1.4 ABSOLUTE MAXIMUM BENDING MOMENT

Absolute maximum bending moment

When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section. The maximum of these bending moments will usually occur near or at the mid span. The maximum of maximum bending moments is called the absolute maximum bending moment.

Absolute maximum bending moment in a simply supported beam

When a series of wheel loads crosses a simply supported beam, the absolute maximum bending moment will occur near mid span under the load Wcr, nearest to mid span (or the heaviest load). If Wcr is placed to one side of mid span C, the resultant of the load system R shall be on the other side of C; and Wcr and R shall be equidistant from C. Now the absolute maximum bending moment will occur under Wcr. If Wcr and R coincide, the absolute maximum bending moment will occur at mid span.

Example:

A girder having a span of 18m is SS at the ends. It is traversed by a train of loads as shown in figure. The 50KN load leading. Find the maximum bending moment which can occurs i)under the 200KN load ii)under 50KN load using influence line diagram.

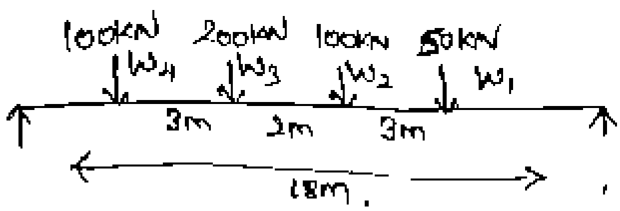

Fig. 1.4.1

Solution:

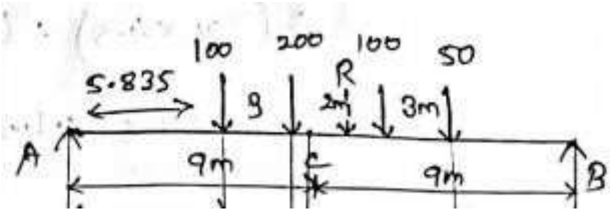

Fig. 1.4.2

Maximum bending moment

i) under 200KN load

Resultant loads =100+200+100+50

=450 KN

Taking moment about W4

(200×3)+(100×5)+(50×8)

1500

=3.33m

Ordinate max =x(l-x)/l

=9(9)/18

=4.5

Distance between C and 200KN

= dist b/w C and R

=0.33/2

=0.165

Ordinate under 100KN =4.5/8.835× 5.385

=2.97

Ordinate under 100KN =4.5/9.165× 7.165

=3.52

Ordinate under 100KN =4.5/9.165× 4.165

=2.05

BM under 200KN load

=(200×4.5)+(100×2.97)+(100×3.51)+(50×2.05)

=1650.5 KNm

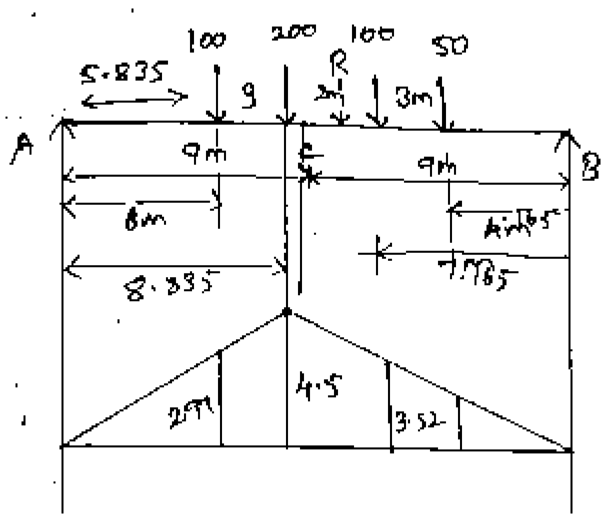

Fig. 1.4.3 Maximum Bending Moment

ii)Bending moment under 50KN load

Centre of span to BM equal distance

=(5-0.33)

Centre =4.67/2

=2.335m

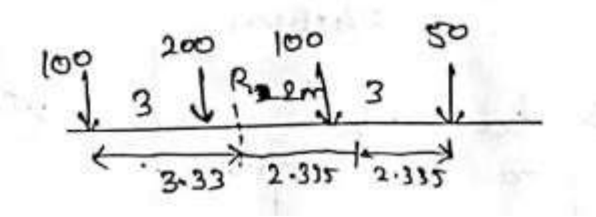

Fig. 1.4.4

Ordinate under 50KN

=x(l-x)/l

=(11.335×6.665)/18

=4.2

Ordinate under 100KN

=4.2/11.335 ×8.335

=3.08

Ordinate under 200KN

=4.2/11.335 ×6.335

=2.35

Ordinate under 100KN

=4.2/11.335 ×3.335

=1.23

Maximum bending moment

$$=(50\times4.2)+(100\times3.09)+(200\times2.35)+(100\times1.24)$$

$$=1113 \text{ KNm}$$

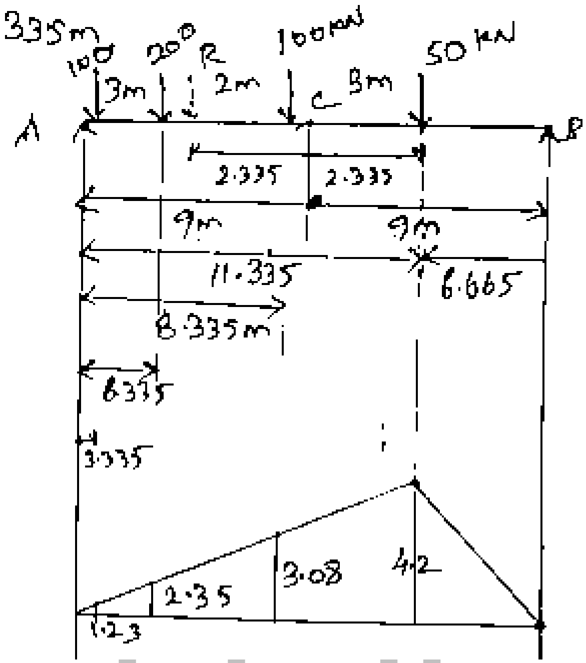

Fig. 1.4.5 Maximum Bending Moment

Example :

Draw the influence line diagram for shear force and bending moment for a section at 5m from the left hand support of a simply supported beam, 20m long. Hence calculate the max bending moment and shear force at the section, due to an uniformly distributed rolling load of length 8m and intensity 10kN/m run

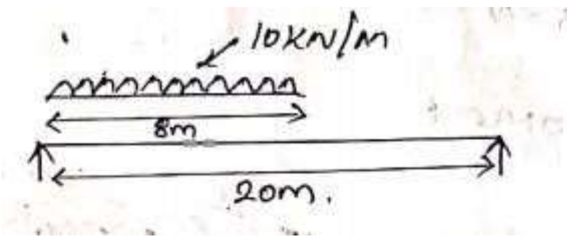

Fig. 1.4.6

Solution :

