## CAPILLARY WATER:

It is held in the inter space of soil due to capillary forces. Capillary action (or) Capillarity is the phenomenon of movement of water is the interstices of a soil due to capillary forces. The minute pores of soil serve as capillary tubes through which the moisture rises above the ground water table.

The capillary force depends on surface tension of water, pressure in water and size of pores.

## CAPILLARY RISE:

The capillary water is held in the intersices of soil due to capillary forces.
Capillary action (or)Capillarity is the phenomenon of movement of water between the voids of the soil due to capillary forces.

The minute pores of soil serve as capillary tubes through which the moisture rises above the groundwater table.
The capillary force depends on surface tension of water, pressure in water and size of pores. The rise of water in the capillary tube is due to the existence of surface tension is called as capillary rise.
ie,
weight of water for the capillary rise height $\mathrm{W}_{\mathrm{w}}=\rho_{\mathrm{w}} \mathrm{gX}$ volume

$$
=\rho_{w} g K d^{\pi d^{2}} \quad X h_{c}-
$$

Vertical component of surface tension $=\mathrm{T}_{5} \cos \theta$
Surface tension force=surface tension X length

$$
=\mathrm{T}_{\mathrm{s}} \cos \theta \mathrm{X} \pi \mathrm{~d}------(2)
$$

At equilibrium (1)\&(2)

$$
\rho_{w} g X \frac{\pi d^{2}}{4} X h_{c}=T \cos \theta \mathrm{x} \pi \mathrm{~d}
$$

Height of capillary water,

$$
\mathrm{h}_{\mathrm{c}}=\frac{4 \mathrm{Ts}}{\gamma_{\mathrm{w}} \mathrm{~d}}
$$

where, $\quad h_{c}=$ height of capillary rise

$$
\text { at } 4^{\circ} \mathrm{C} \text {, of water } \mathrm{h}_{\mathrm{c} \max }=\underline{0.3084 \mathrm{~cm}}
$$

d
at $20^{\circ} \mathrm{C}$, of water $\mathrm{h}_{\mathrm{c} \text { max }}=\underline{0.2975} \mathrm{~cm}$
d

## CAPILLARY TENSION (OR) CAPILLARY POTENTIAL:

It is to be noted that the water in the capillary tube,
$\longrightarrow$ above the G.W.L., (or) free water surface will be in state of Tension. below the G.W.L., (or) free water surface will be in hydrostatic compression as usual.

At any height ' $h$ ' above the W.T.,
Stress, $\mathrm{u}=-\mathrm{h} \gamma_{\mathrm{w}}$ (minus sign for tension)
Max, magnitude of the stress ' $u$ ' will depend on the radius
' $R$ 'of themeniscus.
The relationship between ' $d$ ' and ' $R$ ' is

$$
\underline{\mathrm{d}}=\mathrm{R}
$$


$\cos \alpha 2$

$$
d=2 R \cos \alpha
$$


$u_{c}=$ max., tension at the level of meniscus $=\gamma_{w} h_{c}=\gamma_{w} \underline{4 T s}=\underline{4 T s}$

$$
\gamma_{w} 2 R
$$

2R

$$
\mathrm{u}_{\mathrm{c}}(\max )=\underline{4 \mathrm{Ts}}
$$

d

The tensile stress caused in water is called the capillary tension (or) the capillary potential. It is the pressure deficiency (or) pressure reduction (or) negative pressure in the pore water (pressure below atmospheric) by which water is retained in a soil mass.

It decreases linearly from a max., value of $\gamma_{w} h_{c}$ at the level of meniscus to zero value at the free water surface.

## SOIL SUCTION:

The pressure deficiency in the held water is also termed as soil suction (or) suction pressure. It is measured by the height ' $h_{c}$ ' in ' cm ' to which a water column could be drawn by suction in a soil mass free from external stress. The common logarithm of this height (cm) or pressure $\left(\mathrm{g} / \mathrm{cm}^{2}\right)$ is known asPF value.

$$
\mathrm{PF}=\log _{10}\left(\mathrm{~h}_{\mathrm{c}}\right)
$$

$\mathrm{PF}=2$,Soil suction $=100 \mathrm{~cm}$ of water.
Suction pressure $=100 \mathrm{~g} / \mathrm{cm}^{2}$.

## FACTORS AFFECTING SOIL SUCTION:

## 1) Particle size of soil:

Smaller the size of the particles, smaller will be the pore size with small radii of menisci, resultingin greater capillary rise and hence greater suction.

## 2) Water content:

Smaller the w/c, greater will be the soil suction. Soil suction will attain its maximum value whenthe soil is dry.

## 3) Plasticity Index of soil:

For a given w/c, soil suction will be greater in a soil which has greater plasticity index (Ip) than inthe one which has lower Ip.

## 4) History of drying and wetting:

For the same soil, suction is greater during drying cycle than during wetting cycle.

## 5) Soil structure:

The size of interstices in a soil depends on the structure of the soil. Change in the structure of soil result inthe change in the size of interstices and hence change in soil suction.

## 6) Temperature:

Rise in temperature results in decrease in Surface Tension (Ts) and hence decrease in soil suction. Similarly, fall in temperature results in increase of soil suction

## 7) Denseness of soil:

Increase in denseness of soil results in decrease in the size of the pores of the soil and hence increase in soil suction. At low density, the soil will be relatively loose, with the larger size pores resulting in decrease in soil suction.

## 8) Angle of contact:

The mineralogical composition of soil governs the angle of contact between the soil particles and water. Soil suction decreases with increase in the angle of contact $(\alpha)$. $\alpha=0$, soil suction is maximum depends on $h_{c}$.

## 9) Dissolved salts in pore water:

Impurities such as dissolved salts etc., increase the surface tension resulting in increase of soil suction.

## Problem

1) Compute the maximum capillary tension for a tube be 0.05 mm in diameter

The maximum capillary height at $4^{0} \mathrm{C}$

$$
\begin{gathered}
\left(h_{c \max }\right)=\frac{0.3084}{d} \\
0.3084 \\
=0.005=61.7 \mathrm{~cm}=0.617 \mathrm{~m} \\
\text { capillary tension }=\left(h_{c \max }\right) \gamma_{w}=0.617 \times 9.81=6.05 \mathrm{KN} / \mathrm{m}^{3}
\end{gathered}
$$

2) the internal dia of a tube is 0.1 mm What will be the maximum capillary rise when it held vertical with bottom end dipped in pure water taken in a trough?also compute the maximum capillary tension if the temperature of water is $20^{\circ} \mathrm{C}$.

Given:
$\mathrm{d}=0.1 \mathrm{~mm}=0.1 \times 10^{-3}$
For $20^{\circ} \mathrm{C}, \mathrm{T}_{\mathrm{s}}=72.8 \times 10^{-6} \mathrm{KN} / \mathrm{m}$
Pure water , $\theta=0$

$$
h_{c}=\frac{4 T_{s} \cos \theta}{d \gamma_{w}}
$$

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$$
\frac{4 \times 72.8 \times 10^{-6} \cos 0}{\left(0.1 \times 10^{-3}\right) \times 9.81}-0297 \mathrm{~m}
$$

$$
\left.\begin{array}{r}
\text { Maximum } \\
\text { capillary } \\
\text { tension }=h_{c} \gamma_{w}
\end{array}\right]=0.297 \mathrm{x} 9.81 \text { }=2.91 \mathrm{KN} / \mathrm{m}^{2} .
$$

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## PERMEABILITY:

## Definition:

Permeability is defined as the property of a porous material which permits the passage or seepage of water through its interconnecting voids.

A material having continuous voids is called permeable.

## Gravel highly permeable <br> Stiff clay least permeable (impermeable).

## Laminar flow:

Each fluid particle travels along a definite path which never crosses the path of any other particle.

## Turbulant flow:

The paths are irregular, twisting, crossing and recrossing at random.

## Darcy's Law:

For laminar flow conditions in a saturated soil, the rate of flow (or) the discharge per unit time is proportional to the hydraulic gradient.

$$
\mathrm{q}=\mathrm{kiA}
$$



$$
V=K i={ }^{q} \quad \bar{A}
$$

Where, $\mathrm{q}<$ discharge per unit time
$A \geq$ Total $\mathrm{c} / \mathrm{s}$ area of soil mass, perpendicular to the direction of $\mathbf{6} \mathrm{w}$
i $\because$ Hydraulic gradient
$\mathrm{k} \geq$ Darcy's coefficient of permeability
$\mathrm{V}<$ Velocity of flow (or) average discharge velocity
If soil sample of length ' $L$ ' and $c / s$ area 'A' subjected to differential head of water ( $h_{1}-h_{2}$ ), the hydraulic gradient ' $i$ ' will be equal to $\frac{h 1-h 2}{L}$

$$
q=\frac{K\left(h_{1}-h_{2}\right) A}{L}
$$

If ' i ' is unity ' $k$ ' is equal to ' V '
Thus, coefficient of permeability is defined as the velocity of flow that will occur through the c/s area of
soil under unit hydraulic gradient.
$\mathrm{k} \geqslant \mathrm{V} \geq \mathrm{cm} / \mathrm{sec}$ (or) $\mathrm{m} /$ day (or) feet/day.

## Discharge velocity and seepage velocity:

The velocity of flow ' $v$ ' is the rate of discharge of water per unit of total $c / s$ area ' $A$ ' of soil.

$$
\mathrm{A}=\mathrm{A}_{\mathrm{s}}+\mathrm{A}_{\mathrm{v}}
$$

Since, the flow takes through the voids, the actual (or) true velocity of flow will be more than the discharge velocity. This actual velocity is called the seepage velocity $\left(V_{s}\right)$.

