PH8151 ENGINEERING PHYSICS

UNIT-I PROPERTIES OF MATTER

13-Marks

1. **(a) Define bending moment. Derive an expression for the internal bending moment of a beam in terms of radius of curvature.**

[OR]

(b) What is meant by bending moment of a beam. Derive the expression for the bending moment of a beam.

Definition:

The moment of the couple due to elastic reaction which balances the external couple due to the applied load is called bending moment.

Consider a portion ABCD of a bent beam. P and Q are two points on the neutral axis MN. R is the radius of curvature of the neutral axis. θ is the angle subtended by bent beam at its centre of curvature O. i.e.?$< POQ = \theta$

Consider two corresponding points P' and Q' on a parallel layer at a distance 'X' from the neutral axis,

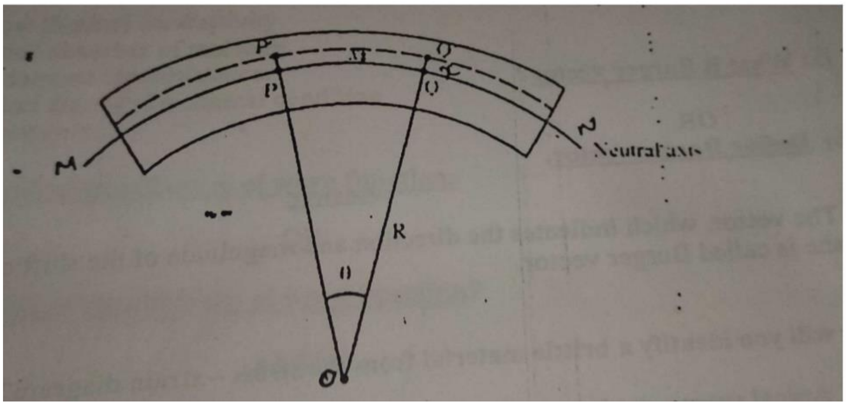
$$PQ = R\theta$$

Corresponding length on the parallel layer,

$$P'Q' = (R + X)\theta$$

Increase in length of P'Q' = P'Q' – PQ $= (R + X)\theta - R\theta$

$$= R\theta + X\theta - R\theta = X\theta$$



Increase in length of P'Q' = xθ

Linear strain produced = $\frac{Increase\ in\ length}{Original\ length} = \frac{X\theta}{R\theta} = \frac{X}{R}$

If 'Y' is the young's modulus of the material, then

Young's modulus, Y = $\frac{Linear\ stress}{Linear\ strain}$

Linear stress = Y x Linear strain = $\frac{YX}{R}$

If $'\delta A'$ is the area of cross-section of the layer, then

Force acting on the area $\delta A = Stress\ x\ area$ $[\therefore Stress = \frac{Force}{Area}$

$$= \frac{YX}{R}\delta A$$

Moment of this force about the neutral axis MN = $\frac{YX}{R}\delta A\ x\ x$

The sum of the moments of forces acting on all the layer = $\sum \frac{Y}{A}\delta A\, x^2$

$$= \frac{Y}{R}\sum \delta A\, x^2$$

$$= \frac{YI}{R}$$

$\sum \delta A\, x^2 = I$ is called geometrical moment of inertia of the cross-section of the beam.

The sum of moments of forces acting on all the layers is the internal bending moment and which comes into play due to elasticity.

Thus, the internal bending moment of the beam = $\frac{YI}{R}$.

Note:

For a rectangular beam of breadth b and thickness d, the geometrical moment of inertia is given by $I = \frac{bd^3}{12}$

Similarly, for a beam of circular cross section,

$$I = \frac{\pi r^4}{4}$$

Where, 'r' is the radius of the rod.

Internal Bending moment of rectangular beam

$$= \frac{Ybd^3}{12R}$$

Internal Bending moment of circular cross-section

$$= \frac{Y\pi r^4}{4R}$$

2. a) What is Torsion Pendulum? Explain how it is used to determine the moment of inertia and rigidity modulus of the material of a thin wire.

