## 2.6 BREAK IN GRADE

Simple situations of a series combination of two channel sections with differing bed slopes are considered. a break in grade from a mild channel to a milder channel is shown. It is necessary to first draw the critical-depth line (CDL) and the normal-depth line (NDL) for both slopes. Since yc does not depend upon the slope for a taken Q = discharge, the CDL is at a constant height above the channel bed in both slopes. The normal depth y01 for the mild slope is lower than that of the of the milder slope (y_{02}). In this case, y_{02} acts as a control, similar to the weir or spillway case and an M1 backwater curve is produced in the mild slope channel. It may be noted that in some situations there can be more than one possible profiles. For example, a jump and S1 profile or an M3 profile and a jump possible. The particular curve in this case depends on the channel and its flow properties. , the section where the grade changes acts a control section and this can be classified as a natural control. It should be noted that even though the bed slope is considered as the only variable in the above examples, the same type of analysis would hold good for channel sections in which there is a marked change in the roughness characteristics with or without change in the bed slope. A long reach of unlined canal followed by a line reach serves as a typical example for the same. A change in the channel geometry (the bed width or side slope) beyond a section while retaining the prismoidal nature in each reach also leads to a natural control section.

(a)

(b)

(c)

(d)
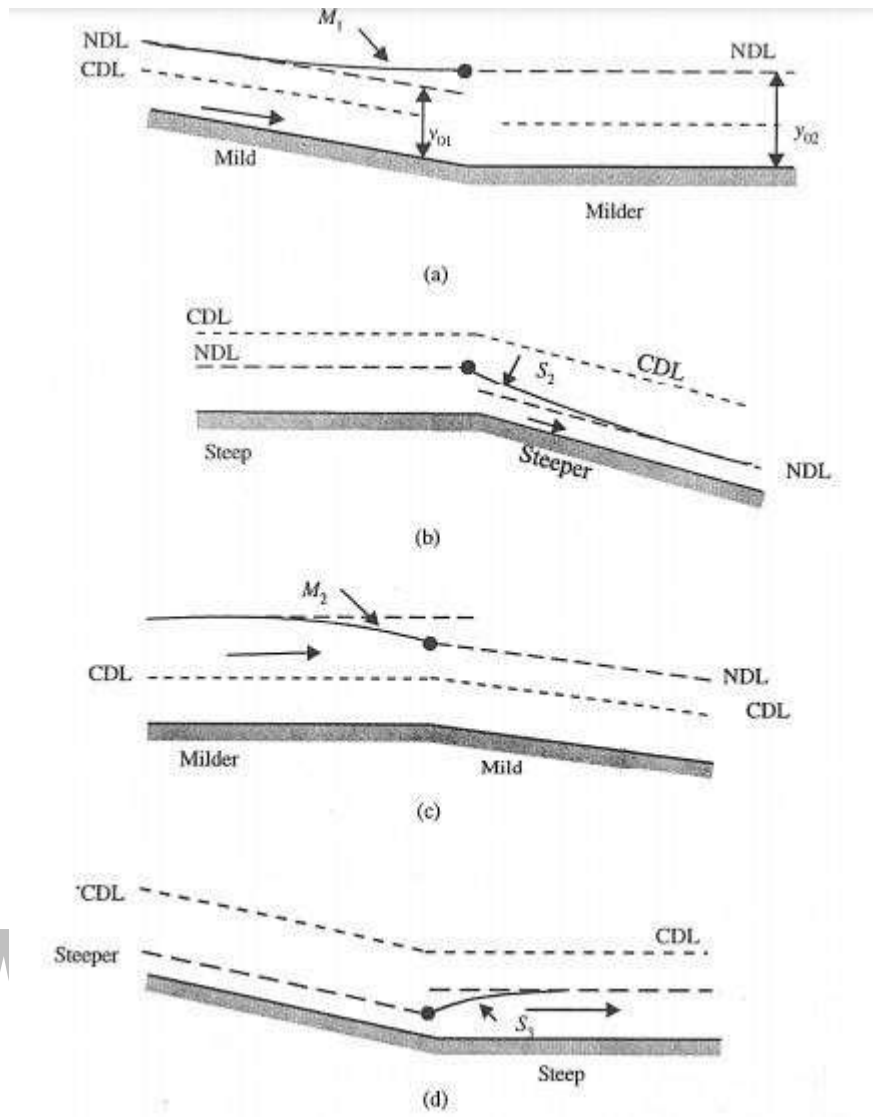
2.3 CLASSIFICATIONS

S.NO CHANNEL	CATEGORY	SYMBOL	CHARACTERISTIC CONDITION	REMARKS
1	Mild	M	$y_0 > y_c$	Subcritical flow at normal depth
`2	Steep	S	$y_0 < y_c$	Supercritical flow at normal depth
3	Critical	C	$y_0 = y_c$	Critical flow at normal depth
4	Horizontal bed	H	$S_0 = 0$	Cannot sustain uniform flow
5	Adverse	A	$S_0 < 0$	Cannot sustain uniform flow

