

Reg. No. :

Question Paper Code : 27327

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Third Semester

Civil Engineering

MA 6351 — TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches except Environmental Engineering, Textile Chemistry, Textile Technology, Fashion Technology and Pharmaceutical Technology)

(Regulation 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Construct the partial differential equation of all spheres whose centres lie on the Z - axis, by the elimination of arbitrary constants.
2. Solve $(D+D'-1)(D-2D'+3)z=0$.
3. Find the root mean square value of $f(x)=x(i-x)$ in $0 \leq x \leq 1$.
4. Find the sine series of function $f(x)=1$, $0 \leq x \leq \pi$.
5. Solve $3x \frac{\partial u}{\partial x} - 2y \frac{\partial u}{\partial y} = 0$; by method of separation of variables.
6. Write all possible solutions of two dimensional heat equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.
7. If $F(s)$ is the Fourier Transform of $f(x)$, prove that $F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$, $a \neq 0$.
8. Evaluate $\int_0^{\infty} \frac{s^2}{(s^2+a^2)(s^2+b^2)} ds$ using Fourier Transforms.
9. Find the Z - transform of $\frac{1}{n+1}$.
10. State the final value theorem. In Z transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find complete solution of $z^2(p^2 + q^2) = (x^2 + y^2)$. (8)
 (ii) Find the general solution of $(D^2 + 2DD' + D'^2)z = 2\cos y - x \sin y$. (8)

Or

- (b) (i) Find the general solution of $(x^2 - y^2 - 2yz)p + (xy + zx)q = (xy - zx)$. (8)
 (ii) Find the general solution of $(D^2 + D'^2)z = x^2y^2$. (8)

12. (a) (i) Find the Fourier series expansion the following periodic function of period 4 $f(x) = \begin{cases} 2+x & -2 \leq x \leq 0 \\ 2-x & 0 < x \leq 2 \end{cases}$. Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}. \quad (8)$$

- (ii) Find the complex form of Fourier series of $f(x) = e^{\alpha x}$ in the interval $(-\pi, \pi)$ where α is a real constant. Hence, deduce that

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{\alpha^2 + n^2} = \frac{\pi}{\alpha \sinh \alpha \pi}. \quad (8)$$

Or

- (b) (i) Find the half range cosine series of $f(x) = (\pi - x)^2, 0 < x < \pi$. Hence find the sum of series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$ (8)
 (ii) Determine the first two harmonics of Fourier series for the following data. (8)

$x:$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
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$f(x):$	1.98	1.30	1.05	1.30	-0.88	-0.25
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13. (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $v = \begin{cases} \frac{2kx}{l} & \text{in } 0 < x < \frac{l}{2} \\ \frac{2k(l-x)}{l} & \text{in } \frac{l}{2} < x < l \end{cases}$. Find the displacement of the string at any distance x from one end at any time t . (16)

Or

- (b) A bar 10 cm long with insulated sides has its ends A and B maintained at temperature 50°C and 100°C , respectively, until steady state conditions prevail. The temperature at A is suddenly raised to 90°C and at the same time lowered to 60°C at B. Find the temperature distributed in the bar at time t . (16)

14. (a) (i) Find the Fourier sine integral representation of the function $f(x) = e^{-x} \sin x$. (8)
- (ii) Find the Fourier cosine transform of the function $f(x) = \frac{e^{-ax} - e^{-bx}}{x}, x > 0$. (8)

Or

- (b) (i) Find the Fourier transform of the function $f(x) = \begin{cases} 1-|x|, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$
- Hence deduce that $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{3}$. (8)
- (ii) Verify the convolution theorem for Fourier transform if $f(x) = g(x) = e^{-x^2}$. (8)

15. (a) (i) If $U(z) = \frac{z^3 + z}{(z-1)^3}$, find the value of u_0, u_1 and u_2 . (8)
- (ii) Use convolution theorem to evaluate $z^{-1} \left\{ \frac{z^2}{(z-3)(z-4)} \right\}$. (8)

Or

- (b) (i) Using the inversion integral method (Residue Theorem), find the inverse Z-transform of $U(z) = \frac{z^2}{(z+2)(z^2+4)}$. (8)
- (ii) Using the Z-transform solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0, u_1 = 1$. (8)