4.2 Compton Effect

When a beam of monochromatic radiation such as x-rays, γ-rays etc., of light frequency is allowed to fall on a small particle then the beam is scattered into two

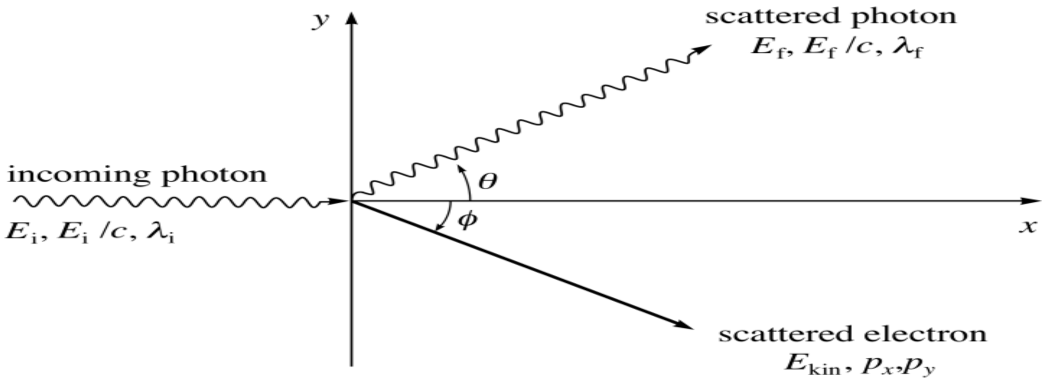


Fig :4.2.1Compton Effect

components. One component has the same wavelength as that of the incident radiation and the other component has a slightly longer wavelength.

This effect of scattering is called Compton Effect. The shift in wavelength is called Compton Shift.

Theory of Compton Effect

Principle

In Compton scattering the collision between a photon and an electron is considered. Then by applying the laws of conservation of energy and momentum, the expression for Compton wavelength is derived.

Assumptions

The collision occurs between the photon and an electron in the scattering material.

The electron is free and is at rest before collision with in incident photon.

With these assumptions let us consider a photon of energy 'hν' colliding with an electron at rest. During the collision process, a part of energy is given to the electron, which in turn increases the kinetic energy of the electron and hence it recoils at an angle of ϕ as shown in figure. The scattered photon moves with an energy hν' (less than hν), at an angle θ with respect to the original direction.

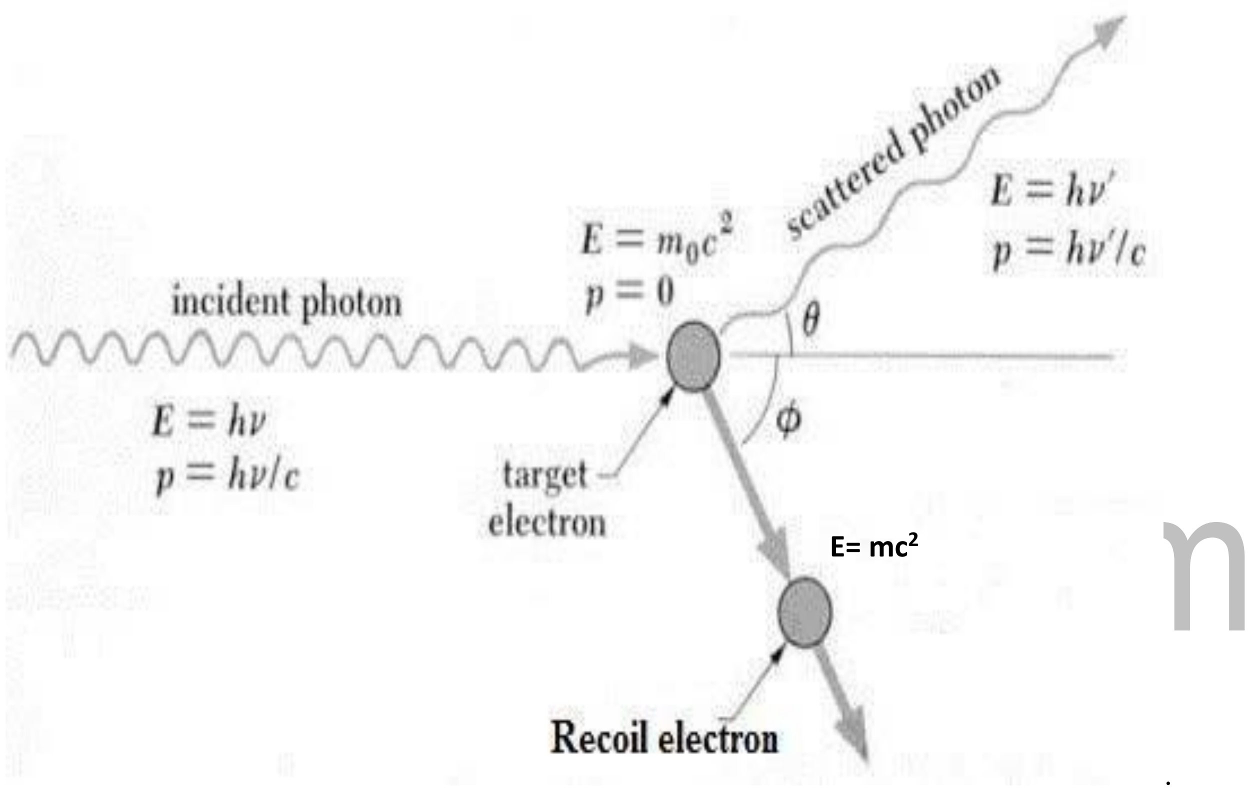


Fig : 4.2.2Compton Scattering

Let us find the energy and momentum components before and after collision process.