For each of the five categories of channels, lines representing the critical

depth (y_c) and normal depth (y_0) (if it exists) can be drawn in the

longitudinal section. These would divide the whole flow space into three regions as:

Region 1: Space above the topmost line,

Region 2: Space between top line and the next lower line,

Region 3: Space between the second line and the bed

CHANNEL SLOPES

Channel slopes are classified based on the relative magnitude of: (i) bed slopes, S_0 with a critical slope, S_c and (ii) normal depth y_n with a critical depth, y_c. A channel bottom slope that falls in the direction of flow is called as sustaining slope. In a sustaining slope, value of S_0 is always +ve and therefore it is also called positive slope. The critical, mild (subcritical), and steep (super critical) slopes may be classified as sustaining slopes. When the slope of the channel bottom is either horizontal or rises in the direction of flow, then it is known as non-sustaining slope. For example, horizontal and adverse slopes are classified as non-sustaining slopes. Normal-depth line (NDL) is a line drawn parallel to bed and at a height of normal depth y_n from the bed. Critical–depth line (CDL) is a line drawn parallel to bed and at a height of critical depth y_c from the bed.

Critical Slope: - The channel bottom slope is termed as critical if the bottom slope S_0 is equal to the critical slope S_c, i.e. $S_0 = S_c$. where S_0 is +ve. For critical slope the normal depth of flow y_n will be equal to the critical depth y_c.
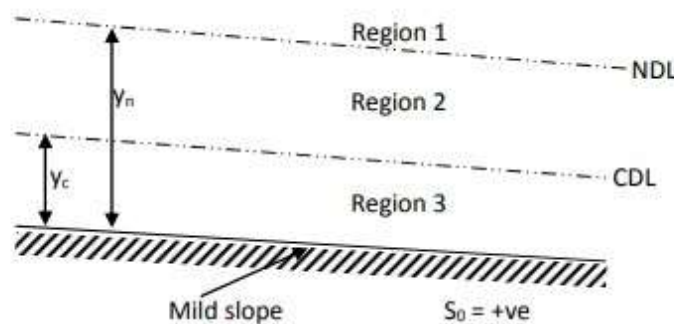
Region 1: Space above the NDL
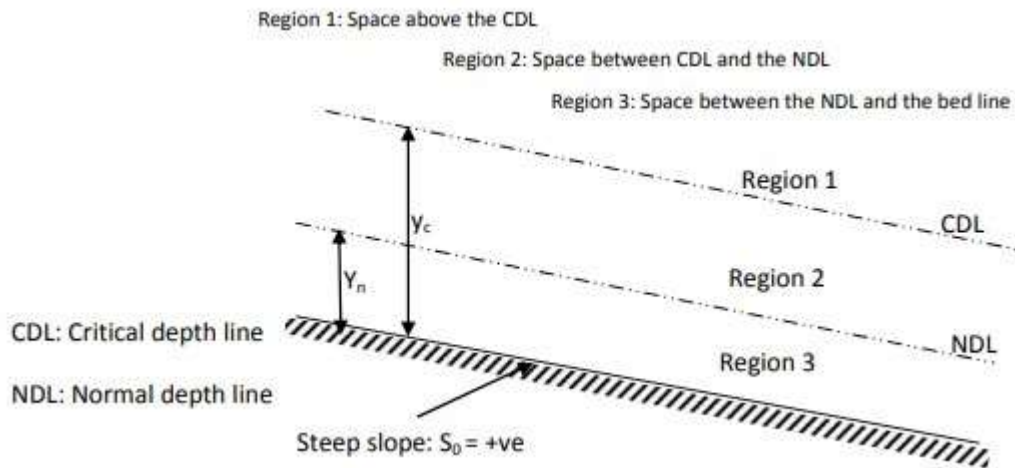Region 3: Space between the CDL and the bed line
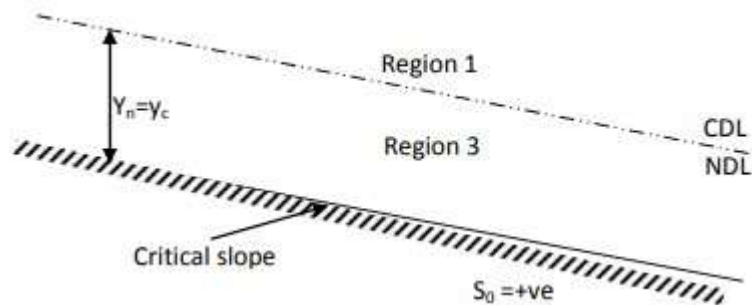
CDL: Critical depth line
NDL: Normal depth line

