 0$$
$$VA-3H-3360 = 0 \qquad 8VA+3H-3360 = 0$$
$$\times4= \quad -4VA+12H+13440 = 0 \qquad - \quad - \quad +$$
$$11H \quad +12960 \quad = 0 \qquad -5H \quad =2400$$

$$H = 480KN$$

$4VA - H - 480 = 0$

$4VA - 480 - 480 = 0$

$VA = 240KN$

$VB = 360\ KN$

Bending moment x= 8m

$BMx = VA(8) - 15\times8\times8/2 - Hy$

$Y = 4r/l^2\ x(l - x)$

$Y = 4\times4/(2\times16)^2\ x8(2\times16 - 8)$

$Y = 3m$

$BM = 240\times8 - 15\times32 - 480\times3$

$= 0$

Radial shear

$R = Vx\cos\theta - H\sin\theta$

$Vx = VA - (15\times8)$

$= 240 - (15\times8)$

$= 120KN$

$\theta = \tan^{-1} [4r/l^2\ (l - 2x)]$

$\theta = \tan^{-1} [4\times4/(2\times16)^2\ (32 - 2\times8)]$

$= 14°2'$

$F = 120\cos 14°2' - 480\sin 14°2'$

$F = 0$

Normal thrust

$$P_N = V x \sin\theta + H \cos\theta$$
$$= 120 \sin 14°2' + 480 \cos 14°2'$$
$$= 494.77 \text{ KN}$$

Example:

A three hinge arch is circular 25m in span with a central rise of 5m.It is loaded with a concentration load of 10KN at 7.5m from the left hand hinge. Find the horizontal thrust, reaction at each end hinge bending moment under the load.