Energy before collision

Energy of the incident photon = hν

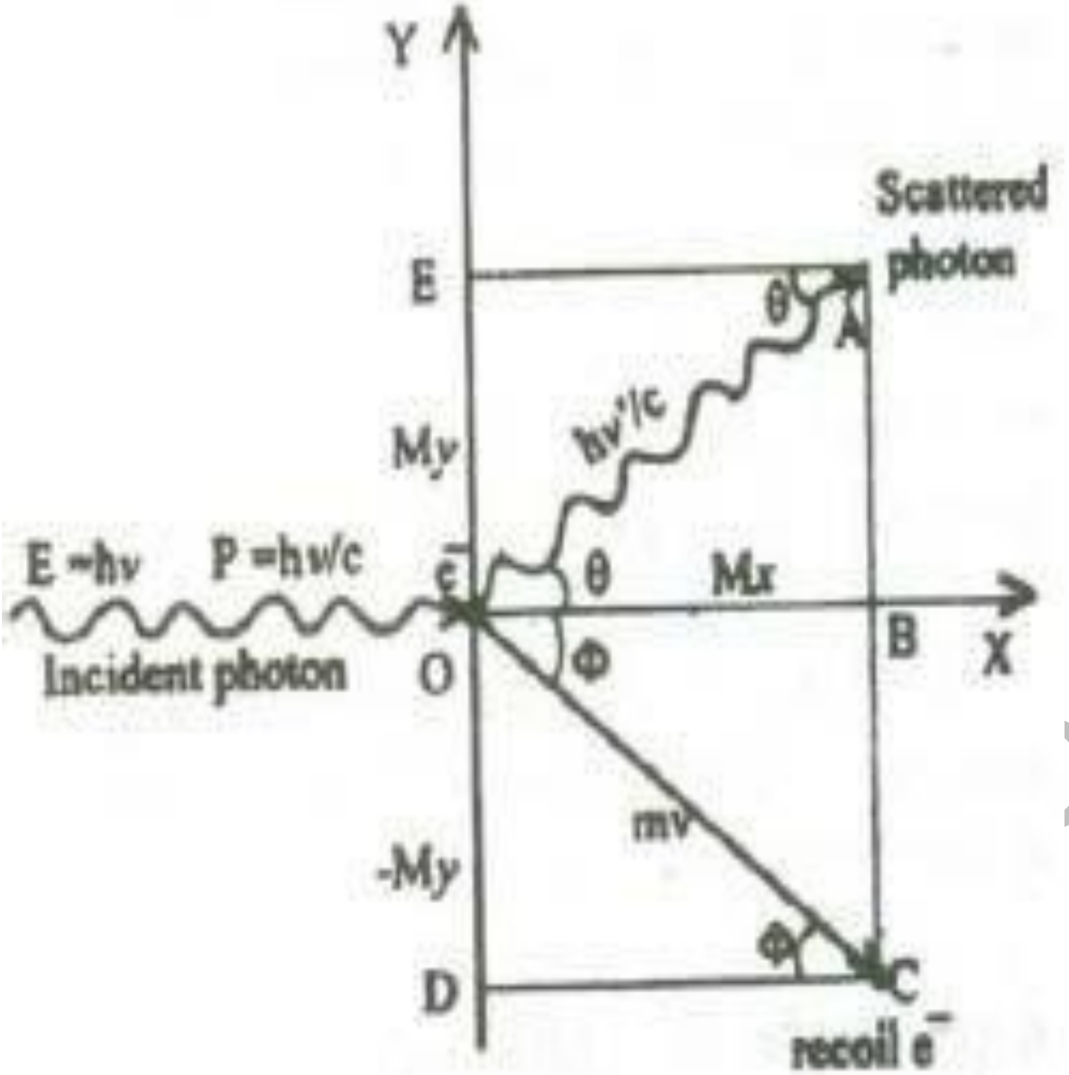
Energy of the electron at rest = m_0c^2

Where, m_0 is the rest mass energy of the electron.

Total Energy before Collision = $h\nu + m_0c^2$ ....(1)

Energy after collision

Energy of the scattered photon = $h\nu'$



Energy of the recoil electron = mc^2

Where, m is the mass of the electron moving with velocity v'

Total energy after collision = $h\nu' + mc^2$ ----------------------**(2)**

According to the **law of conservation of energy**

Total energy before collision = Total energy after collision

Equation (1) = Equation (2)

$$h\nu + m_0c^2 = h\nu' + mc^2 \quad \text{..........................(3)}$$

X – Component of Momentum before Collision

X –component momentum of the incident photon $= \frac{hv}{c}$

X – component momentum of the electron at rest $= 0$

Total X – component of momentum before collision is $= \frac{hv}{c}$ ------- (4)

x- Component of momentum after collision

X- Component momentum of the scattered photon can be calculated from figure.

In Δ OAB

$$\cos\theta = \frac{Mx}{\frac{hv'}{c}}$$

X – Component momentum of the scattered photon is $= \frac{hv'}{c}\cos\theta$

X- Component momentum of the recoil electron can be calculated from figure.

In Δ OBC $\quad \cos\phi = \frac{M_x}{mv}$

X – Component momentum of the recoil electron is $= mv\cos\phi$

Total X- Component of momentum after collision

$$= \frac{hv'}{c}\cos\theta + mv\cos\phi \ldots\ldots\ldots\ldots\ldots (5)$$

According to the **law of conservation of momentum**

Total momentum before collision = Total momentum after collision

Equation (4) = Equation (5)

$$\frac{hv}{c} = \frac{hv'}{c}\cos\theta + mv\cos\phi \ldots\ldots\ldots\ldots (6)$$

Y – Component of Momentum before collision

Y – Component momentum of the incident photon = 0

Y – Component momentum of the electron at rest = 0

Total Y – component of momentum before collision is = 0 .... (7)

Y – Component of momentum after collision

From figure, in Δ OAE,

$$\sin\theta = \frac{M_y}{\frac{hv'}{c}}$$

Y – Component momentum of the scattered photon is = $\frac{hv'}{c}\sin\theta$

From figure, in Δ OCD,

$$\sin\phi = \frac{-M_y}{mv}$$

Y- Component momentum of the recoil electron is = - $mv\sin\phi$

Total Y-Component of momentum after collision is

$$= \frac{hv'}{c}\sin\theta - mv\sin\phi \text{-------------} \quad (8)$$

According to the law conservation of momentum,

Equation (7) = Equation (8)

$$0 = \frac{hv'}{c}\sin\theta - mv\sin\phi \text{------------} \quad (9)$$

From equation (6), we can write

$$\frac{hv}{c} = \frac{hv'}{c}\cos\theta + mv\cos\phi$$

$$mcv\cos\phi = h(v - v'\cos\theta) \text{ ....}(10)$$

From equation (9) we can write

$$mcv \sin\phi = hv' \sin\theta \text{............................ (11)}$$

Squaring and adding equation (10) and (11) we get

$$m^2c^2v^2 (\cos^2\phi + \sin^2\phi) = h^2 [v^2 - 2vv' \cos\theta + (v')^2 \cos^2\theta] + h^2 (v')^2 \sin^2\theta$$

Since , $\cos^2\phi + \sin^2\phi = 1$ and

$h^2 (v')^2[\cos^2\theta + \sin^2\theta] = h^2 (v')^2$ we get

$$m^2c^2v^2 = h^2 [v^2 - 2vv'\cos\theta + (v')^2] \text{........ (12)}$$

From equation (3), we can write

$$mc^2 = m_0c^2 + h (v - v')$$

Squaring on both sides we get

$$m^2c^4 = m_0^2c^4 + 2hm_0c^2 + h(v - v') + h^2[v^2 - 2vv' + (v')^2] \text{..... (13)}$$

