Reg. No. MET ENGG

Question Paper Code : 77191

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Third Semester

Civil Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Form the partial differential equation by eliminating the arbitrary constants a and b from $\log(az-1) = x + ay + b$

2. Find the complete solution of $q = 2px$.

3. The instantaneous current 'i' at time t of an alternating current wave is given by $i = I_1 \sin(\omega t + \alpha_1) + I_3 \sin(3\omega t + \alpha_3) + I_5 \sin(5\omega t + \alpha_5) + \cdots$. Find the effective value of the current 'i'.

4. If the Fourier series of the function $f(x) = x, -\pi < x < \pi$ with period 2π is given by $f(x) = 2\left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \ldots\right)$, then find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$.

5. Classify the partial differential equaiton $(1-x^2)z_{xx} - 2xyz_{xy} + (1-y^2)z_{yy} + xz_x + 3x^2yz_y - 2z = 0$.

6. A rod 30 cm long has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. Find this steady state temperature in the rod.

7. If the Fourier transform of $f(x)$ is $\Im(f(x)) = F(s)$, then show that $\Im(f(x-a)) = e^{ias}F(s)$.

8. Find the Fourier sine transform of $1/x$.

9. If $Z(x(n)) = X(z)$, then show that $Z(a^n x(n)) = X\left(\frac{z}{a}\right)$.

10. State the convolution theorem of Z-transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve: $(x^2 - yz)p + (y^2 - xz)q = (z^2 - xy)$. (8)

(ii) Solve: $(D^2 - 3DD' + 2D'^2)z = (2+4x)e^{x+2y}$. (8)

Or

(b) (i) Obtain the complete solution of $p^2 + x^2y^2q^2 = x^2z^2$. (8)

(ii) Solve $z = px + qy + p^2q^2$ and obtain its singular solution. (8)

12. (a) (i) Find the half-range sine series of $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$. Hence deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (10)

(ii) Find the complex form of the Fourier series of $f(x) = e^{-x}$ in $-1 < x < 1$. (6)

Or

(b) (i) Find the Fourier series of $f(x) = |\sin x|$ in $-\pi < x < \pi$ of periodicity 2π. (8)

(ii) Compute upto the first three harmonics of the Fourier series of $f(x)$ given by the following table: (8)

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

13. (a) Solve $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions: $u(0,t) = 0 = u(l,t), t \geq 0$;

$$u(x,0) = \begin{cases} x, & 0 \leq x \leq l/2 \\ l-x, & l/2 \leq x \leq l \end{cases}.$$ (16)

Or

(b) A string is stretched and fastened to two points that are distant l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time t=0. Find the displacement at any point of the string at a distance x from one end at any time t. (16)

14. (a) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. Using Parseval's identity, prove that $\int_0^\infty \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$. (16)

Or

(b) (i) Show that the function $e^{-x^2/2}$ is self-reciprocal under Fourier transform by finding the Fourier transform of $e^{-a^2x^2}, a > 0$. (10)

(ii) Find the Fourier cosine transform of x^{n-1}. (6)

15. (a) (i) Find $Z(r^n \cos n\theta)$ and $Z^{-1}\left[(1 - az^{-1})^{-2}\right]$. (8)

(ii) Using convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-1/2)(z-1/4)}\right]$. (8)

Or

(b) (i) Using Z – transform, solve the difference equation $x(n+2) - 3x(n+1) + 2x(n) = 0$ given that $x(0)=0, x(1)=1$. (8)

(ii) Using residue method, find $Z^{-1}\left[\frac{z}{z^2 - 2z + 2}\right]$. (8)