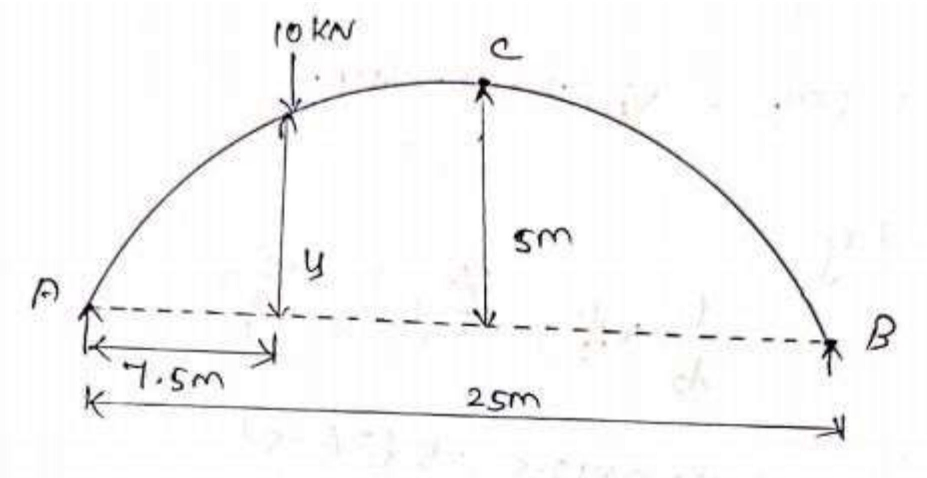

Fig. 3.3.7

solution

Vertical reaction VA and VB

Taking moment about A

$$VB \times 25 - 10 \times 7.5 = 0$$
$$VB = 3KN$$
$$VA = 7KN$$

Horizontal thrust

$$VB \times 12.5 - H \times 5 = 0$$

$$3 \times 12.5 - H(5) = 0$$

$$H = 7.5KN$$

Reaction RA and RB

$$RA = \sqrt{H^2 + VA^2}$$

$$= \sqrt{7^2 + 7.5^2}$$

$$= 10.26 \text{ KN}$$

$$RB = \sqrt{VB^2 + H^2}$$

$$= \sqrt{3^2 + 7.5^2}$$

$$= 8.08 \text{ KN}$$

Bending moment under the load

$$BMD = VA(7.50) - Hy$$

Find y

$$l/l2 \times l/2 = 5\ (2R-5)$$

$$12.5 \times 12.5 = 5\ (2R-5)$$

$$R = 18.125m$$

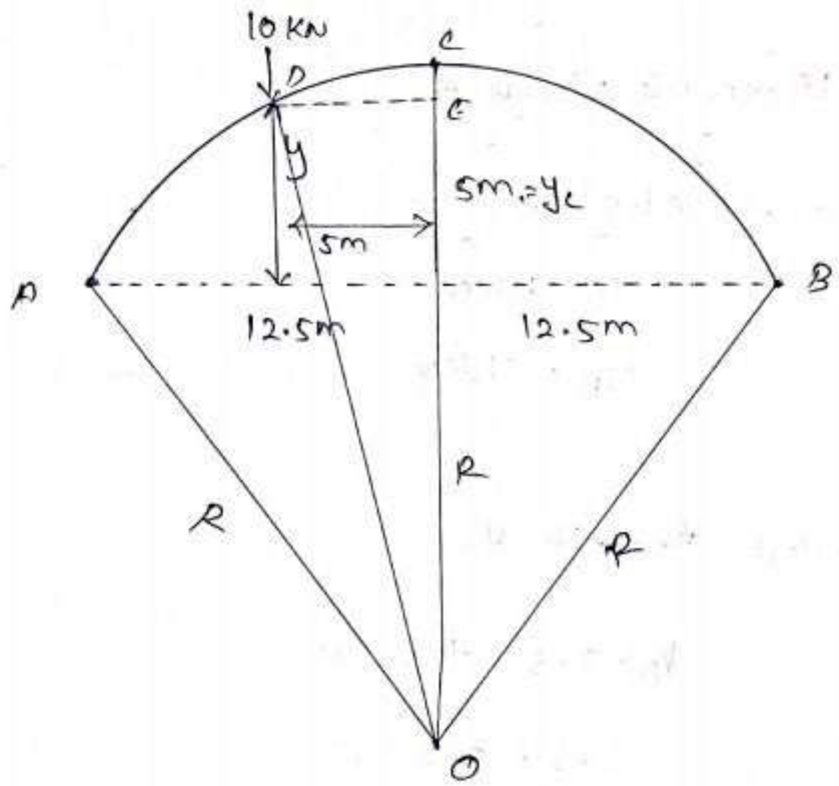

Fig. 3.3.8

$R^2 = (R-Yc+y)^2+x^2$

$18.125^2 = (18.125-5+y)^2+5^2$

$303.515 = (13.125+Y)^2$

$17.421 = 13.125+y$

$Y = 4.3m$

BMD $= 7(7.5)-7.5(4.3)$

$= 20.25KNm$

3.5 ANALYSIS OF FIXED ARCHES SETTLEMENT AND TEMPERATURE EFFECTS

Fixed arches

It is a structure which is statically indeterminate to third degree, due to the presence of three reactions at each support. The fixed arch has three independent static equilibrium equations and the degree if indeterminacy is three. The construction of fixed arch is easy, but the analysis is more complex.

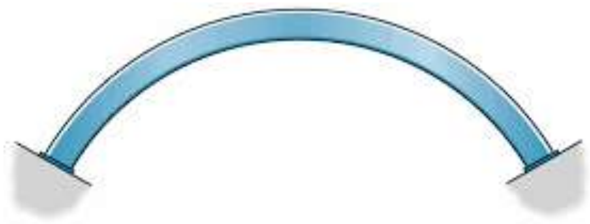

Fig. 3.5.1 Fixed arches

Advantages of fixed arches

a. These kind of arches are taken in application for longer spans, where the rigid foundations are available.
b. The fixed arches can be provided with temporary hinges at the springs, to avoid of shrinkage in reinforcing concrete. This makes the structure statically determinate.
c. The fixed arc are cheap and economical.
d. The fixed arch permits accurate analysis of stresses and therefore help in saving of material.
e. The deflection of the fixed arch is quite lesser than two hinged arches.
f. The positive moment at the centre off the span in minute, when compared with two hinge arches.

Disadvantages of fixed arches

a. Absolute fixity at the ends of the ends of the arches which increases the bending moment at the centre.
b. Fixed arches are not stable, durable compared to arches with hinges.

Settlement in arches

Fixed arches are generally made up of reinforced concrete. These are statically indeterminate to third and therefore require strong abutments. These are affected b he settlement of supports.

Example:

Find the reaction components at the supports of a symmetrical parabolic fixed arch 20m span 3m central rise when it is subjected to a uniformly distributed load of 2KN/m over the left half span .

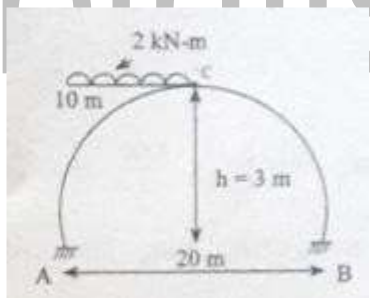

Fig. 3.5.2 Fixed arches

Solution :

Given span of arch (l) = 20m

central rise of the arch (h) = 3m

The equation of the parabolic arch is given by

$$Y = 4hx/l^2 \ (l-x)$$

$= 4\times3/20^2 \times x\times(20\text{-}x)$

$=0.03\times x\times(20\text{-}x)$

$=0.06x\text{-}0.03x^2$

The strain energy U due to bending is given by

U $={}^1\!\int_0 M^2_x /2EI\ dx$

M_x for BC is equal to (V_{Bx}-H_B y- M_B) and the limits between 0-10m

M_x for CA is equal to (V_{Bx}-H_B y- H_B -2(x-10).(x-10)/2 and the limits between 10-20m

Now,

$\partial M/\partial V_B$ =x:

$\partial M/\partial H_B$ =-y:

$\partial M/\partial M_B$ =-1

calculation of reaction components

U $= M^2_x /2EI\ dx$

$\partial U/\partial V_B$ $=2M_x /2EI \times \partial M_x /\partial V_B$

$\int\partial U/V_B$ $= 1/EI \int Mx.\ \partial Mx /\partial VB$

$= 1/EI\ [\ {}^{10}\!\int_0(V_B\ x\text{-}\ H_B\ y\text{-}M_B\ (x).dx + {}^{20}\!\int_{10} (V_B\ x\ \text{-}H_B\ y\ \text{-}M_B\ \text{-}2(x\text{-}2)^2/2\ (x)\ dx]$

