

## 2.3 INFLUENCE LINE FOR SHEARING FORCE, BENDING MOMENT AND SUPPORT REACTION COMPONENTS OF CONTINUOUS BEAMS

### Influence lines

An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

### Uses of influence line diagrams

(i) Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and

(ii) Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

### The principle on which indirect model analysis is based

The indirect model analysis is based on the Muller Breslau principle.

Muller Breslau principle has lead to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.

To get the influence line for any force quantity,

- (i) remove the resistant due to the force,
- (ii) apply a unit displacement in the direction of the
- (iii) plot the resulting displacement diagram. This diagram is the influence line for the force.

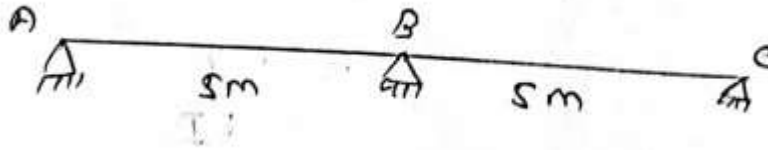
### Similitude

Similitude means similarity between two objects namely the model and the prototype with regard to their physical characteristics:

- a. Geometric similitude is similarity of form
- b. Kinematic similitude is similarity of motion
- c. Dynamic and/or mechanical similitude is similarity of masses and/or forces.

**Example:**

Determine the influence line for  $R_A$  for continuous beam shown in fig. compute the IL ordinate at 1m intervals

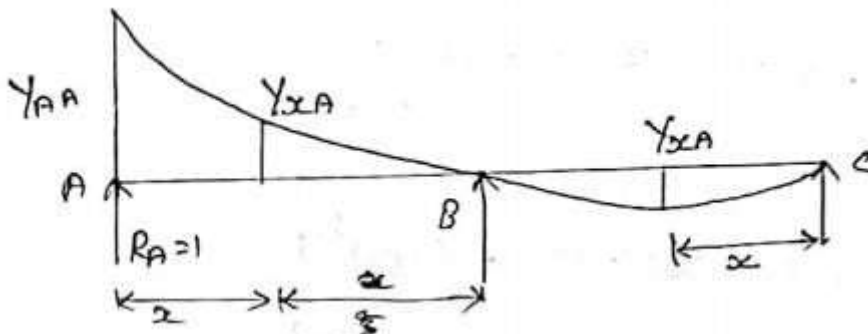


**Fig.2.3.1**

**Solution**

- i) Remove support A
- ii) Apply a unit force at A and compute the deflection at any "x" on the CB and BA
- iii) divide these deflection by the displacement at A

Elastic curve due to  $R_A = 1$



**Fig.2.3.2**

Taking moment about C

$$R_A \times 10 + R_B \times 5 = 0$$

$$10 + R_B \times 5 = 0$$

$$R_B = -10/5$$

$$= -2$$

$$R_A + R_B + R_C = 0$$

$$1 - 2 + R_C = 0$$

$$R_C = 1$$

$$M_x = -EI \frac{d^2y}{dx^2}$$

$$M_x = R_C x + R_B (x-5)$$

$$1x - 2(x-5) = -EI \frac{d^2y}{dx^2}$$

$$-x + 2x - 10 = EI \frac{d^2y}{dx^2}$$

Integrate on both side

$$EI \frac{d^2y}{dx^2} = -x^2/2 + 2x^2/2 - 10x + C_1$$

$$EI \frac{dy}{dx} = -x^2/2 + x^2 - 10x + C_1 \text{_____} (1)$$

Again integrating on both side

$$EI y = (-x^3/6) + (x^3/3) - (10x^2/2) + C_1x + C_2$$

$$EI y = (-x^3/6) + (x^3/3) - (5x^2) + C_1x + C_2 \text{ _____(2)}$$

Apply condition

$$x=0, Y=0$$

$$EI y = (-x^3/6) + (x^3/3) - (5x^2) + C_1x + C_2$$

$$C_2 = 0$$

$$X=5 Y=0$$

$$EI y = (-x^3/6) + (x^3/3) - (5x^2) + C_1x + C_2$$

$$0 = (-5^3/6) + (5^3/3) - (5(5^2)) + C_1(5) + 0$$

$$0 = -104.16 + C_1 \times 5$$

$$C_1 = 20.83$$

Apply C1 and C2

$$Y_{XA} = (-x^3/6) + (x^3/3) - (5x^2) + C_1x + C_2$$

$$= 1/EI (-x^3/6) + (x^3/3) - (5x^2) + 20.86x + 0$$

At x= 10

$$Y_{AA} = 1/EI (-10^3/6) + (10^3/3) - (5(10^2)) + 20.86(10) + 0$$

$$= -125.033$$

$$M_x = -EI \frac{d^2y}{dx^2}$$

$$1x + 2(x-5) = EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = -x + 2(x-5)$$

