PH8151 ENGINEERING PHYSICS

UNIT-II WAVES AND FIBER OPTICS

13-Marks

1. **(a) For atomic transitions derive Einstein's relation and hence deduce the expression for the ratio of spontaneous emission rate to the stimulated emission rate.**

[OR]

b) Derive Einstein's relation for spontaneous and stimulated emission of radiation. Obtain the ratio of stimulated emission rate to stimulated absorption rate and discuss population inversion.

[OR]

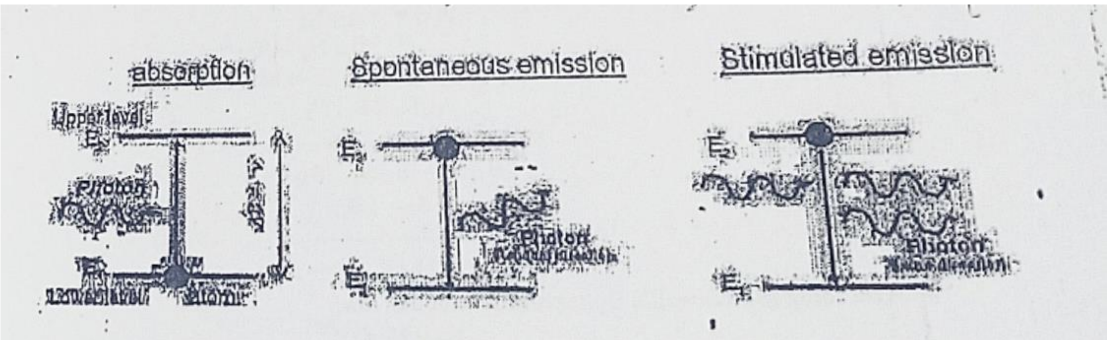
c) Derive an expression for Einstein's coefficients for spontaneous and stimulated emission.

Einstein's coefficients:

Let us consider two level energy system E_1 and E_2 with number of atoms N_1 and N_2 respectively. Let 'n' be the number of photons at frequency 'v' such that $h\nu = E_2 - E_1$.

The energy density of the interacting photons is given by $\rho(\nu) = nh\nu$

When these photons interact with atoms, both upward and downward transitions occur simultaneously.



Upward transition:

1)Stimulated Absorption

An atom in the ground state 'E_1' absorb a photon of energy E_1 absorb a photon of energy 'h v' and gets excited to the higher energy state 'E_2'. This process is known as stimulated absorption.

Stimulated absorption rate $\propto N_1$

$$R_{ab} \propto N_1 \rho(v)$$

$$R_{ab} = B_{12} N_{1\rho(v)} \quad \text{------------- (1)}$$

Where $B_{12} \rightarrow Einstein's coefficient\ of\ stimulated\ absorption.$

Downward transition:

Spontaneous Emission:

The atom in the excited state E_2 returns to the ground state E_1 by emitting a photon of energy $hv = E_2 - E_1$ without the action of an external agency.

Spontaneous emission rate $\propto N_2$

$$R_{sp} = A_{21} N_2 \quad \text{--------------- (2)}$$

Where A_{21} → Einstein's coefficient of spontaneous emission.

Stimulated emission:

The atom in the excited state E_2 returns to the ground state E_1 by interacting with a photon of energy $hv = E_2 - E_1$, thereby emitting two photons of same energy as that of incident photon.

This process of emission is called stimulated emission.

Stimulated emission rate $\propto N_2$

$$\propto \rho(v)$$

$$R_{st} = B_{21} N_2\, \rho(v) \quad \text{------------- (3)}$$

Were, B_{21} → Einstein's coefficient of stimulated emission

For a system of in equilibrium, upward and downward transition rates must be equal. Hence

$$R_{ab} = R_{sp} + R_{st}$$

$$B_{12} N_1\, \rho(v) = A_{21} N_2 + B_{21} N_2\, \rho(v)$$

$$B_{12}N_1\,\rho(v) - B_{21}N_2\,\rho(v) = A_{21}N_2$$

$$\rho(v)\,[B_{21}N_1 - B_{21}N_2] = A_{21}N_2$$

$$\rho(v) = \frac{A_{21}N_2}{B_{21}N_1 - B_{21}N_2}$$

Dividing by $B_{21}N_2, we\ have$

$$\rho(v) = \frac{A_{21}N_2/B_{21}N_2}{B_{12}N_1 - B_{21}N_2/B_{21}N_2}$$

$$\rho(v) = \frac{A_{21}/B_{21}}{B_{12}N_1/B_{21}N_2 - 1} \quad \text{-------------- (4)}$$