$=1/EI\ [{}^{10}\!\int_0 [V_Bx^2\text{-}(H_B \times(0.6x\text{-}0.03x^2)(x)]\text{-}M_B\ x]dx + {}^{20}\!\int_{10}V_B\ x^2\ \text{-}H_B\ (0.6x - 0.03x^2\ )(x)\text{-}M_B\ x\ \text{-}2x\ (x\text{-}2)^2/2$

$=1/EI\ [{}^{10}\!\int_0[V_Bx^2\text{-}(H_B \times(0.6x^2\text{-}0.03x^3)]\text{-}M_B\ x]dx +[{}^{20}\!\int_{10}\{V_Bx^2\ \text{-}H_B\ (0.6x^2 - 0.03x^3\ )\text{-}M_B\ x\ \text{-}x\ (x\text{-}2)^2\}dx)]$

$=1/EI\ \{[V_B\ x^3/3-(H_B\ (0.6 \times x^3/3 \times x^4/4))-M_B\ .X^2/2]_0^{10}+[V_B \times X^3/3-$
$H_B\ (0.6 \times x^3/3\times0.03x^4/4)-M_B\ X^2/2-x^4/4-4 \times x^2/2+4.x^3/3]_{10}^{20}$

$=1/EI\ \{(V_B.1000/3 - 200H_B\ +75H_B\ - 50M_B\ )+(V_B.8000/3 - 1600H_B\ + 1200H_B$
$- 200M_B\ -40000-800+10666.66)-(V_B.1000/3-200H_B\ +75H_B\ -50M_B\ -$
$2500-200+1333.33)\}$

$=1/EI\ \{(V_B.1000/3 - 200H_B\ +75H_B\ - 50M_B\ +V_B.8000/3 - 1600H_B\ + 1200H_B\ -$
$200M_B\ -4000-800+10666.66)-V_B.1000/3+200H_B\ -75H_B\ +50M_B$
$+2500+200-1333.33\}$

$=1/EI\ (8000/3\ V_B- 400H_B\ -200M_B\ -28766.67)$____________(1)

$\partial U/\partial H_B$

$= 1/EI\ {}^1\int_0 Mx.\partial Mx/\partial H_B\ .dx$

$\partial U/\partial H_B$

$=1/EI[\ _0\int^{10} (V_B\ x - H_B\ y -M_B\ ) (-y)\ dx +_{10}\int^{20} (V_Bx-H_B\ y-M_B\ -(x-2)^2\ /2)](-$
$y)dx]$

$y = 0.6x - 0.03x^2$

$=1/EI\ [_0\int^{10} (V_B\ x - H_B\ y -M_B\ ) (-0.6x - 0.03x^2)\ dx +_{10}\int^{20} (V_Bx-H_B\ y-$
$M_B\ -(x-2)^2\ /2)](-0.6x - 0.03x^2)dx]$

$=1/EI\ [_0\int^{10} (-0.6\ V_Bx^2 + 0.03V_Bx^3 + HB\ (0.6x-0.03x^2)^2 +0.6M_B\ x -0.03M_B\ x^2)dx$
$+_{10}\int^{20} (0.6V_B\ x^2 + 0.03V_Bx^3+H_B\ (0.6x-0.03x^2)^2 + 0.6M_B\ x -0.03M_B\ x^2$
$+0.6x^3-0.03x^4-2.4x+0.12x^2+2.4x^2-0.12x^3]$

$=1/EI\ [_0\int^{10} (-0.6\ V_B\ x^2 +0.03V_Bx^3 + H_B\ (0.36x^2-0.0009x^4-0.036x^3) +0.6M_B\ x -$
$0.03M_B\ x^2)dx +_{10}\int^{20} (-0.6V_B\ x^2 + 0.03V_Bx^3+ H_B\ (0.36x^2+0.0009x^4-$
$0.036x^3)\ + 0.6M_B\ x -0.03M_B\ x^2 +0.48x^3-0.03x^4-2.52x^2-2.4x)dx]$

$=1/EI\ [\ (-0.6\ V_B\ x^3\ /3+0.03V_Bx^4\ /4+\ 0.36H_B\ \ x^3\ /3+\ \ 0.0009HBx^5/5-0.036HBx^4/4)$
$+0.6M_B\ x^2\ /2\ -0.03M_B\ x^3/3)]_0^{10}\ +\ (-0.6V_B\ x^3\ /3+\ 0.03V_Bx^4\ /4+\ H_B$
$0.36x^3/3+0.0009x^5-5-0.036HBx^4/4)+\ 0.6M_B\ x^2\ /2\ -0.03M_B\ x^3\ /3\ +0.48x^4\ \ /4$
$-\quad 0.03x^5/5\ -2.52x^3/3-\ 2.4x^2/2)]_{10}^{20}$

$=1/EI\ [(-125V_B+48H_B+20M_B\ )+(-275V_B+48H_B+20M_B\ +4920)]$

$=1/EI\ (-400V_B+96H_B+40M_B\ +4920)$_______(2)

$=\partial U/M_B=\ 1/EI\int\ mx.\partial\ M/\partial M_B.dx\ \&\ \partial M/\partial M_B\ =\ -1$