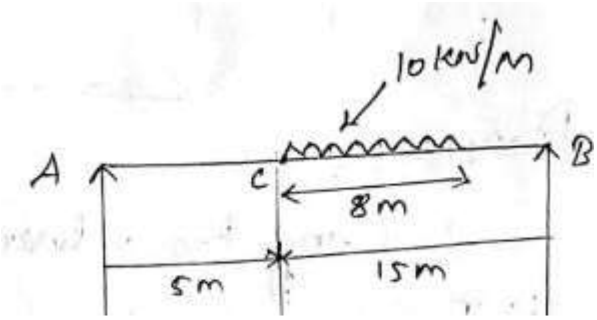

Fig. 1.4.7

a) Maximum shear force

I) Positive shear force

=l-x/l

=15/20

=0.75

Ordinate under C

=0.75/7

=0.35

Maximum positive shear force

=10×[h/2(a+b)]

=10×[0.75+0.35]8 / 2

=44KN

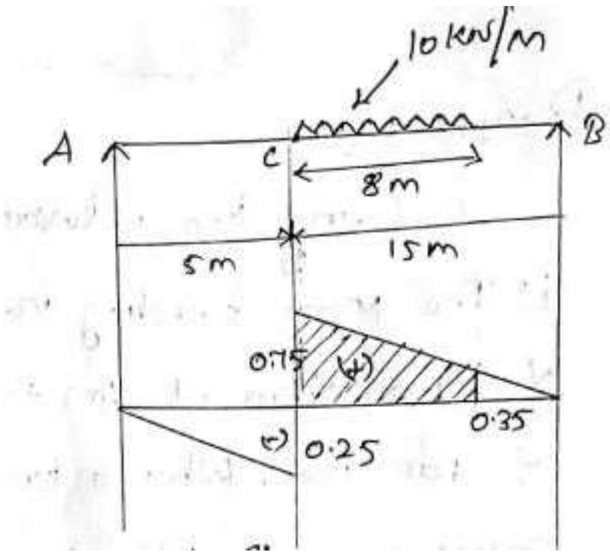

Fig. 1.4.8 ILD For Positive Shear Force

ii)negative shear force

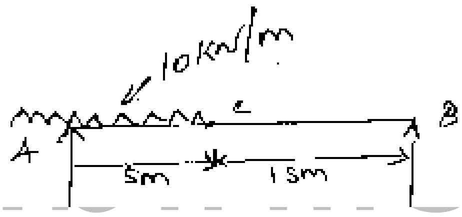

Fig. 1.4.9

Negative shear force

=x/l

=5/20

=0.25

Max Negative shear force

=10×[1/2 ×5×0.5]

=6.25 KN

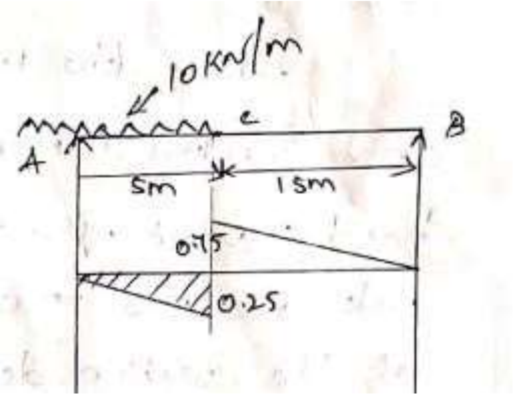

Fig. 1.4.10 ILD For Negative Shear Force

b) Max bending moment

Equal ratio

20/4 =5

8/4 =2

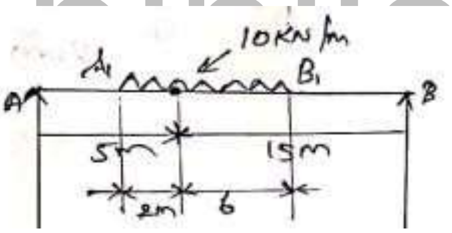

Fig. 1.4.11

Max ordinate

=x(l-x)/l

=5(15)/20

=3.75m

Ordinate under A1

=3.75/5 ×3

=2.25m

Ordinate under B1

$=3.75/15 \times 9$

$=2.25$ m

Max bending moment

$=10[(2.25+3.75)2/2 + (2.25+3.75)6/2]$

$=10(6+18)$

$=240$KNm

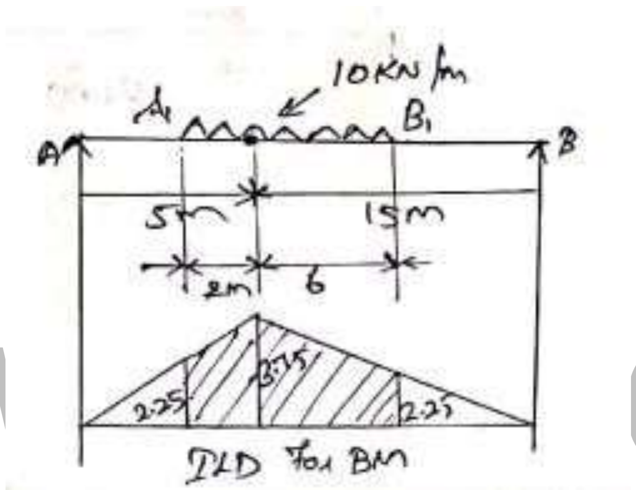

Fig. 1.4.12 ILD For Max Bending Moment

Example:

Four equal loads of 150KN each equally spaced at 2m apart. Followed by a UDL of 60KN/m at a distance of 1.5m from the last 150KN load across a girder of 20m from right to left. Using influence lines. Calculate the shear force and bending moment at a distance of 8m from the left hand support. When the leading 150KN load is at 5m from the left hand support.

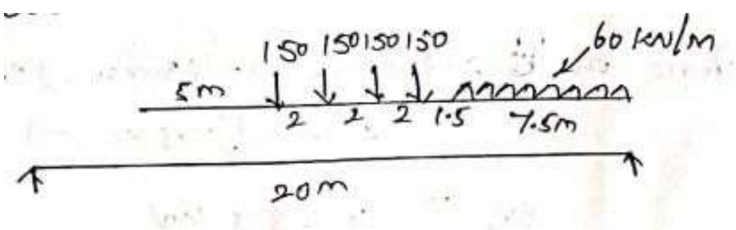

Fig. 1.4.13

Solution :

