### 1.5 BEST HYDRAULIC SECTIONS FOR UNIFORM FLOW

## Most Economical Section of Channels:

A section of a channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction of a channel depends on excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum. This condition is utilized for determining the dimensions of economical sections of different forms of channels.

Most economical section is also called the best section or most efficient section as the discharge, passing through a most economical section of channel for a given cross- sectional area A , slope of the bed S and a resistance coefficient, is maximum.

## Most Economical Rectangular Channel:

Consider a rectangular section of channel as shown

b
wetted perimeter, $\mathrm{P}=\mathrm{b}+2 \mathrm{~d}$

$$
\begin{equation*}
=A d+2 d=A d+2 d------(3) \tag{2}
\end{equation*}
$$

for most economical section, P should be minimum for a
given area.

$$
\frac{\partial p}{\partial d}=0
$$

difference the equation (3) with respect to ' d ' and equating the
same to zero, we get,

$$
\begin{aligned}
& \left.\frac{\frac{d}{d i d f]}}{d i}-2 d\right]=0 \\
& \frac{A}{d^{2}}-2=0 \\
& A=2 d^{2} \\
& \text { But A }=\mathrm{bxd} \\
& \therefore \therefore \mathrm{~b} \times \mathrm{d}=2 \mathrm{~d}^{2} \\
& \quad \mathrm{~b}=2 \mathrm{~d}
\end{aligned}
$$

Now hydraulic mean depth, $\mathrm{m}=\mathrm{A} / \mathrm{P}$

$$
=\frac{b \times d}{b+2 d}
$$

$$
=\frac{2 d \times d}{2 d+2 d}
$$

( ) $=\frac{2 d}{4 d^{2}}$
$\mathrm{~m}=\frac{d}{2}$ com

## problem 1

A rectangular channel of width, 4 m is having a bed slope of 1 in 1500 . Find the maximum discharge through the channel. Take value of $\mathrm{C}=50$

## Given:

$$
\begin{aligned}
& \mathrm{b}=4 \mathrm{~m} \\
& \mathrm{i}=\frac{1}{1500} \\
& \mathrm{C}=50
\end{aligned}
$$

## Solution

$$
\mathrm{b}=2 \mathrm{~d}
$$

$$
\begin{aligned}
& \mathrm{d}=\frac{b}{2} \\
& \mathrm{~d}=\frac{4}{2}=2 \mathrm{~m} \\
& \mathrm{~m}=\frac{d}{2}={ }^{2}=1.0 \mathrm{~m}
\end{aligned}
$$

Area of economical rectangular channel,

$$
\begin{aligned}
& \mathrm{A}=\mathrm{b} \times \mathrm{d}=4 \times 2=8 \mathrm{~m}^{2} \\
& \mathrm{Q}=\mathrm{AC} \mathrm{mi} \\
& \mathrm{Q}=4 \times 2 \times 50 \times \overline{1 \times \frac{1}{1500}}
\end{aligned}
$$

$$
\mathrm{Q}=10.328 \mathrm{~m} 3 / \mathrm{s} .
$$

## problem 2

A rectangular channel carries water at the rate of 400 lt is when bed slope is 1 in 2000. Find the most economical dimension of the channel of $\mathrm{C}=50$

## Given:

$$
\begin{aligned}
& \mathrm{Q}=400 \mathrm{lts} / \mathrm{s}=0.4 \mathrm{~m}^{3} / \mathrm{s} \\
& \mathrm{i}=\frac{1}{2000} \\
& \mathrm{C}=50
\end{aligned}
$$

## Solution

For the rectangular channel to be most economical,
i. Width $\mathrm{b}=2 \mathrm{~d}$.
ii. Hydraulic mean depth $\mathrm{m}=\frac{d}{2}$

$$
\begin{aligned}
& \text { Area }=\mathrm{b} \times \mathrm{d}=2 \mathrm{~d} \times \mathrm{d}=2 \mathrm{~d}^{2} \\
& \mathrm{Q}=\mathrm{AC} \mathrm{mi} \\
& 0.4=2 \mathrm{~d}^{2} \times 50 \overline{\frac{d \times}{2} \frac{1}{2000}} \\
& =2 \times 505 / 4000 \times \mathrm{d}^{5 / 2} \\
& d^{5 / 2}=0.253 \\
& \mathbf{d}=\mathbf{0 . 5 7 7} \mathbf{m} \\
& \mathrm{b}=2 \times \mathrm{d}=2 \times 0.577=1.154 \mathrm{~m}
\end{aligned}
$$

## problem 3

A trapezoidal channel has side slopes 1 to 1 . It is required to discharge $13.75 \mathrm{~m}^{3} / \mathrm{s}$ of water with a bed qradient of 1 in 1000. If unlined the value of chezy's $C$ is 44 . If lined with concrete, its value in 60 . The cost per $\mathrm{m}^{3}$ of excavation is four times the cost per $\mathrm{m}^{2}$ of lining. The channel is to be the most efficient one find whether the lined canal or the unlined canal will be cheaper. What will be the dimension of hat economical canal?