$=1/EI\ {}_0[\int^{10}\ (V_B\ x\ -\ H_B\ y\ -M_B\ )\ (-1)\ dx\ +\ {}_{10}\int^{20}(V_Bx-H_B\ y-M_B\ -(x-2)^2\ /2)](-1)dx]$

$=1/EI[{}_0\int^{10}\ \ (-V_B\ x\ +\ H_B\ \ \ (0.6x-0.03x^2\ \ )+M_B\ )\ \ \ dx\ \ +{}_{10}\int^{20}\ (-V_Bx+H_B\ \ \ (0.6x-0.03x^2$
$)+M_B\ +(x-2)^2dx]$

$=1/EI[\ (-V_B\ x^2\ /2\ +\ H_B\ 0.6x^2\ /2\ -0.03x^3\ /3)+M_B\ x]_0^{10}\ +\ (-V_Bx^2\ /2\ +H_B\ 0.6x^2\ /2\ -$
$0.03x^3/3)+M_B\ x+x^3\ /3+4x\ -4x^2\ /2\ ]_{10}^{20}$

$=1/EI\ [(-50V_B+30H_B-\ 10H_B\ +10M_B\ )+(-150V_B+90H_B-70H_B+10M_B$
$+2333.33+40-600)]$

$=1/EI\ [(-200V_B+40H_B\ +20M_B\ +\ 1773.33)$_________(3)

by equating eqn

$\partial U/\partial V_B = 0$,

$\partial U/\partial H_B = 0$,

$\partial U/\partial M_B = 0$

we get from equation 1,2&3

$2666.66V_B - 400H_B \quad -200M_B = 28766.67)$

$400V_B + 96H_B + 40M_B = -4920$

$-200V_B + 40H_B + 20M_B = -1773.33$

solving these three equation ,we get

V_B =16.55KN

H_B = - 85.83KN

M_B =248.50KN-m

3.4 ANALYSIS OF TWO HINGED ARCHES

Two hinged arches

Arches are curved structural members used to support heavy loads on large spans. When the load is imposed on an arch, it remains primarily under compression. Thus, it receives lower bending moments.

As the name indicates, a two hinged arch is an arch hinged on its two supports. Also, it is a statically indeterminate structures. Since the horizontal thrust cannot be calculated by equilibrium equations.

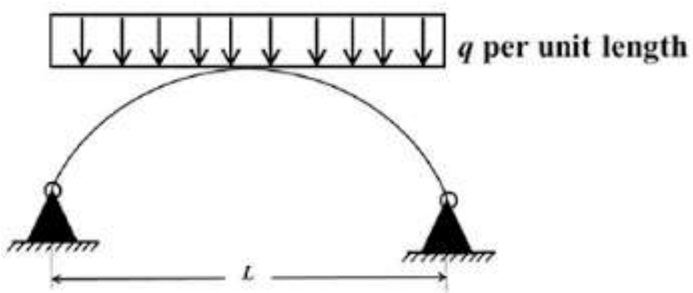

Fig. 3.4.1 Two hinged arches

Reaction locus

Reaction locus is a line that gives the intersection point of two reactions for a particular position of load.

Methods of analysis of two hinged arches.

a. Strain energy principle method or first theorem Castigliono.
b. Consistent deformation method or unit load method

Internal stress resultants induced in arch section

a. Normal thrust
b. Radial shear

Rib-shortening in the case of arches.

In a two hinged arch, the normal thrust which is a compressive force along the axis of the arch will shorten the rib of the arch. This in turn will release part of the horizontal thrust. Normally, this effect is not considered in the analysis (in the case of two hinged arches).

Depending upon the importance of the work we can either take into account or omit the effect of rib shortening. This will be done by considering (or omitting) strain energy due to axial compression along with the strain energy due to bending in evaluating H.

Analysis of two-hinged arch

A typical two-hinged arch is shown in Fig. 33.1a. In the case of two-hinged arch, we have four unknown reactions, but there are only three equations of equilibrium available. Hence, the degree of statically indeterminacy is one for twohinged arch.

