CSE 2013

Reg. No. : [ ]

Question Paper Code : 72078

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fifth Semester

Computer Science and Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the truth table for $p \to q$.

2. Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$.

3. How many different words are there in the word MATHEMATICS?

4. Find the minimum number of students need to guarantee that five of them belongs to the same subject, if there are five different major subjects.

5. How many edges are there in a graph with 10 vertices each of degree 3?

6. Give an example of self complementary graph.

7. Show that every cyclic group is abelian.

8. Let Z be the group of integers with the binary operation $*$ defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of the group $\langle Z, * \rangle$.

9. Let X = {1, 2, 3, 4, 5, 6} and R be a relation defined as $\langle x, y \rangle \in R$ if and only if $x - y$ is divisible by 3. Find the elements of the relation R.

10. Show that the absorption laws are valid in a Boolean algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ by using equivalences. (8)

(ii) Use rules of inferences to obtain the conclusion of the following arguments :

"Babu is a student in this class, knows how to write programmes in JAVA". "Everyone who knows how to write programmes in JAVA can get a high-paying job". Therefore, "someone in this class can get a high-paying job". (8)

Or

(b) (i) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology by using equivalences. (8)

(ii) Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q. (8)

12. (a) (i) Find the number of integers between 1 and 500 that are not divisible by any of the integers 2,3, 5 and 7. (8)

(ii) Solve the recurrence relation $a_n - 7a_{n-1} + 6a_{n-2} = 0$, for $n \geq 2$ with initial conditions $a_0 = 8$ and $a_1 = 6$, using generating function. (8)

Or

(b) (i) Using mathematical induction, show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1}-1}{2}$. (8)

(ii) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (1) they can be male or female, (2) two must be men and two women, (3) they must all are of the same sex. (8)

13. (a) (i) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (10)

(ii) Prove that the complement of a disconnected graph is connected. (6)

Or

(b) (i) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic? (10)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii) Prove that the number of odd degree vertices in any graph is even. (6)

14. (a) State and prove Lagrange's theorem on groups. (16)

Or

(b) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) Let $f : G \to H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G. (8)

15. (a) (i) Show that every chain is a distributive lattice. (8)

(ii) In a distributive complemented lattice, show that the following are equivalent. (8)

(1) $a \le b$

(2) $a \wedge \overline{b} = 0$

(3) $\overline{a} \vee b = 1$

(4) $\overline{b} \le \overline{a}$.

Or

(b) Show that every ordered lattice $\langle L, \le \rangle$ satisfies the following properties of the algebraic lattice (i) idempotent (ii) commutative (iii) Associative (iv) Absorption. (16)

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