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8. State the convolution theorem for Fourier transforms.

9. Find the Z-transform of $\{n\}$.

10. Prove that $Z\{nf(n)\} = -z \frac{d}{dz} F(z)$, where $Z\{f(n)\} = F(z)$.

PART - B

(5×16=80 Marks)

11. a) i) Find the singular solution of the equation $z = px + qy + p^2 + pq + q^2$. (8)

ii) Solve : $x(y - z)p + y(z - x)q = z(x - y)$. (8)

(OR)

b) i) Solve : $(D^2 + 4DD' - 5D'^2)z = \sin(x - 2y) + e^{2x - y}$. (8)

ii) Solve : $(D^2 + DD' - 6D'^2)z = y \cos x$. (8)

12. a) i) Find the Fourier series for $f(x) = x^2$ in $-\pi < x < \pi$. (8)

ii) Find the half range cosine series for $f(x) = x(\pi - x)$ in $(0, \pi)$. (8)

(OR)

b) Find the Fourier series expansion upto the first three harmonics for the function defined in the following table (16)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1	1.4	1.9	1.7	1.5	1.2	1.0

13. a) A string is stretched and fastened to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t . (16)

(OR)

b) A bar, 10 cm long with insulated sides, has its ends A and B kept at 20°C and 40°C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50°C and at the same instant that at B is lowered to 10°C . Find the subsequent temperature at any point of the bar at any time. (16)



14. a) Find the Fourier transform of the function $f(x)$ defined by $f(x) = \begin{cases} 1-x^2 & \text{in } |x| \leq 1 \\ 0 & \text{in } |x| > 1 \end{cases}$.

Hence prove that $\int_0^\infty \frac{\sin s - s \cos s}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$. Also show that

$$\int_0^\infty \frac{(x \cos x - \sin x)^2}{x^6} dx = \frac{\pi}{15}. \quad (16)$$

(OR)

- b) i) Show that the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$. (8)

- ii) Find the Fourier cosine transform of $e^{-a^2 x^2}$ and hence find the Fourier sine transform of $x e^{-a^2 x^2}$. (8)

15. a) i) Find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z+1)}$ using convolution theorem for Z-transforms. (8)

- ii) Find the inverse Z-transform of $\frac{z^2 - 3z}{(z-5)(z+2)}$ using residue theorem. (8)

(OR)

- b) i) Solve : $y_{n+2} - 4y_{n+1} + 4y_n = 0, y_0 = 1, y_1 = 0$, using Z-transform. (10)

- ii) Find the Z-transform of $\{n\}$ and $\left\{\frac{1}{n+1}\right\}$. (6)