According to Maxwell's Boltzmann distribution law,

$$N_n = N_0 e^{-(E_n/KT)}$$

Where K – Boltzmann's constant, T – absolute temperature

$$N_1 = N_0 e^{-(E_1/KT)}$$

$$N_2 = N_0 e^{-(E_2/KT)}$$

$$\frac{N_1}{N_2} = \frac{N_0 e^{-(E_1/KT)}}{N_0 e^{-(E_2/KT)}}$$

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/KT}$$

$$\frac{N_1}{N_2} = e^{hv/KT} \quad \text{------------ (5)}$$

Substituting eq 5 in eq 4 we get,

$$\rho(v) = \frac{A_{21}/B_{21}}{(B_{12}/B_{21})e^{hv/KT} - 1} \quad \text{------------- (6)}$$

Eq (6) agrees with the Planck's radiation formula

$$\rho(v) = \frac{8\pi h v^3}{C^3}\ x\ \frac{1}{e^{hv/KT} - 1} \quad \text{---------- (7)}$$

Comparing eq (6) & (7),

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \theta^3}{C^3} \quad \text{------------ (8)}$$

$B_{12} = B_{21}$ ------------ (9)

Equations (8) and (9) are referred to as the Einstein's relations

Rate of spontaneous and stimulated emission.

From eq (2) & (3)

$$\frac{R_{st}}{R_{sp}} = \frac{B_{21} N_2\,\rho(v)}{A_{21} N_2} = \frac{B_{21}\,\rho(v)}{A_{21}} \quad \text{------------ (10)}$$

But from eq (6)

$$\frac{B_{21}\rho(v)}{A_{21}(B_{12}/B_{21})} = \frac{1}{e^{hv/KT} - 1}$$

Substituting $B_{12} = B_{21},\ we\ get$

$$\frac{B_{21}\,\rho(v)}{A_{21}} = \frac{1}{e^{hv/KT} - 1}$$

Eq (10) becomes,

$$\frac{R_{st}}{R_{sp}} = \frac{1}{e^{hv/KT} - 1}$$

$$\frac{R_{sp}}{R_{st}} = e^{hv/KT} - 1$$

<u>Results:</u>

When the energy of the incident photon is much greater than KT, i.e., h v>>KT, laser action is not possible.

When $N_1 > N_2$, stimulated absorption is predominant than stimulated emission.

When $N_2 > N_1$, stimulated emission will predominant over stimulated absorption.

2. a) Define acceptance angle and Numerical aperture.

Derive expressions for acceptance angle and numerical aperture of Fibre in terms of refractive index of the core and cladding of the Fibre.

[OR]

b) Derive an expression for acceptance angle and numerical aperture of an optical Fibre.

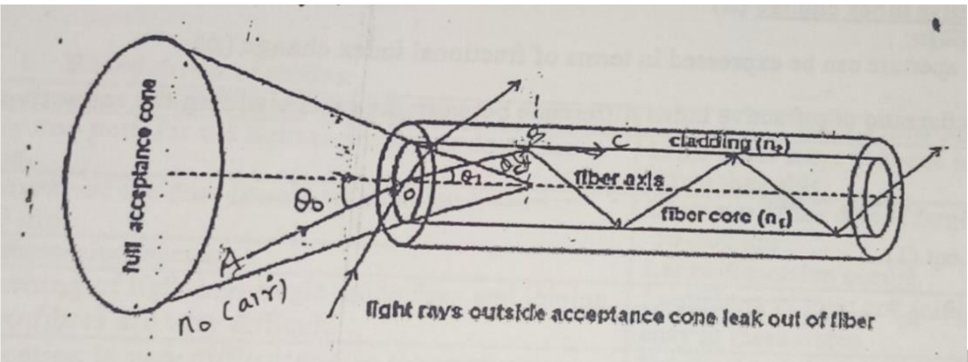
(i) Acceptance angle and Numerical aperture.

Principle:

The maximum angle of incidence with which a light ray of given wavelength can enter an optical fiber so that the total internal reflection takes place inside the fiber is called acceptance angle.

Derivation:

Consider an optical fiber into which the light is launched. Let 'n_1' be the refractive index of the core, 'n_2' be the refractive index of the cladding and 'n_0' be the refractive index of the medium from which light is launched



- AO is the incident ray entering into the core at an angle θ_0 to the fibre axis.
- OB is the refracted ray into the core. The angle of refraction is θ_1.
- At B, the ray falls at the critical angle of incidenceθ_c. At this angle, the ray moves along BC. Applying Snell's law at O,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

Substituting $\theta_1 = 90^\circ - \theta c,\ eqt(1) becomes$

$$n_0 \sin\theta_0 = n_1 \sin(90^\circ - \theta c) = n_1 \cos\theta c = n_1(1 - sin^2\theta c)^{1/2}$$

Applying Snell's law at B,

$$n_1 \sin\theta c = n_2 \sin 90^\circ$$
$$n_1 \sin\theta c = n_2$$
$$\sin\theta c = n_2/n_1$$

Substituting this in eq (2) we get,

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{n_1}{n_0 n_1}\sqrt{n_1^2 - n_2^2}$$

$$n_0 = 1(air)$$

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\theta_0 = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

Thus θ_0 is called the acceptance angle or acceptance cone half angle.

