

25/11/2013

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**Question Paper Code : 80608**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016  
 Third Semester  
 Civil Engineering  
 MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS  
 (Common to all branches except Environmental Engineering, Textile Chemistry,  
 Textile Technology, Fashion Technology and Pharmaceutical Technology)  
 (Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the PDE of all spheres whose centers lie on the  $x$ -axis.
2. Find the complete integral of  $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$ .
3. State the Dirichlet's conditions for a function  $f(x)$  to be expanded as a Fourier series.
4. Expand  $f(x) = 1$ , in  $(0, \pi)$  as a half-range sine series.
5. State the assumptions in deriving one-dimensional wave equation.
6. State the three possible solutions of the one-dimensional heat flow (unsteady state) equation.
7. State change of scale property on Fourier transforms.
8. Find the infinite Fourier sine transform of  $f(x) = \frac{1}{x}$ .
9. State convolution theorem on Z-transform.
10. Find  $Z\left[\frac{1}{n(n+1)}\right]$ .

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the partial differential equations of all planes which are at a constant distance 'k' units from the origin. (8)  
 (ii) Solve the Lagrange's equation  $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ . (8)  
 Or  
 (b) (i) Form the PDE by eliminating the arbitrary functions 'f' and 'φ' from the relation  $z = xf\left(\frac{y}{x}\right) + y\phi(x)$ . (8)  
 (ii) Solve  $(D^2 + DD' - 6D'^2)z = y \cos x$ . (8)
12. (a) (i) Expand  $f(x) = x^2$  as a Fourier series in the interval  $(-\pi, \pi)$  and hence deduce that  $1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots = \frac{\pi^4}{90}$ . (8)

- (ii) Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of  $y$  as given in the following table: (8)

$x$	0	1	2	3	4	5
$y$	9	18	24	28	26	20

Or

- (b) (i) Expand  $f(x) = e^{-ax}$ ,  $-\pi < x < \pi$  as a complex form Fourier series. (8)
- (ii) Expand  $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \end{cases}$  as a series of cosines in the interval  $(0,2)$ . (8)
13. (a) A tightly stretched string of length  $l$  with fixed end points is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity  $y_t(x,0) = v_0 \sin\left(\frac{3\pi x}{l}\right) \cos\left(\frac{\pi x}{l}\right)$ , where  $0 < x < l$ . Find the displacement of the string at a point, at a distance  $x$  from one end at any instant  $t$ . (16)
- Or
- (b) A square plate is bounded by the lines  $x=0, x=20, y=0, y=20$ . Its faces are insulated. The temperature along the upper horizontal edge is given by  $u(x,20) = x(20-x), 0 < x < 20$ , while the other three edges are kept at  $0^\circ\text{C}$ . Find the steady state temperature distribution  $u(x,y)$  in the plate. (16)
14. (a) (i) Find the Fourier transform of  $f(x) = \begin{cases} 1-|x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$  and hence deduce that  $\int_0^\pi \left[\frac{\sin t}{t}\right]^4 dt = \frac{\pi}{3}$ . (8)
- (ii) Find the infinite Fourier sine transform of  $f(x) = \frac{e^{-ax}}{x}$  hence deduce the infinite Fourier sine transform of  $\frac{1}{x}$ . (8)
- Or
- (b) (i) Find the infinite Fourier transform of  $e^{-a^2x^2}$  hence deduce the infinite Fourier transform of  $e^{-x^2/2}$ . (8)
- (ii) Solve the integral equation  $\int_0^\infty f(x) \cos \lambda x dx = e^{-\lambda}$ , where  $\lambda > 0$ . (8)
15. (a) (i) Find (1)  $Z[n^3]$  (2)  $Z[e^{-t^2}]$ . (4+4)
- (ii) Evaluate  $Z^{-1}\left[\frac{9z^3}{(3z-1)^2(z-2)}\right]$ , using calculus of residues. (8)
- Or
- (b) (i) Using convolution theorem, evaluate  $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ . (8)
- (ii) Using Z-transform, solve  $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$  given that  $y_0 = y_1 = 0$ . (8)