Subtracting equation (12) from equation (13), we get

$$m^2c^2 (c^2 - v^2) = m_0^2c^4 + 2hm_0c^2 + h(v - v') - 2h^2vv'(1-\cos\theta) \text{................................(14)}$$

From the theory of relativity, the relativistic formula for the variation of mass with the velocity of the electron is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m^2 = \frac{m_0^2}{1 - \frac{v^2}{c^2}}$$

$$m^2 = \frac{c^2 m_0^2}{c^2 - v^2}$$

$$(c^2 - v^2)m^2 = c^2 m_0^2 \text{ .... } (15)$$

In order to make this equation similar to LHS of equation (14) multiply it by c^2 on both sides,

$$m^2 c^2 (c^2 - v^2) = m_0^2 c^4 \text{ .... } (16)$$

Equating equation (16) and (14), we can write

$$m_0^2 c^4 = m_0^2 c^4 + 2h\, m_0 c^2 + h\,(v - v') - 2h^2 v v' \,(1 - \cos\theta)$$

$$2h\, m_0 c^2 + h\,(v - v') = 2h^2 v v' \,(1 - \cos\theta)$$

$$\frac{v - v'}{v v'} = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

$$\frac{1}{v'} - \frac{1}{v} = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

Multiplying both sides by '*c*', we get

$$\frac{c}{v'} - \frac{c}{v} = \frac{h}{m_0 c^2}(1 - \cos\theta) \text{ .... } (17)$$

Since $\lambda = \frac{c}{v}$ and $\lambda' = \frac{c}{v'}$, we can write equation (17) as

$$\lambda' - \lambda = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

The change in wavelength

$$\Delta\lambda = \frac{h}{m_0 c^2}(1 - \cos\theta) \text{ .... } (18)$$

Equation (18) represents the shift in wavelength or Compton Shift, which is independent of the incident radiation as well as the nature of the scattering substance. Thus the shift in wavelength or Compton Shift depends on the angle of scattering.

Special Cases:

Case:i

When $\theta = 0$; $\cos\theta = 1$

Equation (18) becomes $\Delta\lambda = 0$

This implies that at $\theta = 0$ the scattering is absent and the out coming radiation has the same wavelength or frequency as that of the incident radiation. Thus we get the output as a single peak.

Case (ii)

When $\theta = 90^0$; $\cos\theta = 0$

Equation (18)becomes $\Delta\lambda = \frac{h}{m_0 c}$

Substituting the values of h, m_0 and c we get

$$\Delta\lambda = \frac{6.625x10^{-34}}{9.11x10^{-31}\ 3x10^8}$$

$$\Delta\lambda = 0.02424\ A^0$$

This wavelength is called *Compton wavelength*, which has a good agreement with the experimental results

Case (iii)

When $\theta = 180^0$; $\cos\theta = -1$

Equation (18) becomes $\Delta\lambda = \frac{h}{m_0 c}[1-(-1)]$

$$\Delta\lambda = \frac{2h}{m_0 c}$$

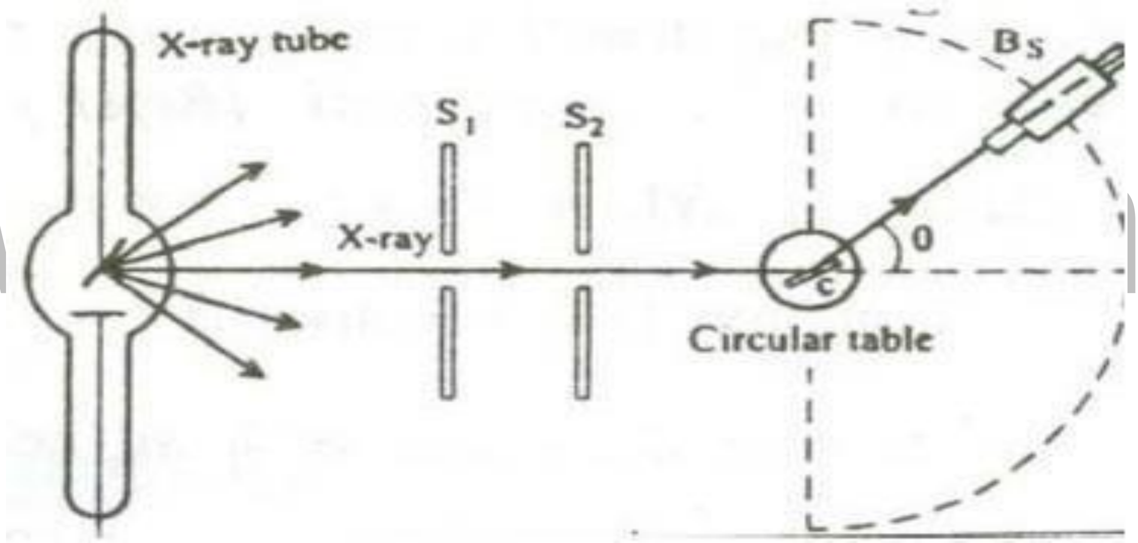
Substituting the values of h, m_0 and c we get

$$\Delta\lambda = 0.04848 \text{ A}^0$$

Thus for $\theta = 180^0$ the shift in wavelength is found to be maximum.

Experimental Verification of Compton Effect Principle:

When a beam of monochromatic radiation such as x-rays, γ-rays etc., of light frequency is allowed to fall on a small particle then the beam is scattered into two components. One component has the same wavelength as that of the incident radiation and the other component has a slightly longer wavelength. This effect of scattering is called Compton Effect. The shift in wavelength is



called Compton Shift.

Fig 4.2.3 Compton Effect

Construction

It consists of an X – ray tube for producing X – rays. A small block of carbon C (scattering element) is mounted on a circular table as shown in figure. A Bragg's spectrometer (B_s) is allowed to freely swing in an arc about the scattering element to catch the scattered photons. Slits S_1 and S_2 helps to focus the X – rays onto the scattering element.

Working

X - Rays of monochromatic wavelength 'λ' is produced from an X - rays tube and is made to pass through the slit S_1 and S_2. These X - rays are made to fall on the scattering element. The scattered X - rays are received with the help of the Bragg's spectrometer and the scattered wavelength is measured. The experiment is repeated for various scattering angles and the scattered wavelengths are measured. The experimental results are plotted.

In this figure when the scattering angle $\theta = 0^0$, the scattered radiation peak will be the same as that of the incident radiation peak 'A'. Now, when the scattering angle is increased, for one incident radiation peak A of wavelength (λ) we get two scattered peals A and B. Here the peak 'A' is found to be of same wavelength as that of the incident wavelength and the peak 'B' is of greater wavelength than the incident radiation.