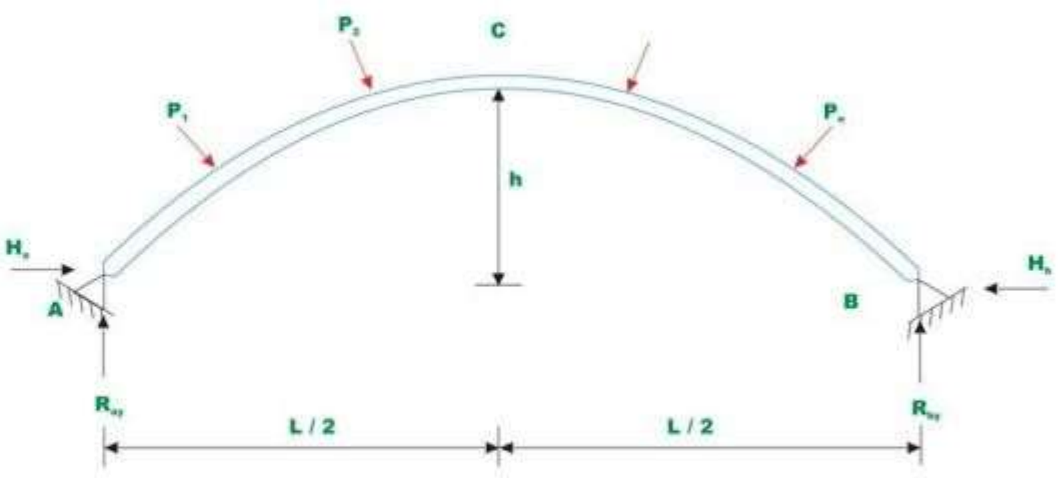

Fig. 3.4.2 Two hinged arches

The fourth equation is written considering deformation of the arch. The unknown redundant reaction is calculated by noting that the horizontal displacement of hinge Hb B is zero. In general the horizontal reaction in the two hinged arch is evaluated by straightforward application of the theorem of least work (see module 1, lesson 4), which states that the partial derivative of the strain energy of a statically indeterminate structure with respect to statically indeterminate action should vanish. Hence to obtain, horizontal reaction, one must develop an expression for strain energy. Typically, any section of the arch (vide Fig 33.1b) is subjected to shear forceV , bending moment M and the axial compression . The strain energy due to bending is calculated from the following expression.

Example:

A parabolic arch hinged at ends has a span of 60m and a rise of 12m.A concentrated load of 8KN act at 15m from the left hinge. The second moment of area varies as the secant of the inclination of arch axis. calculate the horizontal thrust and the reaction at the hinge also calculate the net bending moment of the section.

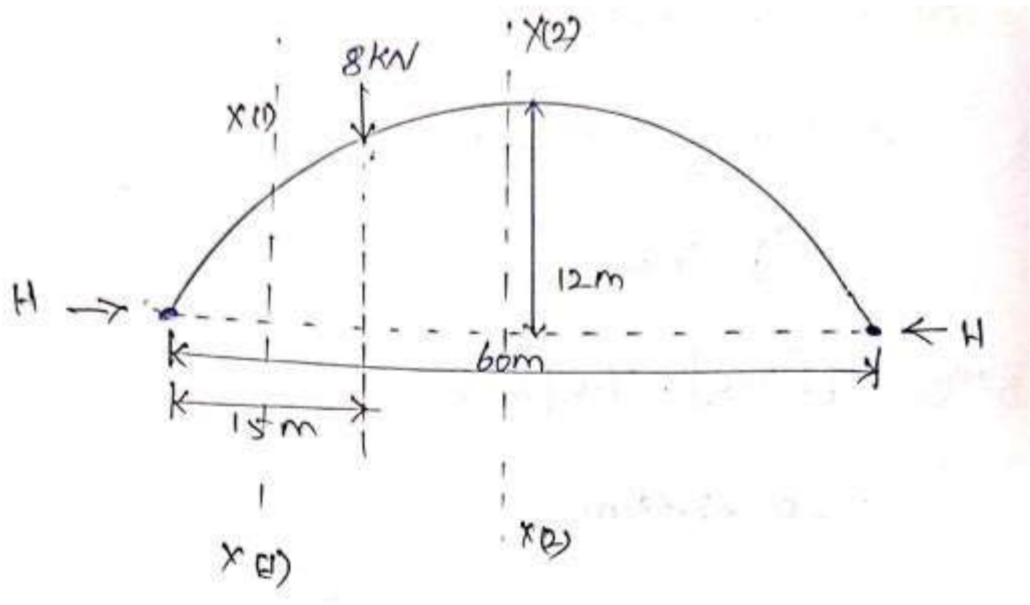

Fig. 3.4.3 Two hinged arches

Solution

Vertical reaction VA and VB

Taking moment about A

VB×60 - 8×15 =0

VB =2KN

Total load =VA+VB

VA = total load -VB

=8 - 2

=6 KN

Horizontal thrust

$H = {}_0\int^{l} \mu\, y\, dx \,/\, {}_0\int^{l} y^2\, dx$

${}_0\int^{l} \mu\, y\, dx = {}_0\int^{15} \mu_1\, y\, dx + {}_{15}\int^{60} \mu_2\, y dx$

$y = 4\, r / l^2\, x\, (l - x)$

${}_0\int^{l} y^2\, dx = {}_0\int^{60} [4\, r / l^2\, x\, (l - x)]^2\, dx$

$= {}_0\int^{60} [4\times12 / 60^2\, x\, (60-x)]^2\, dx$

$= {}_0\int^{60} [(0.0133x\, x\, (60-x)]^2\, dx$

$= {}_0\int^{60} [(0.8x - 0.0133x^2)^2 \quad dx$

$= {}_0\int^{60} [0.64x^2 - 0.0213x^3 + (1.76\times10^{-4})x^4\,]\, dx$

$= {}_0\int^{60} [0.64x^2 - 0.0213x^3 + (1.76\times10^{-4})x^4\,]dx$

$= {}_0[\, (0.64x^3 / 3) - (0.0213x^4 /4) + (1.76\times10^{-4}x^4 /5)]^{60}$

$=46.08\times10^3 - 69.012\times10^3 + 27.37\times10^3$

=4439.52

$_0\int^1 \mu\, y\, dx = {}_0\int^{15} \mu_1\, y\, dx + {}_{15}\int^{60} \mu_2\, y dx$

