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Question Paper Code : 77197

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Civil Engineering

MA 6459 — NUMERICAL METHODS

(Common to Aeronautical Engineering, Electrical and Electronics Engineering, Instrumentation and Control Engineering, Electronics and Instrumentation Engineering, Instrumentation and Control Engineering, Geoinformatics Engineering, Petrochemical Engineering, Production Engineering, Chemical and Electrochemical Engineering, Textile Chemistry and Textile Technology)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Interpret Newton Raphson method geometrically.

2. Which of the iterative methods for solving linear system of equations converge faster? Why?

3. Given $y_0 = 3$, $y_1 = 12$, $y_2 = 81$, $y_3 = 200$, $y_4 = 100$. Find $\Delta^4 y_0$.

4. Distinguish between Newton divided difference interpolation and Lagrange's interpolation.

5. Find $y'(0)$ from the following table.

x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

6. Using two point Gaussian quadrature formula evaluate $I = \frac{\pi}{4}\int_{-1}^{1} \sin\left(\frac{\pi t + \pi}{4}\right) dt$.

7. Find by Taylor's series method, the value of y at $x = 0.1$ from $\frac{dy}{dx} = y^2 + x$, $y(0) = 1$.

8. Distinguish between single step methods and multi-step methods.

9. Classify the following equation : $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$.

10. Express $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ in terms of difference approximation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Using Newton Raphson method find the real root of $f(x) = 3x + \sin(x) - e^x = 0$ by choosing initial approximation $x_0 = 0.5$. (8)

(ii) Determine the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. (8)

Or

(b) (i) Apply Graeffe's method to find all the roots of the equation $x^3 - 2x^2 - 5x + 6 = 0$ by squaring thrice. (8)

(ii) Solve the following system of equations, starting with the initial vector of [0, 0, 0] using Gauss-Seidel method. (8)

$$6x_1 - 2x_2 + x_3 = 11$$
$$-2x_1 + 7x_2 + 2x_3 = 5$$
$$x_1 + 2x_2 - 5x_3 = -1$$

12. (a) (i) Using Lagrange's interpolation find the interpolated value for $x = 3$ of the table. (8)

x :	3.2	2.7	1.0	4.8
$f(x)$:	22.0	17.8	14.2	38.3

(ii) The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface.

x = height :	100	150	200	250	300	350	400
y = distance :	10.63	13.03	15.04	16.81	18.42	19.9	21.27

Find the values of y when $x = 218$ ft using Newton's forward interpolation formula. (8)

Or

(b) (i) Employ a third order Newton polynomial to estimate l_{n2} with the four points given in table. (8)

x :	1	4	6	5
$f(x)$:	0	1.386294	1.791759	1.609438

(ii) The following values of x and y are given in table : (8)

x :	1	2	3	4
y :	1	2	5	11

Find the cubic splines and evaluate $y(1.5)$.

13. (a) The velocity v (km/min) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows :

t :	0	2	4	6	8	10	12
v :	0	10	18	25	29	32	20

(i) Estimate approximately the distance covered in 12 minutes, by Simpson's 1/3rd rule. (8)

(ii) Estimate the acceleration at $t = 2$ seconds. (8)

Or

(b) (i) Given that :

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y :	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ at $x = 1.1$. (8)

(ii) Use the Romberg method to get an improved estimate of the integral from $x = 1.8$ to $x = 3.4$ from the data in table with $h = 0.4$ (8)

x :	1.6	1.8	2.0	2.2	2.4	2.6
$f(x)$:	4.953	6.050	7.389	9.025	11.023	13.464
x :	2.8	3	3.2	3.4	3.6	3.8
$f(x)$:	16.445	20.056	24.533	29.964	36.598	44.701

14. (a) Solve the initial value problem $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ to find $y(0.4)$ by Adam's Bashforth predictor corrector method and for starting solutions, use the information below.

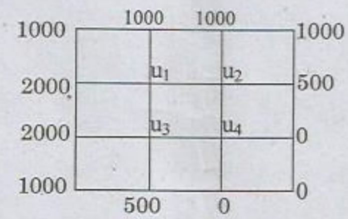
$y(0.1) = 0.9117$, $y(0.2) = 0.8494$. Compute $y(0.3)$ using Runge Kutta method of fourth order. (16)

Or

(b) (i) Employ the classical fourth order Runge-Kutta method to integrate $y' = 4e^{0.8t} - 0.5y$ from $t = 0$ to $t = 1$ using a stepsize of 1 with $y(0) = 2$. (8)

(ii) Given $\frac{dy}{dx} = xy + y^2$ and $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 0.2267$, evaluate $y(0.4)$ by Milne's predictor corrector method. (8)

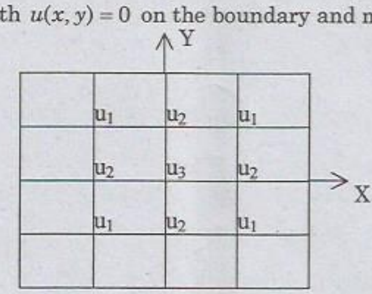
15. (a) (i) Given the values of $u(x, y)$ on the boundary of the square in fig. evaluate the function $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of this fig. by Gauss Seidel method. (8)



(ii) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the condition $u(x,0) = \sin \pi x$, $0 \le x < 1$; $u(0,t) = u(1,t) = 0$ using Crank-Nicolson method. (8)

Or

(b) (i) Solve the Poisson's equation $\nabla^2 u = 8x^2y^2$ for the square mesh of fig. with $u(x, y) = 0$ on the boundary and mesh length = 1. (8)



(ii) Evaluate the Pivotal values of the equation $u_{tt} = 16u_{xx}$ taking $\Delta x = 1$ upto $t = 1.25$. The boundary conditions are $u(0,t) = u(5,t) = u_t(x,0) = 0$ and $u(x,0) = x^2(5 - x)$. (8)