It is defined as the rate of discharge of percolating water per unit $\mathrm{c} / \mathrm{s}$ area of voids perpendicular to the direction of flow.

$$
\begin{gathered}
\mathrm{q}=\mathrm{VA}=\mathrm{V}_{\mathrm{s}} \mathrm{~A}_{\mathrm{V}} \\
V_{S}=\frac{V A}{A_{V}} \\
\frac{A_{V}}{A 1}=\frac{V_{V}}{V}=n \\
V_{s}=V_{n}=-\frac{V}{n}=\frac{1+e}{e} . V
\end{gathered}
$$

The seepage velocity $\mathrm{V}_{\mathrm{s}}$ is also proportional to the hydraulic gradient.
$V_{s}=k_{p} i\left(k_{p}\right.$ is the coefficient of percolation) From Darcy's law, $\mathrm{V}=\mathrm{k} \mathrm{i}$

$$
\begin{gathered}
\frac{V_{S}}{V}=\frac{K_{P}}{K}=\frac{1}{n} \\
K_{P}={ }_{n}-
\end{gathered}
$$

## PERMEABILITY MEASUREMENTS IN THE LABORATORY:

They are two methods

- Constant head permeability test
- Falling head permeability test


## Field methods

- Pumping - out test
- Pumping - in test

Indirect methods of ' $k$ ' involving computations from

- Grain size
- Specific Surface
- Consolidation test data


## Constant head permeability test

$>$ Place the mould assembly in the bottom tank and fill the bottom tank with water up to its outlet.
$>$ Connect the outlet tube of the constant head tank to the inlet nozzle of permeameter.
$>$ Start the stopwatch, and at the same time put a beaker under the outlet of the bottom tank.
$>$ Conduct the test for some convenient time interval. Measure the quantity of water collected in beaker during that time.
$>$ Repeat the test twice more, under the same head and for same time interval.
$>$ This method is used for coarse grained soils.


Fig. 7.2. Schematic diagram of constant head test setup
Let ' $q$ ' be the discharge per unit time.

$$
\mathrm{q}=\mathrm{Q} / \mathrm{t} \longrightarrow 1
$$

From Darcy's law,
$\mathrm{q}=\mathrm{kiA} \longrightarrow 2$
Equate 1 \& 2,

$$
\mathrm{Q} / \mathrm{t}=\mathrm{kiA}
$$

$$
\mathrm{k}=\mathrm{Q} / \mathrm{tiA}
$$

$$
\begin{aligned}
& \mathrm{i}=\mathrm{h} / \mathrm{L} ; \mathrm{k}=\mathrm{Q} /(\mathrm{t}(\mathrm{~h} / \mathrm{L}) \mathrm{A})=\mathrm{QL} / \text { th } \mathrm{A} \\
& \quad K=\frac{Q \cdot L \cdot 1}{\operatorname{th} A}
\end{aligned}
$$

where,
$\mathrm{k} \geqslant$ coefficient of permeability (m)
$\mathrm{Q} \boxtimes$ Dischage $\left(\mathrm{cm}^{2}\right)$
$\mathrm{L} \geqslant$ Length of the sample (cm)
A Totalc/s area of soil specimen (ni)
$\mathrm{h} \geqslant$ Head of water (cm)
$\mathrm{t} \geq$ Time taken ( sec )

## Falling head (or) Variable permeability test:

> Prepare the soil specimen in the permeameter, keep the mould assembly in the bottom tank and fill the bottom tank with water.
$>$ Connect the inlet nozzle of the mould to the stand pipe filled with water. Permit water to flow for some time till steady state of flow is reached.
$>\quad$ With the help of stop watch, note the time interval required for water level in the stand pipe to fall from some convenient initial value to some
 final value.
$>$ Repeat the above step atleast twice and note the time interval and also note the diameter of stand pipe, from which the area of stand pipe calculated.

This method is used for grained soils.
From Darcy's law,

$$
\begin{aligned}
& \mathrm{q}=\mathrm{kiA} \longrightarrow \\
& \mathrm{q}=\mathrm{Q} / \mathrm{t}=\text { Area } x \text { Rate of Velocity } \\
& \mathrm{q}=-\mathrm{adh} / \mathrm{dt} \longrightarrow
\end{aligned}
$$

' - 'sign indicates falling head height decreases, time increases.
Equate $1 \& 2$,
$-\mathrm{a}(\mathrm{dh} / \mathrm{dt})=\mathrm{ki} \mathrm{A}$
$-\mathrm{a}(\mathrm{dh} / \mathrm{dt})=\mathrm{kh} A / \mathrm{L}$
$(\mathrm{dh} / \mathrm{dt})=\mathrm{kAh} / \mathrm{aL}$
$(\mathrm{dh} / \mathrm{h}) \quad=\mathrm{kAdt} / \mathrm{a} \mathrm{L}$
Integrating from ' $h_{1}$ ' to ' $h_{2}$ ' and ' 0 ' to ' $t$ '.

$$
\begin{aligned}
& -\int_{h_{1}}^{h_{2}} \frac{d h}{h}=\frac{k A}{a L} \int_{0}^{t} d t \\
& {\left[\log _{\mathrm{e}}\right]_{h_{1}}^{h_{2}}=-\frac{-k A}{a L}(t]_{0}^{t}}
\end{aligned}
$$

$$
\log _{e} h_{2}-\log _{e} h_{1}=-k A t / a L
$$

$$
\log _{e} h_{1}-\log _{e} h_{2}=k A t / a L k
$$

$$
=a \underline{L} \log _{e}\left(h_{1} / h_{2}\right)
$$

$$
k=2.303 a L \log 10\left(h_{1} / h_{2}\right)
$$

At
$\mathrm{k} \geq$ co-efficient of permeability ( cm /
s) a $\longrightarrow$ area of stand pipe $\left(\mathrm{cm}^{2}\right)$
$L<$ Length of specimen (cm)
A Total c/s area of soil sample (n)
$\mathrm{h}_{1} \geq$ initial head of water (cm)
$\mathrm{h}_{2}<$ final head of water (cm)

## FIELD METHODS:

They are more reliable compared to laboratory methods for the determination of ' k '. Lab tests involves large man of soil with minimum disturbance unlike the small sample is used. The value of ' $k$ ' obtained from field tests represents an average value of ' $k$ ' for the large soil mass over a large area.

## a) Pumping-out tests:

Aguifer: It is a permeable formation which allows a significant quantity of water to move through it under field conditions.

Confined aquifers (or) Artesian aquifers: It is one in which ground water remains entrapped under pressure greater than atmospheric, by overlying relatively impermeable strata.

Unconfined Aguifers: It is one in which the ground water table is the upper surface of the zone of saturation and it lies within the test stratum. It is also called 'free', 'phreatic' or 'non-artesian aquifers'.

When a well is penetrated into an extensive homogeneous aquifer, the water table initially remains horizontal in the well. When the well is pumped, water is removed from the aquifer and the water table or the piezometric surface, depending upon the type of aquifer, is lowered resulting in a parabolic depression in the water table (or) piezometric surface. This depression is called the cone of depression or the draw down curve.

In the pumping - out tests, draw downs corresponding to a steady discharge ' $q$ ', are observed at a number of observation wells.

Pumping must continue at a uniform rate for an adequate time to establish a steady state condition, in which the draw down changes negligibly with time.

## Assumptions:

1) The aquifer is homogeneous with uniform permeability and is of infinite aerial extent.
2) The flow is laminar and Darcy's law is valid.
3) The flow is horizontal and uniform at all points in the vertical section.
4) The well penetrates the entire thickness of the aquifer (and receives water).
5) Natural ground water regime affecting the aquifer remains constant with time.
6) The velocity of flow is proportional to the tangent of the hydraulic gradient (Dupuit's theory).

## Unconfined Aquifer:

Fig., shows a well penetrating an unconfined (or) free aquifer to its full depth of an pumping - out test.

Let $\quad \mathrm{r}=$ radius of main well.
$\mathrm{R}=$ radius of zero drawdown known as max., radius of influence.
$\mathrm{h}=$ depth of water in the main well during pumping, measured above impervious layer.
$\mathrm{H}=$ height of initial water table above impervious layer.
$\mathrm{q}=$ rate at which water is pumped out of well.


Let ' $P$ ' $(x, y)$ be any point on the draw down curve. The point ' $o$ ' (orgin of ordinates) at the bottomof central axis of well is chosen as the orgin of reference.

Applying Darcy's law, for flow through cylindrical surface of radius ' $x$ ' and height ' $y$ ' we have, Discharge, $\quad q=k . A_{x} \cdot i_{x}$

$$
\begin{aligned}
& =k \cdot(2 \pi x y) d y / d x \\
& =q \cdot d x / x \\
& =2 \pi k y d y
\end{aligned}
$$

Integrating between the limits $(\mathrm{R}, \mathrm{r})$ for x , and $(\mathrm{H}, \mathrm{h})$ for y , we get,

$$
\sqrt{\sqrt{k}=\frac{q \log _{e}(\mathrm{R} / \mathrm{r})}{ת\left(\mathrm{H}^{2}-\mathrm{h}^{2}\right)}} \sqrt{ }
$$

In above equation R is found to vary from 150 m to 300 m and can only be estimated crudely, as for example, using the following equation given by Sichardt,


To avoid the use of $R$, an alternative method is to measure drawdowns $s_{1}$ and $s_{2}$ in two observations wells located at radial distances $r_{1}$ and $r_{2}$ from the axis of main well. The depths of water in the two observation wells are,

$$
\mathrm{h}_{1}=\mathrm{H}-\mathrm{S}_{1}
$$

$$
\begin{aligned}
& q \int_{r}^{R} \frac{d x}{x}=2 \pi k \int_{h}^{y} \stackrel{H}{d y} \\
& q_{e}\left[\operatorname{lo\frac {\underset {⿺}{x}}{x}())}{ }_{r}^{R}=2 \pi k[] \frac{y^{2 H}}{2_{h}}\right. \\
& \mathrm{q}=\frac{\pi \mathrm{k}\left(\mathrm{H}^{2}-\mathrm{h}^{2}\right)}{\log _{\mathrm{e}}(\mathrm{R} / \mathrm{r})} \\
& \text { (or) } \\
& \mathrm{q}=\underline{1.36 \mathrm{k}\left(\mathrm{H}^{2}-\mathrm{h}^{2}\right)} \\
& \log _{10}(\mathrm{R} / \mathrm{r})
\end{aligned}
$$

$$
\mathrm{h}_{2}=\mathrm{H}-\mathrm{S}_{2}
$$

We now have,

$$
\begin{aligned}
& y=h_{1}, \text { at } x=r_{1} \\
& y=h_{2}, \text { at } x=r_{2}
\end{aligned}
$$

$$
\begin{gathered}
q \int_{r_{1}}^{r_{2}} d \frac{d x}{x}=2 \pi k \int y y_{h_{1}}^{h_{2}} y \\
q[\operatorname{loge} \underset{x}{1})]_{r_{1}}^{r_{2}}=2 \pi k\left[\quad \frac{y^{2}}{2}\right]_{h_{1}}^{h_{2}}
\end{gathered}
$$

$$
k=\frac{q}{\Pi\left(h_{2}^{2}-h_{1}^{2}\right)} \log _{e}\left(r_{2} / r_{1}\right)
$$