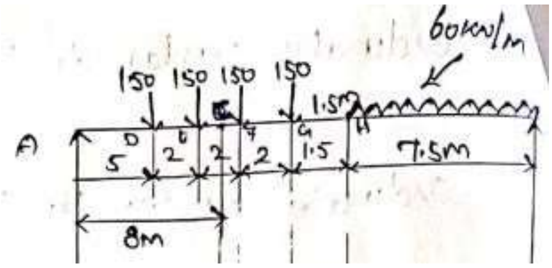

Fig. 1.4.14

a)shear force at this section

Positive shear force

=l-x/l

=20-8/20

=0.6

Ordinate under C left

=x/l

=8/20

=0.4

Ordinate under F

=0.6/12 ×11

=0.55

Ordinate under G

=0.6/12 ×9

=0.45

Ordinate under H

=0.6/12 ×7.5

=0.375

Ordinate under E

=-0.4/8×7

=-0.35

Ordinate under D

=-0.4/8 ×7

=-0.25

Shear force at C

=-(150×0.25)-(50×0.35)+(150×0.55)+(150×0.45)+(60×(1/2×7.5)× 0.35)

SF@c =144.375 KN

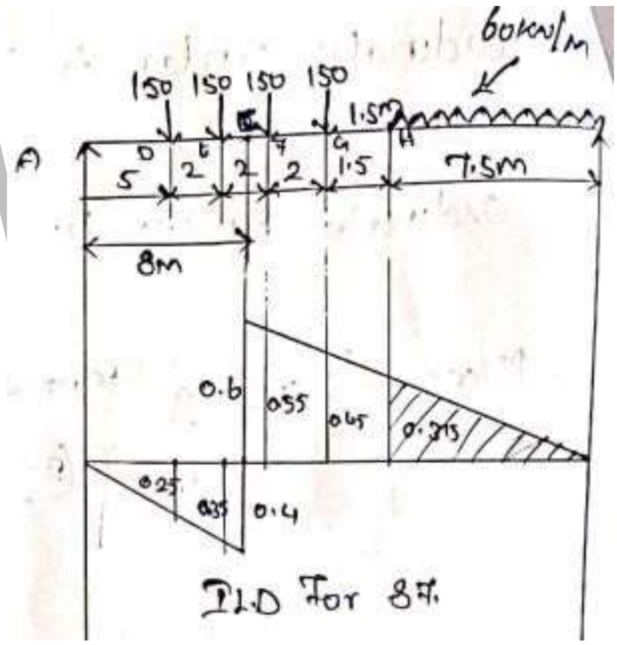

Fig. 1.4.15 ILD For Shear Force

b) Bending moment for given load position

Max Ordinate at C

=x(l-x)/l

=8(12)/20

=4.8

Ordinate under E

=4.8/8 ×7

=4.2m

Ordinate under D

=4.8/8×5

=3m

Ordinate under F

=4.8/12 ×11

=4.4m

Ordinate under G

=4.8/12 ×9

=3.6 m

Ordinate under H

=4.8/12 ×7.5

=3m

Bending moment at C

=(150×3)+(150×4.2)+(150×4.4)+(50×3.6)+(60×(1/2)×7.5×3)

=2955 KNm

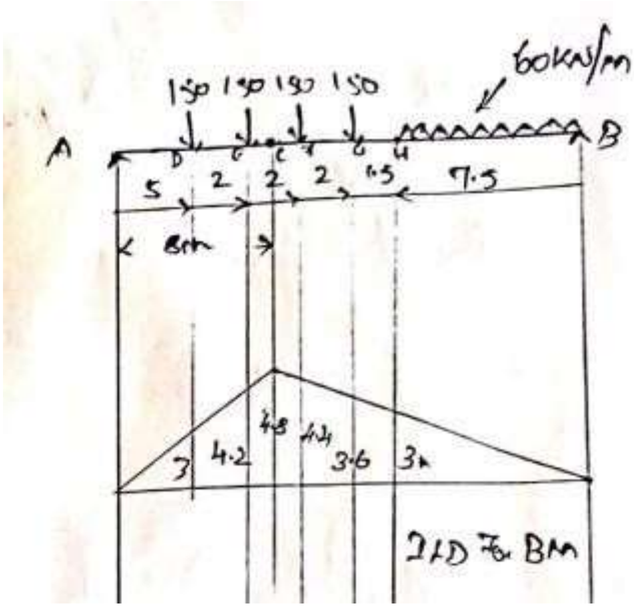

Fig. 1.4.16 ILD For Max Bending Moment

1.3 CALCULATION OF CRITICAL STRESS RESULTANTS DUE TO CONCENTRATED AND DISTRIBUTED MOVING LOADS

Equivalent uniformly distributed load

A girder with live load system is consider, in which the system is replaced by an equivalent uniformly distributed load over the total span so that the shear force at any section due to UDL is equal to or more than the maximum shear force due to live load system or bending moment at any section due to UDL is equal to or maximum bending moment at the section due to live load system. The equivalent uniformly distributed load can be determined for different live load system such as Single wheel load, Uniformly distributed live load shorter than the span, Two wheel loads W1 and W2 spaced apart.

Moving loads

The load moving across the span changes the magnitude of shear force and bending moment at every cross section of the girder. Such loads are known as moving loads or rolling loads.

Maximum shear force diagram

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions. When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram. This diagram yields the 'design shear' for each cross section.

Example:

Two point load of 100KN and 200KN span 3m apart cross a girder of span 15m from the left to right with the 100KN load. Draw the influence for the shear force and bending moment and find the value of max shear force and bending moment at a section D,6m from the left hand support. Also find the absolute max bending moment due to the given load system.

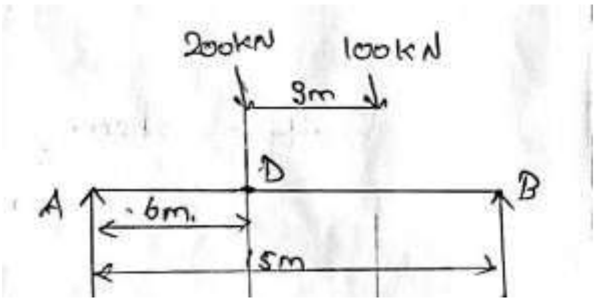

Fig. 1.3.1

Solution:

a) Find the max shear force

shear increament

Si =Wc/l -W1

=300/15 -200

=-180

i) positive shear force

l-x/l =15-6/15

= 0.6

x/l =6/15

= 0.4

ordinate under 200KN

= 0.6

ordinate under 100KN

0.6/9 ×6 = 0.4

max positive shear force

= (200×0.6)+(100×0.4)