$\mu_1 = VA\ x1$

$= 6x$

$\mu_1 = VA\ x_2 - 8\ (x_2 - 15)$

$= 6x - 8x + 120$

$= -\ 2x + 120$

$_0\int^{15} \mu_1\, y\, dx$

$= {}_0\int^{15} 6x(0.8x - 0.0133x^2)dx$

$= {}_0\int^{15} (4.8x^2 - 0.079x^3)dx$

$= {}_0[\ (4.8x^3/3) – (0.079x^4/4)]^{15}$

$= 5400 - 999.84$

$= 4400$

$_{15}\int^{60} \mu_2\, y dx$

$= {}_{15}\int^{60} (120-2x)(0.8-0.013x^2)\ dx$

$= {}_{15}\int^{60} (96x - 1.596x^2 - 1.6x^2 + 0.0266x^3)\ dx$

$= {}_{15}\int^{60} (0.0266x^3 - 3.196x^2 + 96x)dx$

$= {}_{15}[(0.0266x^4/4) – (3.196x^3/3) + (96x^2/2)]^{60}$

$= [(86184 – 230112 + 172800) - (336.6 - 359 + 1080)]$

$_0\int^1 \mu\, y\, dx \quad = {}_0\int^{15} \mu_1\, y\, dx + {}_{15}\int^{60} \mu_2\, y dx$

$= 21330.9$

$H \quad = {}_0\int^1 \mu\, y\, dx \,/\, {}_0\int^1 y^2 dx$

$= {}_0\int^{15} \mu_1\, y\, dx + {}_{15}\int^{60} \mu_2\, y dx \,/\, {}_0\int^{60} y^2\, dx$

$H \quad = 4400 + 21330.9 / 4439.52$

$= 5.79$ KN

Reaction

$RA \quad = \sqrt{H^2 + VA^2}$

$= \sqrt{6^2 + 5.79^2}$

$= 8.18$KN

$RB \quad = \sqrt{H^2 + VB^2}$

$= \sqrt{2^2 + 5.79^2}$

$= 5.91$KN

Max Bending moment

$Mx \quad = VA\,(15) - Hy$

$y \quad = 4 \times 12/60^2 \times 15(60-15)$

$= 9$m

$Mx \quad = 6\,(15) - 5.79\,(9)$

$= 39.87$KNm

3.1 ARCHES

Introduction

Mainly three types of arches are used in practice: three-hinged, two-hinged and hinge less arches. In the early part of the nineteenth century, three-hinged arches were commonly used for the long span structures as the analysis of such arches could be done with confidence. However, with the development in structural analysis, for long span structures starting from late nineteenth century engineers adopted two-hinged and hinge less arches. Two-hinged arch is the statically indeterminate structure to degree one. Usually, the horizontal reaction is treated as the redundant and is evaluated by the method of least work. In this lesson, the analysis of two-hinged arches is discussed and few problems are solved to illustrate the procedure for calculating the internal forces.

Arches

An arch is defined as a curved girder, having convexity upwards and supported at its ends. The supports must effectively arrest displacements in the vertical and horizontal directions. Only then there will be arch action.

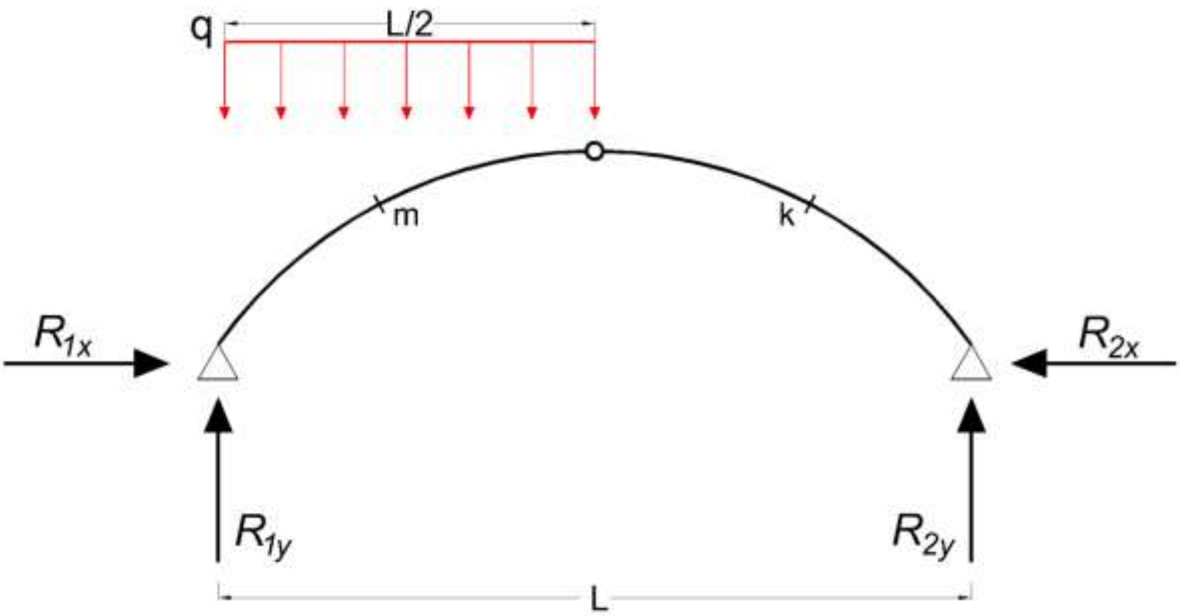

Fig. 3.1.1 Arches

Linear arch

If an arch is to take loads, say W1, W2, and W3 (fig) and a Vector diagram and funicular polygon are plotted as shown, the funicular polygon is known as the linear arch or theoretical arch