The shift in wavelength or difference in wavelength (Δλ) of the two scattered beams is found to increase with respect to the increase in scattering angle. At $\theta = 90^0$, the Δλ is found to be 0.0236 = 0.02424 A^0, which has good agreement with the theoretical results. Hence this wavelength is called Compton wavelength and the shift in wavelength is called Compton shift.

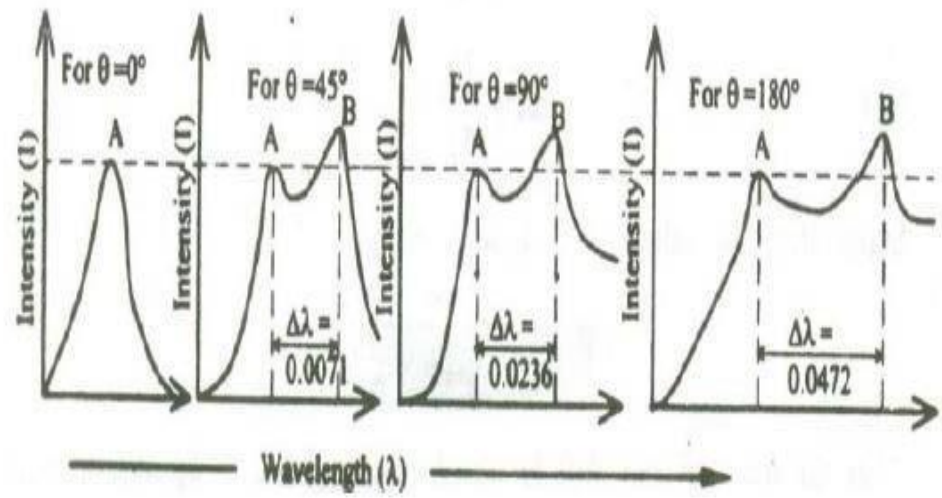


Fig:4.2.4 Compton shift

4.4 ELECTRON DIFFRACTION- G.P.Thomson Experiment

G.P.Thomson made investigations with high speed electrons, accelerated by a potential difference ranging from 10,000 to 50,000 volts and studied the electron diffraction effects.

Thomson found the diffraction patterns exactly an alogous to X-ray patterns.More over he was able to determine the wavelengths associated with electrons.

Experimental arrangement and working:

The experimental arrangement is shown in fig. It consists of a discharge tube in which the electrons are produced from the cathode C. The electrons are accelerated by potential upto 50,000 volts.

These accelerated electrons are passed through a slit S to obtain a fine beam of electrons.Then,they are allowed to fall on a very thin metallic film G of gold foil.

The thickness of the film is of the order of 10^{-6} cm.

The whole apparatus is exhausted to a high vacuum so that the electrons may not lose their energy in collision with the molecules of the gas.

The electron beam coming out of the foil is recorded the photographic plate P. After developing the plate, a symmetrical pattern of concentric rings about central spot diffraction pattern is obtained.This pattern is similar to pattern produced by x-rays.

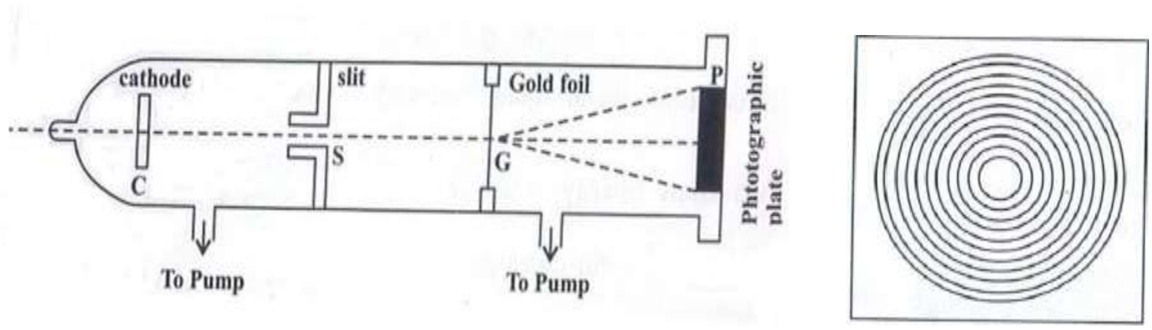


Fig-4.4.1- G.P.Thomson Experiment

The diffraction pattern can only be produced by waves and not by the particles. So Thomson concluded that electrons behaved like waves. He also calculated the associated wavelength of electrons.

It found that the wave length of electron depends only on the accelerating voltage and it is independent of the nature of the target material. Thomson's experiment led to the discovery of electron microscope.

UNIT IV

Quantum Physics

4.6.Particle in a one dimensional box

Application of Schrodinger Wave Equation to a Particle (Electron) Enclosed in a OneDimensional Potential Box

Consider a particle of mass m moving back and forth between the walls of a 1D box. Since the walls are of infinite potential the particle does not penetrate out from the box. Also, the particle has elastic collisions with the walls. Therefore, the potential energy of the electron inside the box is constant and it is taken as zero for simplicity. The potential energy V of the particle is on the wall of the box is infinity.

Thus the potential function is

$$V(x) = 0 \quad 0<x<a$$

V(x)=

This function is known as square well potential

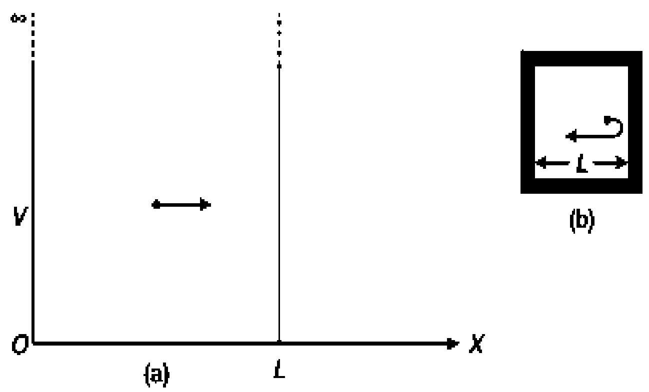


Fig 4.6.1 Particle in 1D box

(Source: "Advanced Engineering Physics" by Sujay Kumar Bhattacharya, Saumen Pal)

Here the particle cannot move outside and the boundary conditions can be written as

$$\Psi = 0 \quad \text{at } x=0 \text{ and } a$$

$$\Psi \neq 0 \quad \text{at } 0 \leq x \leq 0$$

The Schrodinger's equation in 1D is

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \text{------------------------(1)}$$

But V= 0 inside the potential well

(1) Becomes

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(E\Psi) = 0 \text{------------------------(2}$$

Put

$$\frac{2mE}{\hbar^2} = k^2 \text{--------------------(3)}$$

(2) Becomes

$$\frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

Solution for this equation is

$$\Psi(x) = A \sin kx + B\cos Kx \text{------------------(4)}$$

A and B are constants.