Integrate on both sides

$$EI \frac{dy}{dx} = -\frac{x^2}{2} + C_1 + 2\frac{(x-5)^2}{2}$$

$$EI \frac{dy}{dx} = -\frac{x^2}{2} + (x-5)^2 + C_1 \quad (1)$$

Again integrate on both sides

$$EI y = \left(-\frac{x^3}{6}\right) + \frac{(x-5)^3}{3} + C_1x + C_2 \quad (2)$$

Apply condition

i)  $x=0, Y=0$

$$EI y = \left(-\frac{x^3}{6}\right) + \frac{(x-5)^3}{3} + C_1x + C_2$$

$$C_2 = 0$$

ii)  $(X=5, Y=0)$

$$EI y = \left(-\frac{x^3}{6}\right) + \frac{(x-5)^3}{3} + C_1x + C_2$$

$$0 = \left(-\frac{5^3}{6}\right) + \frac{(5-5)^3}{3} + C_1(5) + C_2$$

$$0 = -20.83 + 0 + 5C_1 + 0$$

$$C_1 = 20.83/5$$

$$C_1 = 4.167$$

Apply c1 and c2

$$Y_{XA} = (-x^3/6) + (x-5)^3/3 + C1x + C2$$

$$= 1/EI [(-x^3/6) + 4.167x + (x-5)^3/3]$$

At x= 10

$$Y_{AA} = 1/EI [(-x^3/6) + 4.167x + (x-5)^3/3]$$

$$= 1/EI [(-10^3/6) + 4.167(10) + (10-5)^3/3]$$

$$= 1/EI [- 83.33]$$

$$\text{ILO at x} = Y_{XA}/Y_{AA}$$

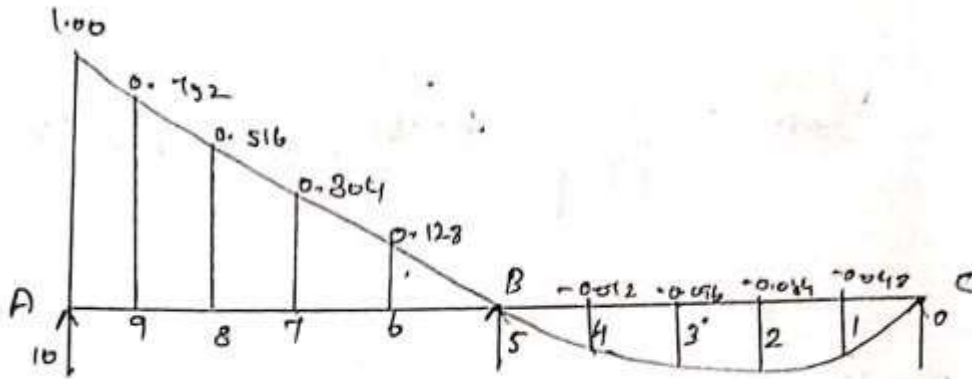
$$= 1/EI [(-x^3/6) + 4.167x + (x-5)^3/3] / (-83.33)$$

$$= [(-x^3/6) + 4.167x + (x-5)^3/3] \times (1/83.33)$$

Ordinate at ILD for RA

x(m)	Support C	1	2	3	4	support B 5	6	7	8	9	support 10
ILO RA	0	-0.048	-0.084	0.096	-0.072	0	0.128	0.128	0.304	0.516	1

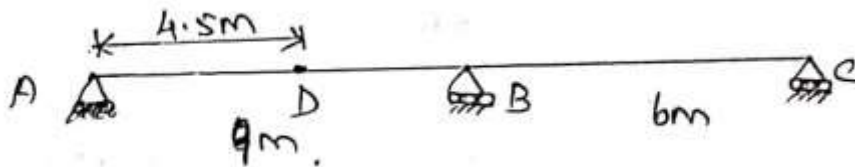
**Table. 2.3.1 Ordinate at ILD for RA**



**Fig.2.3.3 Ordinate at ILD for RA**

**Example:**

Using Muller Breslau principle, draw the influence line for bending moment at the mid-point of span AB of the continuous beam ABC shown in fig, determine the influence line ordinate at suitable intervals and plot them.



**Fig.2.3.4**

**Solution**

To get the influence line for MD

- i) Introduce a hinge at D
- ii) Apply a unit bending moment at D
- iii) Determine the deflection  $Y_{XD}$  and slope  $Q_{DD}$  at D
- iv)  $Y_{XD} / Q_{DD}$  is the influence line ordinate at any x

Bending moment at any x is

$$M_x = -EI \frac{d^2y}{dx^2}$$

$$0.333x - 0.555(x - 6) = -EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = -0.333x + 0.555(x - 6)$$