## CONFINED AOUIFER:

Fig., shows a well fully penetrating a confined (or) artesian aquifer. Let ( $\mathrm{x}, \mathrm{y}$ ) be the coordinates of any point ' p ' on the drawdown curve, measured with respect to the origin ' $\underline{0}$ '.
$\mathrm{q}=$ discharge or rate at which water is pumped out of main well.
$\mathrm{b}=$ thickness of confined aquifer.
Applying Darcy's law for flow through cylindrical surface of radius ' $x$ ' and height ' $b$ '. We have,

$$
\mathrm{q}=\mathrm{k} \mathrm{i}_{\mathrm{x}} \mathrm{~A}_{\mathrm{x}}
$$

where, $A_{x}=c / s$ area of flow, measured at ' $p$ ' $=2 Л x b$
$i_{x}=$ hydraulic gradient at ' $p$ ' $=d y / d x$
$q=k(d y / d x)(2$ Л $x b)($ or $) \quad q(d x / x)=2 Л x b d y$
Integrating between the limits $(R, r)$ for $x$ and $(H, h)$ for $y$, we get,

$q \int_{r}^{R} \frac{d x}{x}=2 \pi k b \int_{h}^{H} d y^{H}$

From which,

$$
\mathrm{q} \log _{\mathrm{e}}(\mathrm{R} / \mathrm{r})=2 Л \mathrm{~kb}(\mathrm{H}-\mathrm{h})
$$


(or)

$$
\mathrm{q}=\frac{2.72 \mathrm{~T} \mathrm{~s}}{\log _{10}(\mathrm{R} / \mathrm{r})}
$$

$\mathrm{T}=$ co-efficient of transmissibility $=b k$
$\mathrm{S}=$ drawdown at the well
Alternatively, if $h_{1}$ and $h_{2}$ are the depths of water measured above bottom impervious stratum in two
observation wells located at radial distances $r_{1}$ and $r_{2}$ from the axis of main well, then we can write,

i) Constant water level method (in open - end pipe) and
ii) Packer method (in section of borehole).

## 1) Constant water level method:

An open end pipe is sunk into the soil to desired depth and the soil is taken out of the pipe till its bottom end. The test is also conducted in a borehole with the pipe casing extending to the desired depth. Fig., illustrates the arrangement for the method.

In fig.,(a) and (c) the bottom end of pipe is above water table and Fig., (b) and (d) it is below water table. Water is pumped into the pipe and the rate of flow, q is adjusted to maintain water level constant in the pipe. In the case of soils of low permeability additional pressure head ' $\mathrm{H}_{\mathrm{p}}$ ' is required to be added to the gravity head ' $\mathrm{H}_{\mathrm{g}}$ ' in order to maintain constant rate of flow. The co-efficient of permeability is computed using the following equation.

$$
\mathrm{K}=\mathrm{q} / 5.5 \mathrm{r} \mathrm{H}
$$

Where,
$\mathrm{r}=$ internal radius of pipe
$\mathrm{q}=$ constant rate of flow
$\mathrm{h}=$ differential head of water (gravity plus pressure, if any)


## 2) Packer method:

A packer is an expandable cylindrical rubber sheeve packers are used as a means of sealing of a section of borehole. Two types of packer methods are used in practice.

## i) Single packer method:

In single packer methods the hole is drilled to the required depth. The packer is fixed at a desired level above the bottom of the hole and the water pumped into the section below the packer. The constant rate of flow ' $q$ ' ie., attained under an applied head ' $H$ ' is found.

## ii) Double packer method:

In double packer method, the hole is drilled to the final depth and cleaned. Two packers are fixed at a distance apart equal to 5 times the diameter of bore hole. Both packers are then expanded and water pumped into the section between the two packers. The constant rate of flow, $q$ that is attained under an applied head, H is found.


The co-efficient of permeability, k is computed using the following.


Where,
$\mathrm{L}=$ length of portion of the hole tested
$r=$ radius of bore hole
$\mathrm{q}=$ constant rate of flow into the test section
$\mathrm{H}=$ differential head for maintaining a constant rate of flow in test section

## PERMEABILITY OF STRATIFIED SOIL DEPOSITS:

In nature soil mass may consists of so many layers deposited one above the other. Each layer is assumed to be homogeneous and isotropic. The average permeability of whole deposit will depend on the direction of flow.

## Average permeability parallel to the direction of flow:

$$
\begin{aligned}
& q=q_{1}+q_{2}+q_{3} \\
& i=i_{1}=i_{2}=i_{3} \\
& k i A=k_{1} i_{1} A_{1}+k_{2} i_{2} A_{2}+k_{3} i_{3} A_{3} \\
& A_{1}=z_{1} x i, A_{2}=z_{2} x i, A_{3}=z_{3} x i, i \text { is equal } \\
& k_{x}=\frac{k_{1} z_{1}+k_{2} z_{2}+k_{3} z_{3}}{z}
\end{aligned}
$$



## Average permeability perpendicular to the direction of flow:

For vertical flow, $\mathrm{v} \& \mathrm{q}$ is equal.

$$
\begin{aligned}
& \therefore \mathrm{i}=\mathrm{v} / \mathrm{k}_{1},[\mathrm{v}=\mathrm{ki}] \mathrm{v} \\
& \quad=\mathrm{ki}=\mathrm{kh} / \mathrm{L}
\end{aligned}
$$



$$
\begin{array}{r}
\mathrm{h}=\mathrm{vL} / \mathrm{k}(\mathrm{or}) \mathrm{vz} / \mathrm{k} \\
\quad \frac{\mathrm{vz}}{=} \stackrel{\frac{\mathrm{vz}_{1}+\mathrm{vz}_{2}+\ldots \ldots .+\mathrm{vz}_{\mathrm{n}}}{\mathrm{k}_{\mathrm{v}}} \frac{\mathrm{k}_{1}}{\mathrm{k}_{2}}}{\mathrm{k}_{\mathrm{n}}} \\
\therefore \mathrm{k}_{\mathrm{v}}=\frac{\mathrm{z}}{\frac{\mathrm{z}_{1}+\frac{\mathrm{z}_{2}+\ldots+\mathrm{z}_{\mathrm{n}}}{\mathrm{k}_{1}} \mathrm{k}_{2}}{\mathrm{k}_{\mathrm{n}}}}
\end{array}
$$

## Factors affecting permeability:

By comparing poiseuille's law with Darcy's law adopted for the flow through the soil pores, we get,

$$
\mathrm{q}=\mathrm{ki} \mathrm{~A}
$$

$$
K=D_{s}{ }^{2}\left(\frac{\gamma_{w}}{\eta}\right)\left(\frac{e 3}{1+e}\right) \cdot c
$$

Thus, the factors affecting permeability are,

1) Grain size
2) Properties of the pore fluid
3) Void ratio of the soil
4) Structural arrangement of the soil particles
5) Entrapped air and foreign matter
6) Adsorbed water in claying soils

## 1) Effect of size and shape of particles:

Permeability varies approximately as the square of the grain size. Since, soil consists of many different sized grains. Some specific grain size has to be used for comparison.
a) Allen Hazen (1892) found in filter sands of particle size between 0.1 and 3 mm .
$\mathrm{k}=\mathrm{CD}_{10}{ }^{2}$
$\mathrm{K}=$ coefficient of permeability $(\mathrm{cm} / \mathrm{sec})$
$\mathrm{D}_{10}=$ Effective diameter (cm)
C $=$ Constant $=100$ (approx.,) When $\mathrm{D}_{10}$ in ' cm '.
b) Attempt have been made to correlate the ' $k$ ' and specific surface of the soil particles by Kozency
(1907).

$$
\mathrm{K}=\frac{1}{\mathrm{~K}_{\mathrm{k} \eta \mathrm{~s}_{\mathrm{s}}^{2}}^{2}} \quad \mathrm{X} \frac{\mathrm{n}^{3}}{1-\mathrm{n}^{2}}
$$

$\mathrm{n}=$ porosity
$\mathrm{s}_{\mathrm{s}}=$ Specific surface of particles $\left(\mathrm{cm}^{2} / \mathrm{cm}^{3}\right)$
$\eta=$ viscosity ( $\mathrm{g} \mathrm{sec} / \mathrm{cm}^{2}$ )
$\mathrm{k}_{\mathrm{k}}=$ constant $=5$ for spherical particles
c) Loudon (1952-53) developed from the basis of his experiment is,

$$
\begin{array}{ll|}
\hline \log 10\left(\mathrm{ks}_{\mathrm{s}}{ }^{2}=\mathrm{a}+\mathrm{bn}\right. \\
\mathrm{a}=1.365 ; & \mathrm{b}=5.150 \text { for ' } 1 \mathrm{c} \text { ' at } 10^{\circ} \mathrm{c} .
\end{array}
$$

## 2) Effect of properties of pore fluid:

From equation 1 , the ' $k$ ' is directly proportional to ' $\gamma_{\mathrm{w}}$ ' and inversely proportional to its viscosity. ' $\gamma_{\mathrm{w}}$ ' does not change with change in temperature. Other factors remain constant. Effect of property of water on ' $k$ '

For a standard temperature at $27^{\circ} \mathrm{c}$

$$
\mathrm{k}_{27}=\mathrm{K} \bar{\eta} \overline{\eta_{27}}
$$

$\mathrm{k}_{27}=$ permeability at $27^{\circ} \mathrm{c}$
$\eta_{27}=$ viscosity at $27^{\circ} \mathrm{c}$
Change of ' $\gamma_{\mathrm{w}}$ 'also taken into account.

$$
\frac{\mathrm{k}_{1}}{\frac{\eta_{2}}{\mathrm{k}_{2}} \cdot \frac{\gamma_{\mathrm{w} 1}}{\eta_{1}} \cdot \overline{\gamma_{\mathrm{w} 2}}=\bar{\eta} \cdot \frac{\rho_{\mathrm{w} 1}}{-\rho_{\mathrm{w} 2}}}
$$

Muscat (1937) is pointed out that in more general co-efficient of permeability called physical permeability ' $\mathrm{k}_{\mathrm{p}}$ ' related to Darcy's co-efficient of permeability ' $k$ '.

$$
\mathrm{k}_{\mathrm{p}}=\frac{\mathrm{K} \eta}{\gamma_{\mathrm{w}}}
$$

## 3) Effect of void ratio:

Void ratio is the opening space between the particles. If the ' $e$ ' is larger means, the water is easily flow inside their by the value of ' $k$ ' is more. $k$ is directly proportional to the void ratio.