Mild slope (M): Channel is called mild (M) when, $S_0 \neg K_c$. since, in general, the value of conveyance increases with the increase in depth. Hence, for a mild slope the normal depth of flow is greater than the critical depth i.e. $y_n > y_c$.
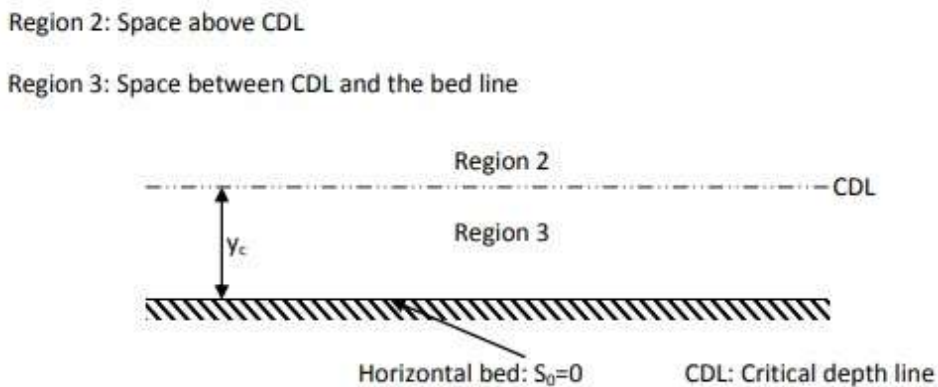
Steep slope(S): In case of a steep channel (S) the, $S_0 > S_c$, where S_0 is +ve, therefore, $K_n < K_c$ and y_n. The normal flow in a steep sloped channel is in supercritical state.

Region 1: Space above the CDL

Region 2: Space between CDL and the NDL

Region 3: Space between the NDL and the bed line

Region 1

CDL

y_c

Region 2

Y_n

NDL

CDL: Critical depth line

Region 3

NDL: Normal depth line

Steep slope: S_0 = +ve

Horizontal slope, (H):

In the case of horizontal slope, $S_0 = 0$;. Hence, the normal depth of flow $y_n = \infty$. Therefore, uniform flow can never occur on a horizontal bed.

Region 2: Space above CDL

Region 3: Space between CDL and the bed line

Region 2

CDL

Region 3

y_c

Horizontal bed: $S_0 = 0$ CDL: Critical depth line

Adverse slope (A): When the channel bottom slope, rises in the direction of flow it is designated as an adverse slope; therefore, S_0

Region 2: Space above CDL

Region 3: Space between the CDL and the bed line

CDL

Region 2

Region 3

y_c

$S_0 = $ -ve

Adverse slope

CDL= Critical depth line

Adverse slope (A): When the channel bottom slope, rises in the direction of flow it is designated as an adverse slope; therefore, S_0
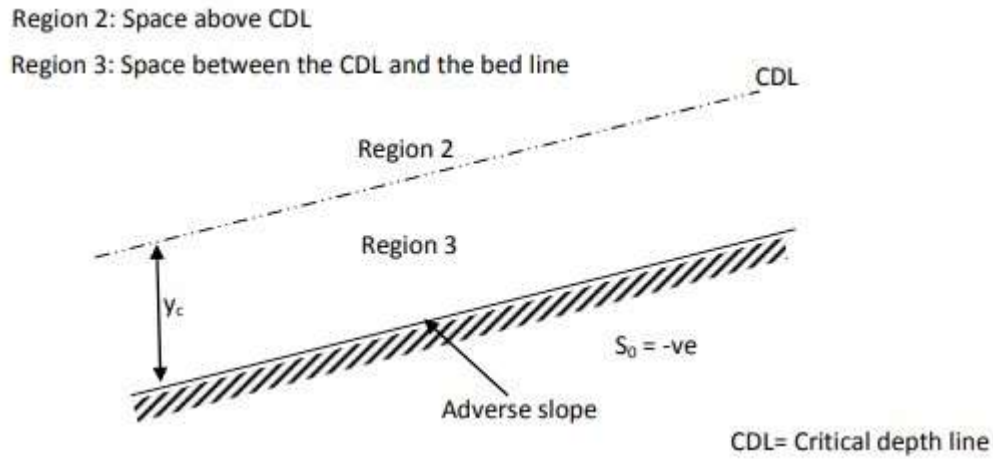
2.4 COMPUTATION BY DIRECT STEP METHOD AND STANDARD STEP METHOD