## Given,

Side slope $\mathrm{n}=1$
Slope of bed $i=\frac{1}{1000}$
$\mathrm{Q}=13.75 \mathrm{~m}^{3} / \mathrm{s}$
For unlined $\mathrm{C}=44$
Lined C $=60$
Cost per $\mathrm{m}^{3}$ of excavation $=4 \mathrm{x}$ cost per $\mathrm{m}^{2}$ of lining.
Let the cost per $\mathrm{m}^{2}$ of lining $=\mathrm{x}$
Cost per $\mathrm{m}^{3}$ of excavation $=4 \mathrm{x}$

For most efficient trapezoidal channel, Hydraulic mean depth $m=\frac{d}{2}$
$d=$ depth of channel
b =width of channel
2. Half of top width $=$ length of sloping side

$$
\begin{gathered}
\frac{b+2 n d}{=d n^{2}+1^{2}} \\
\frac{b+2 \times 1 \times d=d 1^{2}}{}+1^{2} \\
\mathrm{~b}=0.828 \mathrm{~d} \\
\mathrm{~A}=(\mathrm{b}+\mathrm{nd}) \times \mathrm{d}=(0.828 \mathrm{~d}+1 \times \mathrm{d}) \times \mathrm{d} \\
\mathrm{~A}=1.828 \mathrm{~d}^{2}
\end{gathered}
$$

1. For unlined channel:

$$
\mathrm{C}=44
$$

$$
\begin{aligned}
& \mathrm{Q}=\mathrm{AC} \overline{\mathrm{mi}} \\
& 13.75=1.828 \mathrm{~d}^{2} \times 44 \frac{d}{\frac{d}{2} \frac{1}{1000}}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{d}=2.256 \mathrm{~m} \\
& \mathrm{~b}=0.828 \mathrm{~d} \\
& =0.828 \times 2.256=\mathbf{1 . 8 6 8 m}
\end{aligned}
$$

Cost of excavation per running meter Length of unlined channel $=$ Volume of channel x Cost per $\mathrm{m}^{3}$ of excavation

$$
\begin{aligned}
& =(\text { Area of channel } \times 1) x 4 x=[(b+n d) \times d \times 1] \times 4 x \\
& =(1.868+1 \times 2.256) \times 2.256 \times 1 \times 4 x=37.215 x
\end{aligned}
$$

2.For lined channels

Value of $\mathrm{C}=60$

$$
\mathrm{Q}=\mathrm{AC} \overline{\mathrm{mi}}
$$

$$
\begin{aligned}
13.75 & =1.828 \mathrm{~d}^{2} \times 60 \overline{\frac{d x}{2} \frac{1}{1000}} \\
\mathbf{d} & =1.992 \mathrm{~m}
\end{aligned}
$$

## problem 4

A power canal of trapezoidal section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given, discharge equal to $14 \mathrm{~m}^{3} / \mathrm{s}$ bed slope 1:2500 and Manning's $\mathrm{N}=0.02$

## Given:

$$
\begin{aligned}
& \mathrm{Q}=14 \mathrm{~m}^{3} / \mathrm{s} \\
& \mathrm{~N}=0.02 \\
& \mathrm{i}=\frac{1}{2500}
\end{aligned}
$$

The trapezoidal section should be most economical for the excavation of the canal at the least cost. Side slope (Value of $n$ ) is not given. Hence the best side slope for mosteconomical trapezoidal section is given by equation

$$
\mathrm{n}=\frac{1}{\overline{3}}
$$

For most economical section, Half of top width = Length of one of sloping side

$$
\begin{aligned}
& \frac{b+2 n d}{=d n^{2}+1^{2}} \\
& \mathrm{n}=\frac{1}{\overline{3}} \\
& \frac{b+2^{1} d \frac{-}{3^{2}}}{2}=d\left(\quad \overline{\frac{1}{2}^{2}}+1\right. \\
& \mathrm{b}=\frac{2 d}{3}
\end{aligned}
$$

Area of trapezoidal section, $A=(b+2 n d) d$

$$
\mathrm{A}=\mathrm{d}^{2} 3^{-}
$$

Hydraulic mean depth for most economical section,
www:\$inils.

problem 5

A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 1500 . The area of the section is $40 \mathrm{~m}^{2}$. Find the dimensions of the section if it is more economical. Determine the discharge of the most economical section if $\mathrm{C}=50$

## Given:

$$
\begin{aligned}
& \mathrm{c}=50 \\
& \mathrm{i}=\frac{1}{1500}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{m}=\frac{d}{2} \\
& \mathrm{Q}=\mathrm{AC} \overline{\mathrm{mi}} \\
& \text { where } c={ }^{1} \underset{N}{m / 6} \\
& \mathrm{Q}=\mathrm{d}^{2} 3 \times^{1} \frac{m}{N}^{1 / 6} \times m \times \overline{\frac{1}{2500}} \\
& 14=\mathrm{d}^{2} 3 \times \frac{1}{0.02} \times \frac{d^{1 / 6}}{2} \times{ }^{d} \times \overline{-\frac{1}{2500}} \\
& \mathrm{~d}=2.605 \mathrm{~m}
\end{aligned}
$$

$$
\mathrm{A}=40 \mathrm{~m}^{2}
$$

Side slope, $n=1 / 2$

## Solution

For the most economical section

$$
\begin{aligned}
& \frac{b+2 n d}{=d n^{2}+1} \\
& \frac{2}{b+22^{1} d}=d\left(2 \quad \frac{1}{2}\right)^{24} \\
& b=2 \times 1.118 \mathrm{~d}-\mathrm{d} \\
& =1.236 \mathrm{~d}
\end{aligned}
$$