Integrate on both sides

$$EI \frac{dy}{dx} = (-0.333x^2/2) + (0.555(x - 6)^2/2)$$

$$EI \frac{dy}{dx} = -0.1665x^2 + 0.2775(x - 6)^2 + C_1 \quad (1)$$

Again integrate on both sides

$$EI Y = (-0.1665x^3/3) + (0.2775(x - 6)^3/3) + C_1x + C_2$$

$$EI Y = -0.555x^3 + 0.925(x - 6)^3 + C_1x + C_2 \quad (2)$$

Find RA, RB, RC, RD1 and RD2

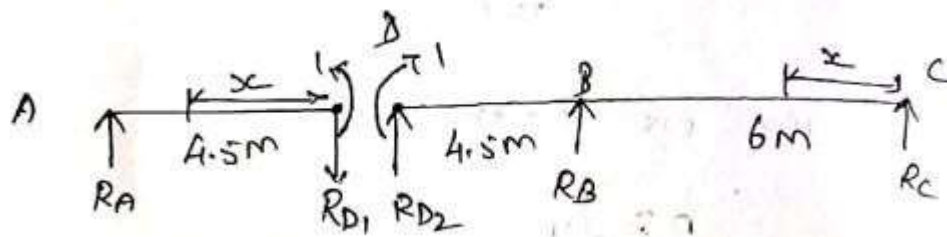
$$M = 1 \text{ at D}$$

$$RA \times 4.5 = 1$$

$$RA = 1/4.5$$

$$= 0.222 \text{ KN}$$





**Fig.2.3.5**

$$R_{D1} = 0.222 \downarrow$$

$$R_{D2} = 0.222 \uparrow$$

Taking moment about C

$$0.222 \times 10.5 + 1 + R_B \times 6 = 0$$

$$R_B = -0.555 \text{ KN}$$

$$R_A + R_B + R_C = 0$$

$$0.222 - 0.555 + R_C = 0$$

$$R_C = 0.333 \text{ KN}$$

Two regions AD and DBC will be considered separately ( because of discontinuity at D)

Boundary condition

I)  $x=0$   $y=0$

$$EI Y = -0.555 x^3 + 0.925 (x - 6)^3 + C_1 x + C_2$$

(2)  $C_2 = 0$

ii)  $x=6$   $Y=0$

(2)  $EI Y = -0.555 x^3 + 0.925 (x - 6)^3 + C_1 x + C_2$

$$0 = -0.555(6)^3 + 0.925 + C_1 \cdot 6 + 0$$

$$0 = -11.988 + C_1 \cdot 6$$

$$C_1 = 2$$

Apply  $C_1$  and  $C_2$  in slope of deflection value

$$(1) \quad EI \frac{dy}{dx} = -0.1665x^2 + 0.2775(x - 6)^2 + C_1$$

$$X = 10.5$$

$$Q_{DC} = \frac{dy}{dx}$$

$$= \frac{1}{EI} [-0.1665(10.5)^2 + 0.2775(10.5 - 6)^2 + 2]$$

$$= \frac{1}{EI} (-10.73)$$

Apply (2)

$$= \frac{1}{EI} [-0.555(10.5)^3 + 0.925(10.5 - 6)^3 + 2(10.5) + 0]$$

$$= \frac{1}{EI} (-34.8)$$

For the Zone AD

$$M_x = 1 - 0.222x$$

$$EI \frac{d^2y}{dx^2} = 0.222x - 1$$

Integrate on both sides

$$EI \frac{dy}{dx} = 0.222 \frac{x^2}{2} - X + C_3$$

$$EI \frac{dy}{dx} = 0.111x^2 - X + C_3 \quad (3)$$

Again integrate on both side

$$EI Y = 0.111 x^3 / 3 - (x^2 / 2) + C3 x + C4$$

$$EI Y = 0.037 x^3 - (x^2 / 2) + C3 x + C4 \text{ (4)}$$

Bounday condition

i)  $x = 0 \quad y = -34.82/EI$

(4)

$$EI Y = 0.037 x^3 - (x^2 / 2) + C3 x + C4$$

$$-34.82 EI / EI = 0.037(0) - 0 + C4$$

$$C4 = - 34.82$$

ii)  $X = 4.5 \quad Y = 0$

(4)

$$EI Y = 0.037 x^3 - (x^2 / 2) + C3 x + C4$$

$$0 = 0.037 (4.5)^3 - (4.5^2 / 2) + C3 (4.5) - 34.82$$

$$0 = - 41.57 + 4.5 C3$$

$$C3 = 9.24$$

Apply c3 and C4 in (3)

(3)