Numerical aperture:

Numerical aperture of the Fiber is the light collecting efficiency of the fiber and is the measure of the number of light rays that can be accepted by the fiber.

Definition:

Numerical aperture is defined as the sine of the acceptance angle. i.e., $NA = \sin\theta_0$

$$NA = \sqrt{n_1^2 - n_2^2}$$

Fractional refractive index change (Δ):

The numerical aperture can be expressed in terms of fractional index change (Δ)

It is defined as the ratio of refractive index difference between core and cladding the refractive index of core.

$Ie, \Delta = n_1 - n_2/n_1, \quad n_1 - n_2 = \Delta n_1$

Now considering eq (1)

$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{n_1 + n_2\ \Delta n_1}$

$Since\ n_1 \sim n_2\ \ and\ \ n_1 + n_2 = 2n_1$

$NA = \sqrt{2n_1^2\Delta}\ \ or\ \ NA = n_1\sqrt{2\Delta}$

Condition for propagation of light:

If θ_1 is the angle of incidence, then the light ray propagates only if $\theta_1 < \theta_0$

$$\sin\theta_i < \sin\theta_0$$

$$\sin\theta_i < NA$$

3. a) Explain in detail how optical fibers are classified according to the material refractive index and modes of propagation.

[OR]

b) How are fibers classified? Explain the classification in detail.

Optical fibers are classified into three major categories based on (i) material (ii) Number of modes (iii) Refractive index profile.

Classification based on material fiber:

(i) Glass fiber
(ii) Plastic fiber

(i) Glass fiber:

If the optical fiber is made by mixtures of metal oxides and silica glasses.

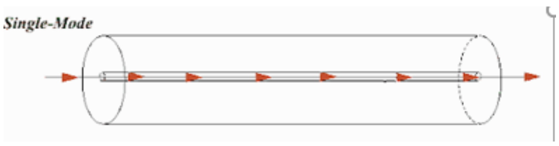
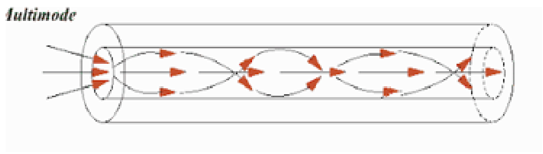
E.g., $SiO_2 - core, \quad B_2O_3 - SiO_2 - cladding$

(ii) Plastic fiber:

If the Fiber is made of plastics Eg: Polystyrene – core, Methyl methacrylate – cladding

Based on no of modes:

SINGLE MODE FIBRE	MULTIMODE FIBRE
Only one path for the light to propagate down the cable.	Light takes more than one path to propagate down the cable.
Diameter of the core is very small about 5-10 µm.	Diameter of the core is large about 50-200 µm.
No dispersion occurs.	Large dispersion occurs.
Launching of light into single mode fires and joining of two fibers are very difficult.	Launching of light and joining of two fibers are easy in these fibers.
Fabrication is very difficult and so the production cost is high.	Fabrication is less difficult and so the production cost is low.
Because of its high bandwidth, they are used in long haul communication systems.	Because of its less bandwidth, it is useful in short haul communication systems.

Diagram	Diagram
 Image source: www.fiber-optical-networking.com	 Image sources: www.fiber-optical-networking.com

Classification based on refractive index profile:

Refractive index profile:

(i) Step index fiber: Step index fiber is further classified into two based on the number of modes.
(ii) Graded Index fiber.

(i) Step index fiber:

SINGLE MODE FIBRE	MULTIMODE FIBRE
It has uniform core of diameter 5-10 µm of high refractive index.	It has a uniform core of diameter 50-200 µm
The core is surrounded by a cladding of diameter 50-125 µm of lesser refractive index than core.	Cladding diameter is about 125-300 µm
Due to its small core diameter, only a single mode of light ray transmission is possible.	Because of large core diameter, propagation of many modes within the fiber is allowed.
The NA and acceptance angle are very small	The NA and acceptance angle is larger as the core diameter of the fiber is larger.
No dispersion	It has large dispersion.
It has high bandwidth	It has low bandwidth

(ii) Graded index fiber:

- If the core has a non-uniform refractive index that gradually decreases from the centre towards the core-cladding interface. The fiber is called a graded index fiber.
- This has only one mode of propagation, which is graded index multimode fiber.
- It has a core diameter of 50-200µm
- Cladding diameter is about 100-250µm
- Single distortion is very less. It has low attenuation
- It has infinite bandwidth.
- The acceptance angle and NA decreases with radial distance from the axis.
- The light propagation is in the form of skew rays and it will not cross the fiber axis.
- The light propagation is in the form of skew rays and it will not cross the fiber axis.
- The path of light propagation is helical (i.e.) spiral manner.