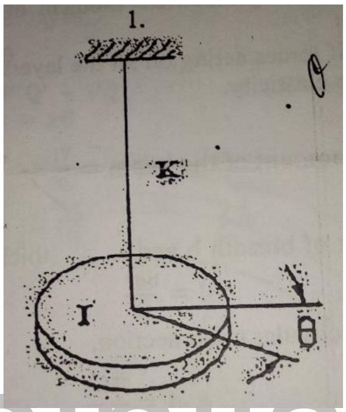
(Or)

b) Derive an expression for the time period of torsional pendulum. How will you find a rigidity modulus of a wire by using this method?

A circular metallic disc suspended using a thin wire that executes torsional oscillation is called torsion pendulum.

<u>Theory:</u>

Torsion pendulum consists of a metallic circular disc D suspended by means of a wire with one end fixed to the torsion head and the other and fixed to the centre of the circular disc. Let 'L' be the length 'r' be the radius of the wire. Let 'n' be the rigidity modulus of the wire and 'c' be the couple per unit twist.



According to the law of conservation of energy, the total energy is conserved.

Total energy = Potential energy + Kinetic energy ------------ (1)

Potential energy is equal to the work done in twisting the disc, thereby creating a restoring couple.

$$Potential\ energy = \int_0^{\theta} moment\ of\ couple\ X d\theta$$

$$= \int_0^{\theta} C\theta\ x\ d\theta$$

$$= C\theta^2/2 \quad \text{---------- (2)}$$

Kinetic energy $= \frac{1}{2} I\omega^2$

$\omega = \frac{d\theta}{dt}$ *is the angular velocity* and

Angular acceleration $= \frac{d}{dt}\left[\frac{d\theta}{dt}\right] = \frac{d^2\theta}{dt^2}$

Kinetic energy $= \frac{1}{2} I \left(\frac{d\theta}{dt}\right)^2$ -------------- (3)

Sub (2) & (3) in (1), we get

Total Energy $= \frac{1}{2} I \left(\frac{d\theta}{dt}\right)^2 + C\theta^2/2$ ------------- (4)

Total energy is constant

Differentiating with respect to time, we get

$\frac{1}{2} I\ x\ 2\frac{d\theta}{dt}\ x\ \frac{d^2\theta}{dt^2} + \frac{1}{2} C\ x\ 2\theta\left(\frac{d\theta}{dt}\right) = 0$

$\frac{d\theta}{dt}\left[I\frac{d^2\theta}{dt^2}+C\theta\right]=0$

Here, $\frac{d\theta}{dt}\neq 0$ [because angular velocity$\neq 0$]

$\left[I\frac{d^2\theta}{dt^2}+C\theta\right]=0$

$\frac{d^2\theta}{dt^2}=\frac{C}{I}\theta$ ------------ (5)

We know, Time period of oscillation is given by

$T=2\pi\sqrt{\frac{Displacement}{Acceleration}}$

$=2\pi\sqrt{\frac{\theta}{\frac{C}{I}\theta}}$

$T=2\pi\sqrt{\frac{1}{C}}$ ------------- (6)

but, Twisting couple, C = $\frac{\pi nr^4}{2L}$

Sub the value of C in eq (6), we get

$T=2\pi\sqrt{\frac{2LI}{\pi nr^4}}$

$T^2=4\pi^2\left(\frac{2LI}{\pi nr^4}\right)$

Rigidity Modulus of the wire, $n=\frac{8\pi Il}{T^2r^4}$

The torsion pendulum is formed by the given circular disc suspended by a steel or bass wire whose rigidity modulus is to be determined.