$$EI dy/dx = 0.111 x^2 - X + C3$$

$$EI dy/dx = 0.111 x^2 - X + 9.24$$

$$Q_{DA} = dy/dx$$

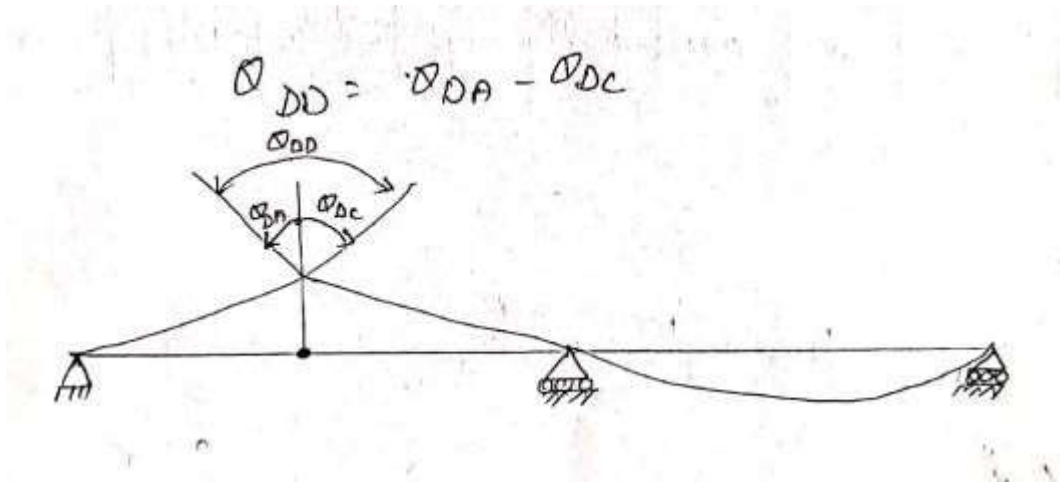
$$= 9.24 / EI$$

At  $x = 0$

(4)

$$EI Y = 0.037 x^3 - (x^2 / 2) + C_3 x + C_4$$

$$EI y = 0.037 x^3 - (x^2 / 2) + 9.24 x - 34.82$$



**Fig.2.3.6**

$$\begin{aligned} Q_{DD} &= Q_{DA} - Q_{DC} \\ &= 9.24 / EI + 10.738 / EI \\ &= 19.978 / EI \end{aligned}$$

For the region CD

ILO for MD

$$= y_{XD} / Q_{DD} [0.33x^3 / 6 + 2x + 0.555(x-6)^3 / 6] / (19.978)$$

For the region D

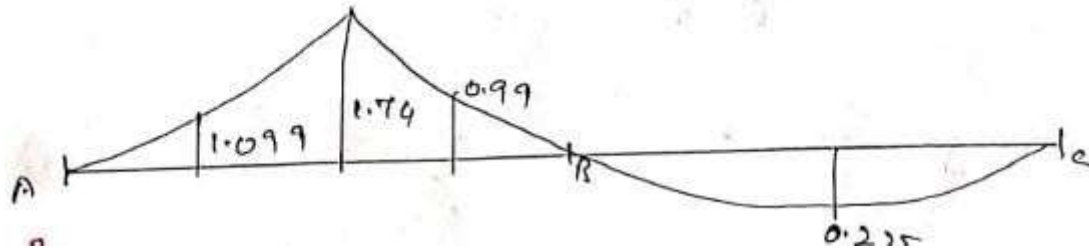
ILO for MD

$$= [(0.222 x^3 / 6) - (x^2 / 2) + 9.24 x - 34.82] / (19.978)$$

Influence line ordinate

x(m)	0	3	6	9	10.5	12	15
ILO	0	0.225	0.0	-0.999	-1.743	-1.099	0

**Table. 2.3.2 Ordinate at ILD**



**Fig.2.3.7 Influence line ordinate**

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## 2.4 INFLUENCE LINE FOR SHEARING FORCE, BENDING MOMENT AND SUPPORT REACTION COMPONENTS OF FIXED BEAMS

### Fixed beam

Consider a fixed beam ABB of span 'L' as shown in figure.

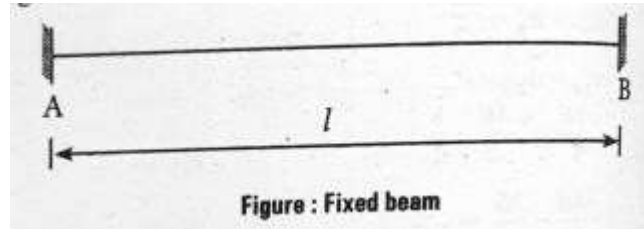


Fig. 2.4.1 Fixed beam

If we consider AB as simply supported beam, the influence lines for reaction at the support can be determined by applying a cut at 'A' and by applying unit displacement at 'A' as shown in figure.

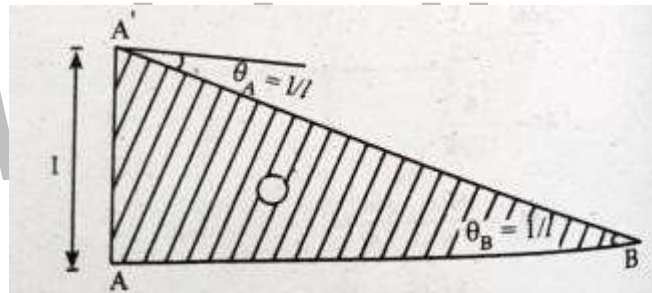


Fig. 2.4.2 Displacement At 'A'

### Maximum shear force diagram

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions. When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram. This diagram yields the 'design shear' for each cross section.

## **Bending moment diagram**

Bending moment diagram represents variation of bending moment. Bending moment diagrams are drawn for only bending moments. If span longer than UDL for a maximum BM, the load on left side is equal to the load on right side in case of bending moment diagram.

## **Several point loads**

The maximum bending moment for a series of moving loads is obtained when the average load on the left of the section is equal to the average load on the right of the section.