To find A and B

Apply boundary conditions

At x=0 $\Psi = 0$

Sub in (4)

$$0 = B$$

$$\therefore \Psi = A \sin kx \text{--------------------(5)}$$

At x =a $\Psi = 0$

0=A sin ka

A sinka=0

A cannot be zero

$\therefore$ Sinka =0

Ka=nπ

$$\therefore k = \frac{n\pi}{a}$$

Sub this in (3)

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2\pi^2\hbar^2}{2ma^2} \text{---------(6)}$$

And we know that

$$\hbar^2 = \frac{h^2}{4\pi^2}$$

Sub this in (6)

$$E = \frac{n^2h^2}{8ma^2} \text{--------------(7)}$$

This is the expression for energy Eigen value for a particle moving in 1D box.

To find energy Eigen function Ψ

$$\Psi = \mathrm{A}\sin kx$$

$$\Psi = \mathrm{A}\sin\frac{n\pi}{a}x \text{-------------(8)} \qquad (\because K = \frac{n\pi}{a})$$

Normalisation of wave function

We know that

Within the potential well the particle is present

Then the probability of finding the particle is

$$\int \Psi\Psi^* d\tau = 1$$

Here it is 1D

$$\int_0^a \Psi\Psi^* dx = 1$$

$$\therefore \int_0^a \mathrm{A}\sin\frac{n\pi}{a}x\,\mathrm{A}\sin\frac{n\pi}{a}x\,dx = 1$$

$$\int_0^a \mathrm{A}^2\sin^2\frac{n\pi}{a}x\,dx = 1$$

$$\mathrm{A}^2\int_0^a \sin^2\frac{n\pi}{a}x\,dx = 1$$

$$\mathrm{A}^2\int_0^a \frac{1-\cos 2\frac{n\pi}{a}x}{2}\,dx = 1$$

$$\mathrm{A}^2\int_0^a dx - \cos 2\frac{n\pi}{a}x dx = 1$$

$$\frac{\mathrm{A}^2}{2}\left(\mathrm{x} - \frac{\sin\frac{2n\pi}{a}x}{\frac{2n\pi}{a}}\right)_0^a = 1$$

Substituting the upper & lower limit

We get

$$\frac{A^2}{2}\left(a - \frac{\sin\frac{2n\pi}{a}a}{\frac{2n\pi}{a}}\right) - 0 = 1$$

$$\frac{A^2 a}{2} = 1$$

$$A^2 = 2/a$$

$$A = \sqrt{\frac{2}{a}}$$

Sub this in (8)

$$\Psi_n = \sqrt{\frac{2}{a}} \sin\frac{n\pi}{a}x$$

This is the expression for Eigen function or wave function of a particle moving in a 1D potential well.

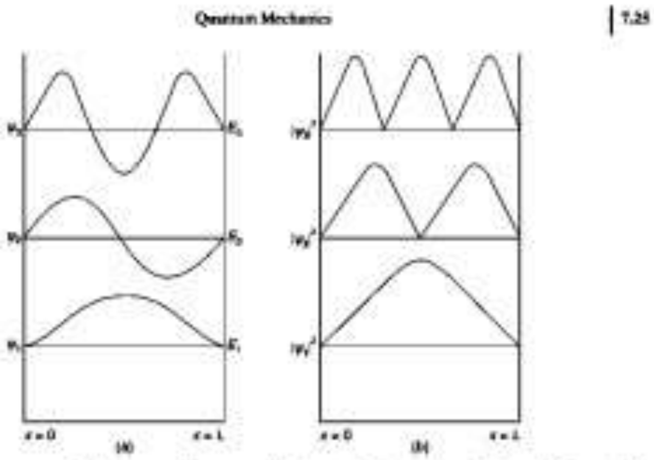


Fig 4.4.2 Particle in a box

Source: "Advanced Engineering Physics" by Sujay Kumar Bhattacharya, Saumen Pal

Result:

1. Energy Eigen value is inversely proportional to the square of width of the potential well.
2. Energy Eigen value is inversely proportional to the mass of the particle.
3. From the figure it is shown that the probability of finding the particle is maximum at the centre for the first energy value.
4. From the figure it is shown that the probability of finding the particle is minimum at the centre for the second energy value.

UNIT IV

Quantum Physics

Introduction:

Black body:

A body which absorbs all the wavelength of radiation and emits all wavelength of radiation is called black body. Sun is a perfect black body.

Black body radiation:

The radiation emitted by a black body is called black body radiation.

Kirchhoff's law:

Ratio of emissive power to the coefficient of absorption, of any given wavelength is the same for all bodies at a given temperature and is equal to the emissive power of the black body at that temperature.

Experiment

In practice a perfect black body is not available. Therefore, let us consider a hollow copper sphere coated with lamp black on its inner surface. A fine hole is made for radiations to enter into the sphere as shown in figure. Now when the radiations are made to pass through the hole it undergoes multiple reflections and are completely absorbed. Thus the black body acts as a perfect absorber. Now when this black body is placed in a temperature bath of fixed temperature, the heat radiations will come out only through the hole in the sphere and not through the walls of the sphere. Therefore, we can conclude that the radiations are emitted only from the inner surface of the sphere and not form the outer surface of the sphere. Thus a perfect black body is a perfect absorber and also a perfect radiator of all wavelengths.

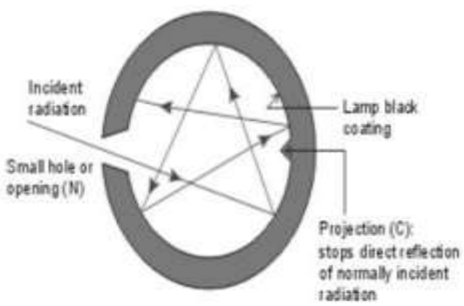


Figure 4.1.1 Perfect black body.

4.1. **Planck's Quantum Theory Of Black Body Radiation**

4.1.1.Assumptions (or) Hypothesis

Planck derived an expression for the energy distribution, with the following assumptions.

(i) A black body radiator contains electrons which are capable of vibrating with all possible frequencies as like simple harmonic oscillator.

(ii) The frequency of radiation emitted by an oscillator is the same as that of the frequency of its vibration.