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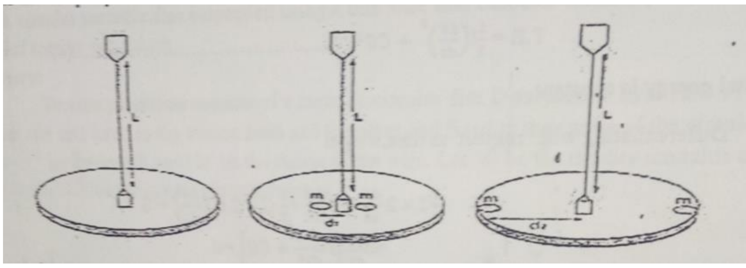
First, the disc is set into torsional oscillations without mass and the time period of oscillations (T) is noted.

$$T=2\pi\sqrt{I/C} \qquad T^2=4\pi^2 I/C$$

Were, I the moment of inertia of the disc about the axis of the wire.

C is the couple per unit twist.



Two equal symmetrical masses (each equal to 'm') are placed along a diameter of the disc at equal distances 'd_1' on either side of the centre on the disc. The disc is rotated through an angle and is then released. The period of oscillations T_1 is determined.

Then $T_1 = 2\pi\sqrt{I/C}$

$T_1^2 = 4\pi^2 I_1/C$

Here I_1 is the moment of inertial of the disc along with the symmetrical masses placed at the distance d_1.

The two masses are kept at equal distance d_2 from the centre of the disc and the corresponding period T_2 is determined.

$$T_2 = 2\pi\sqrt{\frac{I_2}{C}}$$

$$T_2^2 = 4\pi^2 I_2/C$$

Let I be the moment of inertia of the disc along the axis of the wire and 'I ' be the moment of inertia of each mass about a parallel axis passing through its centre of gravity. Then by the parallel axes theorem.

$$I_1 = I + 2I_m + 2md_1^2 \qquad I_2 = I + 2I_m + 2md_2^2$$

$$I_2 - I_1 = I + 2I_m + 2md_2^2 - I - 2I_m - 2md_1^2$$

$$I_2 - I_1 = 2m(d_2^2 - d_1^2)$$

$$T_2^2 - T_1^2 = \frac{4\pi^2}{C} I_2 - \frac{4\pi^2}{C} I_1 = \frac{4\pi^2}{C}(I_2 - I_1)$$

Substituting equation (4), we get

$$T_2^2 - T_1^2 = \frac{4\pi^2}{C} 2m(d_2^2 - d_1^2)$$

$$\frac{T^2}{T_2^2 - T_1^2} = \frac{\frac{4\pi^2}{C} I}{\frac{4\pi^2}{C}(I_2 - I_1)}$$

$$\frac{T^2}{T_2^2 - T_1^2} = \frac{I}{2m(d_2^2 - d_1^2)}$$

$I = \frac{2m(d_2^2 - d_1^2)T^2}{T_2^2 - T_1^2}$

We know, the rigidity modulus $n = \frac{8\pi IL}{T^2 r^4}$

Substituting the value of I from equation (5), we get

$n = \frac{16\pi Lm(d_2^2 - d_1^2)}{r^4(T_2^2 - T_1^2)}$

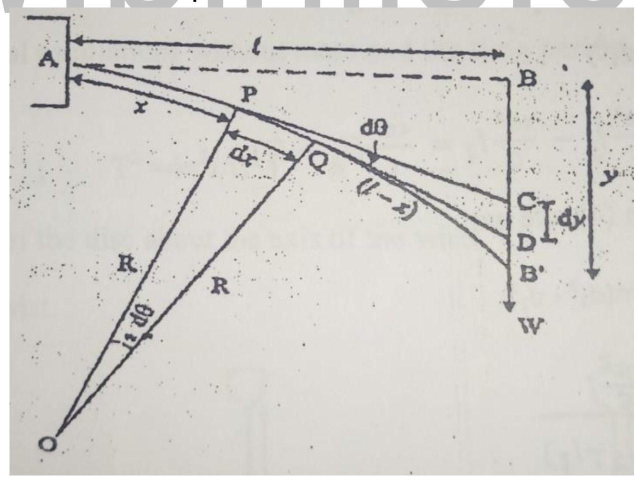
Using this relation, the rigidity modulus is determined.