=160KN

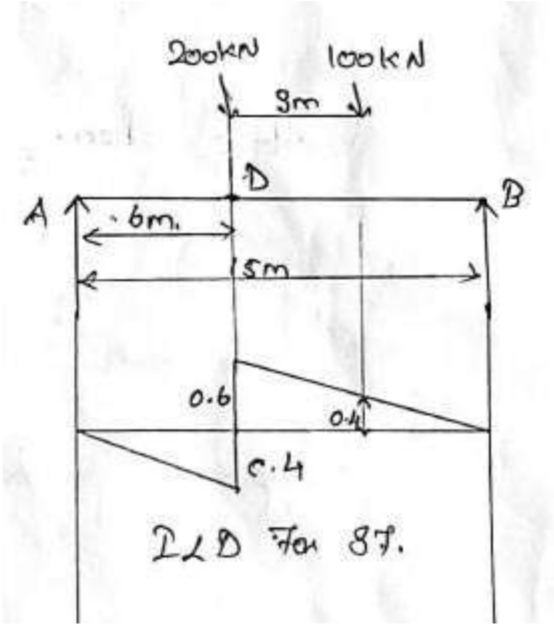

Fig. 1.3.2 ILD For Positive Shear Force

ii) Negative shear force

shear increament

Si =wl/l -w1

=300×3/15 -100

= -40

Ordinate under 200KN

=0.4/6 ×3

=0.2

max negative shear force

=(200×0.2)+(100×0.4)

=80KN(neg)

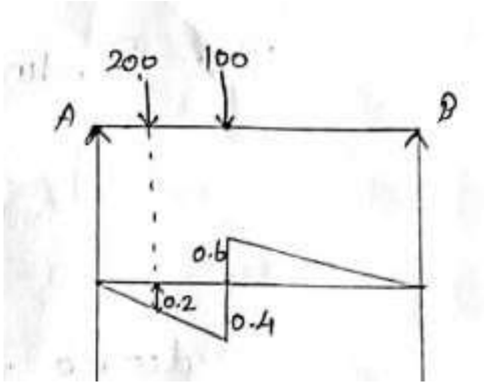

Fig. 1.3.3 ILD For Negative Shear Force

b)max bending moment

find critical load

loading rate Lr = W leff/x -W right/(l-x)

=200/6 -100/9

= 22 (+)

Loading rate Lr =0/6 - 300/9

= - 33.33 (_ve)

ordinate under 100KN

=3.6/9 × 6

=2.4 m

max bending moment

= load × ordinate

=(200×3.6)+(100×2.4)

= 960 KNm

ordinate of ILD =x(l-x)/l

= 9× 6/ 15

= 3.6 m

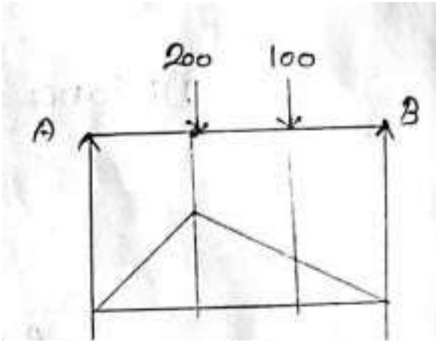

Fig. 1.3.4 ILD For Max Bending Moment

C) Absolute max bending moment

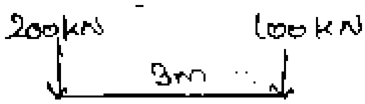

Fig. 1.3.5

Taking moment about 200KN

100×3 x

300 x

x = 1 m

Distance of this 200KN from C

x/2

=1/2

=0.5

Max ordinate under 200KN

=(l-x)x/l

=8×7/15

=3.73 m

Ordinate under 100KN

=3.73/8 × 5

=2.33 m

Absolute max bending moment

= (200×3.73) +(100×2.33)

=979.3 KNm.

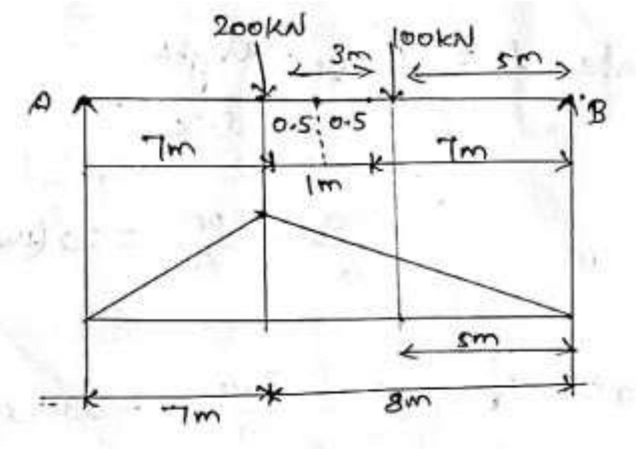

Fig. 1.3.5 ILD For Absolute Max Bending Moment

1.5 INFLUENCE LINES FOR MEMBER FORCES IN PIN JOINTED PLANE FRAMES

Forces in the members of truss

Based on the loading conditions the members of truss experience torsion and compression. The diagonal members are in tension and vertical members are in compression. When the unit load is transmitted along the bottom chord members the influence l9ine diagrams will be drawn for different chord members.

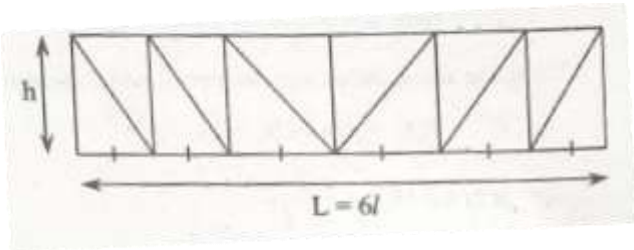

Fig. 1.5.1 Truss

Top chord members

Consider a top chord member U1, U2 Let the support reactions be RA and RB. When a unit load is taken, three conditions are taken into considerations which are as follows

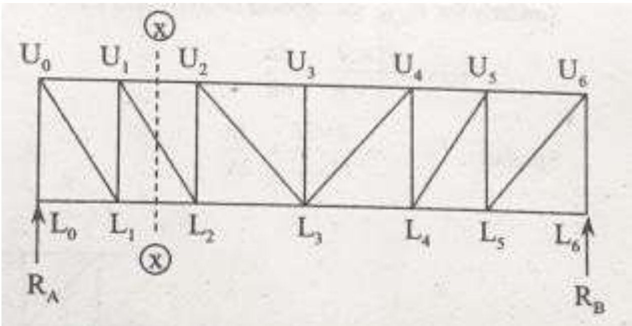

Fig. 1.5.2 Top Chord Members

I. When unit load is on the left side off L1
II. When unit load is on the right side off L2
III. When unit load is between L1 and L2

Rolling loads

Shifting of load positions is common enough in buildings. But they are more pronounced in bridges and in gantry girders over which vehicles keep rolling.

Reversal of stresses

In certain long trusses the web members can develop either tension or compression depending upon the position of live loads. This tendency to change the nature of stresses is called reversal of stresses.