From the relation permeability varies as a square of void ratio.
$\mathrm{k}_{1}=$ permeability at void ratio ' $\mathrm{e}_{1}$ '
$\mathrm{k}_{2}=$ permeability at void ratio ' $\mathrm{e}_{2}$ '

## 4) Structural Arrangements:

The structure of soil becomes changed depends on the method of compaction. Due to compaction particles come closer compact mass, there by pore size get reduced. So the value of ' $k$ ' get reduced. If effect of compaction is more means, the ' k ' is less.


$$
\begin{array}{cc}
\text { Plate structure } & \text { Flocculated structure } \\
(\mathrm{k} \text { is less }) & (\mathrm{k} \text { is more })
\end{array}
$$

Fine grained soils $\longrightarrow$ more

$$
K_{H}>K_{V}
$$

## 5) Effect of degree of saturation:

When the degree of saturation increases, the ' $k$ ' is increased. At a lower percentage of degree of saturation, the ' $k$ ' is increased.

Partly saturated soils $\longrightarrow$ reduces ' $k$ '
Fully saturated soils $\longrightarrow$ increases ' k '.

## 6) Effect of presence of foreign matter:

Presence of foreign matter such as dust, reduces the value of ' $k$ '. Because it sometimes closes the open space between the particles, there by the movement of water get reduced.

## 7) Effect of adsorbed water:

The adsorbed water, which is held by soil particles, is not free to move and therefore reduces the effective pore space available for passage of water.

## PROBLEMS

1) Calculate the coefficient of permeability of a soil sample, 6 cm in height and $50 \mathrm{~cm}^{2}$ in $\mathrm{c} / \mathrm{s}$ area, if a quantity of water equal to 430 ml passes down in 10 minutes, under an effective constant head of 40 cm . On over drying, the test specimen has mass of 498 g . Taking the specific gravity of soil solids as 2.65 , calculate the seepage velocity of water during the test.

## Given:

$$
\mathrm{Q}=430 \mathrm{ml} ; \mathrm{t}=10 \times 60=600 \text { seconds; } \mathrm{A}=50 \mathrm{~cm}^{2} ; \mathrm{L}=6 \mathrm{~cm} ; \mathrm{h}=40 \mathrm{~cm}
$$

## Solution:

We know, co-efficient of permeability $\mathrm{K}=(\mathrm{Q} / \mathrm{t})$. ( $\mathrm{L} / \mathrm{h}) \cdot(1 / \mathrm{A})$

$$
\text { Now, velocity, } \mathrm{v}=\mathrm{ki}=2.15 \times 10^{-3} \times(40 / 6)
$$

$$
\begin{aligned}
& v=\frac{1.435 \times 10^{-2} \mathrm{~cm} / \mathrm{sec}}{(\text { or })} \\
& v=(q / A)=(430 /(600 \times 50))=\underline{1.435 \times 10^{-2} \mathrm{~cm} / \mathrm{sec}}
\end{aligned}
$$

$$
\text { Now, } p_{d}=\left(M_{d} / v\right)=(498 /(50 \times 6))=\underline{1.66 \mathrm{~g} / \mathrm{cm}^{3}}
$$

$$
e=\frac{6 \rho_{w}}{\rho_{d}}-1=\frac{265 \times 1}{-1=0.595}
$$

$$
n=\frac{e}{1+e}=\frac{0.595}{1.595}=0.373
$$

Seepage velocity, $\mathrm{v}_{\mathrm{s}}=\mathrm{v}=1.435 \times 10^{-2}$

$$
\begin{array}{cc}
\mathrm{n} \quad 0.373 \\
\mathrm{v}_{\mathrm{s}}= & \begin{array}{c}
0.85 \times 10^{-2} \mathrm{~cm} / \mathrm{sec}
\end{array}
\end{array}
$$

2) In a falling head permeameter test, the initial head $(t=0)$ is 40 cm . The head drops by 5 cm in 10 minutes. Calculate the time required to run the test for the final head to be at 20 cm . If the sample is 6 cm in height and $50 \mathrm{~cm}^{2}$ in $\mathrm{c} / \mathrm{s}$ area, calculate the co-efficient of permeability, taking area of stand pipe $=0.5 \mathrm{~cm}^{2}$.

## Given:

Falling head permeability test
$\mathrm{h}_{1}=40 \mathrm{~cm} ; \mathrm{t}_{1}=0$
Time, $\mathrm{t}=10 \min \times 60=600$ seconds. 5 cm drop., so $\mathrm{h}_{2}=40-5$
$\mathrm{L}=6 \mathrm{~cm} ; \mathrm{a}=0.5 \mathrm{~cm}^{2} ; \mathrm{A}=50 \mathrm{~cm}^{2}$


## Solution:

Co-efficient of permeability, $\quad K=2.303\left(\frac{\mathrm{aL}}{\mathrm{At}}\right) \log _{10} \hbar_{1}\left(\frac{-}{h_{2}}\right)$


$$
\text { If } \mathrm{h}_{1}=40 \mathrm{~cm} ; \mathrm{h}_{2}=35 \mathrm{~cm} ; \mathrm{t}=600 \mathrm{sec}
$$

$600=m \log _{10}(40 / 35)$

$$
\therefore \mathrm{m}=10.363 \times 10^{3}
$$

If $\mathrm{h} 1=40 \mathrm{~cm} ; \mathrm{h} 2=20 \mathrm{~cm}$

$$
\begin{aligned}
& \therefore \mathrm{t}=10.363 \times 10^{3} \log _{10}[40 / 20] \\
&=3.12 \times 10^{3} \mathrm{sec} / 60 \\
& t=52 \text { minutes } \\
& \text { Now, } \mathrm{m}=2.3\left(\frac{\mathrm{aL}}{\mathrm{AK}}\right)
\end{aligned}
$$

$10.363 \times 10^{3}=0.5 \times 6$
$50 \times \mathrm{K}$
$\therefore \mathrm{k}=1.33 \times 10^{-5} \mathrm{~cm} / \mathrm{sec}$
3) A stratified soil deposit is shown in figure along in the co-efficient of permeability of the individual strata. Determine the ratio of $\mathrm{k}_{\mathrm{H}}$ and $\mathrm{k}_{\mathrm{v}}$. Assume an average hydraulic gradient of 0.3 in both horizontal and vertical seepage. Find discharge and discharge velocity for each layer, for horizontal flow and hydraulic gradient and loss in head in each layer for vertical flow.

## Given data:



$$
\mathrm{k}_{\mathrm{H}}=\text { ? }
$$

$$
\mathrm{k}_{\mathrm{H} /} \mathrm{k}_{\mathrm{v}}=\text { ? }
$$

$$
\mathrm{v}_{1}=\mathrm{v}_{2}=\mathrm{v}_{3}=?
$$

$$
\mathrm{i}_{1}=\mathrm{i}_{2}=\mathrm{i}_{3}=?
$$

$$
\mathrm{h}_{1}=\mathrm{h}_{2}=\mathrm{h}_{3}=?
$$

## Solution:

We know, for horizontal seepage, $\mathrm{k}_{\mathrm{H}}=\mathrm{k}_{1} \mathrm{Z}_{1}+\mathrm{k}_{2} \mathrm{Z}_{2}+\mathrm{k}_{3} \mathrm{Z}_{3}$

$$
\begin{aligned}
& \mathrm{z}_{1}+\mathrm{z}_{2}+\mathrm{z}_{3} \\
= & \frac{5 \times 10^{-4}(200)+5 \times 10^{-4}(500)+5 \times 10^{-4}(200)}{200+500+200} \\
\mathrm{k}_{\mathrm{H}}= & 5 \times 10^{-4} \mathrm{~cm} / \mathrm{sec}
\end{aligned}
$$

$$
\left.\begin{array}{l}
\text { for vertical seepage, } \mathrm{k}_{\mathrm{v}}=\frac{\mathrm{z}_{1}+\mathrm{z}_{2}+\mathrm{z}_{3}}{} \\
\qquad \begin{array}{r}
\frac{\mathrm{z}_{1}+\mathrm{z}_{2}+\mathrm{z}_{3}}{\mathrm{k}_{1}} \overline{\mathrm{k}_{2}} \overline{\mathrm{k}_{3}}
\end{array} \\
=\begin{array}{r}
200+500+200 \\
5 \times 10^{-4} \quad 5 \times 10^{-4} \quad 5 \times 10^{-4}
\end{array} \\
\mathrm{~K}_{\mathrm{V}}=5 \times 10^{-4} \mathrm{~cm} / \mathrm{Sec}
\end{array}\right\} \begin{aligned}
& K_{H}=\begin{array}{l}
5 \times 10^{-4} \\
K_{V}
\end{array} \\
& 5 \times 10^{-4}=1
\end{aligned}
$$

## i) Discharge velocity (V):(for horizontal flow)

Let $\mathrm{q}_{1}, \mathrm{q}_{2}, \mathrm{q}_{3}$ be the discharge through the layer $1,2,3$.
$\underline{\mathrm{V}_{1}, \mathrm{~V}_{2}, \mathrm{~V}_{3}}$ be the velocity through the layer $1,2,3$.
We know,

For layer 1,

$$
\begin{aligned}
& \begin{array}{l}
\mathrm{q}=\mathrm{kiA} \\
\mathrm{q}_{1}=\mathrm{k}_{1 \mathrm{i}_{1} \mathrm{~A}_{1}} \\
=5 \times 10^{-4} \times 0.3 \times(200 \times 1)
\end{array} \\
& \mathrm{q}_{1}=0.03 \mathrm{~cm}^{3} / \mathrm{sec}
\end{aligned}
$$