The above statement exists in a system of moving point loads. In such cases, each load is passed over the section and average load on each side is calculated. The load, when it crosses the section makes the heavier side lighter and lighter side heavier and gives the maximum bending moment at the section.

**Types of connections possible with the model used with Begg's deformer.**

- (i) Hinged connection
- (ii) Fixed connection
- (iii) Floating connection

## **Principle on which indirect model analysis is based.**

The indirect model analysis is based on the Muller Breslau principle.

Muller Breslau principle has led to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.

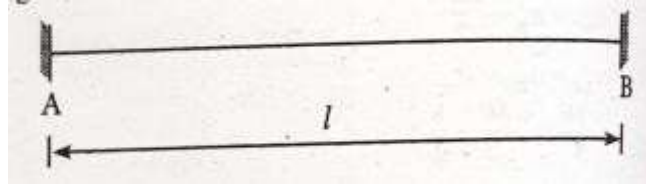
To get the influence line for any force quantity,

- (i) remove the resistant due to the force,
- (ii) apply a unit displacement in the direction of the

(iii) plot the resulting displacement diagram. This diagram is the influence line for the force.

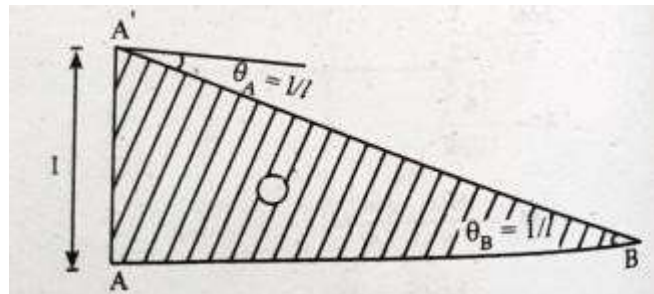
**Example:**

Sketch the influence line diagram for reaction at support of the fixed beam



**Fig. 2.4.3**

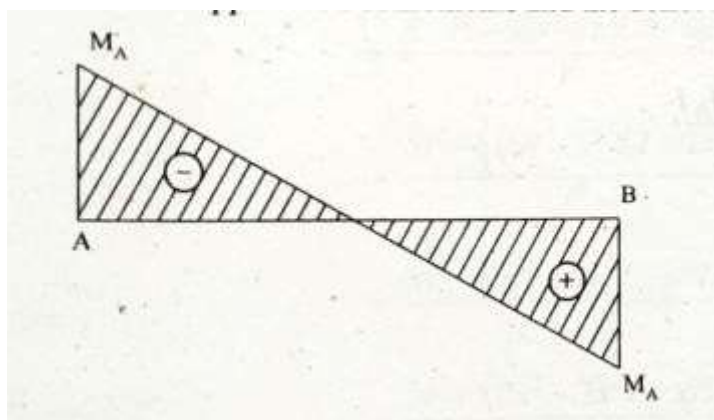
**Solution :**



**Fig. 2.4.4**

Reaction at  $R_A = M_A \frac{1}{3} + M_B \frac{1}{6}$

Reaction at  $R_B = M_A \frac{1}{6} + M_B \frac{1}{3}$



**Fig. 2.4.5 ILD for Reaction at AB**



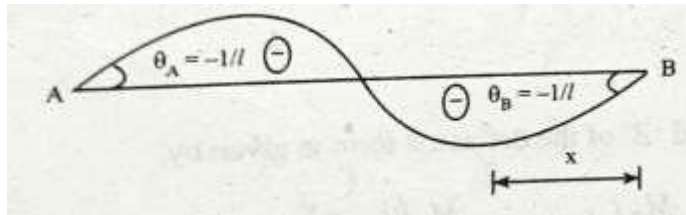
$$\theta_A = \theta_B$$

$$\frac{1}{l} = M_A \frac{1}{3} + M_B \frac{1}{6}$$

$$= 0$$

$$\frac{1}{l} = -M_A \frac{1}{3} + M_B \frac{1}{6}$$

$$\frac{1}{l} = -(M_A \frac{1}{3} + M_B \frac{1}{6}) \quad (1)$$



**Fig. 2.4.6**

$$\theta_B = -\theta_A$$

$$\frac{1}{l} = -(M_A \frac{1}{6} + M_B \frac{1}{3})$$

$$= 0$$

$$\frac{1}{l} = M_A \frac{1}{6} + M_B \frac{1}{3} \quad (2)$$

equating 1 & 2

$$-(M_A \frac{1}{3} + M_B \frac{1}{6}) = M_A \frac{1}{6} + M_B \frac{1}{3}$$

$$-M_A \frac{1}{3} - M_A \frac{1}{6} = M_B \frac{1}{3} + M_B \frac{1}{6}$$

$$-3M_A \frac{1}{6} = 3M_B \frac{1}{6}$$

$$-M_A = M_B$$

$$M_A = -M_B \quad (3)$$

Sub 3 in 2

$$\frac{1}{l} = -M_B \frac{1}{6} + M_B \frac{1}{3}$$

$$\frac{1}{l} = -M_B \frac{1}{6} + 2M_B \frac{1}{6}$$

$$1/l = -M_B / 6$$

$$M_B = 6/l^2$$

$$M_A = -M_B$$

$$= -6/l^2$$

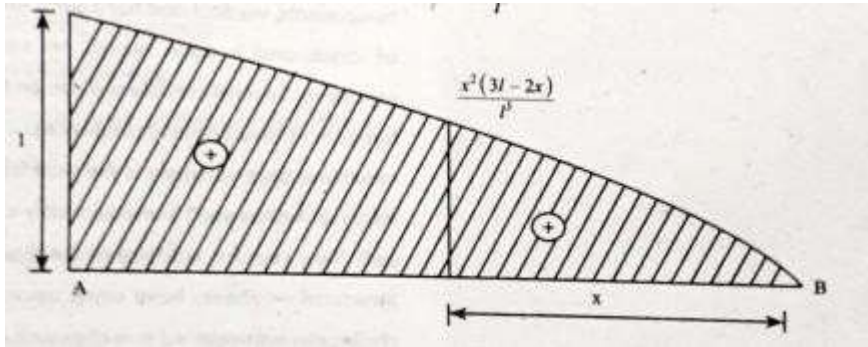