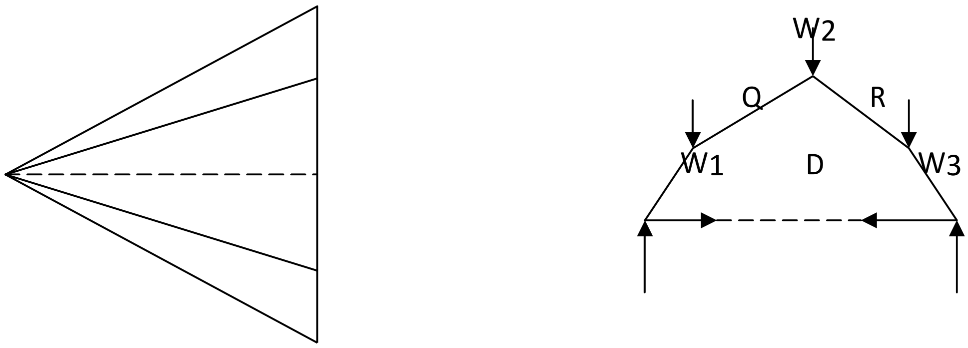

Fig. 3.1.2 Linear Arches

The polar distance 'ot' represents the horizontal thrust. The links AC, CD, DE, and EB will be under compression and there will be no bending moment. If an arch of this shape ACDEB is provided, there will be no bending moment.

For a given set of vertical loads W1, W2…..etc., we can have any number of linear arches depending on where we choose 'O' or how much horizontal thrust (ot) we choose to introduce.

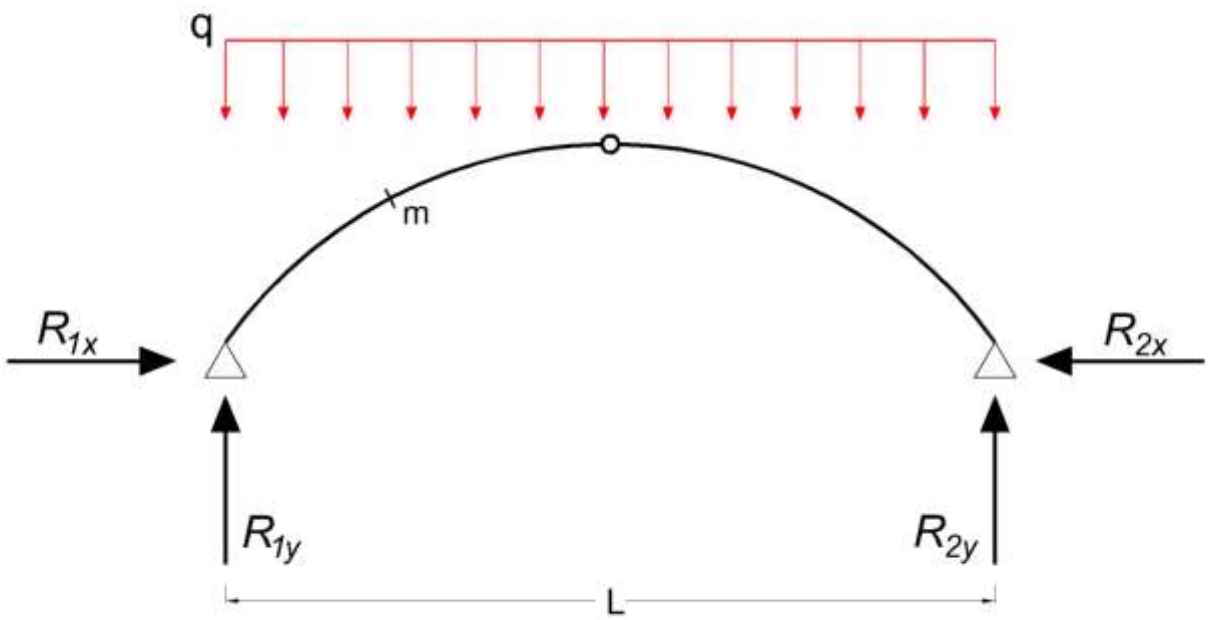

Fig. 3.1.3 Linear Arches

State Eddy's theorem.

Eddy's theorem states that " The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch (or theoretical arch) and the centre line of the actual arch."