Also,

$$
\begin{aligned}
& \mathrm{q}_{2}=\mathrm{k}_{2} \mathrm{iA}_{2}=5 \times 10^{-4} \times 0.3 \times(500 \times 1)=0.075 \mathrm{~cm}^{3} / \mathrm{sec} \\
& \mathrm{q}_{3}=\mathrm{k}_{3} \mathrm{iA}_{3}=5 \times 10^{-4} \times 0.3 \times(200 \times 1)=0.03 \mathrm{~cm}^{3} / \mathrm{sec} \\
& \therefore \mathrm{~V}_{1}=\mathrm{k}_{1} \mathrm{i}=5 \times 10^{-4} \times 0.3=1.5 \times 10^{-4} \mathrm{~cm} / \mathrm{sec} \\
& \therefore \mathrm{~V}_{2}=\mathrm{k}_{2} \mathrm{i}=5 \times 10^{-4} \times 0.3=1.5 \times 10^{-4} \mathrm{~cm} / \mathrm{sec} \\
& \therefore \mathrm{~V}_{3}=\mathrm{k}_{3} \mathrm{i}=5 \times 10^{-4} \times 0.3=1.5 \times 10^{-4} \mathrm{~cm} / \mathrm{sec}
\end{aligned}
$$

## ii) Hydraulic gradient and loss in head in vertical flow:

We know,

$$
\mathrm{V}=\mathrm{Ki}
$$

Hydraulic gradient,

$$
\begin{aligned}
& i_{1}=V_{1} \overline{K_{1}}=\frac{1.5 \times 10^{-4}}{5 \times 10^{-4}}=0.3 \\
& i_{2}=\frac{V_{2}}{} \overline{K_{2}}=\frac{15 \times 10^{-4}}{5 \times 10^{-4}}=0.3 \\
& i_{3}=\frac{V_{3}}{K_{3}}=\frac{15 \times 10^{-4}}{5 \times 10^{-4}}=0.3
\end{aligned}
$$

Loss of head

$$
\mathrm{h}=\mathrm{iz}
$$

$$
\therefore \mathrm{i}=\mathrm{h} /(\mathrm{L} . \mathrm{z}) \quad[\mathrm{L}=1 \mathrm{~m}]
$$

$\therefore \underline{\mathrm{h}_{1}}=\mathrm{i}_{1} \mathrm{XZ}_{1}=0.3 \times 2=\underline{0.6 \mathrm{~m}}$
$\therefore \underline{\mathrm{h}_{2}}=\mathrm{i}_{2} \mathrm{XZ}_{2}=0.3 \times 5=\underline{1.5 \mathrm{~m}}$
$\therefore \underline{\mathrm{h}_{3}}=\mathrm{i}_{3} \mathrm{XZ}_{3}=0.3 \times 2=\underline{0.6 \mathrm{~m}}$
4).An aquifer of 20 m average thickness is overlain by an impermeable layer of 30 m thickness .A test well of 0.5 m diameter and two observation wells at distances of 10 m and 60 m from the test well are drilled through the aquifer. After pumping at a rate of $0.1 \mathrm{~m} 3 / \mathrm{sec}$ for a long time, the following draw downs is stabilized in these wells: First observation well 4 m ; second observation well, $m$. Show the arrangement in a diagram. Determine the coefficient of permeability and draw down in the test well.


Given:

$$
\begin{aligned}
& \mathrm{h}_{1}=50-4=46 \mathrm{~m} \\
& \mathrm{~h}_{2}=50-3=47 \mathrm{~m} \\
& \text { dia } \mathrm{D}=0.5 \mathrm{~m} \\
& \mathrm{q}=0.1 \mathrm{~m}^{3} / \mathrm{s} \\
& \mathrm{~b}=20 \mathrm{~m}
\end{aligned}
$$

$$
k=\frac{2303 q}{2 \pi b\left(h_{2}-h_{1}\right)} \log _{10}\left(C_{r}\right) \frac{r_{2}}{1}
$$

$$
\begin{aligned}
& k=\frac{2303 \times 0.1}{2 \pi .20(47-46)^{\operatorname{logiot}} 10} \quad- \\
& =1.43 \times 10^{-3} \mathrm{~m} / \mathrm{sec}
\end{aligned}
$$

Draw down:

$$
\begin{aligned}
& \binom{h-h}{w}=\frac{2303}{2 \pi b k \log _{10}(r)} \quad \frac{r_{2}}{1} \\
& \left(47-h_{\omega}-\frac{2303 \times 0.1}{2 \pi \times 20 \times 143 \times 10^{-3}}-\operatorname{lofit10}(10) \quad 60\right. \\
& h_{w}=43.94 \mathrm{~m}
\end{aligned}
$$

Draw down, $\mathrm{S}=50-\mathrm{h}_{\mathrm{w}}$

$$
=50-43.94=6.06 \mathrm{~m}
$$

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## Seepage Pressure:

By virtue of the viscous friction exerted on water flowing through soil pores, an energy transfer is effected between the water and the soil. The force corresponding to this energy transfer is called the seepage force (or) seepage pressure. Thus, seepage pressure is the pressure exerted by water on soil through which it percolates.

Seepage pressure is responsible for the phenomenon known as quick sand, and is of vital importance in the stability analysis of earth structures subjected to seepage action.

If ' $h$ ' is the hydraulic head or head lost due to frictional drag of water flowing through soil mass of thickness'z'.
$\therefore$ seepage pressure, $\mathrm{p}_{\mathrm{s}}=\mathrm{h} \gamma_{\mathrm{w}}$
(or) $\mathrm{p}_{\mathrm{s}}=\mathrm{hz} \gamma_{\mathrm{w}}=\mathrm{iz} \gamma_{\mathrm{w}}$
Z


Seepage force per unit volume $j=i z \gamma_{w} A$

$$
\frac{\mathrm{z} \mathrm{~A}}{\mathrm{j}=\mathrm{i} \gamma_{\mathrm{w}}}
$$

The vertical effective pressure may be decreased or increased due to seepage pressure depending on the direction of flow.

Effective pressure, $\sigma^{\prime}=\mathrm{z} \gamma^{\prime} \pm \mathrm{p}_{\mathrm{s}}=\mathrm{z} \gamma^{\prime} \pm \mathrm{iz} \gamma_{\mathrm{w}}$
$\downarrow$ Flow $\rightarrow \sigma^{\prime} \rightarrow$ increased $\rightarrow$ use" +ve ".
$\uparrow$ Flow $\rightarrow \sigma^{\prime} \rightarrow$ decreased $\rightarrow$ use"-ve".

## UPWARD FLOW: OUICK SAND CONDITION:

When flow takes place in an upward direction, the seepage pressure also acts in the upward direction and the effective pressure is reduced.

Flow $\uparrow \longrightarrow \mathrm{p}_{\mathrm{s}} \uparrow \longrightarrow \sigma^{\prime} \longrightarrow$ decreased


If $\quad \mathrm{p}_{\mathrm{s}}=\gamma_{\text {sat }} \quad \therefore \underline{\sigma=0}$
In such cases, a cohesion less soil loses all its shear strength and the soil particles have a tendency to move up in the direction of flow. This phenomenon of lifting of soil particles is called, quick condition, Boiling condition, (or) quick sand.
$\sigma^{\prime}=\mathrm{z} \gamma^{\prime}-\mathrm{p}_{\mathrm{s}}=0$
(or) $\mathrm{p}_{\mathrm{s}}=\mathrm{z} \gamma^{\prime}$

From which,

$$
\mathrm{i}=\mathrm{i}_{\mathrm{c}}=\frac{\gamma^{\prime}}{\gamma^{\prime}}=\frac{\mathrm{G}-1}{1+\mathrm{e}}\left(\frac{\left(\gamma_{\mathrm{sat}}-\gamma_{\mathrm{w}}\right)}{\gamma_{\mathrm{w}}}\right)
$$



The hydraulic gradient at such a critical state is called the critical hydraulic gradient.
For loose sand (or) silt, $\mathrm{e}=0.67$ and $\mathrm{G}=2.67$ ie., $\quad \mathrm{i}_{\mathrm{c}}=1$

## Ouick sand condition:

It should be noted that quick sand is not a type of sand but a flow condition occurring within a cohesion less when effective pressure is reduced to zero due to upward flow of water.

Fig., shows setup of phenomenon of quick sand of thickness z , and hydraulic head, h.


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When the soil particles are in the state of critical equilibrium,
The total upward force at the bottom of soil (aa) ( $\uparrow$ ) = Total weight of all the materials above the surfaceEquating them, at the level $\mathrm{a}-\mathrm{a}$,

$$
\begin{aligned}
& (\mathrm{h}+\mathrm{z}) \gamma_{\mathrm{w}} \mathrm{~A}=\mathrm{z} \gamma_{\mathrm{sat}} \mathrm{~A} \\
& \mathrm{~h} \gamma_{\mathrm{w}}=\mathrm{z}\left(\gamma_{\mathrm{sat}}-\gamma_{\mathrm{w}}\right)=\mathrm{z} \gamma^{\prime} \\
& \mathrm{h}=\mathrm{i}_{\mathrm{c}}=\gamma^{\prime}=\mathrm{z} \\
& -\frac{\mathrm{G}-1}{\gamma_{\mathrm{w}}} \\
& \frac{1+\mathrm{e}}{}
\end{aligned}
$$

## TWO DIMENSIONAL FLOW: LAPLACE EOUATION

The quantity of water flowing through a saturated soil mass, as well as the distribution of water pressure can be estimated by the theory of fluids through porous medium.

For computing these quantities with the help of theoretical analysis the following assumptions are made;

1) The saturated porous medium is incompressible. The size of the pore spaces does not change with time, regardless of water pressure.
2) The seepage water flows under 'I' (hydraulic gradient) which is due to only to gravity head loss (or) Darcy's law for flow through porous medium is valid.
3) There is no change in the degree of saturation (s) in the zone of soil through which water seeps.
4) The quantity of water flowing into element of volume $=$ Quantity which flows out in the same length of time.
5) The hydraulic boundary conditions at entry and exit are known.
6) Water is incompressible.

