

5. Classify the partial differential equation $u_{xx} + u_{yy} = f(x, y)$.
6. Write down all the possible solutions of one dimensional heat equation.
7. State Fourier integral theorem.
8. Find the Fourier transform of a derivative of the function $f(x)$ if $f(x) \rightarrow 0$ as $x \rightarrow \pm \infty$.
9. Find $Z \left\{ \frac{1}{n!} \right\}$
10. Find $Z \{ (\cos \theta + i \sin \theta)^n \}$.

PART - B (5 × 16 = 80 Marks)

11. (a) (i) Solve the equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. (8)
- (ii) Find the singular integral of the equation $z = px + qy + \sqrt{1 + p^2 + q^2}$. (8)

OR

- (b) (i) Solve : $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$. (8)
 - (ii) Solve : $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$ (8)
12. (a) (i) Find the Fourier series of $f(x) = x$ in $-\pi < x < \pi$. (6)
 - (ii) Find the Fourier series expansion of $f(x) = |\cos x|$ in $-\pi < x < \pi$. (10)

OR

- (b) (i) Find the half range sine series of $f(x) = x \cos \pi x$ in $(0, 1)$. (8)

- (ii) Find the Fourier cosine series up to third harmonic to represent the function given by the following data : (8)

$$x: 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

$$y: 4 \quad 8 \quad 15 \quad 7 \quad 6 \quad 2$$

13. (a) Find the displacement of a string stretched between two fixed points at a distance of $2l$ apart when the string is initially at rest in equilibrium position and points of

$$\text{the string are given initial velocities } v \text{ where } v = \begin{cases} \frac{x}{l} & \text{in } (0, l) \\ \frac{2l-x}{l} & \text{in } (l, 2l) \end{cases}, x \text{ being the}$$

distance measured from one end. (16)

OR

- (b) A long rectangular plate with insulated surface is l cm wide. If the temperature along one short edge is $u(x, 0) = k(lx - x^2)$ for $0 < x < l$, while the other two long edges $x = 0$ and $x = l$ as well as the other short edge are kept at 0°C , find the steady state temperature function $u(x, y)$. (16)

14. (a) Find the Fourier cosine and sine transform of $f(x) = e^{-ax}$ for $x \geq 0$, $a > 0$. Hence

$$\text{deduce the integrals } \int_0^{\infty} \frac{\cos sx}{a^2 + s^2} ds \text{ and } \int_0^{\infty} \frac{s \sin sx}{a^2 + s^2} ds. \quad (16)$$

OR

- (b) (i) Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$ in $(-\infty, \infty)$. (8)

- (ii) Find the Fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$ and hence find the

$$\text{value of } \int_0^{\infty} \frac{\sin^4 t}{t^4} dt. \quad (8)$$

15. (a) (i) Find the Z-transforms of $\cos \frac{n\pi}{2}$ and $\frac{1}{n(n+1)}$. (8)

(ii) Using convolution theorem, evaluate $Z^{-1} \left\{ \frac{z^2}{(z-a)^2} \right\}$. (8)

OR

(b) (i) Find the inverse Z-transform of $\frac{z}{z^2 - 2z + 2}$ by residue method. (8)

(ii) Solve the difference equation $y_{n+2} + y_n = 2$, given that $y_0 = 0$ and $y_1 = 0$ by using Z-transforms. (8)