Reg. No. :

Question Paper Code : 80615

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016

Fifth Semester

Computer Science Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Give the contra positive statement of the statement "If there is rain, then I buy an umbrella".

2. Construct the truth table for $P \to \sim Q$.

3. How many permutations are there on the word "MALAYALAM"?

4. Find the recurrence relation of the sequence $s(n) = a^n : n \geq 1$.

5. Define a complete graph.

6. Draw the graph with the following adjacency matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

7. Define a semi group.

8. Find the idempotent elements of $G = \{1, -1, i, -i\}$ under the binary operation multiplication.

9. Define a lattice.

10. State the De Morgan's laws in a Boolean Algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the PDNF and PCNF of $(P \wedge Q) \vee (\sim P \wedge R)$. (8)

(ii) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \rightarrow R, P \rightarrow M, \sim M$. (8)

Or

(b) (i) Show that $(x)\,[P(x) \rightarrow Q(x)] \wedge (x)[Q(x) \rightarrow R(x)] \Rightarrow (x)[P(x) \rightarrow R(x)]$. (8)

(ii) Show that $(\urcorner P \wedge (\urcorner Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$, without using truth table. (8)

12. (a) (i) Prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$ for $n \geq 2$ using principle of mathematical induction. (8)

(ii) Find the number of integers between 1 to 250 that are not divisible by any of the integers 2, 3, 5 and 7. (8)

Or

(b) (i) Solve $G(k) - 7G(k-1) + 10G(k-2) = 8k + 6$, for $k \geq 2$. (8)

(ii) How many bits of string of length 10 contain

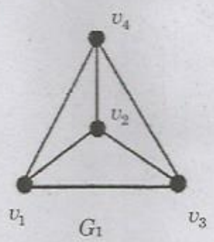
(1) exactly four 1's

(2) at most four 1's

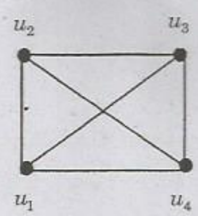
(3) at least four 1's

(4) an equal number of 0's and 1's.

13. (a) (i) Define Isomorphism. Establish an isomorphism for the following graphs. (8)



G_1



G_2

(ii) Let G be a graph with exactly two vertices has odd degree. Then prove that there is a path between those two vertices. (8)

Or

(b) (i) State and prove hand shaking theorem. Also prove that maximum number of edges in a connected graph with n vertices is $\frac{n(n-1)}{2}$. (8)

(ii) Give an example of a graph which is

(1) Eulerian but not Hamiltonian

(2) Hamiltonian but not Eulerian

(3) Hamiltonian and Eulerian

(4) Neither Hamiltonian nor Eulerian. (8)

14. (a) (i) Show that $(Q^{+},*)$ is an abelian group, where $*$ is defined by

$$a*b=\frac{ab}{2}, \forall a,b\in Q^{+}.$$ (8)

(ii) Let $f:(G,*)\to(G',\Delta)$ be a group homomorphism. Then prove that

(1) $[f(a)]^{-1}=f(a^{-1})\ \forall a\in G$,

(2) $f(e)$ is an identity of G', when e is an identity of G. (8)

Or

(b) (i) Prove that the intersection of two normal subgroups of a group G is again a normal subgroup of G. (8)

(ii) State and prove Lagrange's theorem in a group. (8)

15. (a) (i) In a complemented and distributive lattice, prove that complement of each element is unique. (8)

(ii) Prove that every chain is a distributive Lattice. (8)

Or

(b) (i) Consider the Lattice D_{105} with the partial ordered relation divides, then

(1) Draw the Hasse diagram of D_{105}.

(2) Find the complement of each elements of D_{105}.

(3) Find the set of atoms of D_{105}.

(4) Find the number of subalgebras of D_{105}. (8)

(ii) Show that in a Boolean algebra $a\le b \Leftrightarrow a\wedge\overline{b}=0 \Leftrightarrow \overline{a}\vee b=1 \Leftrightarrow \overline{b}\le\overline{a}$. (8)