Consider an element of soil of size $\Delta x, \Delta y$ and of unit thickness. Let $\mathrm{v}_{\mathrm{x}}, \mathrm{v}_{\mathrm{y}}$ are entry velocity components in ' x ' and ' y ' directions

$$
\begin{array}{ll}
\mathrm{v}_{\mathrm{x}}+\frac{\partial \mathrm{v}_{\mathrm{x}} \Delta \mathrm{x}}{\partial \mathrm{x}} & \text { exit velocity } \\
\mathrm{v}_{\mathrm{y}}+\frac{\partial \mathrm{v}_{\mathrm{y}} \Delta \mathrm{y}}{\partial \mathrm{y}} & \text { components }
\end{array}
$$

According to assumption no., 4, Entry ' $v$ ' $=$ Exit ' $v$ '

$$
\mathrm{v}_{\mathrm{x}}(\Delta \mathrm{y} .1)+\mathrm{v}_{\mathrm{y}}(\Delta \mathrm{x} .1)=\mathrm{v}_{\mathrm{x}}+\frac{\partial \mathrm{v}_{\mathrm{x}}}{\partial \mathrm{x}} \Delta \mathrm{x} \quad(\Delta \mathrm{y} .1)+\mathrm{v}_{\mathrm{y}}+\frac{\partial \mathrm{v}_{\mathrm{y}}}{\partial \mathrm{y}} \Delta \mathrm{y}
$$

From which, $\left.\begin{array}{|cc|}\hline \frac{\partial v_{x}}{\partial x} & +\partial v_{x} \\ \partial x\end{array}\right] 1$ (continuity equation)
According to assumption no., 2 ,
$\mathrm{v}_{\mathrm{x}}=\mathrm{k}_{\mathrm{x}} \cdot \mathrm{i}_{\mathrm{x}}=\mathrm{k}_{\mathrm{x}} \cdot \frac{\partial \mathrm{h}}{\partial \mathrm{x}}$
$\mathrm{v}_{\mathrm{y}}=\mathrm{k}_{\mathrm{y}} \cdot \mathrm{i}_{\mathrm{y}}=\mathrm{k}_{\mathrm{y}} . \partial \mathrm{h}$
$\overline{\partial y}$
Where, $\mathrm{h} \rightarrow$ hydraulic head.
$k_{x}, k_{y} \rightarrow$ co-efficient of permeability in ' $x$ ' and ' $y$ ' directions.
Substitute the value of $\mathrm{v}_{\mathrm{x}}$, $\mathrm{v}_{\mathrm{y}}$ in eqn., 1
We get, $\partial^{2}\left(\mathrm{k}_{\mathrm{x}} . \mathrm{h}\right)+\partial^{2}\left(\mathrm{k}_{\mathrm{y}} . \mathrm{h}\right)=0$
$\partial \mathrm{x}^{2} \quad \frac{\partial \mathrm{y}^{2}}{}$
For an isotropic soil, $\mathrm{k}_{\mathrm{x}}=\mathrm{k}_{\mathrm{y}}=\mathrm{k}$

$$
\frac{\partial^{2} \mathrm{~h}}{\partial \mathrm{x}^{2}}+\frac{\partial^{2} \mathrm{~h}}{\partial \mathrm{y}^{2}}=0
$$

Substitute $\phi=\mathrm{kh}=$ velocity potential, we get,

$$
\frac{\partial^{2} \phi}{\partial \mathrm{x}^{2}}+\frac{\partial^{2} \phi}{\partial \mathrm{y}^{2}}=0
$$

This is the Laplace equation of flow in two directions.

## Velocity potential ( $\phi$ ):

It is defined as a scalar function of space and time such that its derivative with respect to any direction gives the fluid velocity in that direction.

We know $\phi=$ kh

$$
\frac{\partial \phi}{\partial \mathrm{x}}=\mathrm{k} \frac{\partial \mathrm{~h}}{\partial \mathrm{x}}=\mathrm{k} \mathrm{i}_{\mathrm{x}}=\mathrm{v}_{\mathrm{x}}
$$

Similarly,

$$
\frac{\partial \phi}{\partial \mathrm{y}}=\mathrm{k} \frac{\partial \mathrm{~h}=\mathrm{k} \mathrm{i}_{\mathrm{y}}=\mathrm{v}_{\mathrm{y}}}{\partial \mathrm{y}}
$$

The solution of Laplace equation can be obtained by,

1) Analytical methods
2) Graphical methods
3) Experimental methods.

Solution gives two sets of curves known as,
a) Equipotential lines (E.L)
b) Stream lines (Flow lines) (S.L (or)F.L)

Mutually orthogonal to each other.

## Equipotential line(E.L.)

It represents contours of equal head (potential).
Direction of seepage always perpendiculartoE.L.

## Stream lines (Flow lines)(S.L. or F.L.)

The path along which the individual particles of water seep through the soil.

## Properties of flow net:

1) The flow line and equipotential line meet at right angles to one another.
2) The fields are approximately squares, so that a circle can be drawn touching all the four sides of the square.
3) The quantity of water flowing through each flow channel is the same. Similarly, the same potential drop occur between two successive equipotential lines.
4) Smaller the dimensions of the field, greater will be the hydraulic gradient and velocity of flow through it.
5) In a homogeneous soil, every transition in the shape of the curves is smooth, being either elliptical (or) parabolic in shape.

## APPLICATIONS OF FLOW NET:

A flow net can be utilized for the following purposes.

## 1) Determination of seepage:

The portion between any two successive flow lines is known as at flow channel. The portion enclosed between two successive equipotential lines and successive flow lines is known as field (latched in figure).
Let , ' $b$ ' and 'l' be the width and length of the field.
$\Delta \mathrm{h}=$ head drop through the field
$\Delta q=$ discharge passing through the flow channel
$\mathrm{H}=$ total hydraulic head causing flow $=$ diff., $\mathrm{b} / \mathrm{w} \mathrm{v} / \mathrm{s}$ and $\mathrm{D} / \mathrm{s}$ heads
From Darcy's law, of flow through soils, $\Delta \mathrm{q}=\mathrm{k} . \Delta \mathrm{h}(\mathrm{bx} 1) \quad$ (unit thickness)
$+$
If $\mathrm{N}_{\mathrm{d}}=$ Total no., of potential drops in the complete flow net, then
$\Delta \mathrm{h}=\frac{\mathrm{H}}{\mathrm{N}_{\mathrm{d}}} ; \quad \Delta \mathrm{q}=\mathrm{k} \cdot \underline{\mathrm{H}} \mathrm{N}_{\mathrm{d}}\left(\frac{\mathrm{b}}{\mathrm{g}}\right)$
Total discharge, $\mathrm{q}=\sum \Delta \mathrm{q}=\mathrm{k} \cdot \frac{\mathrm{H}}{\mathrm{N}_{\mathrm{d}}}\binom{\mathrm{b}}{\mathrm{l}} \mathrm{N}_{\mathrm{f}}$


$$
\mathrm{q}=\mathrm{kH} \underset{\mathrm{~N}_{\mathrm{f}}}{\mathrm{~N}_{\mathrm{d}}}\left(\frac{\mathrm{~b}}{\mathrm{I}}\right)
$$

Where, $\mathrm{N}_{\mathrm{f}}=$ Total number of flow channels in the net.
The field is square, hence $b=1$
Thus,


Discharge expression for passing through a flow net

For isotropic soils $\mathrm{k}_{\mathrm{x}}=\mathrm{k}_{\mathrm{y}}=\mathrm{k}$

## 2) Determination of hydrostatic pressure:

The hydrostatic pressure at any point with the soil mass is given by,


The hydrostatic pressure interms if ' $h_{w}$ ' is,

$$
\mathrm{h}_{\mathrm{w}}=\mathrm{h}-\mathrm{z}
$$

Where $\mathrm{h}=$ hydraulic potential at that point

$$
\begin{aligned}
& z=\text { position head of the point above datum }(\uparrow+v e) \\
& \text { Hence, } H=h_{w}+h+z(\text { interms of } \%)
\end{aligned}
$$

Used to plot a pressure net representing lines of equal water pressure (piezometric head).

## For example:

Take $\mathrm{h}_{\mathrm{w}}=20 \% \mathrm{~h}_{\mathrm{w}}=20=\mathrm{h}-\mathrm{z}$
if, $h=30 \% \mathrm{H}, \mathrm{z}=30-20=10 \% \mathrm{H}$
if, $\mathrm{h}=40 \% \mathrm{H}, \mathrm{z}=40-20=20 \% \mathrm{H}$
The results obtained from various points are plotted and joined by a smooth
curve to get a contour ofh ${ }_{w}=20 \%$.
3) Determination of seepage pressure:

The hydraulic pressure ' $h$ ' at any point located after ' $n$ ' potential drops each of value $\Delta \mathrm{h}$ is given
by,

$$
\mathrm{h}=\mathrm{H}-\mathrm{n} \Delta \mathrm{~h}
$$

We know, seepage pressure, $\mathrm{p}_{\mathrm{s}}=\mathrm{h} \gamma_{\mathrm{w}}$

$$
P_{s}=(H-n \Delta h) \cdot \gamma_{w}
$$

The pressure acts in the direction of flow

## 4) Determination of exit gradient:

It is the hydraulic gradient at the $\mathrm{d} / \mathrm{s}$ end of the flow line where percolating water leaves the soilmass and emerges into free water at $\mathrm{d} / \mathrm{s}$.

$$
i_{e}={ }_{l}-\frac{\Delta h}{}
$$

Where, $\Delta \mathrm{h}=$ potential drop
$1=$ average length of last field in the flow net at exit

1) A homogeneous anisotropic earth dam, which is 20 m high is constructed on an impermeable foundation. The coefficients of permeability of soil used for the construction of the dam. in the horizontal and vertical direction are $4.8 \times 10^{-8} \mathrm{~m} / \mathrm{s}$ and $1.6 \times 10^{-8} \mathrm{~m}$ is respectively. The water level on the reservoir side is 18 m from the base of the dam: downstream side is dry.lt is seen that there are 4 flow channels and 18 equipotential drops is a square flow net drawn in the transformed dam section. Estimate the quantity of seepage per unit length in $\mathrm{m}^{3} / \mathrm{s}$ through the dam.

$$
\begin{array}{r}
q=k \cdot H \cdot \frac{N_{f}}{N_{d}} \\
k=\sqrt{k_{H} K_{V}} \\
=\overline{\sqrt{4} 48 \times 1.6 \times 10^{-8}}
\end{array}
$$

$=277 \times 10^{-8} \mathrm{~m} / \mathrm{s}$

$$
q=2.77 \times 10^{-8} .18 .{ }^{4} \overline{18}
$$

$=11.085 \times 10^{-8} \mathrm{~m}^{3} / \mathrm{s} / \mathrm{m}$ run

## Soil water:

Water present in the voids of soil mass is called soil water .It can be classified in several ways given below:
(a) Broad classification

1. Free water or gravitational water
2.Held water
(i) Structural water
(ii) Adsorbed water
(iii)Capillary water
(b) classification on phenomenological basis
(i)Groundwater
(ii) Capillary water
(iii)Adsorbedwater
(iv)In filtered water
c) Classification on structural aspect
i)Pore water
ii)Solvate water
(iii)Adsorbed water
iv)Structural water

## Soil Water



Free water (or) Gravitational water
Held water
Water that is free to move through a soil mass Water held in soil pores by some forces existingunder the influence of gravity. within the pores \& not free to move under gravitational forces.