3. **(a) Derive an expression for the deflection produced at the free end of a rectangular cantilever subjected to point load at free end. What will be the deflection produced at the free end with same load, if the cantilever is of circular cross section.**

[OR]

(b) Derive an expression for depression at the free end of a cantilever due to load. Describe an experiment to determine the young's modulus of the cantilever material using this depression.

- A cantilever is a beam fixed horizontally at one end and loaded at the other end.
- Consider a cantilever of length 'l' fixed at one end A and loaded at other end B by a weight W the end B is depressed to B^1.



Let BB^1 = y (depression) due to the load W applied at the free end

The moment of force = force X perpendicular distance

The moment of force [the external bending moment]

=W (1-X) ----------- (1)

We have,

The internal beading moment = YI/R --------- (2)

Here, I → geometrical moment of inertia

R → radius of curvature

Y → young's modulus

At equilibrium,

External bending moment = internal bending moment

W (l-x) = YI/R

R = YI/W (l-x) ----------- (3)

Let Q be another point at a distance dx from p, the segment PQ males an angle dθ with the centre of curvature o.

In POQ

Here, PQ =dx

And dx = Rdθ ------------ (4) [arc length = radius x angle]

Tangent drawn from P and Q meeting at C&D in CDP

CD = d y

So, d y = (1-x) dθ ---------- (5) [Arc length = radius x angle]

(4)/ (5) $\frac{dx}{dy} = R\theta/(1-x)d\theta$

$$\frac{dx}{dy} = R/(1-x)$$

$$R = (1-x)\frac{dx}{dy} \quad \text{------------- (6)}$$

From equation (3) and (6)

$$(1-x)dx/dy = YI/w(1-x)$$

$$dy = \frac{W}{YI}(1-x)(1-x)dx$$

$$dy = \frac{W}{YI}(1-x)^2dx$$

The net depression of the cantilever

$$y = \frac{W}{YI}\int_0^1(1-x)^2dx$$

$$= \frac{W}{YI}\int_0^1(1-x)^2dx$$

$$y = WI^3/3YI$$ ---------------- (7)

This is the expression for depression of a cantilever

In case of rectangular beam

$$I = \frac{bd^3}{12}, \qquad w = mg$$

$$Y = \frac{4mgl^3}{bd^3y}$$

$$Y = \frac{4mgl^3}{bd^3y}$$

The young's modulus of the material of a cantilever beam

$$Y = \frac{4mgl^3}{bd^3y} \quad Nm^{-2}$$

Experiment:

- The given beam is fixed rigidly at one end and loaded at the other end.
- A pin is fixed at thee loaded end
- A travelling microscope Is focused the tip of the pin
- Microscope is adjusted so that the horizontal crosswire coincides with the image of the tip of the pin.
- The reading on the vertical scale is noted.
- The weights are added in steps of 50g (loading)
- The microscope reading is noted in each case.
- The experiment is repeated by decreasing the weight by 50g (un loading)
- The mean depression "y" for a load of M kg is found out from the tabular column.
- The length of the cantilever "l" between the fixed end and loaded end.
- The breadth "b" and thickness "d" of the cantilever are measured using vernier caliper and screw gauge respectively.
- The value is substituted in to the equation. (young's modulus)

$$Y = \frac{4mgl^3}{bd^3y} \qquad Nm^{-2}$$

Load	Microscope reading			Depression (y) for a load of M kg
	Loading	Unloading	Mean	

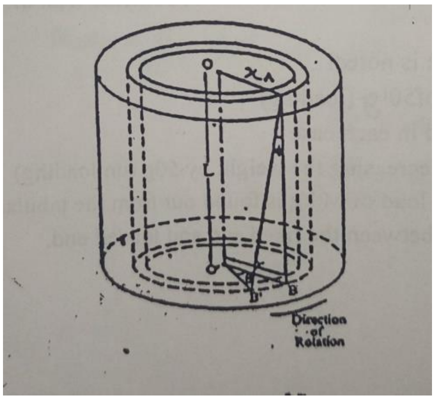
4. (a) Derive an expression for the torsion couple per unit angular twist when a cylinder is twisted.

[OR]

(b) Derive an expression for couple per unit twist for a cylinder.