(iii) The oscillators (electrons) radiate energy in a discrete manner and not in a continuous manner

(iv) The oscillator's exchanges energy in the form of either absorption or emission within the surroundings in terms of quanta of magnitude 'hv'

(v) E = n hv Where, n = 0, 1, 2, 3.... r This implies that the exchange of energy will not take place continuously but are limited to discrete set of values say 0, hv, 2hv, 3hv, 4hv ..... rhv. (Or) 0, E, 2E, 3E..... rE, for r-oscillators

4.1.2Planck's law of Radiation (Derivation)

Energy density of radiation at a particular temperature between the energy interval E and E+ d is given by

$$E_\lambda d\lambda = \frac{8\pi hc\, d\lambda}{\lambda^5\left(e^{\frac{hc}{\lambda kT}}-1\right)}$$

This is called Planck's radiation formula.

Consider a black body with 'N' Number of oscillators with their total energy as E_T. Then the average energy of an oscillator is given by

If $N_0, N_1, N_2, N_3 \ldots N_r$, are the oscillators of energy 0, E, 2E, 3E….. *r*E respectively then we can write

(i) The total number of oscillators

$$N = N_0 + N_1 + N_2 + N_3 \ldots + N_r \text{ ---------(2)}$$

(ii) Total energy of oscillators

$$E_r = 0\, N_0 + EN_1 + 2EN_2 + 3EN_3 \ldots + rEN_r \text{---------} \quad (3)$$

According to Maxwell's distribution formula, the number of oscillator having an energy *r*E given by

$$N_r = N_0\, e^{-r/KBT} \text{ ------------(4)}$$

Where, K_B is Boltzmann constant and r = 0, 1, 2, 3....

For several of r, i.e. r = 0, 1, 2.... r, the number of oscillators N_0, N_1, N_2, N_3.... N_r can be got as follows:

(i) For r = 0; $N_0 = N_0\, e^{0}$

(ii) For r = 1; $N_1 = N_0\, e^{-1E/K_BT}$

(iii) For r = 2; $N_2 = N_0\, e^{-2E/K_BT}$

(iv) For r = 3; $N_3 = N_0\, e^{-3E/K_BT}$

Similarly for r = r; $N_r = N_0\, e^{-rE/K_BT}$

The total number of oscillators can be got by substituting the values of N_0, N_1, N_2, N_3 --------- N_r in equation (2)

$$N = N_0\, e^{0} + N_0\, e^{-1E/KBT} + N_0\, e^{-2E/KBT} + N_0\, e^{-3E/KBT} + \quad + N_0\, e^{-rE/KBT}$$

$$N = N_0\ [1 + e^{-1E/K_BT} + e^{-2E/K_BT} + e^{-3E/K_BT} + \ldots\ldots\ldots + e^{-rE/K_BT}] \ldots\ldots(5)$$

Put $x = e^{-E/K_BT}$ We know that,

$$1 + x + x^2 + x^3 + \ldots\ldots\ldots = \frac{1}{1-x}$$

Therefore, we can write equation (5) as

The total number of oscillators is,

$$N = \frac{N_0}{1-x} \text{---------}(6)$$

Similarly, by substituting the values of N_0, N_1, N_2, N_3 N_r in equation (3), the total energy can be written as

$$E_r = 0N_0\, e^{0} + E\, N_0\, e^{-1E/KBT} + 2EN_0 e^{-2E/KBT} + 3EN_0\, e^{-3E/KB} + rEN_0\, e^{-rE/KBT}$$

$$E_r = N_0\, Ee^{-E/KBT}\, [1 + 2e^{-E/KBT} + 2Ee^{-2E/KBT} + \ldots + rEe^{-(r-1)E/KBT}] \ldots\ldots (7)$$

Put $x = e^{-E/\,KBT}$

We know that $1 + 2x + 3x^2 + \ldots + rx^{r-1} = \frac{1}{(1-x)^2}$

$$E_r = \frac{N0\,xE}{(1-x)^2}$$

$$E_r = \frac{N0\,xh\gamma}{(1-x)^2} \text{----------------(7)}$$

Substituting (6) & (7) in (1)

$$E = \frac{N0\,xh\gamma}{(1-x)^2} \div \frac{N_0}{1-x}$$

$$E = \frac{xh\gamma}{1-x}$$

$$E = \frac{h\gamma}{\frac{1}{x}-1}$$

$$E = \frac{h\gamma}{e^{\frac{h\gamma}{KT}}-1}$$

$$\text{Energy density } = \frac{\text{Total Energy}}{\text{No of oscillating particle}} \times \frac{\text{No of oscillating particle}}{\text{Volume}}$$

$$E_\lambda = \frac{h\gamma}{e^{\frac{h\gamma}{KT}}-1} \times \frac{8\pi d\lambda}{\lambda^4}$$

We know that

$$c = \nu\lambda$$

$$\nu = \frac{c}{\lambda}$$

$$\text{Energy density } E_\lambda d\lambda = \frac{8\pi hc\, d\lambda}{\lambda^5 e^{\frac{hc}{\lambda KT}}-1} \text{----------------(8)}$$

This is Planck's radiation formula

4.1.3.Wien's Displacement Law

Statement:

The wavelength corresponding to maximum energy density is inversely proportional to th absolute temperature of the black body.

$$\textbf{(ie)}\boldsymbol{\lambda_m}\ \alpha\ \frac{1}{T}$$

We know that the Wien's displacement law holds good only for shorter wavelengths.

Since, $e^{hc/\lambda K_B T} \gg 1$, By neglecting 1 we can write

$$e^{hc/\lambda KBT}-1 = e^{hc/\lambda KBT}$$

Therefore, equation (8) becomes

$$E_\lambda = \frac{8\pi hc}{\lambda^5 e^{\frac{hc}{\lambda kT}}} \text{---------}(9)$$

Equation (9) represents the Wien's displacement law from Planck's radiation law using Quantumtheory of black body radiation.

4.1.4.Rayleigh-Jean law

Statement:

The energy density of a black body is directly proportional to the absolute temperature and inversely proportional to the fourth power of the wavelength.

We know that the Rayleigh-Jeans law holds good only for longer wavelength.

If λ is greater; 1/λ will be lesser.

$$\therefore e^{\frac{hc}{\lambda kT}} = 1 + \frac{hc}{\lambda kT} + \left(\frac{hc}{\lambda kT}\right)^2 + \left(\frac{hc}{\lambda kT}\right)^3 + \left(\frac{hc}{\lambda kT}\right)^3$$

Neglecting higher terms

$$\text{We get } e^{\frac{hc}{\lambda kT}} = 1 + \frac{hc}{\lambda kT}$$

$$\therefore E_\lambda = \frac{8\pi hc}{\lambda^5 \frac{hc}{\lambda kT}}$$

$$E_\lambda = \frac{8\pi kT}{\lambda^4}$$

This represents Rayleigh Jeans law.