Water chemically combined in the crystal structure of soil $\quad$ Structural watermineral and can be removed only by breaking structure.

Adsorbed water


Capillary water

## ADSORBED WATER (OR) HYGROSCOPIC WATER:

It is the part where the soil particles freely absorbed from atmosphere by the physical forces ofattraction and is held by the forces of adhesion.

The soil particles having negative charge, due to this charge they attract water. The attractive forcesbetween soil and water forms soil water forces.

If an oven-dried soil sample is placed in moist air the sample absorb moisture, till its water contentreaches some constant value.

The quantity of adsorbed water for a given soil varies with the temperature and the
relativehumidity of air, and the characteristics of soil particles.


Fig : ADSORBED WATER (After Lambe)

## CAPILLARY WATER:

It is held in the interspace of soil due to capillary forces. Capillary action (or) Capillarity is the phenomenon of movement of water is the interstices of a soil due to capillary forces. The minute pores of soil serve as capillary tubes through which the moisture rises above the ground water table.

The capillary force depends on surface tension of water, pressure in water and size of pores.

## In filtered water :

It is that portion of surface precipitation which soaks into ground, moving downwards through air-containing zones. It is subject to capillary forces.

## Porewater:

From the point of view of inter-particle forces, soil water can also be divided into two heads: the adsorbed water which is attracted by forces within the soil strong enough to influence its behavior , and pore water which is essentially free of strong soil attractive forces (Lambe,1953).The capillary water and the gravitational water may be considered as the two types of pore water. It exhibits the physical and
chemical properties of ordinary liquid water .It is capable of moving under Hydro dynamic forces unless restricted in its free movement, such as when entrapped between air bubbles or by retention due to capillary forces that in fine pores may overcome the hydrodynamic forces.

Solvate water : It is that water which forms a hydration shell (presumably not more than 200 molecules thick) around soil grains. It is subject to polar, electrostatic and ionic binding forces. It remains mobile under hydro dynamic forces, though its density and viscosity are greater than those of ordinary water.

Stuctural water :It is the water chemically combined in the crystal structure of the soil mineral. It refers to hydroxyl groups that constitute parts of crystal lattice. Under loading encountered in soil engineering, the structural water cannot be separated or removed and is, therefore, unimportant. It can also not removed by oven drying at $105^{\circ} \mathrm{C}$ $110^{\circ} \mathrm{C}$.However, it can only be driven off at such high temperatures as would cause the destruction of the crystal structure. We will therefore, consider the structural water as part and parcel of the soil particle

## STRESS CONDITIONS IN SOIL:

## Total stress (or) Unit pressure ( $\sigma$ ):

Total load per unit area. This pressure may be due to

1) Self weight of soil (saturated weight, if soil is saturated).
2) Over-burden on the soil.

## Consists of two components:

1) Inter granular pressure (or) effective pressure (or) effective stress ( $\sigma^{\prime}$ ):

- It is the pressure transmitted from particle through their point of contact through their soil mass abovethe plane.
- It is effective in decreasing the voids ratio of the soil mass and in mobilizing its shear strength.

2) Neutral pressure (or) pore water pressure (u):
$>\quad$ It is the pressure transmitted through the pore fluid.
$>$ It is equal to water load per unit area above the plane.
$>$ It does not have any influence (measurable) on the voids ratio or any other mechanical property ofthe soil such as shearing resistance.
Total vertical pressure $=$ Effective pressure + pore pressure


Pore pressure, $u=$ piezometric head $\left(h_{w}\right) x$ unit weight of water $\left(\gamma_{w}\right)$

$$
\mathrm{u}=\mathrm{h}_{\mathrm{w}} \cdot \gamma_{\mathrm{w}}
$$

To find the value of effective pressure we shall consider different conditions of soil water system.

## 1) Submerged soil mass:



Fig., shows saturated soil mass of depth z , submerged under water of height $\mathrm{z}_{1}$ above its top level. If a piezometric tube is inserted at level AA, water will rise in
it upto level CC. Now, total pressure at AA isgiven by,

$$
\sigma=\mathrm{Z} \gamma_{\mathrm{sat}}+\mathrm{Z}_{1} \gamma_{\mathrm{w}}
$$

pore pressure, $\mathrm{u}=\mathrm{h}_{\mathrm{w}} . \gamma_{\mathrm{w}}$

$$
\begin{aligned}
& \sigma^{\prime}=\sigma-\mathrm{u} \\
&=\mathrm{Z} \gamma_{\mathrm{sat}}+\mathrm{Z}_{1} \gamma_{\mathrm{w}}-\mathrm{h}_{\mathrm{w}} . \gamma_{\mathrm{w}} \\
&=\mathrm{Z} \gamma_{\mathrm{sat}}+\mathrm{Z}_{1} \\
& \gamma_{\mathrm{w}}-\left(\mathrm{Z}+\mathrm{Z}_{1}\right) \gamma_{\mathrm{w}} \sigma^{\prime} \\
&= \mathrm{Z}\left(\gamma_{\mathrm{sat}}-\gamma_{\mathrm{w}}\right) \\
& \sigma^{\prime}=\mathrm{Z} \gamma^{\prime}
\end{aligned}
$$

@B-B

$$
\begin{aligned}
& \sigma=\gamma_{\mathrm{W}} \cdot \mathrm{Z}_{2} \\
& \mathrm{U}=\gamma_{\mathrm{W}} \cdot h_{\mathrm{W}} \\
& \sigma^{\prime}=\sigma-\mathrm{U} \\
& =0
\end{aligned}
$$

@C-C
The total, effective and pore water pressure are zero

2) Soil mass with surcharge:

Let us now consider a moist soil mass of height $Z_{1}$ above the saturated mass of height Z. Soil mass supports a surcharge pressure of intensity ' $q$ ' per unit area.
At level AA, the pressure are,


$$
\begin{gathered}
\sigma=\mathrm{q}+\mathrm{Z}_{1} \gamma+\mathrm{Z} \gamma_{\mathrm{sat}} \\
\mathrm{u}=\mathrm{h}_{\mathrm{w}} \cdot \gamma_{\mathrm{w}} \\
=\mathrm{Z} \gamma_{\mathrm{w}}
\end{gathered}
$$

$$
\begin{gathered}
\sigma^{\prime}=\sigma-\mathrm{u} \\
=\mathrm{q}+\mathrm{Z}_{1} \gamma+\mathrm{Z} \gamma_{\mathrm{sat}}-\mathrm{Z} \gamma_{\mathrm{w}} \sigma^{\prime} \\
=\mathrm{q}+\mathrm{Z}_{1} \gamma+\mathrm{Z} \gamma^{\prime} \\
\text { At the plane } \mathrm{BB},
\end{gathered}
$$

$$
\begin{aligned}
& \sigma=\mathrm{q}+\mathrm{Z}_{1} \gamma \\
& \mathrm{u}=\mathrm{h}_{\mathrm{w}} \cdot \gamma_{\mathrm{w}}
\end{aligned}
$$

$$
=0
$$

At the plane C-C

$$
\sigma=\mathrm{q}
$$

$\mathrm{U}=0$

$$
\sigma^{\prime}=\sigma-u
$$

$=\mathrm{q}$


## 3) Partially saturated soil:

In a partially saturated soil, a part of void space is occupied by air. Hence, in addition to pore waterpressure ( $\mathrm{u}_{\mathrm{w}}$ ) pore air pressure $\left(\mathrm{u}_{\mathrm{a}}\right)$ will also to there.

Bishop (1959) based on his intuition gave the following expression for the effective stress.

$$
\sigma^{\prime}=\sigma-u_{a+x}\left(u_{a}-u_{w}\right)
$$

where,

$$
\mathrm{u}_{\mathrm{a}}=\text { pore air pressure }
$$


$s \rightarrow$
$\mathrm{u}_{\mathrm{w}}=$ pore water pressure
$x=$ factor of unit $\mathrm{c} / \mathrm{s}$ area
occupied by waterx $=\underline{A}_{w}$
A
$\mathrm{Aw}=$ Area of water $\mathrm{A}=$ Area of $\mathrm{c} / \mathrm{s}$ of soil

If ( $\mathrm{S}>=90 \%$ ) near unity it is recommended to
take ' $x$ ' as unity (ie., 1). $\sigma$ ' $=\sigma-u_{w}$

$$
\sigma^{\prime}=\sigma-u
$$

## 4) Saturated soil with capillary fringe:

Fig shows a saturated soil mass of height $Z$ above this, there is a soil mass of height $Z_{1}$ saturated by capillary water. If we insert a piezometric tube at AA, water will rise to a height corresponding to the free water level BB


Hence, at the level $A A$,

$$
\begin{aligned}
& \sigma^{\prime} \\
& =\left(Z+Z_{1}\right) \gamma^{\prime} \times Z_{1} \gamma_{w} \\
& =Z \gamma^{\prime}+Z_{1} \gamma^{\prime}+Z_{1} \gamma_{w}=Z \gamma^{\prime}+Z_{1} \gamma_{s a t} \\
\text { Alternatively, at } A A, \quad \sigma & \sigma=Z \gamma_{s a t}+Z_{1} \gamma_{\text {sat }} \\
& u
\end{aligned}
$$

Alternatively, at $A A$,

Similarly, at the level $B B, \sigma^{\prime}=Z_{1} \gamma^{\prime}+Z_{1} \gamma_{w}=Z_{1} \gamma_{\text {st }}$
At the level CC

$$
\begin{aligned}
& \quad \sigma^{1}=0 \\
& \quad \mathrm{U}=\mathrm{h}_{\mathrm{w}} \gamma_{\mathrm{w}}=\left(-\mathrm{Z}_{1}\right) \mathrm{xh}_{\mathrm{w}} \\
& \sigma=\sigma^{1}-\mathrm{U} \\
& =0-\left(-\mathrm{h}_{\mathrm{w}} \mathrm{Z}_{1}\right) \\
& =\mathrm{h}_{\mathrm{w}} \mathrm{Z}_{1}
\end{aligned}
$$

Problem 1) The water table in a deposit of sand 8 m thick, is at a depth of 3 m below the surface. Above the W.T. the sand is saturated with capillary water. The bulk density of sand is $19.62 \mathrm{kN} / \mathrm{m}^{3}$. Calculate the effective pressure at $1 \mathrm{~m}, 3 \mathrm{~m}$ and 8 m below the surface. Hence plot the variation of total, neutral pressure and effective pressure over the depth of 8 m .
a) Stresses at D, 8m below ground:

If we insert a piezometric tube at $D$, water will be rise through a height $h_{w}=5 \mathrm{~m}$ in it.