4. (a)Compare the homojunction semiconductor laser with a heterojunction semiconductor laser and detail their features.

[OR]

b) Explain the principal construction and working of n semiconductor laser diode and mention its merits and demerits.

[OR]

c) With suitable diagram, explain how the laser action is achieved in homojunction and hetrojunction lasers.

Semiconductor laser diode:

Semiconductor laser is classified into

(i) Homojunction semiconductor laser
(ii) Hetrojunction semiconductor laser

(i) Homojunction semiconductors laser:

It is specially fabricated p – n junction diode. This diode emits laser light when it is forward biased.

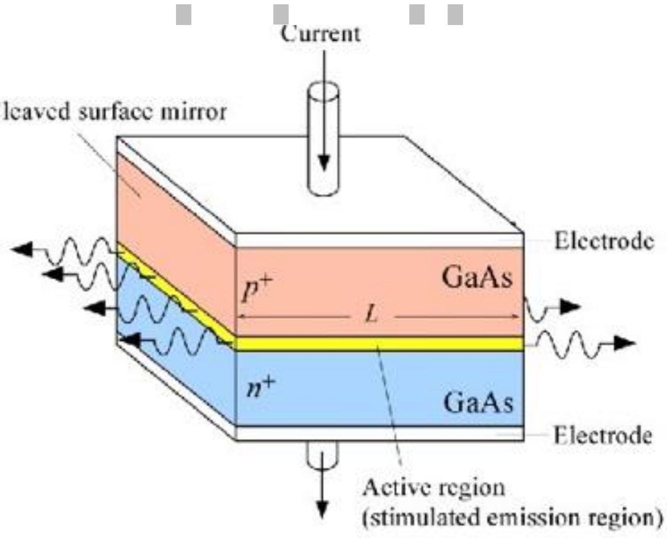
<u>Principle:</u>

When the p – n junction diode is forward biased, the electrons from n – region and holes from p – region cross the junction and recombine with each other. During the recombination process, the light radiation is related from a certain specified direct band gap semiconductors like GA As.

The photon emitted during recombination stimulate other electrons and holes to recombine. As a result, stimulated emission takes place and laser light is produced.

<u>Construction:</u>

- The active medium is a p – n junction diode made from a single crystal of GA As. This crystal is cut in the form of a platelet having a thickness of 0.5mm. This platelet consists of two regions n – type and p – type.
- The natural electrode is connected to both upper and lower surfaces of the semiconductor diode.
- The forward biased voltage is applied through metal electrodes.
- Now the photon emission is stimulated in a very thin layer of p – n junction.
- The end faces of the p – n junctions are well polished and parallel to each other. They act as an optical resonator through which the emitted light comes out.



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<u>Working:</u>

- When the p – n junction is forward biased, the electrons and holes are injected into junction region.
- The region around junction contains large number of electrons in the conduction band and holes in the valence band.

- Now the electrons and holes recombine with each other. During recombination, light photons are produced.
- When the forward biased voltage is increased, more light photons are emitted. These photons trigger a chain of stimulated recombination resulting in the emission of more light photons in phase.
- These photons moving at the plane of the junction travel back and forth by reflection between two polished surfaces of the junction. Thus, the light photons grow in strength.
- After gaining enough strength, laser beam of wavelength $8400A^0$ is emitted from the junction.
- The wavelength of the laser light is given by $E_g = hv = hc/\lambda$

$$\lambda = h\, c/E_g$$

E_g – band gap energy in joule

Advantages:

- This laser has high efficiency.
- The laser output can be easily increased by increasing the junction current.
- It is operated with less power than ruby and CO_2 laser.
- It emits a continuous wave output or pulsed output.

Disadvantages:

- Laser output beam has large divergence.
- The purity and monochromaticity are poor.
- It has poor coherence and stability.

Heterojunction semiconductor laser:

A diode laser with a p – n junction made up of different semiconductor materials in two regions i.e., n – type and p – type is known as hetrojunction semiconductor laser.

