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**Question Paper Code : 91781**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Second Semester

Civil Engineering

MA 6251 – MATHEMATICS – II

(Common to All Branches Except Marine Engineering)

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Prove that  $3x^2y\bar{i} + (yz - 3xy^2)\bar{j} - \frac{z^2}{2}\bar{k}$  is a solenoidal vector.
2. State Green's theorem.
3. Solve :  $(D^3 + D^2 + 4D + 4)y = 0$ .
4. Transform the equation  $(2x + 3)^2 y'' - 2(2x + 3) y' + 2y = 6x$  into a linear differential equation with constant coefficients.
5. Prove that  $L\left(\int_0^t f(t)dt\right) = \frac{F(s)}{s}$ , where  $L(f(t)) = F(s)$ .
6. Find  $L^{-1}\left(\log\frac{s}{s-a}\right)$ .
7. The real part of an analytic function  $f(z)$  is constant, prove that  $f(z)$  is a constant function.
8. Find the critical points of the transformation  $w = z^2 + \frac{1}{z^2}$ .
9. State Cauchy's integral theorem.
10. Find the residue of  $f(z) = \tan z$  at  $z = \frac{\pi}{2}$ .

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PART - B

(5×16=80 Marks)

11. a) i) Show that  $\nabla^2(r^n) = n(n+1)r^{n-2}$  where  $r^2 = x^2 + y^2 + z^2$ . Hence find the value of  $\nabla^2\left(\frac{1}{r}\right)$ . (8)

ii) Using Green's theorem, evaluate  $\int_C (y - \sin x)dx + \cos x dy$  where C is the triangle formed by  $y = 0$ ,  $x = \frac{\pi}{2}$ ,  $y = \frac{2x}{\pi}$ . (8)

(OR)

b) Verify the Gauss divergence theorem for  $\vec{A} = (2x - z)\mathbf{i} + x^2y\mathbf{j} - xz^2\mathbf{k}$  taken over the region bounded by  $x = 0$ ,  $x = 1$ ,  $y = 0$ ,  $y = 1$ ,  $z = 0$ ,  $z = 1$ . (16)

12. a) i) Solve  $(D^3 + 2D^2 + D)y = e^{-x} + \cos 2x$ . (8)

ii) Solve  $\frac{dx}{dt} + 4x + 3y = t$ ,  $\frac{dy}{dt} + 2x + 5y = e^t$ . (8)

(OR)

b) i) Solve  $y'' + y = \sec x$ . (6)

ii) Solve  $(2x + 7)^2 y'' - 6(2x + 7) y' + 8y = 8x$ . (10)

13. a) i) Find Laplace transform of  $t^2 e^{-3t} \cos t$  and  $\int_0^t \frac{\sin t}{t} dt$ . (8)

ii) Using convolution theorem evaluate  $\int_0^t \sin u \cos(t-u) du$ . (8)

(OR)

b) i) Find the Laplace transform of  $f(t) = \begin{cases} \frac{4E}{T}t - E; & 0 \leq t \leq \frac{T}{2} \\ 3E - \frac{4E}{T}t; & \frac{T}{2} \leq t \leq T \end{cases}$  and

$f(t+T) = f(t)$  and E is a constant. (8)

ii) Solve using Laplace transform,  $x'' - 2x' + x = e^t$  when  $x(0) = 2$ ,  $x'(0) = -1$ . (8)



14. a) i) If  $f(z)$  is a regular function of  $z$ , prove that  $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log|f(z)| = 0$ . (8)
- ii) Show that the transformation  $w = \frac{1}{z}$  transforms in general, circles and straight lines into circles and straight lines. (8)
- (OR)
- b) i) Find the analytic function  $f(z) = u + iv$ , given that  $2u + 3v = e^x(\cos x - \sin y)$ . (8)
- ii) Find the bilinear transformation which maps the point  $-1, 0, 1$  of the  $z$ -plane into the points  $-1, -i, 1$  of the  $w$ -plane respectively. (8)
15. a) i) Using Cauchy's integral formula, evaluate  $\int_C \frac{z dz}{(z-1)^2(z+2)}$ , where  $C$  is the circle  $|z-1|=1$ . (8)
- ii) Using Contour integration, evaluate  $\int_0^\infty \frac{\cos mx dx}{x^2 + a^2}$ . (8)
- (OR)
- b) i) Find the Laurent's series expansion of  $f(z) = \frac{1}{z^2 + 5z + 6}$  valid in the region  $1 < |z+1| < 2$ . (8)
- ii) Evaluate  $\int_C \frac{z dz}{(z^2+1)^2}$  where  $C$  is the circle  $|z-i|=1$ , using Cauchy's residue theorem. (8)