Gradually varied flow profiles shown above may be quickly solved by simple numerical techniques. One computer program can be written to solve most situations.

There are two basic numerical methods that can be used

i. Direct step – distance from depth

ii. Standard step method – depth from distance

1. The direct stepmethod – distance from depth

This method will calculate (by integrating the gradually varied flow equation) a distance for a given change in surface height.

$$\Delta x = \Delta y \frac{1 - f^2}{s_0 - s_f}$$

The steps in solution are

1. Determine the control depth as the starting point

2. Decide on the expected curve and depth change if possible

3. Choose a suitable depth step Δy

4. Calculate the term in brackets at the "mean" depth (y initial $+ \Delta y/2$)

5. Calculate Δ x

6. Repeat 4 and 5 until the appropriate distance / depth changed reached.

2. The standard step method – depth from distance

This method will calculate (by integrating the gradually varied flow equation) a depth

at a given distance up or downstream.

$$\Delta E = \Delta x \; s_0 - s_f$$

The steps in solution are similar to the direct step method shown above but for each x there is the following iterative step

1. Assume a value of depth y (the control depth or the last solution depth)

2. Calculate the specific energy EsG

3. Calculate S f

4. Calculate Δ Es using equation 1.30

5. Calculate E s(x+ Δx) s = E + Δ E

6. Repeat until ΔE s (x+ Δx) = E sG

The Standard step method – alternative form

This method will again calculate a depth at a given distance up or downstream but this time

$$\Delta H = -\Delta x \; -s_f$$

$$\text{where H} = \text{y+z+}\frac{\alpha v^2}{2g}$$

The strategy is the same as the first standard step method, with the same necessity to iterate for each step.

2.5 CONTROL SECTION

A control section is defined as a section in which a fixed relationship exists between the discharge and depth of flow. Weirs, spillways, sluice gates are some typical examples of structures which give rise to control sections. The critical depth is also a control point. However, it is effective in a flow profile which changes from subcritical to supercritical flow. In the reverse case of transition from supercritical flow to subcritical flow, a hydraulic jump is usually formed bypassing the critical depth as a control point. Any GVF profile will have at least one control section.
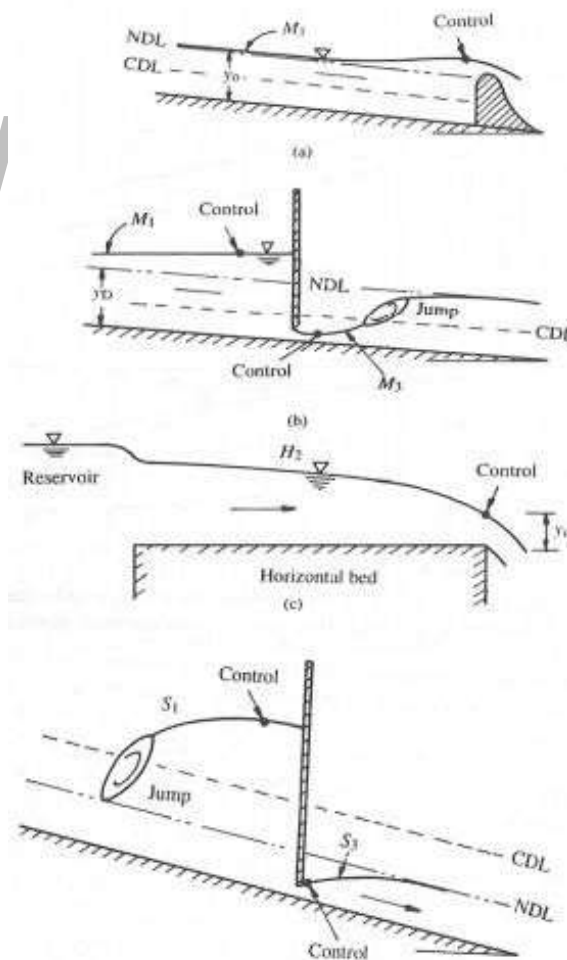
In the synthesis of GVF profiles occurring in a serially connected channel elements, the control sections provide a key to the identification of proper profile shapes. It may be noted that subcritical flows have controls in the downstream end, while supercritical flows are governed by control sections existing at the upstream end of the channel section. for the M1 profile, the control section is just at the upstream of the spillway and sluice gate respectively. for M3 and S3 profiles respectively, the control point is at the vena contracta of the sluice gate flow