Area of trapezoidal section

$$
A=\stackrel{b+(b+2 n d)}{2} \times d
$$

## WWW <br> 

$\mathrm{d}=4.80 \mathrm{~m}$

$$
\mathrm{b}=1.236 \mathrm{~d}
$$

$$
\mathrm{b}=5.933 \mathrm{~m}
$$

Discharge for most economical section $m=d / 2$

$$
\begin{aligned}
\mathrm{Q} & =\mathrm{AC} \overline{\mathrm{mi}} \\
& =40 \times 50 \overline{24 \times \frac{1}{1500}} \\
\mathrm{Q} & =80 \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

## problem 6

A trapezoidal channel has side slopes of 3 horizontal to 4 vertical and slope of its bed is 1 in 2000. Determine the optimum dimensions of the channel, if it is to carry water at $0.5 \mathrm{~m}^{3} / \mathrm{s}$. Take chezy's constant 80 .

## Given:

$$
\begin{aligned}
& \mathrm{c}=80 \\
& \mathrm{i}=\frac{1}{2000} \\
& \mathrm{Q}=0.5 \mathrm{~m}^{3} / \mathrm{sec}
\end{aligned}
$$



The condition for most economical section,

$$
\begin{aligned}
& \xrightarrow{b+2 n d}=d n^{2}+12 \\
& \stackrel{\begin{array}{l}
b+2-3 \\
4 \\
4
\end{array}=d\left(2 \quad \frac{3}{4}\right)^{2+1}}{ } \\
& \mathrm{~b}=2 \times 1.25-1.5 \mathrm{~d}=\mathrm{d} \\
& \mathrm{~b}=\mathrm{d} \\
& \mathrm{Q}=\mathrm{AC} \overline{\mathrm{mi}} \\
& 0.5=\mathrm{A} \times 80 \frac{\overline{d \times x}}{\frac{1}{2}} \frac{1}{2000}
\end{aligned}
$$

Area of trapezoidal section

$$
\begin{aligned}
& \mathrm{A}=(\mathrm{b}+\mathrm{nd}) \times \mathrm{d} \\
& \mathrm{~A}=d+{ }^{3} d_{4}^{-d} \\
& \mathrm{~A}=1.75 \mathrm{~d}^{2} \\
& 0.5=1.75 \mathrm{~d}^{2} \times 80 \frac{\sqrt{d_{\underline{\mathbf{x}}}^{2}} \frac{1}{2000}}{} \\
& \mathbf{d}=\mathbf{0 . 5 5} \mathbf{~ m} \\
& \mathbf{b}=\mathbf{0 . 5 5} \mathbf{~ m}
\end{aligned}
$$

### 1.8 CRITICAL FLOW

The critical state of flow has been defined as the condition for which the Froude number is equal to unity. A more common definition is that it is the state of flow at which the specific energy is a minimum for a given discharge.

When the depth of flow of water over a certain reach of a given channel is equal to the critical depth yc, the flow is called critical flow.A

Critical slope is a slope such that normal flow occurs with Froude number, $\mathrm{F}=1$. The smallest critical slope for a specified channel shape, discharge and roughness is termed as limiting slope. Furthermore, by adjusting the slope and discharge, critical uniform flow may be obtained at the given normal depth Scn.

The equation for specific energy in channel of small slope with $\alpha=1$, may be written

$$
E=y+\frac{Q^{2}}{2 g A^{2}}
$$

Differentiating with respect to y and noting that Q is constant,

$$
L_{d y}^{d E}=1 \frac{Q^{2}}{g A^{3}} \quad \frac{d A}{d y}=1-\frac{v^{2}}{g A} \frac{d A}{d y}
$$

dA can be written as Tdy . Therefore $\mathrm{dA} / \mathrm{dy}=\mathrm{T}$ and the hydraulic depth $\mathrm{D}=\mathrm{A} / \mathrm{T}$ so the above equation becomes

$$
\begin{gathered}
\frac{d E}{d y}=1-\frac{v^{2} T}{g A}=1-\frac{v^{2}}{g D} \\
\frac{v^{2} T}{g A}
\end{gathered}
$$

At the critical state of flow the specific energy is a minimum or $\mathrm{dE} / \mathrm{dy}=0$. The above equation, therefore, gives

$$
\frac{Q^{2} T}{g A^{3}}=1
$$

At critical state of flow, velocity head is equal to half hydraulic depth. $\neg$ A flow at or near the critical state is unstable. This is because a minor change in specific energy at or close to critical state will cause a major change in depth.

## DEFINITION

An open channel is a natural or a man made structure in which liquid flows with a free surface at atmospheric pressure.For example, flow in rivers, streams, flow in sanitary and storm sewers flowing partially full. The section of a channel may be uniform or non-uniform. For example, a canal, a sewer, an aqueduct, etc. are channels of uniform section, while rivers and streams are channels of non-uniform section.Open channels may be either natural channels or man-made channels. All water ways formed by natural causes are natural channels. Channels constructed for various purposes are artificial channels. For example, canals, flumes, culverts etc. are artificial channels.Rectangular, trapezoidal and circular sections are the usually adopted sections for channels. Though for some special reasons other geometrical sections may be used. For sewers for example, oval or egg shaped sections are often selected, because in these cases there are large fluctuations in the rate of discharge.