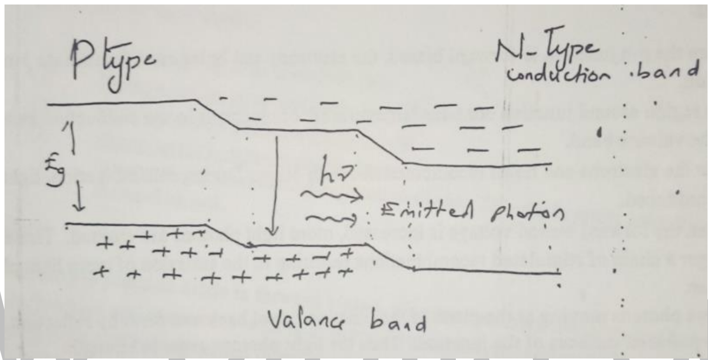
Principle:

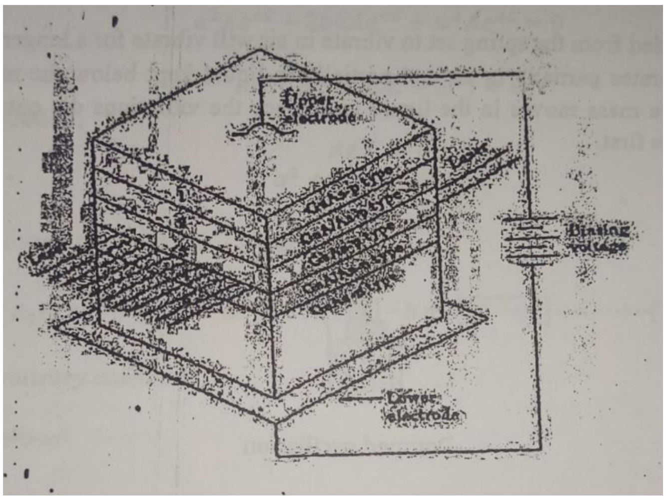
When the p – n junction diode is forward biased the electrons from n – region and holes from p – region recombine with each at the junction. During recombination process, light photon is released.

Construction:

- The laser consists of five layers. A layer GaAs p – type (3^{rd} layer) acts as active region. This layer is kept between two layers having wider band gap Ga AIAs p – type (2^{nd} layer) and Ga AIAs n – type (4^{th} layer)
- The bias voltage is applied through the metal electrodes fixed on top and bottom layer of heterojunction semiconductor laser.
- The end faces of the junctions of 3^{rd} and 4^{th} layers are well polished and parallel to each other. They act as an optical resonator.

Diagram:





Working:

- When the p – n junction is forward biased, the electrons and holes are injected into the junction region. The region around the junction contains large number of electrons in the conduction band and holes in the valence band.
- Now some of the injected charge carriers recombine and produce light radiation.
- When the forward biased voltage is increased, more light photons are emitted and hence the intensity is more.
- The light photons trigger a chain of stimulated recombination resulting in the release of photons are in phase.
- These photons moving at the plane of the junction travel back and forth by reflection between two sides and grow in strength and produces coherent laser beam of wavelength $8000A^0$.

Advantages:

- It produces continuous wave output
- The power output is very high.

5. a) i) Explain forced and damped oscillations.

ii) Derive the equation of motion. With appropriate figures.

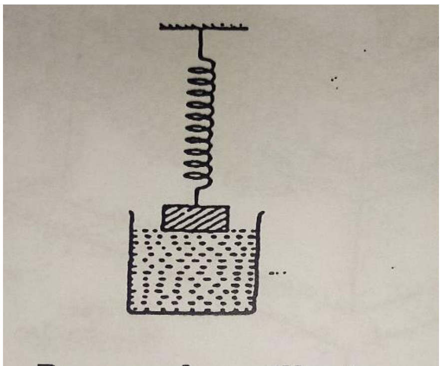
[OR]

b) What is meant by forced and damped oscillations? Write the differential equation for damped and forced oscillations.

When a body execute vibrations, if the amplitude keeps on decreasing due to the frictional resistance of the motion and hence the vibrations die out after some time. The motion is damped by friction and is called damped vibrations. The resisting force is proportional to the velocity of the body.

Differential equation and its solution to damped oscillations:

A mass suspended from the spring set to vibrate in air will vibrate for a longer time as compared to the mass which vibrates partially in air and partially in liquid kept below the mass. The damping force is more when the mass moves in the liquid and hence the vibrations die out quickly in second case as compared to the first.



. Damped oscillation

Expression for the period and amplitude of damped harmonic motion

The damped system is subjected to

(i) a restoring force which is proportional to displacement but oppositely directed. This is written as -μy, where μ is a constant of proportionality or force constant

(ii) a frictional force proportional to the velocity but oppositely directed. This may be written as $-r\frac{dy}{dt'}$ where 'r' is the frictional force per unit velocity

$m\frac{d^2y}{dt^2} = -\mu y - r\frac{dy}{dt}$

$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{\mu}{m}y = 0$

$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = 0$ ------------- (1)

Where r/m = 2b and $\mu/m = \omega^2$

Equation (1) is a differential equation of second degree. Let its solution is

$y = Ae^{\alpha t}$ --------- (2)