2.1 DYNAMIC EQUATIONS OF GRADUALLY VARIED FLOWS

VARIED FLOW

Flow properties, such as depth of flow area of cross section and velocity of flow vary with respect to distance is called Non-uniform flow.

It is, otherwise, called as varied flow. The varied flow is broadly classified into two types:

1) Rapidly varied flow (R.V.F)

2) Gradually varied flow (G.V.F)

If the depth of flow changes quickly over a small length of the channel, the flow is said to be gradually varied flow (GVF). Example: Back water in a dam.

The following assumptions are made for analyzing the gradually varied flow:

1. The flow is steady

2. The pressure distribution over the channel section is hydrostatic, i.e., streamlines are practically straight and parallel.

3. The head loss is same as for uniform flow.

4. The channel slope is small, so that the depth measured vertically is the same as depth measured normal to the channel bottom.

5. A channel is prismatic.

6. Kinetic energy correction factor is very close to unity.

7. Roughness coefficient is constant along the channel length

8. The formulae, such as Chezy's formula, Manning's formula which are applicable, to the uniform flow are also applicable for the gradually varied flow for determining slope of energy line.

DYNAMIC EQUATION OF GVF
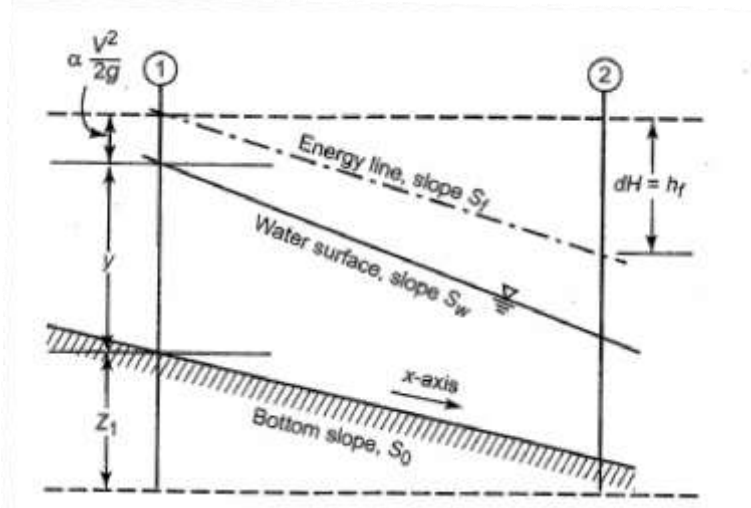
let

Z = height of bottom of channel above datum

h = depth of flow

V = Mean velocity of flow

i b = slope of the channel bed.

i e = slope of the energy line

b = width of channel

Q = discharge through the channel

The energy equation at any section is given by Bernoulli's equation.

$$E = Z + \square + \frac{v^2}{2g}$$

Differentiating this equation with respect to x is where x is measured along the bottom of the channel in the direction of flow, we get.

Equation of Non-uniform flow (Slope of free water surface)

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{d\Box}{dx} + \frac{d}{dx} \times \frac{v^2}{2g}$$

$$\frac{d}{dx} \times \frac{v^2}{2g} = \frac{d}{dx} \times \frac{Q^2}{2gA^2}$$

$$\frac{d}{dx} \times \frac{Q^2}{2gh^2b^2} = \frac{Q^2}{2gb^2} \frac{d}{dx} \times \frac{1}{h^2}$$

$$= \frac{Q^2}{2gb^2} \frac{d}{dh} \times \frac{1}{h^2} \times \frac{dh}{dx}$$

$$= \frac{Q^2}{2gb^2} \times \frac{-2}{h^3} \times \frac{dh}{dx}$$

$$= \frac{Q^2}{2gb^2} \times \frac{-2}{h^3} \times \frac{dh}{dx}$$

$$\frac{d}{dx} \times \frac{v^2}{2g} = \frac{Q^2}{2gb^2} \times \frac{-2}{\Box^2} \times \frac{1}{\Box} \times \frac{d\Box}{d x} = -\frac{v^2}{g\Box} \frac{d\Box}{dx}$$