DIFFERENCE BETWEEN PIPE FLOW AND OPEN CHANNEL FLOW

| BASIS OF COMPARISON | PIPE FLOW CHANNEL | OPEN FLOW CHANNEL |
| :---: | :--- | :--- |
| Description | The pipe flow is a type of flow <br> within a closed conduit. | Open channel is a type of fluid <br> flow in a conduit with a free <br> surface open to the <br> atmosphere. |
| Nature | Pipe flow is confined within a <br> closed conduit; therefore it's <br> subjected to atmospheric <br> pressure but hydraulic | Open channel flow has a free <br> surface and it's only subjected <br> to atmospheric pressure. |


|  | pressure. |  |
| :---: | :---: | :---: |
| Flow | Flow occurs due to difference in pressure. | Flow occurs due to gravity. |
| Velocity | The maximum velocity occurs at the center of the pipe. | The maximum velocity occurs at a little distance below the water surface. |
| Cross-section | Cross section of pipe flow is generally round or circular. | Cross section of open channel can be trapezoidal, triangular, rectangular, circular etc. |
| Hydraulic Gradient Line <br> (HGL) | Hydraulic Gradient Line (HGL) do not coincide top surface of the water. | Hydraulic Gradient Line (HGL) coincides with water surface line. |
| Surface Roughness | Surface roughness varies with the type of pipe material. | Surface roughness varies with depth of flow. |

## UNIT I UNIFORM FLOW

## PROPERTIES OF OPEN CHANNEL

## Artificial channels

These are channels made by man. They include irrigation canals, navigation canals, spillways, sewers, culverts and drainage ditches. They are usually constructed in a regular cross-section shape throughout - and are thus prismatic channels (they don't widen or get narrower along the channel.

In the field they are commonly constructed of concrete, steel or earth and have the surface roughness' reasonably well defined (although this may change with age - particularly grass lined channels.) Analysis of flow in such well defined channels will give reasonably accurate results.

## Natural channels

Natural channels can be very different. They are not regular nor prismatic and their materials of construction can vary widely (although they are mainly of earth this can possess many different properties.) The surface roughness will often change with time distance and even elevation. Consequently it becomes more difficult to accurately analyse and obtain satisfactory results for natural channels than is does for man made ones. The situation may be further complicated if the boundary is not fixed i.e. erosion and deposition of sediments.

Geometric properties necessary for analysis
For analysis various geometric properties of the channel cross-sections are required. For artificial channels these can usually be defined using simple algebraic equations given y the depth of flow.

|  | Rectangle | Trapezoid | Circle |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| Area, A | $b y$ | $(b+x y) y$ | $\frac{1}{\frac{1}{2}(\phi-\sin \phi) D^{2}}$ |
| Wetted perimeter P | $b+2 y$ | $b+2 y \sqrt{1+x^{2}}$ | $\frac{1}{2} \phi D$ |
| Top width B | $b$ | $b+2 x y$ | $(\sin \phi / 2) D$ |


| Hydraulic radius R | $b y /(b+2 y)$ | $\frac{(b+x y) y}{b+2 y \sqrt{1+x^{2}}}$ | $\frac{1}{4}\left(1-\frac{\sin \phi}{\phi}\right) D$ |
| :---: | :---: | :---: | :---: |
| Hydraulic mean <br> depth $\mathrm{D}_{\mathrm{m}}$ | $y$ | $\frac{(b+x y) y}{b+2 x y}$ | $\frac{1}{8}\left(\frac{\phi-\sin \phi}{\sin (1 / 2 \phi)}\right) D$ |

Depth(y)-the vertical distance from the lowest point of the channel section to the free surface.
Stage (z) - the vertical distance from the free surface to an arbitrary datum
Area (A) - the cross-sectional area of flow, normal to the direction of flow
Wetted perimeter $(\mathrm{P})$ - the length of the wetted surface measured normal to the direction of flow.
Surface width (B) - width of the channel section at the free surface
Hydraulic radius (R) - the ratio of area to wetted perimeter ( $\mathrm{A} / \mathrm{P}$ )
Hydraulic mean depth (Dm) - the ratio of area to surface width ( $\mathrm{A} / \mathrm{B}$ )

## VELOCITY DISTRIBUTION IN OPEN CHANNEL

An open channel is a conduit which has free water surface exposed to the atmosphere. Rivers, canals etc come under open channel category. Because of free water surface and frictional resistance along the channel boundary velocity distribution is non-uniform in open channels. To measure velocity of open channel at required depth, Pitot tube or current meter are used. In general, to find average velocity of a particular open channel, velocity at a depth of 0.6 m from free water surface is measured. In the other case, velocity at 0.2 m depth, 0.8 m depth from free water surface is taken and average velocity of these two values is considered as channel average velocity.

### 1.7 SPECIFIC ENERGY AND SPECIFIC FORCE

The total energy of a channel flow referred to datum is given by,

$$
\mathrm{H}=\mathrm{z}+\mathrm{y}+{ }_{2 \mathrm{~g}} \frac{\mathrm{v}^{2}}{-}
$$

If the datum coincides with the channel bed at the cross-section, the resulting expression is know as specific energy and is denoted by E. Thus, specific energy is the energy at a cross-section of an open channel flow with respect to the channel bed. The concept of specific energy, introduced by Bakmeteff, is very useful in defining critical water depth and in the analysis of open channel flow. It may be noted that while the total energy in a real fluid flow always decreases in the downstream direction, the specific energy is constant for a uniform flow and can either decrease or increase in a varied flow, since the elevation of the bed of the channel relative to the elevation of the energy line, determines the specific energy.