Example :

Draw the influence line diagram for forces in the member of warren truss as shown below

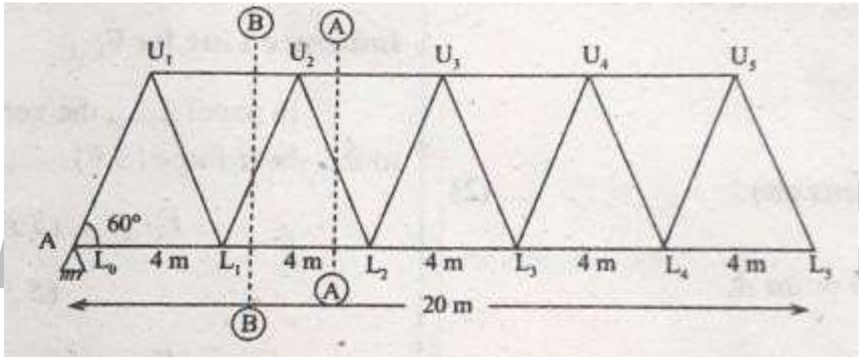

Fig. 1.5.3

Solution

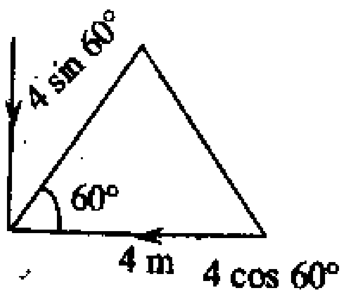

Fig. 1.5.4

The truss height 'h' is

$$h = 4 \sin 60$$

h =3.464 m

Influence line for F U_2 U_3

considering a section AA

$P_{U2\ U3} = Ml_2 / h$

$P_{U2\ U3} = Ml_2 /3.464$ (compression)

=1/3.464 (8×12/20)

=1.386 (under point L2)

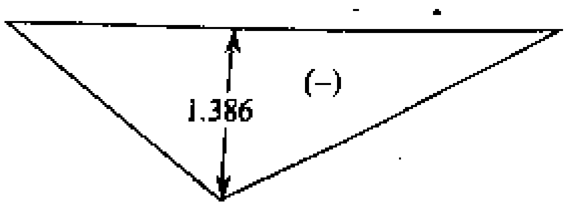

Fig. 1.5.5 ILD for F U_2 U_3

Influence line for the Force in member L_1 L_2 (F L_1L_2)

F L_1 L_2 =M_{U2} /h

F L_1 L_2 =M_{U2} /3.464 (Tension)___2

When the load is acting on point A

RB =0

M_{U2} and F L_1 L_2 = 0

When the load is acting on point L_1

RB = 1×4 / 20

= 1/5

M_{U2} = RB × 14

M_{U2} = 1/5 × 14

=2.8 KN-m

sub the value of M_{U2} in eqn 2

$F\,L_1\,L_2$ = 1 /3.464(1/5×14)

$F\,L_1\,L_2$ = 0.808 (Tension)

When the load is acting on point L2 then

RA = 1×12/ 20

= 3/5

M_{U2} = RA ×6

M_{U2} = 3/5 ×6

sub the value of M_{U2} in eqn 2

$F\,L_1\,L_2$ =M_{U2}/3.464

= 1/3.464 (3/5 ×6)

$F\,L_1\,L_2$ =1.039 (tension)

when the load is acting on point B then

RA =0

and

M_{U2} and $F\,L_1\,L_2$ =0

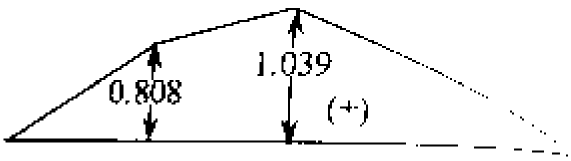

Fig. 1.5.6 ILD for L_1 L_2 (F L_1L_2)

Influence line for F U_2 L_2

=(s.f) L_1 L_2 Cosec Ø

=(s.f) L_1 L_2 Cosec 60

F U_2 L_2 =1.155 (s.f) L_1 L_2

When the load is acting on point A

RA = 1

(s.f) L_1 L_2 and

F U_2 L_2 = 0

when the load is acting on point load L1

F U_2 L_2 =cosec Ø × m/n

= cosec 60 × 1/5

F U_2 L_2 = 0.231(compression)

F U_2 L_2 = 1.155 × n-m-1 / n

= 1.155 × 5-1-1/5

F U_2 L_2 = 0.693 (tension)

When the load is acting on point B

RA =0

and

(s.f) L_1 L_2 and

F U_2 L_2 = 0

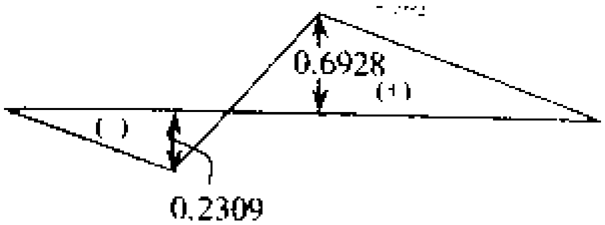

Fig. 1.5.7 Influence line for F U_2 L_2

Influence line for F U_2 L_1

F $U_2 L_2$ $=$(s.f) $L_1 L_2$

Cosec Ø $=$(s.f) L_1 L_2 Cosec 60

$=1.155$ (s.f) L_1 L_2

When the load is acting on point L1

F $U_2 L_1$ = tensile

F $U_2 L_1$ = compressive

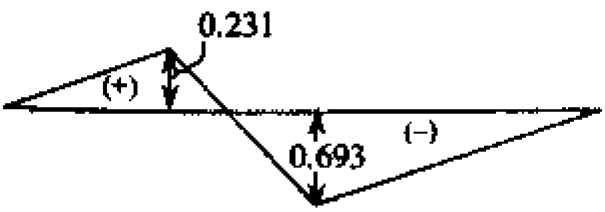

Fig. 1.5.8 Influence line for F U_2 L_1

Example:

Draw the IL for force in member BC and CI for the truss shown in figure, the of the height truss is 8m and each segments is 8m long