Moment at a distance x from end B of deflection from given by

$$\begin{aligned}M_x &= M_A \cdot x / 6 - M_A \cdot x^3 / 6l + M_B \cdot (1-x) / 6 - M_B \cdot (1-x)^3 / 6l \\&= [-6/l^2 \cdot lx / 6] - [-6/l^2 \cdot x^3 / 6l] + [6/l^2 \cdot 1 / 6 (1-x)] - [6/l^2 (1-x)^3 / 6l] \\&= -6/l^2 (lx / 6 - x^3 / 6l) + 6/l^2 [1(1-x) / 6 - (1-x)^3 / 6l] \\&= -6/l^2 (l^2 x - x^3 / 6l) + 6/l^2 (l^2 - lx) / 6 - (1-x)^3 / 6l \\&= 1/l^3 [-x (l^2 - x^2) + l^2 (1-x) - (1-x)^3] \\&= 1/l^3 [-x (1+x)(1-x) + l^2 (1-x) - (1-x)^3] \\&= (1-x) / l^3 [-x (1+x) + l^2 - (1-x)^2] \\&= (1-x) / l^3 [-lx - x^2 + l^2 - 1 + 2x - x^2 + 2lx] \\&= (1-x) / l^3 [-lx - 2x^2] \\&= (1-x) / l^3 [x (1-2x)] \\&= x (1-x) (1-2x) / l^3\end{aligned}$$

Influence line ordinate for reaction at support A

$$\begin{aligned}&= x/l - x(1-x) (1-2x) / l^3 \\&= x \cdot l^2 - x(1-x) (1-2x) / l^3 \\&= xl^2 - [x(1-x^2) (1-2x)] / l^3 \\&= xl^2 - [xl^2 - 2x^2l - x^2l + 2x^3] / l^3\end{aligned}$$

$$\begin{aligned} 1/l &= -M_B / 6 \\ &= x l^2 - [ 2x^3 - 3x^2 l + x^2 l / 3 \\ &= x l^2 - 2x^3 + 3x^2 l - x^2 l / 3 \\ &= - 2x^3 + 3x^2 l / 3 \\ &= x^2 ( 3l - 2x ) / 3 \end{aligned}$$



**Fig. 2.4.7**ILD for Moment at a distance x from end B

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## 2.2 INFLUENCE LINE FOR SHEARING FORCE, BENDING MOMENT AND SUPPORT REACTION COMPONENTS OF PROPPED CANTILEVER

### Influence line for bending moment at any point in propped cantilever beam

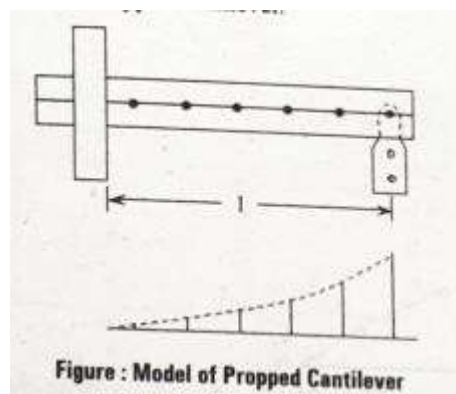
Let us consider a point 'C' at a distance 'b' from the propped end. In order to get the influence line for bending moment at 'C' by assuming that the beam is simply supported, a hinge is introduced and displacement is provided at 'CC' such that unit rotation is produced at 'C'. The slopes at A and B are  $-b/l$  and  $a/l$  respectively.

**The indirect model analysis is based on the Muller Breslau principle.**

Muller Breslau principle has led to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.

To get the influence line for any force quantity,

- (i) remove the resistant due to the force,
- (ii) apply a unit displacement in the direction of the
- (iii) plot the resulting displacement diagram. This diagram is the influence line for the force.