A and α are arbitrary constants

Differentiating equation (2) with respect to 't' we get

$\frac{dy}{dt} = \alpha A e^{\alpha t}$

$\frac{d^2y}{dt} = \alpha^2 A e^{\alpha t}$

Substituting these values in equation (1), we have

$\alpha^2 Ae^{\alpha t} + 2b\alpha Ae^{\alpha t} + \omega^2 Ae^{\alpha t} = 0$

$Ae^{\alpha t}(\alpha^2 + 2b\alpha + \omega^2) = 0$

In the above equation

$Ae^{\alpha t} \neq 0$

$\alpha^2 + 2b\alpha + \omega^2 = 0$

This general solution of equation is given by

$y = A_1 exp[-b \pm \sqrt{b^2 - \omega^2}]t + A_2 exp[-b \pm \sqrt{b^2 - \omega^2}]$ ------------ (3)

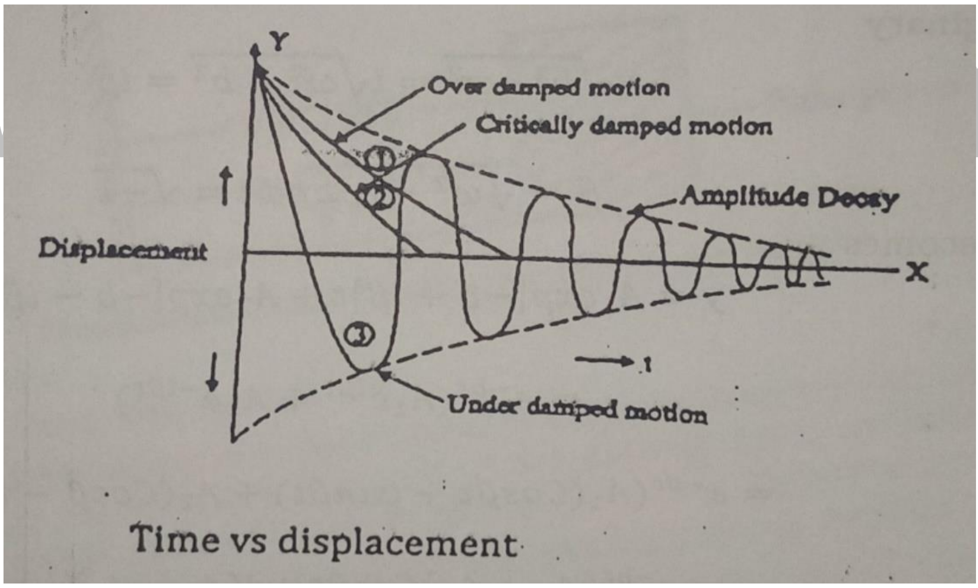
$A_1\ and A_2$ are arbitrary constants.

Case (i): When $b^2 > \omega^2$

$\sqrt{b^2 - \omega^2}$ is real and less than b

Now the powers $[-b + \sqrt{b^2 - \omega^2}]$ and $[-b - \sqrt{b^2 - \omega^2}]$ in equation 3 are –ve

Therefore, the displacement y consists of two terms both dying off exponentially without performing any oscillation. The rate of decrease of displacement is governed by the term $[-b + \sqrt{b^2 - \omega^2}]t$



This type of motion is called as over damped or dead beat

(Eg) Motion shown by a pendulum moving in thick oil

By a dead-beat moving coil galvanometer

Case (ii): When $b^2 = \omega^2$

If we put $b^2 = \omega^2$ in equation (3) then this solution does not satisfy the differential equation (1). Let us consider $\sqrt{b^2 - \omega^2}$ is not zero but equal to a small quantity h ie, $\sqrt{b^2 - \omega^2} = h$

Equation (3) reduces to

$y = A_1 exp[-b + h]t + A_2 exp[-b - h]t$

$= e^{-bt}[A_1 e^{ht} + A_2 e^{-ht}]$

$= e^{-bt}[A_1(1 + ht + \cdots) + A_2(1 - ht + \cdots)]$

$= e^{-bt}[(A_1 + A_2) + ht(A_1 - A_2) + \cdots]$

$y = e^{-bt}(p + qt)$ -------------- (4)

Equation (4) represents possible form of solution as 't' increases but e^{-bt} decreases. Hence the displacement due to (p +qt) increases at first and decreases by e^{-bt} then, which approaches to zero as 't' increases.

In this case exponential term is –b t where as in first case its more than –b t, hence the particle tends to acquire its position of equilibrium much more rapidly than case (i). Such motion is called critical damped motion.

(Eg) This motion is exhibited by many pointer instruments such as voltmeter, ammeter etc., in. Which the pointer moves to the correct position and comes to rest without any oscillation.