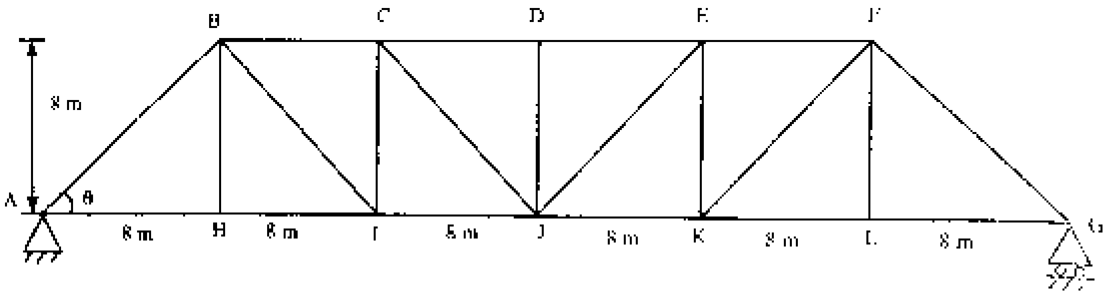

Fig. 1.5.9

Solution

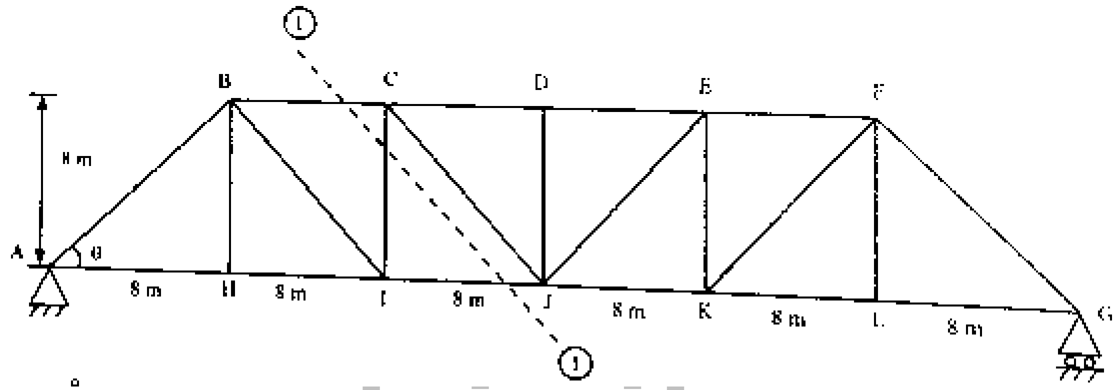

Fig. 1.5.10

height of the truss = 8m

length = 8m

$\tan Ø = 8/8$

$= 1$

$Ø = 45$

$\sin Ø = \sin 45$

$= 1/\sqrt{2}$

$\cos Ø = \cos 45$

$= 1/\sqrt{2}$

Influence line for force in the Member BC (P_{Bc})

$P_{Bc} = M_I/9$

Moment at I = a(l - a) /l

=16 (48-16) / 48

= 16×32 / 48

= 10.46

Force in member BC

= 10.66/8

= 1.33 KN

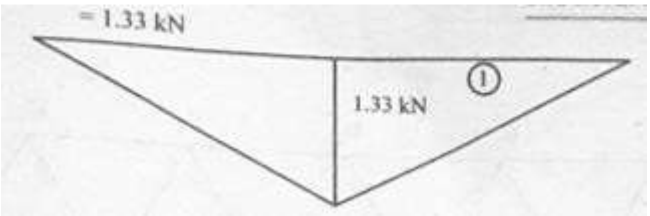

Fig. 1.5.11 Influence line for force in the Member BC (P_{Bc})

Influence line Diagram for force in Member CI (P_{CI})

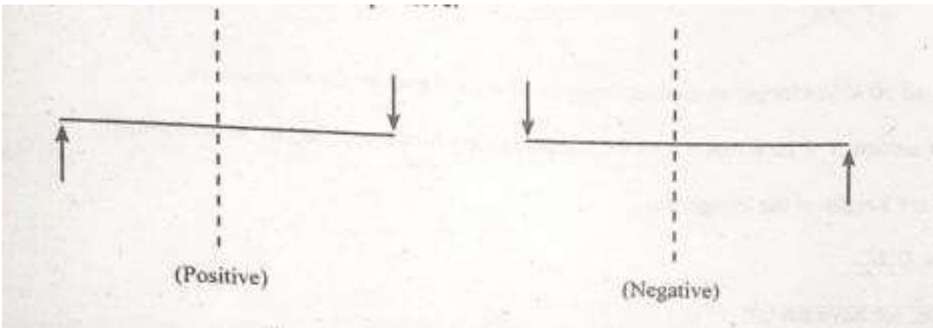

Fig. 1.5.12 Influence line for force in the Member CI (P_{CI})

At point I ordinate of I.LD

= a/l

= 16 /48

= 1/3

At point J ,ordinate of I.L.D

= - a/ l (Compressive)

= -24/48

= -1 /2

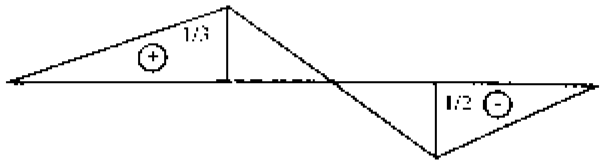

Fig. 1.5.13 Influence line for force in the Member CI (P_{CI})

1.1 INFLUENCE LINES FOR REACTIONS IN STATICALLY DETERMINATE BEAMS

Influence lines

An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

Uses of influence line diagrams

(i) Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and

(ii) Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

Influence line diagram

Influence line diagram represents variation of bending moment at one particular section at a unit load moves along the length of the member.

Influence line diagram are drawn for shear force, reaction and for bending moment..

If span longer than UDL for the maximum and negative shear force, the load should cover only either positive or negative portion of the influence line diagram.

Example:

A single rolling load of 100KN moves on a girder of span 20m. a) Construct the influence line for shear force and bending moment for a section 5m from the left support. b) construct the influence line for point at which absolute maximum shear and absolute maximum bending moment develop. Determine absolute maximum value.

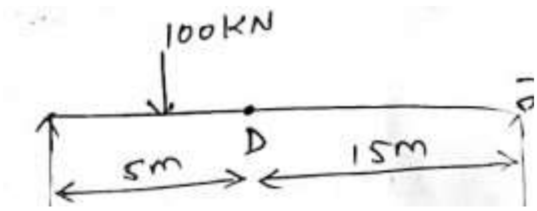

Fig. 1.1.1

Solution :

a) To find max shear force and bending moment at 5m from left support.

Influence line diagram for shear force

IL ordinate to the right of D

=l-x/l

=20-5/20

=0.75

IL ordinate to the left of D

=x/l

=5/20

=0.25

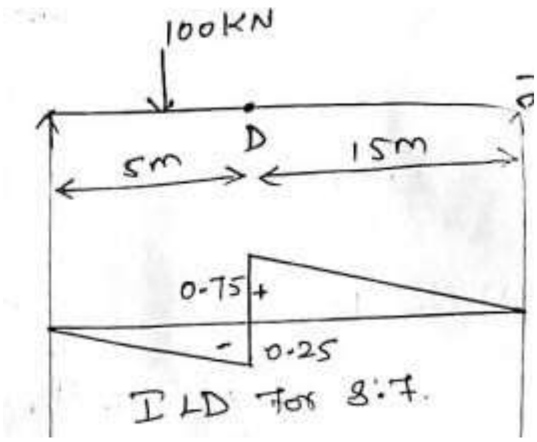

Fig. 1.1.2 Influence Line Diagram For Shear Force

lL for BM

IL ordinate at D

x(l-x)/l =5(20-5) / 20

= 3.75 m

Max positive shear force

= load × ordinate

=100×0.75

=75KN(+)

Max negative shear force

=load ×Ordinate

=100×0.25

=25KN(-)