**Fig. 2.2.1 Propped Cantilever**

### Maximum shear force diagram

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions. When

these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram. This diagram yields the 'design shear' for each cross section.

### **Bending moment diagram**

Bending moment diagram represents variation of bending moment. Bending moment diagrams are drawn for only bending moments. If span longer than UDL for a maximum BM, the load on left side is equal to the load on right side in case of bending moment diagram.

### **Location of maximum shear force**

In a simple beam with any kind of load, the maximum positive shear force occurs at the left hand support and maximum negative shear force occurs at right hand support.

### **Influence lines**

An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

### **Uses of influence line diagrams**

(i) Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and

(ii) Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

### **Example:**

Draw the influence line for reaction at B and for the support moment  $M_A$  at A for the propped cantilever as shown in fig, compute the IL ordinate at 1.5 m intervals

$$1/l = -M_B/l$$

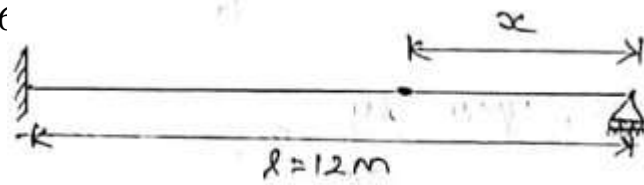


Fig. 2.2.2

**Solution :**

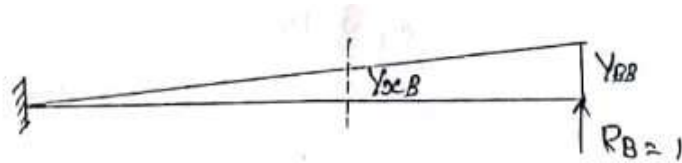


Fig. 2.2.3

when  $R_B = 1$

$Y_{xB}$  is displacement at x section

due to unit load applied at B

$$M_x = EI \frac{d^2y}{dx^2}$$

$$R_B x = -EI \frac{d^2y}{dx^2}$$

$$1 \cdot x = -EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = -x$$

Integrating on both sides

$$EI \frac{dy}{dx} = -x^2/2 + C_1 \quad (1)$$

Again Integrating both sides

$$EI Y = -x^3/6 + C_1x + C_2 \quad \text{---(2)}$$

Sub at  $x=12$   $dy/dx = 0$

$$EI dy/dx = -x^2/2 + C_1$$

$$0 = -12^2/2 + C_1$$

$$C_1 = 72$$

sub  $x=12$   $Y=0$  in 2

$$EI Y = -x^3/6 + C_1x + C_2$$

$$0 = -12^3/6 + (72 \times 12) + C_2$$

$$C_2 = -576$$

apply  $C_1$  and  $C_2$  in 2

$$EI Y = -x^3/6 + C_1x + C_2$$

$$Y_{xB} = 1/EI [ -x^3/6 + 72x - 576 ]$$

At  $x=0$

$$Y_{BB} = 1/EI [ -0^3/6 + 72 \times 0 - 576 ]$$

$$Y_{BB} = -576/EI$$

$$1/1 = -M_B / 6$$

ILO for RB at x

$$X = Y_{xB} / Y_{BB}$$

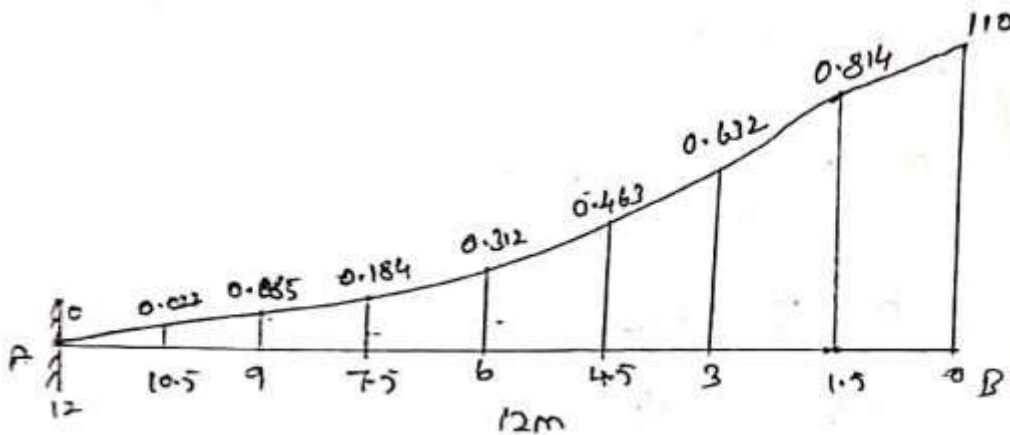
$$= [1/EI [ -x^3/6 + 72x - 576 ] ] / (-576/EI)$$

$$= [ -x^3/6 + 72x - 576 ] / (-576)$$

Ordinate of ILD for RB at 1.5m interval

x(m)	0	1.5	3	4.5	6	7.5	9	10.5	12
RB	1	0.814	0.632	0.463	0.312	0.184	0.085	0.022	0.0