Case (iii): when $b^2 < \omega^2$

$\sqrt{b^2 - \omega^2}$ is imaginary

$\sqrt{b^2 - \omega^2} = i\sqrt{\omega^2 - b^2} = i\beta$

Equation (3) becomes

$y = A_1 exp[-b + i\beta]t + A_2 exp[-b - i\beta]t$

$= e^{-bt}\left(A_1 e^{i\beta t} + A_2 e^{-i\beta t}\right)$

$= e^{-bt}(A_1(\cos\beta t + i\sin\beta t) + A_2(\cos\beta - i\sin\beta t)$

$= e^{-bt}(A_1 + A_2)\cos\beta t + i(A_1 - A_2)\sin\beta t$

$= e^{-bt}[a\sin\varphi\cos\beta t + a\cos\varphi\sin\beta t]$

Where $\sin\varphi = (A_1 + A_2); a\cos\varphi = i(A_1 - A_2)$

$y = e^{-bt}a\sin(\beta t + \varphi)$

$y = ae^{-bt}\sin\left[\left(\sqrt{\omega^2 - b^2}\right)t + \varphi\right]$ -------------- (5)

This equation represents simple harmonic with amplitude ae^{-bt} and time period

$T = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2 - b^2}}$

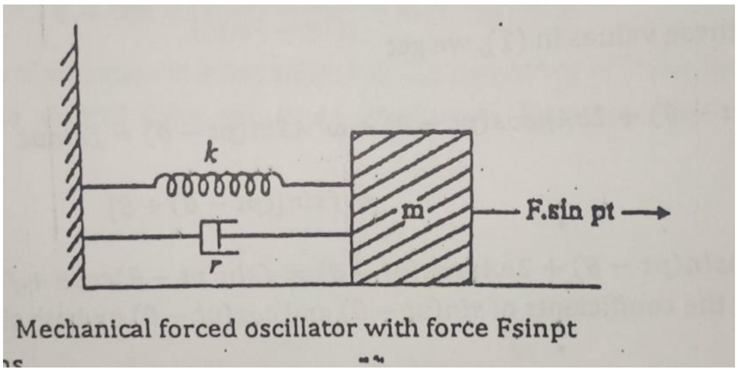
e^{-bt} is called the dampling factor

The value of $\sqrt{\omega^2 - b^2)t} + \varphi$ varies between +1 and -1. Hence the amplitude also varies betweenae^{-bt} and $-ae^{-bt}$. The decay of amplitude depends upon damping coefficient b. It is called "under damped" motion. In this case the period is slightly increased or frequency decreased because the period is now $\frac{2\pi}{\sqrt{\omega^2 - b^2}}$ which is in the absence of damping it was $\frac{2\pi}{\omega}$

The example for this type of pendulum is the motion of a pendulum in air. The motion of the coil of ballistic galvanometer or the electric oscillation of the L-C-R circuit.

Forced Oscillations:

Forced vibrations can be defined as the vibrations in which the body vibrates with a frequency other than its natural body frequency under the action of an external periodic force.



Mechanical forced oscillator with force Fsinpt

Theory of forced vibration's:

The forces acted upon the particle are

(i) A restoring force which is proportional to displacement but oppositely directed. This is written as -µy, where µ is a constant of proportionality or force constant.

(ii) A frictional force proportional to the velocity but oppositely directed. This may be written as $-r\frac{dy}{dt}$, where 'r' is the frictional force per unit velocity

(iii) The external periodic force, represented by F sin p t where F is maximum value of this force and $\frac{p}{2\pi}$ is its frequency.

Total force acting on the particle is given by

$$-\mu y - r\frac{dy}{dt} + F\sin pt$$

By Newton's second law of motion this must be equal to the product of mass 'm' of the particle and its instantaneous acceleration i.e.

$$m\frac{d^2y}{dt^2} = -\mu y - r\frac{dy}{dt} + F\sin pt$$

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{\mu}{m}y = \frac{F}{m}\sin pt$$

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f\sin pt \quad \text{------------ (1)}$$

Where $\frac{r}{m} = 2b;\ \frac{\mu}{m} = \omega^2\ and\ \frac{F}{m} = f$

Equation (1) is the differential equation of motion of the particle.

At steady state the particle vibrates with the frequency of applied force and not with its own natural frequency. The solution of differential equation (1) must be

$$y = A\sin(pt - \theta) \quad \text{----------- (2)}$$

Displacement 'y' lags behind the applied force $F\sin pt$. $A\ and\ \theta$ are arbitrary constants.