Max bending moment

=load × ordinate

= 100×3.75

=375 KNm

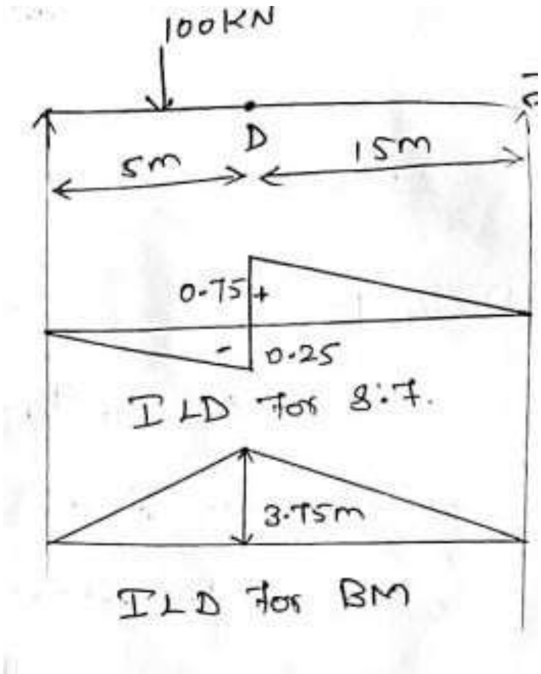

Fig. 1.1.3 Influence Line Diagram For Bending Moment

b) Absolute max shear force and bending moment

For shear force

IL ordinate at A

=20/20

=1

IL ordinate of B

=20/20

=1

IL ordinate at BM at mid span

=l/4

=20/4

=5

positive shear force = load × ordinate

=100×1

=100KN(+)

positive shear force = load × ordinate

=100×1

=100KN(-)

Absolute max bending moment

=load ×Ordinate

=100×5

=500 KNm

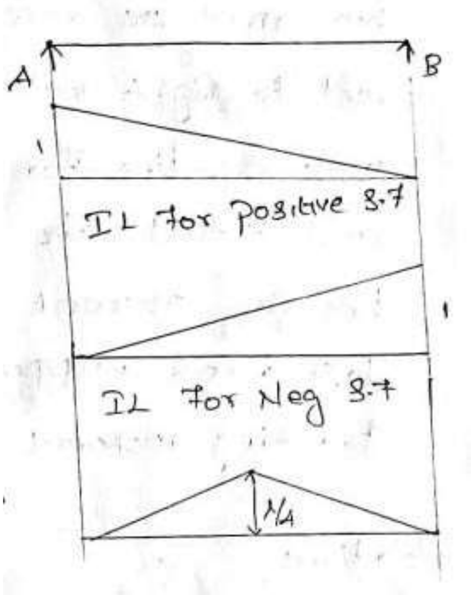

Fig. 1.1.4 Absolute Max Shear Force And Bending Moment

1.2 INFLUENCE LINES FOR SHEAR FORCE AND BENDING MOMENT

Location of maximum shear force

In a simple beam with any kind of load, the maximum positive shear force occurs at the left hand support and maximum negative shear force occurs at right hand support.

Maximum shear force diagram

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions. When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram. This diagram yields the 'design shear' for each cross section.

Bending moment diagram

Bending moment diagram represents variation of bending moment. Bending moment diagrams are drawn for only bending moments. If span longer than UDL for a maximum BM, the load on left side is equal to the load on right side in case of bending moment diagram.

Several point loads

The maximum bending moment for a series of moving loads is obtained when the average load on the left of the section is equal to the average load on the right of the section.

The above statement exists in a system of moving point loads. In such cases, each load is passed over the section and average load on each side is calculated. The load, when the crosses the section makes the heavier side lighter and lighter side heavier and gives the maximum bending moment at the section.

Example:

A train of 5 wheel loads crosses a ss beam of span 22.5m.using influence lines, calculate the max positive and negative shear forces at mid span and absolute max bending moment anywhere in the span.

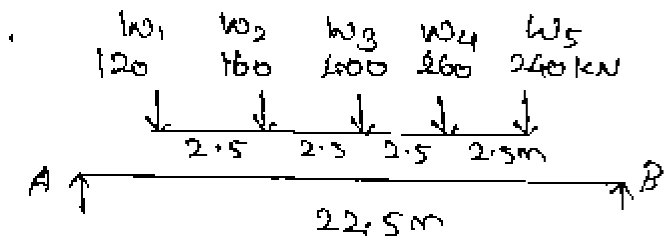

Fig. 1.2.1

Solution:

a)max shear force

find shear increment

w =1180KN

c =2.5

Si =Wc/l -W1

=(1180×2.5)/(22.5)-120

=11.11(+)

Si =Wc/l -W1

=(1180×2.5)/(22.5)-160

=- 28.8(-)

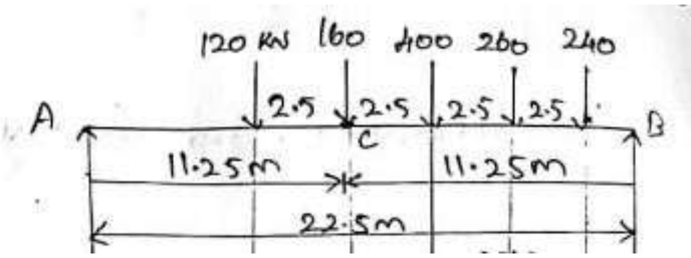

Fig. 1.2.2

Ordinate under C

right side =l-x/l

=11.25/22.5

=0.5

left side =x/l

=11.25/22.5

= 0.5

Ordinate under 400KN

=0.5/11.25 × 8.75

=0.38

Ordinate under 260KN

=0.5/11.25 ×3.75

= 0.16

Ordinate under 120KN

=-0.5/11.25 ×8.75

= -0.38(-)

Max positive shear force

= (-120×0.38)+(400×0.38)+(260×0.27)+(240×0.16)+(160×0.5)