Specific energy at a cross-section is,

$$
E=y+\frac{v^{2}}{2 g}=y+\frac{Q^{2}}{2 g A^{2}}
$$

Here, cross-sectional area A depends on water depth y and can be defined as, $\mathrm{A}=\mathrm{A}(\mathrm{y})$. show us that, there is a functional relation between the three variables as,

$$
f(E, y, Q)=0
$$

In order to examine the functional relationship on the plane, two cases are introduced

1. $\mathrm{Q}=$ Constant $=\mathrm{Q} 1 \rightarrow \mathrm{E}=\mathrm{f}(\mathrm{y}, \mathrm{Q} 1)$.

Variation of the specific energy with the water depth at a cross-section for a given discharge Q1.
2. $\mathrm{E}=$ Constant $=\mathrm{E} 1 \rightarrow \mathrm{E} 1=\mathrm{f}(\mathrm{y}, \mathrm{Q})$ Variation of the discharge with the water depth at acrosssection for a given specific energy E1.

## Constant Discharge Situation

Since the specific energy,

$$
\mathrm{E}=\mathrm{y}+\mathrm{v}^{2} \frac{\mathrm{E}}{2 \mathrm{~g}} \mathrm{y}+\mathrm{Q}^{2} \frac{}{2 g A^{2}}
$$



For a channel of known geometry, $\mathrm{E}=\mathrm{f}(\mathrm{y}, \mathrm{Q})$. Keeping $\mathrm{Q}=$ constant $=\mathrm{Q} 1$, the variation of E with $y$ is represented by a cubic parabola. (Figure 5.1). It is seen that there are two positive roots for the equation E indicating that any particular discharge Q1 can be passed in a given channel at two depths and still maintain the same specific energy E1. The depths of flow can be either $\mathrm{PR}=$ y 1 or $\mathrm{PR}=\mathrm{y}^{`} 1$. These two possible depths having the same specific energy are known as alternate depths. In Fig. (5.1), a line (OS) drawn such that $\mathrm{E}=\mathrm{y}$ (i.e. at 450 to the abscissa) is the asymptote of the upper limb of the specific energy curve. It may be noticed that the intercept $P^{\prime} R$ and $P^{\prime} R$ represents the velocity head. Of the two alternate depths, one $(P R=y 1)$ is smaller and has a large velocity head while the other $\left(\mathrm{PR}^{\prime}=\mathrm{y}^{\prime} 1\right)$ has a larger depth and consequently a smaller velocity head. For a given Q , as the specific energy is increased the difference between the two alternate depths increases. On the other hand, if $E$ is decreased, the difference ( $\mathrm{y}^{`} 1-\mathrm{y} 1$ ) will decrease and a certain value $\mathrm{E}=\mathrm{Ec}$, the two depths will merge with each other (point C in Fig. 5.1). No value for y can be obtained when $\mathrm{E}<\mathrm{Ec}$, denoting that the flow under the given conditions is not possible in this region. The condition of minimum specific energy is known as the critical flow condition and the corresponding depth yc is known as critical depth.

## problem 1

Calculate the Specific energy,Critical depth and the velocity of the flow of $10 \mathrm{~m}^{3}$ in a cement lined rectangular channel 2.5 m wide with 2 m depth of water. Is the given flowis sub critical or super critical

## Given Data

$$
\begin{aligned}
& \mathrm{Q}=10 \mathrm{~m} 3 / \mathrm{s} \\
& \mathrm{~b}=2.5 \mathrm{~m} \\
& \mathrm{y}=2 \mathrm{~m}
\end{aligned}
$$

## To find

1. Specific Energy
2. Critical Depth
3. Velocity for the flow


$$
\begin{aligned}
& E=y+\frac{v^{2}}{2} \\
& V=\frac{Q}{A} \quad, V=\frac{10}{102.5} \\
& v=2 \mathrm{~m} / \mathrm{s} \\
& E=2+\frac{2^{2}}{2 \times 9.81} \\
& E=2.20 \mathrm{~m}
\end{aligned}
$$

## STEP 2 : Critical Depth

$$
\begin{aligned}
& y_{c}={ }_{g}^{q^{2}}{ }^{1 / 3} \\
& \mathrm{q}=\frac{Q}{b} \\
& \mathrm{q}=\frac{10}{2.5}=5 \mathrm{~m}^{2} / \mathrm{sec}
\end{aligned}
$$

$$
\begin{gathered}
y=5^{2}{ }^{9.81} 1 / 3 \\
y_{c}= \\
1.18 \mathrm{~m}
\end{gathered}
$$

## STEP 3: Velocity of flow

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{c}}=\mathrm{y}_{\mathrm{c}} \overline{\times \mathrm{g}} \\
& \mathrm{v}_{\mathrm{c}}=1.18 \times 9.81 \\
& \mathrm{v}_{\mathrm{c}}=3.4 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

STEP 4: To find weather the flow is Sub critical of Super critical

$$
\begin{aligned}
\mathrm{F} & =\frac{v}{g \times D} \\
& =\frac{2}{9.81 \times 2} \\
& =0.45<1.0
\end{aligned}
$$

Hence the flow is Sub critical

## WWW binils com

### 1.4 STEADY UNIFORM FLOW: CHEZY EQUATION, MANNING EQUATION



When uniform flow occurs gravitational forces exactly balance the frictional resistance forces which apply as a shear force along the boundary (channel bed and walls).