Let us consider a cylindrical wire of length 'L' and radius 'r'. The wire is fixed at its upper end and twisted through an angle $'\theta'$ by applying a torque at the lower end. The wire can be assumed to be made up of a number of hollow cylindrical tubes (co-axial) whose radii varies from 0 to r.

Let us consider one such cylinder of radius x and thickness dx.



Due to the twisting torque the line AB which is initially parallel to the axis OO' of the cylinder is displaced to a position AB' through an angle.

The result of twisting the cylinder is shearing strain

The angle of shear = $< BAB = \varphi$

$In\ \frac{BAB'}{BB'} = L\emptyset$ and $In\ \frac{BOB'}{BB'} = x\theta$

So, $L\emptyset = x\theta$, $\varphi = \frac{x\theta}{L}$

$Rigiditymodulus(n) = \frac{tangentialstress}{Shearingstrain} = \frac{shearingstress}{angleofshear}$

$Shearing\ stress = n\emptyset$

Substituting for ∅ from eqn (1) in eqn (2), we have

$Shearing\ stress = \frac{nx\theta}{L}$

We know, $Shearingstress = \frac{Shearing\ force}{area}$n

Shearing force = Shearing stress

x area on which the shearing force is acting

$F = \frac{nx\theta}{L}.2\pi x dx$

Where $2\pi x dx$ is the area over which the shearing force acts as shown in figure.

Therefore, Moment of the force about the OO' axis of the cylinder

= Shearing force x distance

$= \frac{nx\theta}{L}.2\pi x.dx.x$

$= \frac{2\pi n\theta}{L}.x^3.dx$

Twisting couple of the whole wire can be derived by integrating eqn. (5)

Within the limits 0 to r (since the radii varies from 0 t0 r)

$Twisting couple\ on\ the\ wire\ C = \int_0^r \frac{2\pi n\theta}{L}.x^3 dx$

(or) $C = \frac{2\pi n\theta r^4}{L}$ $(0r)$ $C = \frac{\pi n\theta r^4}{2L}$

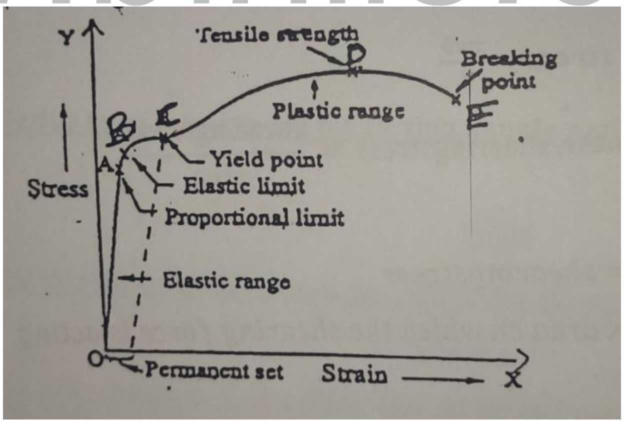
If twist θ is unity i.e., $\theta = 1$ radian then we can wire

The couple per unit twist $C = \frac{n\pi r^4}{2L}$

5. (a)Draw the typical stress-strain diagram of a wire and mention its uses.

[OR]

(b) Discuss the behaviour of ductile material under loading using stress-strain diagram.



The above graph is the stress-strain diagram of an elastic wire under loading.

<u>OA region:</u>

The portion OA of the curve is straight line which shows that stress is proportional to the strain, i.e., Hooke's law is obeyed upto A.

<u>AB region:</u>

Point B is the elastic limit of the material. This point lies near the point A and upto this point the wire return back to its original length, when the load is removed.

BC region:

If the stress is further increased beyond the elastic limit, the stress-strain curve takes a bend. In this region, if wire is unloaded, it does not regain its original length.

CD region:

The material of the wire flows beyond C, which is known as plastic flow.

DE region:

Beyond point D, when load is further increased, the thinning of wire and diameter of a wire decreases further a breaking point E is reached.