=295 KN

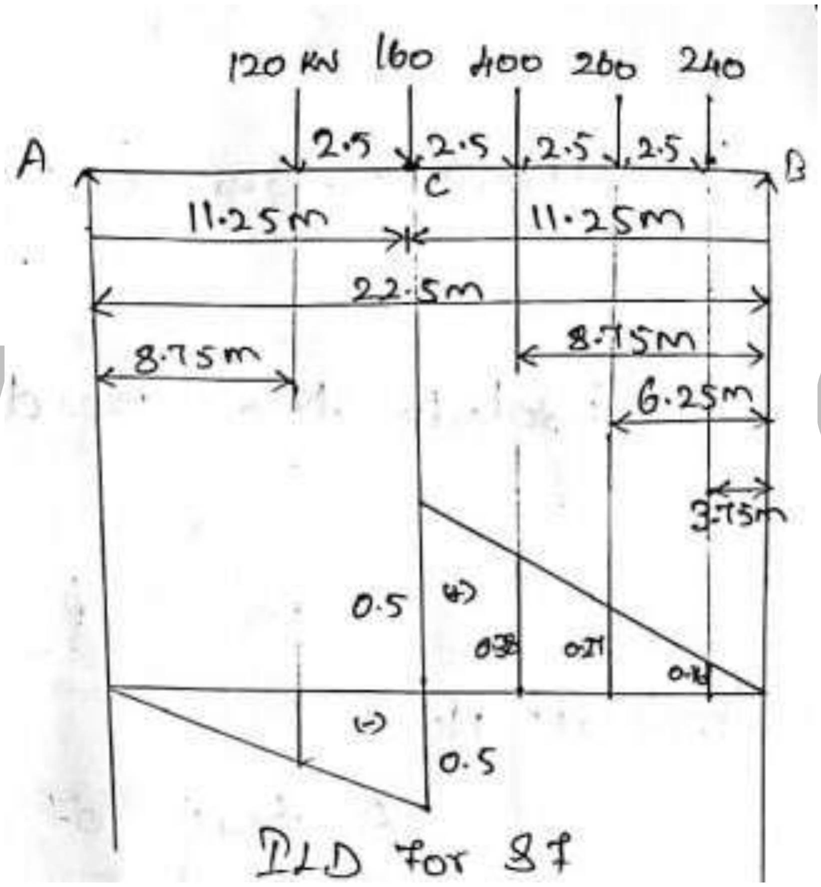

Fig. 1.2.3 Influence Line Diagram For Positive Shear Force

ii) Negative shear force

Find shear increment

Si =Wc/l - W1

=(1180×2.5)/(22.5) - 240

= -108.89(-)

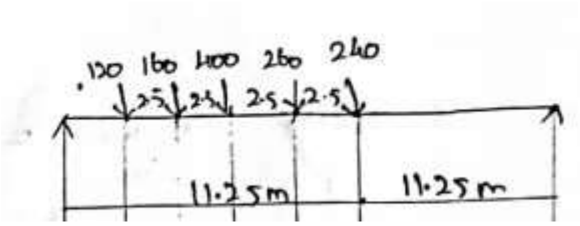

Fig. 1.2.4

Ordinate under 400KN

=0.5/11.25 × 6.25

=0.27

Ordinate under 260KN

=0.5/11.25 ×8.75

= 0.39

Ordinate under 160KN

=0.5/11.25 × 3.75

=0.167

Ordinate under 120KN

=0.5/11.25 ×1.25

= 0.056

Max negative shear force

=(240×-0.5)+(260×-0.39)+(400×- 0.27)+(160×0.167)+(120×0.056)

=-366.04 KN (-ve)

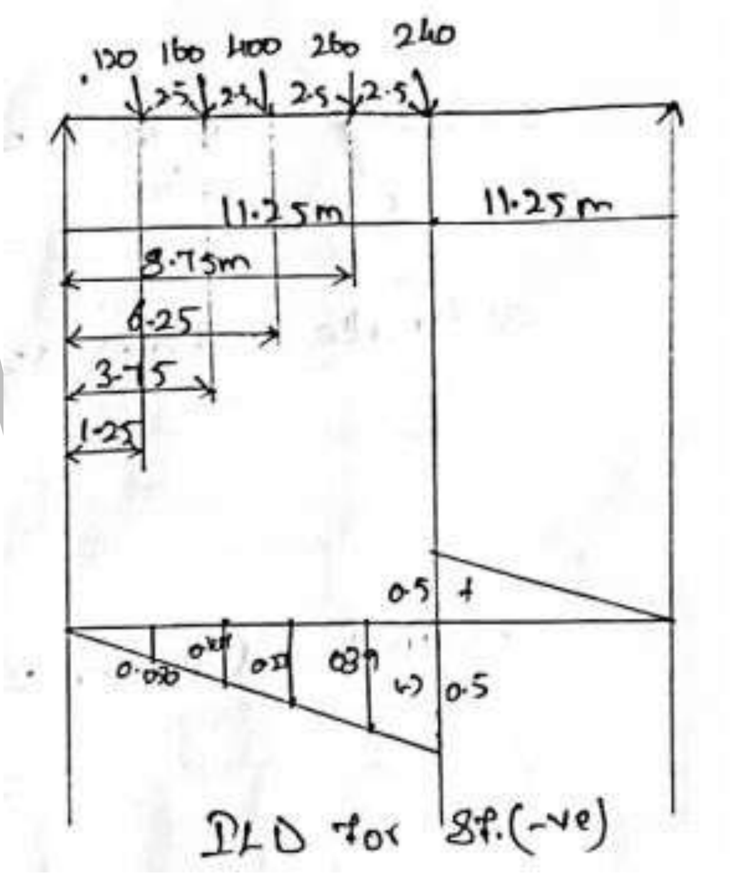

Fig. 1.2.5 Influence Line Diagram For Negative Shear Force

b) Absolute max bending moment

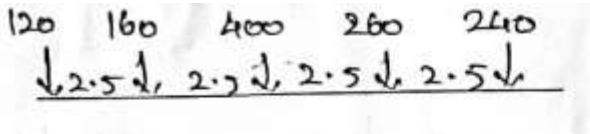

Fig. 1.2.4

Taking moment about 120KN

. . x

6750 x

x =5.72 m

Max ordinate of lLD

=x(l-x) /l

=10.89(22.5-10.39)/22.5

=5.62

Ordinate under 160KN

=5.62/10.89 × 8.39

=4.33

Ordinate under 120KN

=5.62/10.89 × 5.39

=3.04

Ordinate under 160KN

=5.62/10.89 × 9.11

=4,41

Ordinate under 160KN

=5.62/10.89 × 6.61

=3.20

Absolute maximum bending moment

=120(3.04)+160(4.33)+400(5.62)+260(4.41)+(240×3.2)

=5220.2 KNm

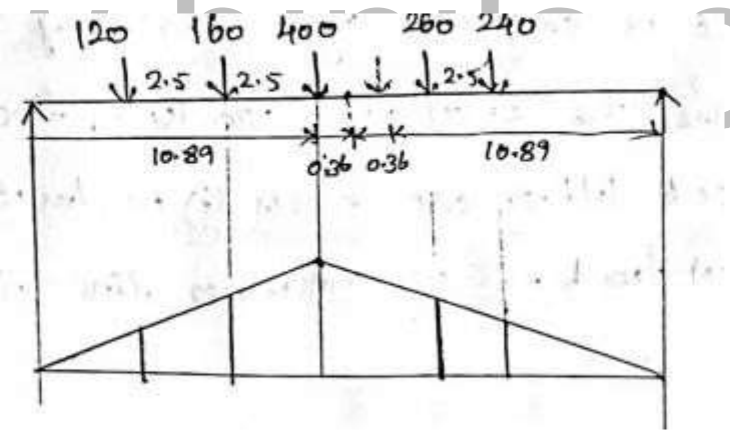

Fig. 1.2.4 Absolute Maximum Bending Moment