4.7 Scanning Tunneling Microscope Introduction (STM)

A microscope is a device which is used to view smaller object which cannot see through naked eye.

In 1981 Gerd Binning and Heinrich Rohrer developed the scanning microscope (STM)significantly superior tool for observing surface atom by atom.

STM is the highest resolution imaging and nano fabrication technique available. It depends on quantum tunneling of electron from sharp metal tip to a conducting surface.

Principle

The basic principle used in Scanning Tunneling Microscope (STM) is the tunneling of electron between the Sharpe metallic tip of the probe and surface of the sample.

STM has a metal needle that scans a sample by moving back and forth and gathering information about the curvature of the surface. It follows the smallest changes in the contours of a sample.

The needle does not touch the sample,however,but stays about the width of two atoms about it.

Instrumentation

A schematic of STM is shown in figure. It has the following components. Piezo electric tube with the tip and electrode. Capable of moving in X,Y, Z direction. Fine needle tip for scanning the sample surface. A macro scale image of an etched tungsten STM tip is shown in figure.

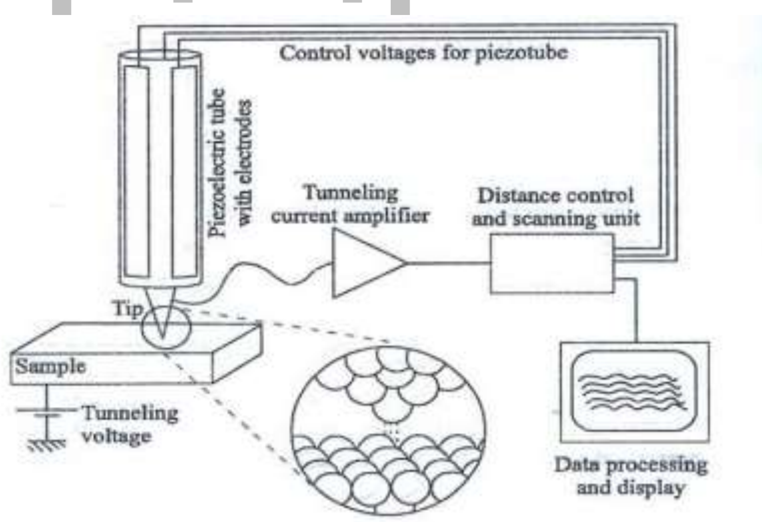
Tip is affixed to the piezoelectric tube in order to control it position and movement on an atomic scale. Piezoelectric materials exhibit an elongation or contraction along their length when an electric field is applied.

Working

The tip is mechanically connected to the scanner, an XYZ positioning device. The sharp metal needle is brought close to the surface to be imaged. The distance is of the order of a few angstroms.

A bias voltage is applied between the sample and the tip. When the needle is at a potential with respect to the surface, electrons can tunnel through the gap and set up a small tunneling current in the needle. This feeble tunneling current is amplified and measured.

With the help of the tunneling current the feedback electronics keeps the distance between tip and sample constant. The sensitivity of the STM is so large that electronic corrugation of the surface atoms and the electron distributions around them can be detected.



4.7.1 Scanning Tunneling Microscope

Applications of STM

1. The STM shows the positions of atoms more preciously.
2. STMs are versatile.
3. STMs give the 3 dimensional profile of a surface, which allows researchers to examine a multitude of characteristics , including roughness, surface defects and molecule size.

4. STM is used in study of structure , growth ,morphology, electronic structure ,thin films and nano structures.

5. Lateral resolution of 0.1nm to 0.01nm of resolution in depth can be achieved.

Disadvantages

1. It is very expensive
2. It needs specific training to operate effectively.
3. A single dust particle could damage the needle.
4. A small vibration, even a sound, could smash the tip and sample together.

4.5 Schrodinger wave equation

4.5.1 Schrodinger Time Independent wave equation

Consider a wave associated with a moving particle. Let *x*, *y*, *z* be the coordinate of the particle and Ψ is a wave function for de – Broglie at any instant of time t.

The classical differential equation for wave motion is given by

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} \text{----------------------(1)}$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

(1) gives $\nabla^2 \Psi = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$ ------------------ (2)

The solution of equation (2) becomes

$$\Psi(x, y, z, t) = \Psi_0(x, y, z)e^{-i\omega t} \text{------------(3)}$$

Differentiating (3) twice w.r.t time't'

$$\frac{\partial \Psi}{\partial t} = -i\,\Psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \Psi}{\partial t^2} = (-i\,w)(-i\,w)\Psi_0 e^{-i\omega t} \text{---------------(4)}$$

$$\frac{\partial^2 \Psi}{\partial t^2} = -w^2\,\Psi \text{------------------------------(5)}$$

Substitute (5) in(2)

$$\nabla^2 = -\frac{1}{v^2}(w^2\ \Psi)$$

$$\nabla^2 = -\frac{w^2}{v^2}\Psi \text{--------------} (6)$$

w.k.t

$\omega = 2\pi\upsilon$; but v = $\upsilon\lambda$

$\omega = 2\pi\frac{\upsilon}{\lambda}$ $\upsilon = \frac{\upsilon}{\lambda}$

$$\frac{\omega}{v} = \frac{2\pi}{\lambda}$$

$$\frac{w^2}{v^2} = \frac{4\pi^2}{\lambda^2} \text{--------------------} (7)$$

Sub (7) in (6)

$$\nabla^2\ \Psi = -\frac{4\pi^2}{\lambda^2}\Psi$$

$$\nabla 2\ \Psi + \frac{4\pi^2}{\lambda^2}\ \Psi = 0 \text{--------} (8)$$

According to De-Broglie's theory $\lambda = \frac{h}{mv}$----------(9)

Where m - mass of particle

v - velocity

sub (9) in (8)

$$\nabla^2 \Psi + \frac{4\pi^2}{(\frac{h}{mv})^2} \Psi = 0$$

$$\nabla^2 \Psi + \frac{4\pi^2 \; v^2 m^2}{h^2} = 0 \text{--------} (10)$$

Taking $\hbar = \frac{h}{2\pi}$; $\frac{1}{\hbar} = \frac{2\pi}{h}$

$$\frac{1}{\hbar^2} = \frac{4\pi^2}{h^2} \text{-------------------------} (11)$$

Sub (11) in (10)

$$\nabla 2 \; \Psi + \frac{v^2 m^2}{h^2} = 0 \text{--------} (12)$$

Total Energy $E = V + \frac{1}{2} mv^2$

$2(E-V) \;=\; mv^2$

Multiply 'm' on both sides

$2m(E-V) \;=\; m^2v^2$ ----------- (13)

Sub (13) in (12)

$$\nabla^2 \Psi + \frac{2m(E-V)}{\hbar^2} \Psi = 0$$

This is the final expression of Schrodinger time independent wave equation.