Uses:

Determine elastic strength, yield strength and tensile strength.
Estimate the working stress
The area under the curve gives the energy required to deform it.

6. a) Compare uniform and non-uniform bending.

[OR]

b) Derive an expression for finding the young's modulus of a material using uniform and non-uniform bending.

a) Appraise the properties and applications of I shape grinders.

[OR]

b) Explain I shaped grinders. Give its application, merits and demerits.

b) i) Uniform Bending:

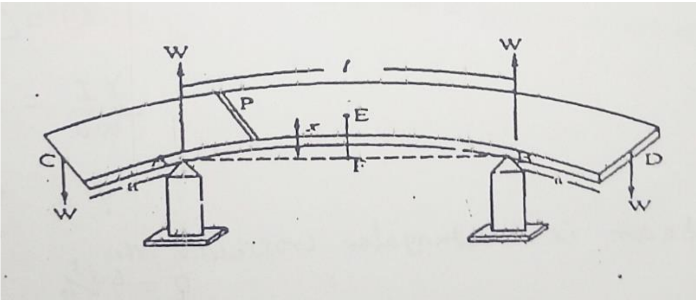
The beam is loaded uniformly on its both ends, the bending of the beam forms an arc of a circle. The elevation is produced in the beam. This type of bending is known as uniform bending.

Expression:

Consider a beam 'AB' arranged horizontally on two knife edges 'C' and 'D' symmetrically so that AC = BD = a.

The beam is loaded with equal weights 'W' at each end's 'A' and 'B'.

The reactions on the knife edges at 'C' and 'D' are equal to 'W' and they are acting vertically upwards.



The external bending moment on the part 'AF' of the beam is = WA

Internal bending moment $= \frac{YI}{R}$

Were,

Y → Young's modulus of the beam

I → Moment of inertia of the beam

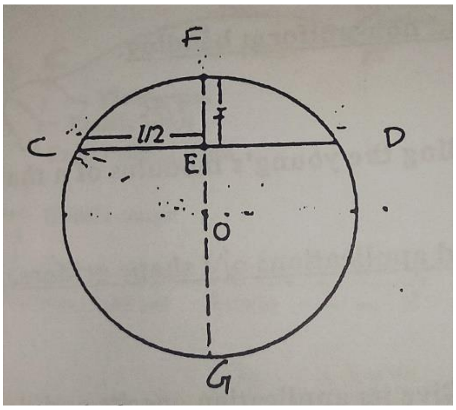
R → Radius of curvature of the beam at F.

In the equilibrium position,

External bending moment = Internal bending moment

$Wa = \frac{YI}{R}, \quad R = \frac{YI}{wa}$ ----------- (1)

Since for a given value of 'W', the value of 'a', '.' And 'I' are constant, 'R' is constant so that the beam bends uniformly into an, arc of a circle of radius 'R'



Intersecting chord theorem of a circle

'CD = 1', 'y' is the elevation of the midpoint 'E' of the beam y = EF

Then, from the property of the circle (figure), [Law of segment]

EF x EG = CE x ED

$$y(2R - y) = \left(\frac{1}{2}\right)^2$$

$$2Ry - y^2 = \frac{l^2}{4}$$ [y^2 neglected]

$$2Ry = \frac{l^2}{4}$$

$$R = \frac{l^2}{8y} \text{ ----------- (2)}$$

(1) = (2)

$$\frac{YI}{wa} = \frac{l^2}{8y}$$

$$Y = \frac{wal^2}{8yI}$$

If the beam is rectangular cross-section, then $I = \frac{bd^3}{12}$

Where b is breadth and d thickness of the beam. If M is the mass, the corresponding weight W = Mg where, M is the mass suspended at the free end g is acceleration due to gravity.