3.2 TYPES OF ARCHES

Types of arches according to the support conditions.

a. Three hinged arch
b. Two hinged arch
c. Single hinged arch
d. Fixed arch (or) hinge less arch

Types of arches according to their shapes.

a. Curved arch
b. Parabolic arch
c. Elliptical arch
d. Polygonal arch

Based on material used

a. Steel arch
b. Reinforced concrete arch
c. Timber arch
d. Brick or stone or masonry arch

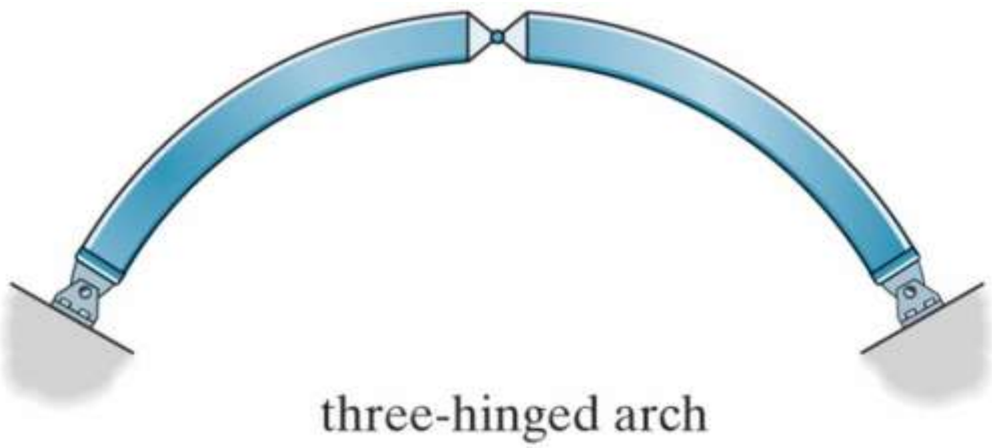

Fig. 3.2.1

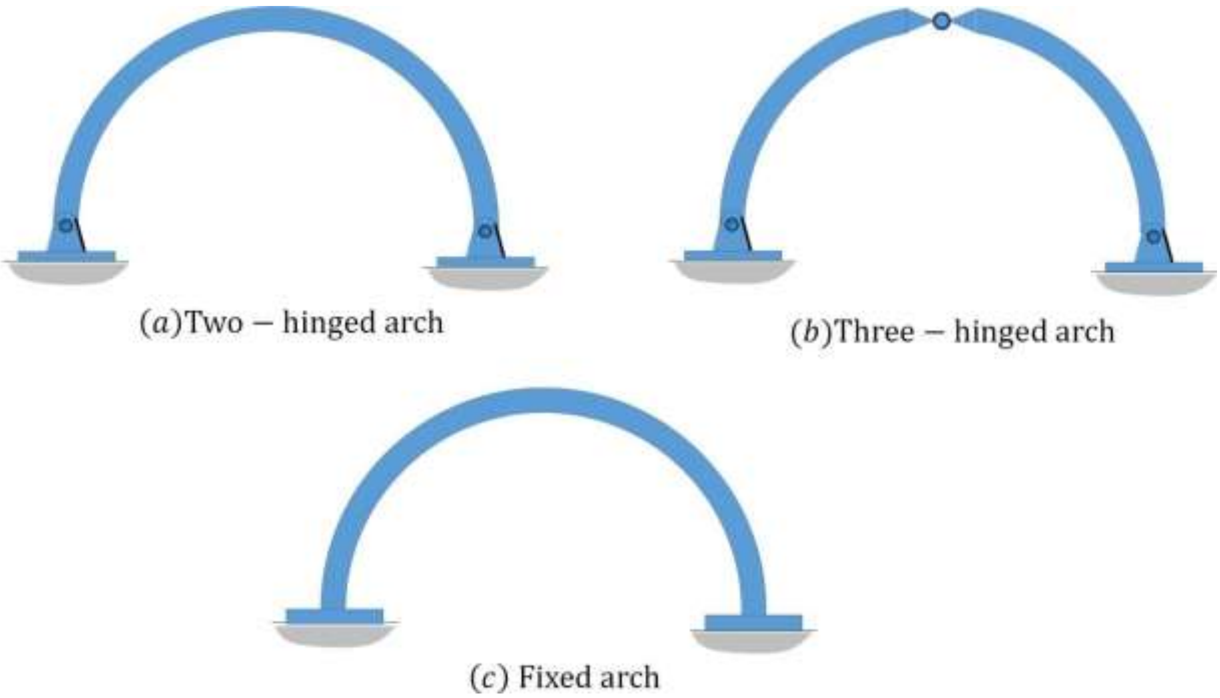

Fig. 3.2.2 Arches

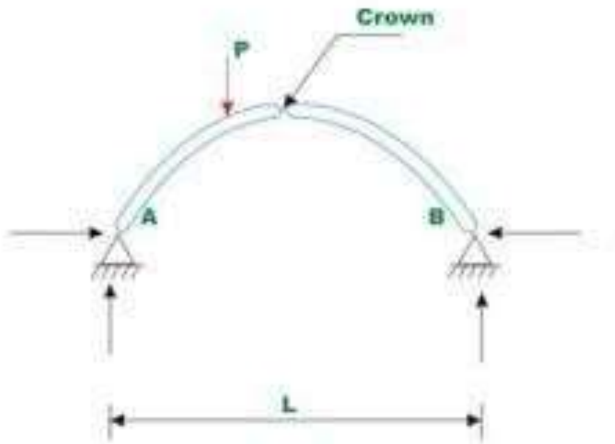

Fig. 3.2.3 Hinged Arches