$\sigma=(3+5) \gamma_{\text {sat }}=8 \times 19.62=156.96 \mathrm{kN} / \mathrm{m}^{2}$
$\mathrm{u}=\mathrm{h}_{\mathrm{w}} \cdot \gamma_{\mathrm{w}}=5 \times 9.81=\underline{49.05} \mathrm{kN} / \mathrm{m}^{2}$
$\sigma^{\prime}=\sigma-\mathrm{u}=156.96-49.05=\underline{107.91} \mathrm{kN} / \mathrm{m}^{2}$
Alternatively, $\quad \sigma^{\prime}=5 \gamma^{\prime}+3 \gamma_{\text {sat }}$

$$
=5 \times 9.81+3 \times 19.62=\underline{107.91} \mathrm{kN} / \mathrm{m}^{2}
$$

## b) Stresses at C. 3 m below G.L.:

$\sigma=3 \gamma_{\mathrm{sat}}=3 \times 19.62$

$$
=\underline{58.86} \mathrm{kN} / \mathrm{m}^{2}
$$

$\mathrm{u}=0$ (zero)
$\sigma^{\prime}=58.86 \mathrm{kN} / \mathrm{m}^{2}$
Alternatively, $\sigma=\mathrm{hx} \gamma_{\mathrm{sat}}=3 \times 19.62$

$$
\begin{aligned}
= & \underline{58.86} \mathrm{kN} / \mathrm{m}^{2}(\mathrm{or}) \\
\sigma^{\prime} & =3 \mathrm{x} \gamma^{\prime}+\mathrm{h}_{\mathrm{c}} \cdot \gamma_{\mathrm{w}} \\
& =3(19.62-9.81)+3 \times 9.81 \\
& =\underline{58.86} \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

c) Stresses at B, 1m below G.L:
$\sigma=1 \gamma_{\text {sat }}=1 \times 19.62=\underline{19.62} \mathrm{kN} / \mathrm{m}^{2}$
$u=-2 \gamma_{w}=-2 \times 9.81=-19.62 \mathrm{kN} / \mathrm{m}^{2}$
(ie., pressure due to weight of water hanging below that level) $\sigma^{\prime}$
$=(\sigma-\mathbf{u})$
$=19.62-(-19.62)$
$=\underline{39.24} \mathrm{kN} / \mathrm{m}^{2}$

## d) Stresses at A, at Ground Level.:

$\sigma=0$
$\mathrm{u}=-\mathrm{h}_{\mathrm{c}} \cdot \gamma_{\mathrm{w}}=-3(9.81)$
$=\underline{-29.43} \mathrm{kN} / \mathrm{m}^{2}$
$\sigma^{\prime}=0-(-29.43)$
$\sigma^{\prime}=\underline{29.43} \mathrm{kN} / \mathrm{m}^{2}$

Problem 2) The water table in a certain area is at a depth of 4 m below the ground surface, to a depth of 12 m , the soil consists of very fine sand having an average voids ratio of 0.7. Above the water table the sand has an average degree of saturation of $50 \%$. Calculate the effective pressure on a horizontal plane at a depth 10 m below the ground surface. What will be the increase in the effective pressure if the soil gets saturated by capillary upto a height of 1 m above the W.T.? (Assume $\mathrm{G}=2.6$ ).


## Solution:

Height of sand layer above the w.t $=\mathrm{Z}_{1}=4 \mathrm{~m}$
Height of saturated layer $=12-4=8 \mathrm{~m}$

Depth of point x , where pressure is to be computed $=10 \mathrm{~m}$
Height of saturated layer above $x=Z_{2}=10-4=6 \mathrm{~m}$

$$
\gamma_{\mathrm{d}}=\underline{\mathrm{G} \gamma_{\mathrm{w}}}=\frac{2.65 \times 9.81}{1+\mathrm{e}}=\underline{15.29} \mathrm{kN} / \mathrm{m}^{3}
$$

i) For sand above water table,
$\gamma_{1}=\frac{(\mathrm{G}+\mathrm{eS})}{1+\mathrm{e}} \gamma_{\mathrm{w}}, \quad \gamma_{2}=\gamma_{\mathrm{sat}}=\frac{(\mathrm{G}+\mathrm{e})}{1+\mathrm{e}} \gamma_{\mathrm{w}}$

$$
e=\frac{w G}{S}
$$

$$
w=\frac{e S}{G}
$$

$$
w=\frac{0.7 \times 0.5}{2.65}
$$

$$
=0.132
$$

$\gamma_{1}=\gamma_{\mathrm{d}}(1+\mathrm{w})=15.29 \mathrm{x}(1+0.132)=\underline{17.31} \mathrm{kN} / \mathrm{m}^{3}$
ii) For saturated sand below water table,
$W_{\text {sat }}=\frac{\mathrm{ex} 1}{\mathrm{G}}=\frac{0.7}{2.65}=0.264$
$\gamma_{2}=\gamma_{\mathrm{d}}\left(1+\mathrm{w}_{\text {sat }}\right)=15.29 \times 1.264=19.33 \mathrm{kN} / \mathrm{m}^{3}$
$\gamma_{2}{ }^{\prime}=19.33-9.81=\underline{9.52} \mathrm{kN} / \mathrm{m}^{3}$

$$
\left(\gamma^{\prime}=\gamma_{\mathrm{ref}}=\gamma_{\mathrm{sat}}-\gamma_{\mathrm{w}}\right)
$$

Effective pressure at x ,
$\sigma=\mathrm{Z}_{1} \gamma_{1}+\mathrm{Z}_{2} \gamma_{2}{ }^{\prime}=(4 \times 17.31)+(6 \times 19.33)$
$\sigma=\underline{185.222} \mathrm{kN} / \mathrm{m}^{2}$
$\mathrm{u}=\mathrm{h}_{\mathrm{w}} . \gamma_{\mathrm{w}}=6 \mathrm{x} 9.81$
$=\underline{58.86} \mathrm{kN} / \mathrm{m}^{2}$
$\sigma^{\prime}=\sigma-u=185.22-58.86$
$=\underline{126.36} \mathrm{kN} / \mathrm{m}^{2}$
Effective stress at x after capillary rise,

$$
\begin{aligned}
\sigma^{\prime}= & 3 \gamma_{1}+(6+1) \gamma_{2}^{\prime}+\mathrm{h}_{\mathrm{c} \cdot} \cdot \gamma_{\mathrm{w}} \\
& =(3 \times 17.31)+(7 \times 9.52)+(1 \times 9.81) \\
& =\underline{128.38} \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

Increase in pressure $=128.38-126.36$
$=\underline{2.02} \mathrm{kN} / \mathrm{m}^{2}$
(or)

$$
\begin{aligned}
\sigma & =\mathrm{Z}_{1} \gamma_{1}+\mathrm{Z}_{2} \gamma_{2}^{\prime}+\mathrm{h}_{\mathrm{c}} \cdot \gamma_{\mathrm{w}} \\
& =3 \times 17.31+7 \times 19.33+1 \times 9.81=\underline{197.05} \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

$$
\mathrm{U}=7 \times 9.81=\underline{68.67} \mathrm{kN} / \mathrm{m}^{2}
$$

$$
\sigma^{\prime}=\sigma-u
$$

$$
=197.05-68.67
$$

$$
=\underline{128.38} \mathrm{kN} / \mathrm{m}^{2}
$$

3) At a construction site a 3 m thick clay layer is followed by a 4 m thick gravel layer which is resting on impervious rock. A load of $25 \mathrm{kN} / \mathrm{m}^{2}$ is applied suddenly at the surface. The saturated unit weight of the soils is $19 \mathrm{kN} / \mathrm{m}^{3}$ and $20 \mathrm{kN} / \mathrm{m}^{3}$ for the clay and gravel layers, respectively. The water table is at the surface. Draw diagrams showing variation with depth of total, neutral and effective stress in the layers.

(a) Soil profile

(b) $\sigma$-dia.

(c) u-dia.

(d) $\sigma^{\prime}-d i a$.

At A-A
$\sigma=25 \mathrm{KN} / \mathrm{m}^{2}$
$\mathrm{U}=25 \mathrm{KN} / \mathrm{m}^{2}$ (Since load is applied suddenly the entire load is taken by pore water)

$$
\begin{aligned}
\sigma^{\prime} & =\sigma-\mathrm{u} \\
& =25-25 \\
& =0 \mathrm{KN} / \mathrm{m}^{2}
\end{aligned}
$$

At B-B
$\sigma=25+3 \times 19=82 \mathrm{KN} / \mathrm{m}^{2}$
$\mathrm{U}=25+9.81 \times 3$
$=54.43 \mathrm{KN} / \mathrm{m}^{2}$ (Since load is applied suddenly the entire load is taken by pore water)

$$
\begin{aligned}
& \begin{aligned}
\sigma^{\prime} & =\sigma-\mathrm{u} \\
& =82-54.43 \\
& =27.57 \mathrm{KN} / \mathrm{m}^{2}
\end{aligned} \\
& \text { At C-C }
\end{aligned}
$$

$$
\sigma=25+3 \times 19+4 \times 20=162 \mathrm{KN} / \mathrm{m}^{2}
$$

$\mathrm{U}=25+9.81 \times 3+9.81 \times 7=93.67 \mathrm{KN} / \mathrm{m}^{2}$ (Since loa is applied suddenly the entire load is taken by pore water)

$$
\begin{aligned}
\sigma^{\prime} & =\sigma-\mathrm{u} \\
& =162-93.67=68.33 \mathrm{KN} / \mathrm{m}^{2}
\end{aligned}
$$

