Reg. No. :

Question Paper Code : 72074

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fourth/Fifth/Sixth/Seventh Semester

Civil Engineering

MA 6459 — NUMERICAL METHODS

(Common to Aeronautical Engineering, Agriculture Engineering, Electrical and Electronics Engineering, Electronics and Instrumentation Engineering, Geoinformatics Engineering, Instrumentation and Control Engineering, Manufacturing Engineering, Mechanical and Automation Engineering, Petrochemical Engineering, Production Engineering, Chemical Engineering, Chemical and Electrochemical Engineering, Handloom and Textile Technology, Petrochemical Technology, Plastic Technology, Polymer Technology, Textile Chemistry, Textile Technology)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the Newton-Raphson formula and the criteria for convergence.

2. Find the dominant eigenvalue of $A=\begin{pmatrix}2 & 3\\ 5 & 4\end{pmatrix}$ by power method upto 1 decimal place accuracy. Start with $X^{(0)}=\begin{pmatrix}1\\ 1\end{pmatrix}$.

3. Find the Lagrange's interpolating polynomial passing through the points $(0, 0)$, $(1, 1)$, $(2, 20)$.

4. Define a cubic spline.

5. Find $\frac{dy}{dx}$ at $x = 50$ from the following table :

x :	50	51	52
y :	3.6840	3.7084	3.7325

6. Evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ by using two point Gaussian formula.

7. Using Euler's method, compute $y(0.1)$, given $\frac{dy}{dx} = 1 - y$, $y(0) = 0$.

8. State Adam-Bashforth predictor and corrector formulae to solve first order ordinary differential equation.

9. Write down the finite difference scheme for solving $y'' + x + y = 0$; $y(0) = y(1) = 0$.

10. Derive explicit finite difference scheme for $u_t = u_{xx}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a real root of the equation $\cos x = 3x - 1$ correct to three decimal places using fixed point iteration method. (8)

(ii) Find the solution of the system of following equations by Gauss-Seidal method (Upto 4 iterations). (8)

$$x - 2y + 5z = 12$$
$$5x + 2y - z = 6$$
$$2x + 6y - 3z = 5.$$

Or

(b) (i) Using Gauss-Jordan method, find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & -1 \\ 4 & 1 & 0 \\ 2 & -1 & 3 \end{pmatrix}$. (8)

(ii) Solve the following system of equations by Gauss Elimination method (8)

$$x + 2y - 5z = -9$$
$$3x - y + 2z = 5$$
$$2x + 3y - z = 3.$$

12. (a) (i) Find $f(1)$ by using divided difference interpolation from the following data : (8)

x :	–4	–1	0	2	5
$f(x)$:	1245	33	5	9	1335

(ii) Find a polynomial of degree two for the data by Newton's forward difference formula. (8)

x :	0	1	2	3	4	5	6	7
y :	1	2	4	7	11	16	22	29

Or

(b) Find the cubic spline in the interval $1 \le x \le 2$ and hence evaluate $y(1.5)$ and $y'(1.5)$ by using the following data : (16)

x :	1	2	3	4
y :	1	2	5	11

13. (a) (i) Using backward difference, find $y'(2.2)$ and $y''(2.2)$ from the following table : (6)

x :	1.4	1.6	1.8	2.0	2.2
y :	4.0552	4.9530	6.0496	7.3891	9.0250

(ii) The following table gives the values of $y = \frac{1}{1+x^2}$. Take $h = 0.5$, 0.25, 0.125 and use Romberg's method to compute $\int_0^1 \frac{1}{1+x^2}\,dx$. Hence deduce an approximate value of π. (10)

x :	0	0.125	0.25	0.375	0.5	0.675	0.75	0.875	1
y :	1	0.9846	0.9412	0.8767	0.8	0.7191	0.64	0.5664	0.5

Or

(b) (i) Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^1 \int_0^1 \frac{dx\,dy}{1+xy}$ with $h = k = 0.25$. (8)

(ii) Evaluate $\int_0^5 \log_{10}(1+x)\,dx$ by three points Gauss quadrature formula. (8)

14. (a) (i) Find the value of $y(0.1)$, $y(0.2)$ with $h=0.1$, given $\frac{dy}{dx}=x^2y-1$, $y(0)=1$ by Taylor's series method upto four terms. (8)

(ii) Derive the Milne's predictor-corrector formula for solving first order differential equation $y'=f(x,y)$, $y_0=y(x_0)$. (8)

Or

(b) (i) Using Runge-Kutta method of order four, solve $y''=xy'^2-y^2$, $y(0)=1$, $y'(0)=0$ for $x=0.2$ correct to 4 decimal places with $h=0.2$. (8)

(ii) Given $\frac{dy}{dx}=y-x^2+1$, $y(0)=0.5$. Find $y(0.2)$ by modified Euler's method. (8)

15. (a) (i) Using Crank-Nicholson scheme, solve $u_{xx}=16u_t$, $0<x<1$, $t>0$ given $u(x,0)=0$, $u(0,t)=0$ and $u(1,t)=100t$. Take $\Delta x=\frac{1}{4}$ and $\Delta t=1$. Compute u for one time step at the interior mesh points. (8)

(ii) Solve the Poisson equation $u_{xx}+u_{yy}=-81xy$, $0<x<1$; $0<y<1$ $u(0,y)=0$, $u(1,y)=100$, $u(x,0)=0$, $u(x,1)=100$ and $h=\frac{1}{3}$.

Or

(b) (i) Solve numerically, $4\frac{\partial^2u}{\partial x^2}=\frac{\partial^2u}{\partial t^2}$ with the boundary conditions $u(0,t)=0$, $u(4,t)=0$ and the initial condition $u_t(x,0)=0$ and $u(x,0)=x(4-x)$ taking $h=1$ (for 4 times steps). (8)

(ii) Solve : $y''-y=x$, $0<x<1$, given $y(0)=y(1)=0$ using finite differences dividing the interval into 4 equal parts. (8)

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