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Question Paper Code : 91783

28/11/19

BN

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019
Third Semester
Civil Engineering
MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS
(Common to All Branches)
(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A

(10×2=20 Marks)

1. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x - a)^2 + (y - b)^2 + 1$.
2. Find the complete integral of $p + q = x + y$.
3. State the Dirichlet's conditions for a function $f(x)$ to be expanded as a Fourier series.
4. Expand $f(x) = 1$, in $(0, \pi)$ as a half-range sine series.
5. Write all possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.
6. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.
7. If the Fourier transform of $f(x)$ is $\mathfrak{F}(f(x)) = F(s)$, then show that $\mathfrak{F}(f(x - a)) = e^{ias} F(s)$.
8. Find the Fourier sine transform of $1/x$.
9. Find the Z – transform of $\frac{1}{n+1}$
10. State the final value theorem of Z transforms.

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PART - B

(5×16=80 Marks)

11. a) i) Find the general solution of $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$. (8)
- ii) Find the general solution of $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$. (8)
- (OR)
- b) i) Find the general solution of $z = px + qy + p^2 + pq + q^2$. (8)
- ii) Find the general solution of $(D^2 - 3DD' + 2D'^2 + 2D - 2D')z = \sin(2x + y)$. (8)
12. a) i) Find the Fourier series expansion of the following periodic function
 $f(x) = \begin{cases} 2+x & -2 \leq x \leq 0 \\ 2-x & 0 < x \leq 2 \end{cases}$ of period 4 Hence deduce that
 $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$. (8)
- ii) Find the complex form of Fourier series of $f(x) = e^{ax}$ in the interval $(-\pi, \pi)$ where a is a real constant. Hence, deduce that $\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{a^2 + n^2} = \frac{\pi}{a \sinh a\pi}$. (8)
- (OR)
- b) i) Find the half range cosine series of $f(x) = (\pi - x)^2$, $0 < x < \pi$. Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$. (8)
- ii) Determine the first two harmonics of Fourier series for the following data.
- | | | | | | | |
|--------|------|-----------------|------------------|-------|------------------|------------------|
| x | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | π | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ |
| $f(x)$ | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 |
- (8)
13. a) A tightly stretched string of length 'l' with fixed end points is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $y_t(x, 0) = v_0 \sin\left(\frac{3\pi x}{l}\right) \cos\left(\frac{\pi x}{l}\right)$, where $0 < x < l$. Find the displacement of the string at a point, at a distance x from one end at any instant 't'. (16)
- (OR)
- b) A square plate is bounded by the lines $x = 0$, $x = 20$, $y = 0$, $y = 20$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 20) = x(20 - x)$, $0 < x < 20$, while the other three edges are kept at 0°C . Find the steady state temperature distribution $u(x, y)$ in the plate. (16)



14. a) i) Find the Fourier Transform of $f(x)$ if $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate the integral $\int_0^{\infty} \left(\frac{\sin t}{t} \right)^4 dt$. (10)

ii) State and prove convolution theorem for Fourier transforms. (6)

(OR)

b) i) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ using transforms. (6)

ii) Find the Fourier cosine transform of $f(x) = e^{-a^2 x^2}$ and hence find $F_s [xe^{-a^2 x^2}]$. (10)

15. a) i) Find $Z(r^n \cos n\theta)$ and $Z^{-1}[(1 - az^{-1})^{-2}]$. (8)

ii) Using convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z - 1/2)(z - 1/4)}\right]$. (8)

(OR)

b) i) Using Z-transform, solve the difference equation $x(n + 2) - 3x(n + 1) + 2x(n) = 0$ given that $x(0) = 0, x(1) = 1$. (8)

ii) Using residue method, find $Z^{-1}\left[\frac{z}{z^2 - 2z + 2}\right]$. (8)