Substitute the value of $\frac{d}{dx} \times \frac{v^2}{2g}$

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{d\Box}{dx} - \frac{v^2}{2g\Box} \frac{d\Box}{dx}$$

$$\frac{dE}{dx} = \frac{d Z}{dx} + \frac{d \Box}{dx} \left[1 - \frac{v^2}{2g\Box} \right]$$

$\frac{dE}{dx} =$ Slope of the energy line = -ie.

$$\frac{dZ}{dx} = \text{Slope of the bed of the channel} = - \text{ib}.$$

-Ve sign with ie & ib us taken with the increase of x, the value of E and Z decreases.

Substituting the value of $\frac{dE}{dx}$ $and \frac{dZ}{dx}$

$$-i_b = -i_e + \frac{d□}{d_x} \left[1 - \frac{v^2}{2g□} \right]$$

$$i_b - i_e = \frac{d□}{d_x} \left[1 - \frac{v^2}{2g□} \right]$$

$$\frac{d□}{dx} = \frac{i_b - i_e}{1 - \frac{v^2}{2g□}}$$

$$\frac{d□}{dx} = \frac{i_b - i_e}{1 - f^2}$$

As h is the depth of flow and x is the distance measured along the bottom of the

channel

$\frac{d}{x}$ represents the variation of the water depth along the bottom of the channel.
$\frac{d}{□}$

This is also called the slope of the free water surface.

1. If Sf = So, then dy/dx = 0, the surface profile of flow is parallel to the

   bottom of the channel.

2. If Sf < So, then dy/dx is positive which means the flow profile is rising

   gradually.

3. If Sf > So, then dy/dx is negative which means the flow profile is lowering

   gradually.

problem 1

In a rectangular channel 12 m wide, depth 3.6 m with a velocity of 12m/s. The bed slope of channel is 1 in 4000. If the flow of water through the channel is regulated in such away the energy line having a slope of 0.00004. Find the rate of change of depth of water in the channel.

Given

$$b = 12m,$$

$$h = 3.6m$$

$$V = 1.2 \text{ m/s},$$

$$i_b = 1/4000$$

$$i_e = 0.00004$$

To find

1. the rate of change of depth of water in the channel.

Solution

$$\frac{dh}{dx} = \frac{i_b - i_e}{1 - \frac{v^2}{2gh}}$$

$$\frac{dh}{dx} = \frac{i_b - i_e}{1 - f^2}$$

$$= 2.189 \times 10^{-4}$$

2.2 TYPES OF FLOW PROFILES

The water surface profile is a measure of how the flow depth changes longitudinally. The profiles are classified based on the relationship between the actual water depth (y), the normal depth (y_n) and the critical depth (y_c). Normal depth is the depth of flow that would occur if the flow was uniform and steady, and is usually predicted using the Manning's Equation. Critical depth is defined as the depth of flow where energy is at a minimum for a particular discharge.

Flow profiles are classified by the slope of the channel (So), y_n, and y_c. There are five slope classifications designated by the letters C, M , S, A, and H (critical, mild, steep, adverse, and horizontal) respectively.

- **Mild (M) if $y_n > y_c$**
- **Steep (S) if $y_n < y_c$**
- **Critical (C) if $y_n = y_c$**
- **Adverse (A) if So < 0 (if slope is positive in the downstream direction)**
- **Horizontal (H) if So = 0**

The profile is further classified according to the relative position of the actual flow depth to normal and critical depth as designated by the numbers 1, 2, and 3.

- Type 1 curve: Actual depth is greater than y_c and y_n, flow is subcritical
- Type 2 curve: actual depth is between y_c and y_n, flow can be either subcritical or supercritical
- Type 3 curve: actual depth is less than both y_c and y_n, flow is supercritical

Gradually Varied Flow profiles

Channel	Region	Condition	Type
Mild slope	1	$y > y_0 > y_c$	M1
	2	$y_0 > y > y_c$	M2
	3	$y_0 > y_c > y$	M3
Steep slope	1	$y > y_c > y_0$	S1
	2	$y_c > y > y_0$	S2
	3	$y_c > y_0 > y$	S3
Critical slope	1	$y > y_0 = y_2$	C1
	3	$y < y_0 = y_c$	C3
Horizontal bed	2	$y > y_c \; y < y_c$	H2
	3		H3
Adverse slope	2	$y > y_c \; y < y_c$	A2
	3		A3