4.5.2 Schrodinger Time dependent wave equation:

The differential equation for wave motion is given by

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}$$

$$\nabla^2\Psi=\frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}\text{------------------- (1)}$$

The solution of equation (1) becomes

$$\Psi(x,y,z,t)=\Psi_0(x,y,z)e^{-i\omega t}\text{---------(2)}$$

Differentiating(2)twicew.r.t time't'

$$\frac{\partial\Psi}{\partial t}=-i\,\Psi_0 e^{-i\omega t}$$

$$\frac{\partial\Psi}{\partial t}=(-i\,w\,\Psi)\text{ -------------- (3)}$$

w.k.t

$\omega=2\pi\upsilon$; but $E=h\upsilon$; $\upsilon=\frac{E}{h}$

$$\omega=2\pi\frac{E}{h}\text{ --------------(4)}$$

Substitute (4) in (3)

$$\frac{\partial\Psi}{\partial t}=\frac{-i2\pi E\Psi}{h}=\frac{-ii2\pi E\Psi}{ih}\text{ (multiply \& divide by i)}$$

$$\frac{\partial \Psi}{\partial t} = \frac{-2\,\pi E\Psi}{ih} = \frac{E\Psi}{i\hbar}$$

$$\frac{\partial \Psi}{\partial t} i\hbar = E\Psi$$

$$E\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

Substitute EΨ in time independent wave equation

$$\nabla^2 \Psi + \frac{2m(E-V)}{\hbar^2} \Psi = 0$$

$$\nabla^2 \Psi + \frac{2m(E\Psi - V\Psi)}{\hbar^2} = 0$$

$$\nabla^2 \Psi = \frac{-2m(E\Psi - V\Psi)}{\hbar^2}$$

$$\frac{-\hbar^2}{2m} \nabla 2 = E\Psi - V\Psi$$

$$\frac{-\hbar^2}{2m} \nabla 2 + V\Psi = E\Psi \text{ ----------------(6)}$$

Substitute (5) in (6)

$$\frac{-\hbar^2}{2m} \nabla 2 + V\Psi = i\hbar \frac{\partial \Psi}{\partial t} \text{ -----------(7)}$$

$$\left[\frac{-\hbar^2}{2m} \nabla 2 + V\right] \Psi = i\hbar \frac{\partial \Psi}{\partial t} \text{ -----------(8)}$$

(8) is the Schrodinger Time dependent wave equation

Here

Hamiltonion operator H= $\left[\frac{-\hbar^2}{2m}\nabla^2 + V\right]$

Energy operator E = $i\hbar \frac{\partial \Psi}{\partial t}$

(8) gives $H\,\Psi = E\,\Psi$

4.3 Dual Nature of Radiation (Light) and Matter (Particles) – Matter Waves

The universe is made of Radiation (light) and matter (particles). The light exhibits the dual nature (i.e.) it can behave both as a wave (interference, diffraction phenomenon) and as a particle (Compton Effect, photo-electric effect etc.). Since the nature loves symmetry, in 1924 Louis de- Broglie suggested that an electron or any other material particle must exhibit wave like properties in addition to particle nature.

The waves associated with a material particle are called as Matter waves or De-Braglie wave.

4.3.1 De-Broglie Wavelength

From the theory of light, considering a photon as a particle the total energy of the photon is given by

$$E = mc^2 \quad \text{......................................(1)}$$

Where, m - mass of the particle

c - velocity of light

Considering the photon as a wave, the total energy is given by

$$E = h\nu \quad \text{.................. (2)}$$

Where, h - Planck's constant

ν - frequency of radiation

From equations (1) and (2) we can write

$$E = mc^2 = h\nu \quad \text{........ (3)}$$

We know momentum = mass x velocity

$$P = mc$$

Equation (3) becomes

$$h\nu = pc$$

$$P = \frac{h\nu}{c}$$

Since $\frac{c}{v} = \lambda$ we can write p = $\frac{h}{\lambda}$

The wave length of photon $\lambda = \frac{h}{mv}$ ------------------------- (4)

De-Broglie suggested that equation (4) can be applied both for photons and material particles. If *m* is the mass of the particle and 'v' is the velocity of the particle, then Momentum $p = mv$.

De-Broglie wavelength $\lambda = \frac{h}{mv}$

-

OTHER FORMS OF DE-BROGLIE WAVELENGTH :

i) De-Broglie wavelength in terms of Energy:

We know kinetic energy $E = \frac{1}{2}mv^2$

Multiplying by '*m*' on both sides we get

$$Em = \frac{1}{2}v^2m^2$$

(Or)

$$m^2v^2 = 2Em$$

$$mv = \sqrt{2Em}$$

de-Broglie wave length $\lambda = \frac{h}{\sqrt{2E}}$

ii) de-broglie wavelength in terms of voltage :

If a charged particle of charge '*e*' is accelerated through a potential difference 'V'

Then the kinetic energy $= \frac{1}{2}mv^2$ --------------(1)

Also, we know that energy = eV -------------(2)

Equating (1) and (2)

$$\frac{1}{2}mv^2 = eV$$

Multiplying by '*m*' on both sides we get

$$m^2v^2 = 2meV$$

$$mv = \sqrt{2meV}$$

substituting in mv

$$\lambda = \frac{h}{mv}$$

de-Broglie wave length $\lambda = \frac{h}{\sqrt{2meV}}$

iii)De-Broglie wavelength in terms of Temperature

When a particle like neutron is in thermal equilibrium at temperature T, then they possess Maxwell distribution of velocities.

Therefore kinetic energy E $= \frac{1}{2}mv_{rms}^2$ ------------------- (1)

Where v_{rms} is the root mean square velocity of the particle

Also ,we know energy $= \frac{3}{2}K_B T$ ---------------------- (2)

K_B - Boltzmann constant.

Equating (1) and (2) we get

$$\frac{1}{2}mv^2 = \frac{3}{2}K_B T$$

$$m^2v^2 = 3mK_B T$$

$$mv = \sqrt{3mK_B T}$$

De-Broglie wavelength $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mK_B T}}$

PROPERTIES OF MATTER WAVE:

1. Matter wave are not an electromagnetic wave.
2. It motion due to the charge particles.
3. The wave and particle aspects cannot appear together.
4. Locating exact the position of the particle in the wave is uncertain.
5. Lighter particles will have high wavelength.
6. Particles moving with less velocity will have high wavelength.
7. Velocity of matter wave depends on the velocity of the particle.

The velocity of matter wave is greater than the velocity of light