Substituting for W and I in equation, we get,

$$Y = \frac{Mgal^2}{8\frac{bd^3}{12}y}$$

$$Y = \frac{Mgal^2}{\frac{2bd^3y}{3}}$$

$$Youngs\ Modulus, Y = \frac{3Mgal^2}{2bd^3y}$$

<u>Experiment:</u>

A rectangular beam AB of uniform-section is supported horizontally on two knife edges A and B as shown in Figure. Two weight hangers of equal masses are suspended at the ends of the beam. A pin is arranged vertically at the midpoint of the beam. A microscope is focused on the tip of the pin. A travelling microscope is focused on the image of the tip of the pin such that the horizontal crosswire coincides with the tip of the pin.

The readings in the vertical scale of microscope are noted. The weights are added in steps of 50gm to the weight hanger. The microscope is adjusted each time so that the horizontal cross-wire coincides with the image of the tip of the pin. The microscope reading is noted in each case. The experiment is repeated for decreasing loads and the readings are tabulated in the tabular column.

Load	Microscope reading			Elevation (y) for a load of M kg
	Increasing load	Decreasing load	Mean	

The mean elevation for a load of M kg is found out from the tabular column. The length of the beam 'l' between the knife edges & the distance AC = BD = a is noted.

The breadth 'b' and the thickness'' of the beam are measured using a vernier caliper & screw-gauge respectively.

$$Y = \frac{3Mgal^2}{2bd^3y} \qquad Nm^{-2}$$

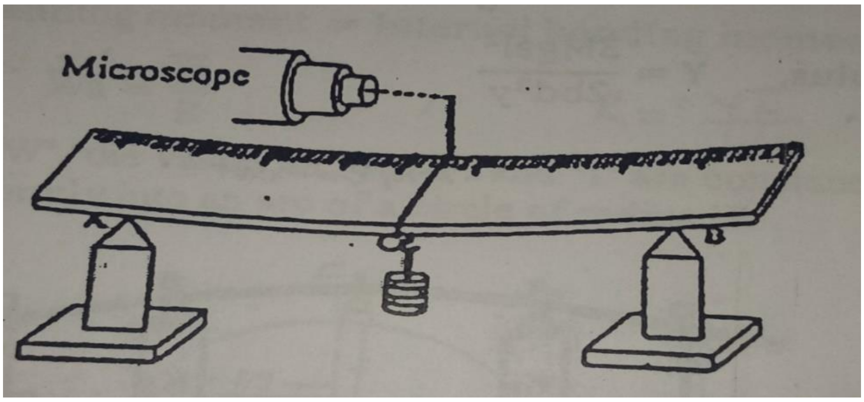
M – mass, g – acceleration due to gravity, l – length between two knife edges, y – elevation.

The Young's modulus of the material of the beam is determined.

Non-uniform Bending Experiment:

Description:

It consists of a beam, symmetrically supported on the two knife edges A and B. A weight hanger is suspended at the centre (C) of the beam by means of loop or thread. A pin is fixed vertically at 'C' by some wax.



In order to focus the tip of the pin a travelling microscope (M) is placed in front of this arrangement.

Procedure:

Taking the weight hanger as the dead load (W) the microscope is adjusted and the tip of the pin is made to coincide with the horizontal cross wire. The readings are noted from the vertical scale of the microscope.

The weights are added in steps of m, 2m, 3m, kilograms and the corresponding reading are noted in the tabular column as shown. The mean depression Y is found for a load of M kg.

S.no	Load (M)	Microscopic reading			Elevation
		loading	Unloading	Mean	
Unit	x 10^{-3} m	x 10^{-2} m	x 10^{-2} m	x 10^{-2} m	Meter

Theoretically, we know the depression produced is $y = \frac{WI^3}{48II_g}$

Were, 'I' be the length of the beam. If 'b' is the breadth of the beam and 'd' is the thickness of the beam, then geometrical moment of inertia.