Considering the above diagram, the gravity force resolved in the direction of flow is Gravity force $=\rho \mathrm{gAL} \sin \theta$
boundary shear force resolved in the direction of flow is

$$
\text { shear force }=\tau_{0} \mathrm{PL}
$$

In uniform flow these balance

$$
\tau_{0} \mathrm{PL}=\rho \mathrm{gAL} \sin \theta
$$

Considering a channel of small slope, (as channel slopes for unifor and gradually varied flow seldom exceed about 1 in 50) then

## 1.The Chezy equation

$$
\sin q \sim \operatorname{tanq}=S_{o}
$$

If an estimate of $\tau_{0}$ can be made then we can make use of Equation.
If we assume the state of rough turbulent flow then we can also make the assumption the shear force is proportional to the flow velocity squared i.e.

$$
\begin{gathered}
\tau_{\mathrm{o}} \alpha \mathrm{~V}^{2} \\
\tau_{\mathrm{o}}=\mathrm{KV}^{2}
\end{gathered}
$$

Substituting this into equation gives

$$
V=\sqrt{\frac{\rho g}{K} R S_{0}}
$$

Or grouping the constants together as one equal to C

$$
V=C \sqrt{R S_{0}}
$$

This is the Chezy equation and the C the 'Chezy $\mathrm{C}^{\prime}$

## The Manning equation

A very many studies have been made of the evaluation of C for different naturaland manmade channels.

$$
c=\frac{R^{i \cdot 6}}{n}
$$

## Problem 1

Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide and 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take chezy's constant $\mathrm{C}=55$.

## Given:

Width of rectangle channel, $b=6 \mathrm{~m}$.
Depth d $=3 \mathrm{~m}$
Bed Slope, $i=1$ in $2000=1 / 2000$


Area $=b \times d=6 \times 3=18 \mathrm{~m}^{2}$
Perimeter $\mathrm{P}=\mathrm{b}+2 \mathrm{~d}=6+2 \times 3=12 \mathrm{~m}$
Hydraulic mean depth, $\mathrm{m}=\mathrm{A} / \mathrm{P}=18 / 12=1.5 \mathrm{~m}$
$\mathrm{V}=\mathrm{C} \overline{\mathrm{m}=55} \times 15 \times 1 / 2000=1.506 \mathrm{~m} / \mathrm{s} \mathrm{Q}=\mathrm{V} \times$
Area $=1.506 \times 18=\mathbf{2 7 . 1 0 8 m} \mathbf{m}^{3} / \mathbf{s}$.

## Problem 2

Find the slope of the bed of a rectangular channel 5 m when depth of water is 2 m and rate of flow is given as $20 \mathrm{~m}^{3} / \mathrm{s}$. Take chezy's constant, $\mathrm{C}=50$.

## Given:

Width of channel $b=5 \mathrm{~m}$.

Depth of water $\mathrm{d}=2 \mathrm{~m}$
Rate of flow $\mathrm{Q}=20 \mathrm{~m} 3 / \mathrm{s}$.
$\mathrm{C}=50$
Bed Slope $=\mathrm{i}$

## Solution:

$$
\begin{aligned}
& \mathrm{Q}=\mathrm{AC} \overline{\mathrm{mi}} \\
& \mathrm{~A}=\text { Area }=\mathrm{b} \times \mathrm{d}=5 \times 2=10 \mathrm{~m}^{2}
\end{aligned}
$$

$$
\text { Perimeter } \mathrm{P}=\mathrm{b}+2 \mathrm{~d}=5+2 \times 2=9 \mathrm{~m}
$$

Hydraulic mean depth, $\mathrm{m}=\mathrm{A} / \mathrm{P}$

$$
=10 / 9 \mathrm{~m}
$$



## Problem 3

Find the discharge through a trapezoidal channel of width 8 m and side slope of 1 horizontal to 3 vertical. The depth of flow of water is 2.4 m and value of Chezy's constant, $\mathrm{C}=50$. The slope of the bed of the channel is given 1 in 4000.

## Given:

Width $b=8 \mathrm{~m}$

Side Slope $=1$ horizontal to 3 vertical

Depth d $=2.4 \mathrm{~m}$

Chezy's constant $C=50$, Bed Slope $I=1 / 4000$


## Solution

Horizontal distance BE $=2.4 \times 1 / 3=0.8 \mathrm{~m}$

Therefore Top Width of the channel,
$\mathrm{CD}=\mathrm{AB}+2 \times \mathrm{BE}=8.0+2 \times 0.8=9.6 \mathrm{~m}$

Therefore Area of trapezoidal channel, ABCD is given as,
$\mathrm{A}=(\mathrm{AB}+\mathrm{CD}) \times \mathrm{CE} / 2=(8+9.6) \times 2.4 / 2=17.6 \times 1.2=21.12 \mathrm{~m}^{2}$
1 Wetted Perimeter, $P=A B+B C+A D=A B=2 B C$

$$
\begin{gathered}
=(0.8)^{2}+(2.4)^{2}=2.529 \\
\mathrm{~m} \\
\mathrm{P}=8+2 \times 2.529=13.058 \mathrm{~m}
\end{gathered}
$$

