Reg. No. :

Question Paper Code : 40918

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018
Seventh/Eighth Semester
Computer Science and Engineering
CS 6702 – GRAPH THEORY AND APPLICATIONS
(Common to : Information Technology)
(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Define the terms with respect to graph : walk and path.
2. State two properties of binary tree.
3. Define fundamental circuit in a graph.
4. State Kuratowski's theorem.
5. Let a graph G is 2 – chromatic, then prove that it is bipartite.
6. Define minimal covering.
7. Find the number of ways in which the letters of the word TRIANGLE can be arranged such that vowels occur together ?
8. Find the number of non-negative integral solutions to $x_1 + x_2\ x_3 + x_4 = 20$.
9. Find the exponential generating function of the sequence 0!, 1!, 2!, 3!, . . .
10. Determine the coefficient of x^{15} in $f(x) = (x^2 + x^3 + x^4 + . . .)^4$.

PART – B (5×16=80 Marks)

11. a) i) Prove that the number of vertices of odd degree in a graph is always even. (6)

ii) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. (10)

(OR)

b) i) Prove that a tree with n vertices has (n – 1) edges. (6)

ii) State and prove Dirac's theorem. (10)

12. a) i) Prove that every circuit has an even number of edges in common with a cut-set. (8)

ii) Prove the following :

With respect to the given spanning tree T, a branch b_i that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S and in no others. (8)

(OR)

b) i) Explain max-flow min-cut theorem. (8)

ii) Define 2-isomorphism and prove that the rank and nullity of a graph are invariant under 2-isomorphism. (8)

13. a) i) If G is a tree with n vertices, then prove that its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$. (8)

ii) Define chromatic number. Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length. (8)

(OR)

b) i) State and prove five-colour theorem. (8)

ii) Discuss about any four types of digraph with suitable examples. (8)

14. a) i) Using the principle of inclusion and exclusion find the number of prime numbers not exceeding 100. (8)

ii) Show that if n and k are positive integers, then $C(n+1, k) = \frac{n+1}{k} C(n, k-1)$. Use this identity, construct an inductive definition of the binomial co-efficient. (8)

(OR)

b) i) A survey of 150 college students reveals that 83 own cars, 97 own bikes, 28 own motorcycles, 53 own a car and a bike, 14 own a car and motorcycle, 7 own a bike and a motorcycle and 2 own all the three. How many students own a bike and nothing else and how many students do not own any of the three ? (8)

ii) Five professors P_1, P_2, P_3, P_4, P_5 are to be made class advisor for five sections C_1, C_2, C_3, C_4, C_5, one professor for each section. P_1 and P_2 do not wish to become the class advisors for C_1 or C_2, P_3 and P_4 for C_4 or C_5 and P_5 for C_3 or C_4 or C_5. In how many ways can the professors be assigned the work (without displacing any professor) ? (8)

15. a) i) Obtain the fractional de-composition and identify the sequence having the expression $\frac{3-5z}{1-2z-3z^2}$ as a generating function. (8)

ii) Find the generating function of the sequence 7, 8, 9, 10, . . . (4)

iii) Find the number of distinct summands of the integer 6. (4)

(OR)

b) i) Solve the recurrence relation $y_{n+2} - 6y_{n+1} + 8y_n = 3n+5$. (8)

ii) If a_n denotes the sum of the first n positive integers, find a recurrence relation for a_n and then solve it. (8)