**Table. 2.2.1 Ordinate of ILD for RB**



**Fig. 2.2.4 ILD for RB**



We have to apply unit rotation at A



Fig. 2.2.5

$$M_A = 1$$

$$R_B = -R_A = 1/12$$

$$M_x = -EI \frac{d^2y}{dx^2}$$

$$EI \frac{d^2y}{dx^2} = -X/12$$

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Integrate on both side

$$EI \frac{dy}{dx} = -x^2 / 24 + C_1 \quad (1)$$

Again integrating

$$EIY = -x^3 / 72 + C_1x + C_2$$

At  $x=0, y=0$

$$EI \frac{dy}{dx} = -x^2 / 24 + C_1$$

At  $x=12, y=0 = -M_B / 6$

$$EIY = -x^3 / 72 + C_1x + C_2$$

Hence  $C_2 = 0, C_1 = 2$

$$Y_{XA} = 1/EI [-x^3 / 72 + 2x]$$

$$\begin{aligned} \theta_{AA} &= dy/dx \\ &= 1/EI [-x^2 / 24 + 2] \end{aligned}$$

$\theta_{AA}$  at  $x = 12$

$$\begin{aligned} \theta_{AA} &= [-12^2 / 24 + 2] \\ &= -4/EI \end{aligned}$$

when we divide  $Y_{XA}$  by  $\theta_{AA}$  We get the ILO at X

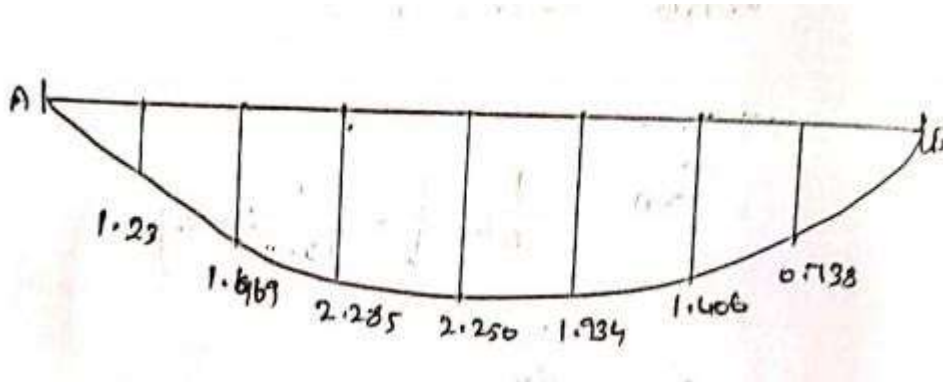
$$\begin{aligned} \text{ILO from MA} &= 1/EI [-x^2 / 24 + 2] / -4/EI \\ &= [-x^3 / 72 + 2x] / (-4) \\ &= [+x^3 / 288 - x/2] \end{aligned}$$

Ordinate of the ILD for MA at 1.5 m

x(m)	0	1.5	3	4.5	6	7.5	9	10.5	12
ILO	0	-0.788	-1.406	-1.934	-2.250	-2.285	-1.069	-1.23	0

**Table. 2.2.2 Ordinate of the ILD for MA**

$$1/1 = -M_B / 6$$



**Fig. 2.2.6 Ordinate of the ILD for MA**

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## 1/1 = - M<sub>B</sub> 1/2.1 MULLER BRESLAU'S PRINCIPLE

### Introduction

The analysis of determinate structure is what discussed in the previous unit and it is evident that influence line diagrams play a key role in it. But there are also beam such as fixed beams, continuous beams and propped cantilever that are hard to analyze by using equilibrium equations. Such structures are called indeterminate structure and they can be determined by various deformations and by application of compatibility condition.

Influence line on such structures could be improvised based on principle suggested by Muller-Breslau which is based on Maxwell's theorem of reciprocal displacement. The benefits of such principle and the analysis in indeterminate structure is the main matter of this unit..

### Muller-Breslau principle.

Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,

(i) We remove from the structure the resistant to that force quantity

(ii) We apply on the remaining structure a unit displacement corresponding to that force quantity. The resulting displacements in the structure are the influence line ordinates sought

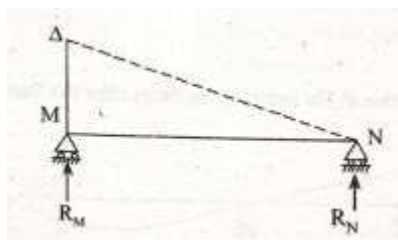


Fig. 2.1.1

**Statement** = -  $M_B$  l/6

If an internal stress component like shear force, bending moment etc, or a reaction component is allowed to act through a small distance thereby causing deformation of the structure, the curve of the deformed shape represents to some scale, the influence lines for that stress or the reaction component.

**State Maxwell-Betti's theorem.**

In a linearly elastic structure in static equilibrium acted upon by either of two systems of external forces, the virtual work done by the first system of forces in undergoing the displacements caused by the second system of forces is equal to the virtual work done by the second system of forces in undergoing the displacements caused by the first system of forces.

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