Hydraulic mean depth $\mathrm{m}=\mathrm{A} / \mathrm{P}$

$$
\begin{aligned}
& =12.12 / 13.058=1.617 \mathrm{~m} \\
& \begin{aligned}
\mathrm{Q}= & \mathrm{AC} \overline{\mathrm{ml}} \\
& =21.12 \times 501.617 \times 1 / 4000 \\
= & \mathbf{2 1 . 2 3} \mathbf{~ m}^{3} / \mathbf{s} .
\end{aligned}
\end{aligned}
$$

## Problem 4

Find the bed slope of trapezoidal channel of bed width 6 m , depth of water 3 m and side slope of 3 horizontal to 4 vertical, when the discharge through the channel is $30 \mathrm{~m}^{3} / \mathrm{s}$. Take Chezy's Constant, $\mathrm{C}=70$

## Given:

Bed width, $b=6.0 \mathrm{~m}$
depth of flow, $\mathrm{d}=3.0 \mathrm{~m}$
side slope $=3$ horizontal to 4 vertical
discharge $\mathrm{q}=30 \mathrm{~m}^{3} / \mathrm{s}$

Chezy s Constant $=70$

## Solution <br> Depth of flow $\mathrm{CE}=3 \mathrm{~m}$ <br> $\mathrm{BE}=3 \times 3 / 4=2.25 \mathrm{~m}$

Therefore Top Width, $\mathrm{CD}=\mathrm{AB}+2 \times \mathrm{BE}$

$$
=6.0+2 \times 2.25=10.50 \mathrm{~m}
$$

$$
\text { Wetted Primeter, } \begin{aligned}
\mathrm{P} & =\mathrm{AD}+\mathrm{AB}+\mathrm{BC} \\
& =\mathrm{AB}+2 \mathrm{ABC}(\ldots \mathrm{BC}=\mathrm{AD}) \\
& =\mathrm{AB}+2 B E^{2}+C E^{2} \\
& =6.0+2225^{2}+3^{2}=13.5 \mathrm{~m}
\end{aligned}
$$



$$
\mathrm{A}=\text { Area of trapezoidal } \mathrm{ABCD}
$$

$$
\begin{aligned}
& =(\mathrm{AB}+\mathrm{CD}) \times \mathrm{CE} / 2 \\
& =(6+10.50) / 2 \times 3=24.75 \mathrm{~m}^{2}
\end{aligned}
$$

Hydraulic mean depth, $\mathrm{m}=\mathrm{A} / \mathrm{P}=24.75 / 13.50=1.833$

$$
\begin{gathered}
\mathrm{Q}=\mathrm{AC} \overline{\mathrm{mi}} \\
30.0=24.75 \times 70 \quad 1833 \times \mathrm{l}=2345.6 \quad \\
\mathrm{i}=(30 / 2345.6)^{2}=1 /(2345.6 / 30)^{2}=1 / 6133 \\
\mathbf{i}=\mathbf{1} / \mathbf{6 1 3 3}
\end{gathered}
$$

## problem 5

Find the discharge of water through the channel shown in the fig. Take thevalue of Chezy's constant $=60$ and slope of the bed as 1 in 2000

## Given:

Chezy s Constant C $=60$

Solution:



$$
\begin{aligned}
\mathrm{A} & =\text { Area } \mathrm{ABCD}+\text { Area BEC } \\
& =(1.2 \times 3.0)+\pi \mathrm{R}^{2} / 2 \\
= & 3.6+(1.5)^{2} \pi / 2=7.134 \mathrm{~m}^{2}
\end{aligned}
$$

Wetted Perimeter, $\mathrm{P}=\mathrm{AB}+\mathrm{BEC}+\mathrm{CD}$

$$
\begin{aligned}
& =1.2+\pi \mathrm{R}+1.2=1.2+\pi 1.5+1.2 \\
& =7.1124 \mathrm{~m}
\end{aligned}
$$

Hydraulic mean depth, $\mathrm{m}=\mathrm{A} / \mathrm{P}$

$$
=7.134 / 7.1124=1.003
$$

$$
\begin{aligned}
\mathrm{Q} & =\mathrm{AC} \overline{\mathrm{ml}} \\
& =7.134 \times 60 \times 1.003 \times \quad \frac{1}{2000} \\
& =\mathbf{9 . 5 8 5} \mathbf{~ m}^{3} / \mathbf{s}
\end{aligned}
$$

## problem 6

Find the rate of flow of water through a V- Shaped channel as shown in the fig. Take the value of $\mathrm{C}=55$ and slope of the bed 1 in 2000

## Given:

$$
C=55
$$

Bed Slope $\mathrm{i}=1 / 1000$

Depth of flow, $\mathrm{d}=4.0 \mathrm{~m}$

Angle made by each side with vertical i.e $\angle \mathrm{ABD}=\angle \mathrm{CBD}=30^{\circ}$