Differentiating equation (2) we get

$$\frac{dy}{dt} = Ap\cos(pt - \theta)$$

$$\frac{d^2y}{dt^2} = -Ap^2\sin(pt - \theta)$$

Substitute these values in (1), we get

$$-Ap^2\sin(pt-\theta)+2bA\cos(pt-\theta)+\omega^2 A\sin(pt-\theta)=f\sin pt$$

$$=f\sin[(pt-\theta)+\theta]$$

$A(\omega^2-p^2)\sin(pt-\theta)+2bA\cos(pt-\theta)=f\sin(pt-\theta)\cos\theta+f\cos(pt-\theta)\sin\theta$

Comparing the coefficients of $\sin(pt-\theta)$ *and* $\cos(pt-\theta)$ *on both sides*

$$A(\omega^2-p^2)=f\cos\theta \quad \text{---------- (3)}$$

$$2bAp=f\sin\theta \quad \text{----------- (4)}$$

Squaring and adding equation (3) and (4)

$$A^2(\omega^2-p^2)^2+4b^2A^2P^2=f^2$$

$$A^2[(\omega^2-p^2)^2+4b^2p^2]=f^2$$

$$A=\frac{f}{\sqrt{[(\omega^2-p^2)^2+4b^2p^2]}} \quad \text{----------- (5)}$$

While on dividing equation (4) by (3), we get

$$\tan\theta=\frac{2bAp}{a(\omega^2-p^2)}$$

$$\theta=\tan^{-1}\left[\frac{2bAp}{a(\omega^2-p^2)}\right] \quad \text{--------- (6)}$$

Equation (5) gives the amplitude of forced vibration while equation (6) gives its phase.

Depending upon the relation values of p and ω the following cases are possible

Case (i) When driving frequency is low i.e., $P<<\omega$

$$A=\frac{}{\sqrt{[(\omega^2-p^2)^2+4b^2p^2]}}=\frac{f}{\omega^2}=\cos st$$

$$\theta=\tan^{-1}\left[\frac{2bAp}{a(\omega^2-p^2)}\right]=\tan^{-1}(0)=0$$

Amplitude of vibration is independent of the frequency of force. Amplitude depends on the magnitude of the applied force and force constant µ. The force and displacement are always in phase

Case (ii) when $p=\omega$

$$A = \frac{f}{2bp} = \frac{f}{r\omega}$$

$$\theta = \tan^{-1}\left[\frac{bp}{o}\right] = \tan^{-1}(\infty) = \frac{\pi}{2}$$

Amplitude of the vibration is governed by damping and for small damping forces, the amplitude of vibration is quite large. The displacement lags behind the force by a phase$\frac{\pi}{2}$.

Case (iii): When $P >> \omega$

The frequency of the force is greater than the natural frequency ω of the body.

$$A = \frac{f}{\sqrt{[(\omega^2-p^2)^2+p^2]}} = \frac{f}{p^2} = \frac{F}{mp^2}$$

$$\theta = \tan^{-1}\left[\frac{2bAp}{a(\omega^2-p^2)}\right]$$

$$= \tan^{-1}\left[\frac{2bp}{p}\right]$$

$$= \tan^{-1}(-0) = \pi$$

The amplitude A goes on decreasing and phase difference tends towards π.

6. a) Demonstrate the working of any one type of fiber optic pressure sensor.

[OR]

b) Explain the construction and working of pressure sensors.

1. Temperature sensor:

Principle:

It is based on the principle of interference.

Description:

It consists of a laser source which emits powerful beam of light. A beam splitter is kept inclined at an angle of 45^0 to the inclined beam. It has two bundles of fibers. One is used as reference fiber and the other as test fiber. Separate lens system is provided to collect and focus the beams.

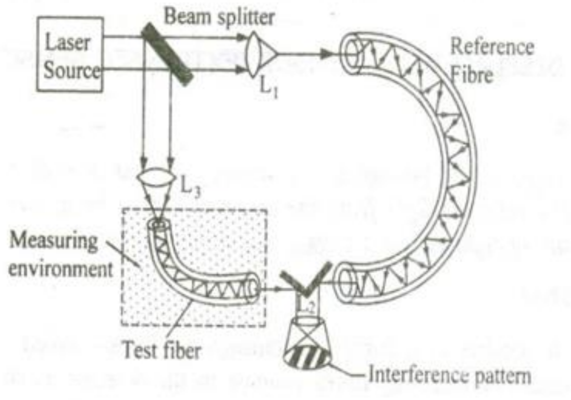


Image source: https://www.brainkart.com

Working:

A monochromatic beam of light from a laser source is allowed to fall on the beam splitter. The incident beam is split into two beams of equal intensities. One is reflected and the other is transmitted. The reflected beam is focused by means of lens L_2 on to the test fiber, where the parameters to be measured is applied. The transmitted beam is focused by lens L_1 on to the reference fiber. The light beam passing through the test fiber is subjected to a path difference due to changes in temperature of pressure. The beams emerging from both the fibers are reflected by mirrors M_1 and M_2 are made to fall on the lens L_3. A path difference is produced between the two beams which forms an interference pattern. With the help of interference pattern, the change in temperature can be measured easily.