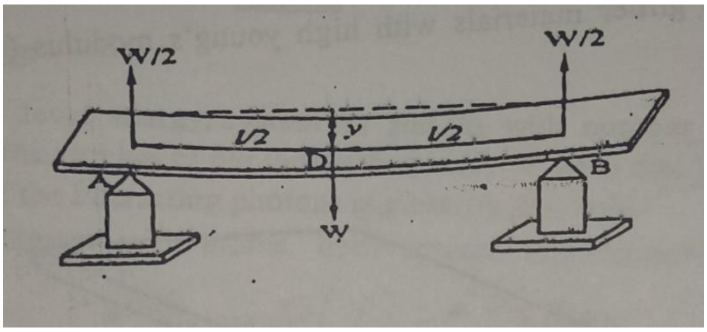
Also, the weight W = Mg, $I_y = \frac{bd^3}{12}$

$$y = \frac{Mgl^3}{4ybd^3}$$

$$Young's modulus\ Y = \frac{Mgl^3}{4bd^2y} \quad Nm^{-2}$$

Substituting the mean value of (M/y) from the tabular column, the young's modulus Y can be determined.

Let us consider a beam of length 'l' supported on the two knife edges A and B. The load 'W' is suspended at the centre 'C'. It is found that the beam bends and the maximum displacement is at the point 'D', where the load is given.



Due to the load (W) applied, at the middle of the beam the reaction W/2 is acted vertically upwards at each knife edges. The bending is called as non-uniform bending.

The beam may be considered as two cantilevers, whose free end carries a load W/2 each of length 1/2 and fixed at the point 'D'.

Hence, we can say the elevation of A above D as the depression of D below 'A'

We Know depression of a cantilever

$$y = \frac{Wl^3}{3YI}$$

Therefore, substituting the value of l as l/2 and W is W/2 in the expression for the depression of a cantilever,

$$depression = y = \frac{(w/2)\left(\frac{1}{2}\right)^3}{3Yi}$$

$$y = \frac{Wl^3}{48YI}$$

For rectangular cross-section, $I = \frac{bd^3}{12}$

i.e., The depression $y = \frac{Wl^3}{48Y\frac{bd^3}{12}}$

$Y = \frac{Wl^3}{4Ybd^3}$

Here, W = mg, so, $y = \frac{mgl^3}{4bd^3y}$

The young's modulus of the material of the beam $y = \frac{mgl^3}{4bd^3y}$

ii) I shaped girder:

A girder is a metallic beam supported at its ends by pillars or an opposite wall.

A metallic beam with upper & lower cross-sections broadened and the middle section tapered. So that it can withstand heavy loads over it is called I-shaped girder.

It is bent non uniformly under its own weight to the form of an inverted double cantilever.

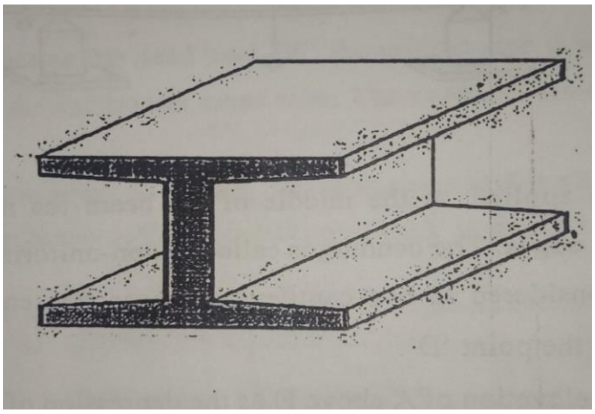
$$Y = \frac{Mgl^3}{4bd^3y}$$

Where, y is the depression, b is the breadth of the beam, d is the thickness of the beam, Y is the young's modulus, M is the mass, g is the acceleration due to gravity.

To reduce the depression for any load, y, b & d should be large l and m should be small, depression y is inversely proportional to $d^{3,}$ by increasing the thickness d, the increasing breadth 'b' of the beam. So, the beam is designed to have a large thickness to minimize bending.

By selecting the girder materials with high young's modulus (steel) one can get the small depression.

Diagram



Advantages:

1) The cost of material is low
2) This type of cross-section provides a high bending moment

3) Lot of material is saved
4) It has high young's modulus, more strength & durability

Applications:

1)In large apartments, the beam of pillars has a shape of I.
2)In construction of bridges, iron rails in railway tracks and in machine bases.
3)In support frames & columns for trolley way.