## Solution:

$$
\begin{aligned}
& \text { Area } \begin{aligned}
\mathrm{A} & =\text { Area of } \mathrm{ABC} \\
& =2 \times \mathrm{Area} \text { of } \mathrm{ABCD}=(2 \times \mathrm{AD} \times \mathrm{BD}) / 2=\mathrm{AD} \times \mathrm{BD} \\
& =\mathrm{BD} \tan 3^{\circ} 0 \times \mathrm{BD} \\
& =4 \tan 30^{\circ} \times 4=9.2376 \mathrm{~m}^{2} \\
& \text { Wetted Perimeter, } \mathrm{P}=\mathrm{AB}+\mathrm{BC}=2 \mathrm{AB} \\
& =2 B D^{2}+A D^{2} \\
& =2 \frac{4^{2}+(4 \tan 30)^{2}}{}
\end{aligned} . \begin{array}{l} 
\\
\end{array} \\
& \\
& \\
&
\end{aligned}
$$

$$
=2(\overline{16.0+5.333)=9.2375} \mathrm{~m}
$$

Hydraulic mean depth, $\mathrm{m}=\mathrm{A} / \mathrm{P}$

$$
=9.2376 / 9.2375=1.0 \mathrm{~m}
$$

$$
\begin{aligned}
Q & =A C \overline{m l} \\
& =9.2376 \times 55(1 * \overline{1 / 1000}) \\
& =\mathbf{1 6 . 0 6 6} \mathbf{m} \mathbf{m}^{3} \mathbf{s}
\end{aligned}
$$

## TYPES OF FLOW

## 1. Steady and Unsteady flows:

The steady flow is defined as that type of flow in which the fluid characteristics like velocity, density, pressure, etc at a point do not change with the time.

The Unsteady flow is defined as that type of flow in which the fluid characteristics like velocity, density, pressure, etc at a point change respected to time.

## 2.Uniform and Non-uniform fluid flow:

This uniform fluid flow is defined as the type of flow in which the velocity at any given time does not change with respect to space (i.e length of direction of the flow).

This non-uniform fluid flow is defined as the type of flow in which the velocity at any given time changes with respect to space (i.e length of the direction of the flow).

## 3. Laminar, and Turbulent fluid flow:

This laminar fluid flow is defined as the type of flow in which the fluid particles move along well-defined paths or streamline and all the streamlines are straight and parallel.

Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of fluid is also called as streamline flow or viscous flow.

Turbulent fluid flow is defined as the type of flow in which the fluid particles move in a zigzag way, the eddies formation takes place which is responsible for high energy loss.

For pipe flow, The type of flow is determined by a non-dimensional number [(VD) / (v) nuo] called the Reynolds number.

Where,

- $\mathrm{D}=$ Diameter of pipe
- $\mathrm{V}=$ Mean velocity flow in a pipe
- $\quad v=$ Kinematic viscosity of the fluid.
- If the Reynold Number is less than 2000, the flow is called Laminar flow.
- Reynold Number is more than 4000 , the flow is called Turbulent flow.
- If the Reynold Number is lies between 2000-4000, the flow may be laminar or turbulent.


## 4. compressible and Incompressible fluid flow:

Compressible fluid flow is defined as the flow in which the density is not constant which means the density of the fluid changes from point to point.

Incompressible fluid flow is defined as the flow in which the density is constant which means the density of the fluid does not change from point to point.

## 5. Rotational and irrotational Fluid flow:

The rotational fluid flow is defined as the type of fluid flow in which the fluid particles while flowing along streamline and also rotate about there own axis.

Whereas, The Ir-rotational fluid flow is defined as the type of fluid flow in which the fluid particles while flowing along streamline and do not rotate about there own axis.
6. One, Two and Three-dimensional fluid Flow:

One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say $x$.

$$
u=f(x), v=0 \text { and } w=0
$$

Where $\mathrm{u}, \mathrm{v}$ and w are velocity component in $\mathrm{x}, \mathrm{y}$ and z directions respectively.
Two-dimensional fluid flow is the type of flow in which velocity is a function of time and two rectangular space co-ordinate say $\mathrm{x}, \mathrm{y}$.

$$
\mathbf{u}=\mathbf{f}_{1}(\mathbf{x}, \mathbf{y},), \mathbf{v}=\mathbf{f}_{2}(\mathbf{x}, \mathbf{y},) \text { and } \mathbf{w}=\mathbf{0} .
$$

Three-dimensional fluid flow is the type of flow in which velocity is a function of time and three mutually perpendicular directions. The function of 3 space coordinates ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ).

$$
\mathbf{u}=\mathbf{f}_{1}(\mathbf{x}, \mathbf{y}, \mathbf{z}), \mathbf{v}=\mathbf{f}_{\mathbf{2}}(\mathbf{x}, \mathbf{y}, \mathbf{z}) \text { and } \mathbf{w}=\mathbf{f}_{\mathbf{3}}(\mathbf{x}, \mathbf{y}, \mathbf{z}) .
$$

### 1.6 WIDE-OPEN CHANNELS

- In very wide-open channels the velocity distribution in the central region of the section is essentially the same as it would be in a rectangular channel of infinite width.
- In wide-open channels, the sides of the channel have practically no influence on the velocity distribution in the central region
- A wide-open channel can safely be defined as a rectangular channel whose width is greater than 10 times the depth of flow
- For either experimental or analytical purposes, the flow in the central region of a wideopen channel may be considered the same as the flow in a rectangular channel of infinite width.
- ${ }^{\mathrm{H}}$
- Consider a rectangular channel; for a rectangular channel we know that

$$
R=\frac{A}{\overline{\bar{P}}} \frac{B y}{2 Y+B}
$$

For a wide rectangular channel, the denominator $p=B+2 Y \approx B$ as y is very small. $R=\frac{B y}{B}=y$

