UNIT III

STATIC MAGNETIC FIELDS

Biot -Savart Law, Magnetic field Intensity, Estimation of Magnetic field Intensity for straight and circular conductors, Ampere's Circuital Law, Point form of Ampere's Circuital Law, Stokes theorem, Magnetic flux and magnetic flux density, The Scalar and Vector Magnetic potentials, Derivation of Steady magnetic field Laws.

3. INTRODUCTON

A **magnetic field** is the magnetic effect of electric currents and magnetic materials. The magnetic field at any given point is specified by both a *direction* and a *magnitude* (or strength); as such it is a vector field. The term is used for two distinct but closely related fields denoted by the symbols **B** and **H**, where **H** is measured in units of amperes per meter (symbol: $A \cdot m^{-1}$ or A/m) in the SI. **B** is measured in tesla (symbol) and Newton's per meter per ampere (symbol: $N \cdot m^{-1} \cdot A^{-1}$ or N/(m.A)) in the SI. **B** is most commonly defined in terms of the Lorentz force it exerts on moving electric charges.

Magnetic fields can be produced by moving electric charges and the intrinsic magnetic moments of elementary particles associated with a fundamental quantum property, their spin. In special relativity, electric and magnetic fields are two interrelated aspects of a single object, called the electromagnetic tensor; the split of this tensor into electric and magnetic fields depends on the relative velocity of the observer and charge. In quantum physics, the electromagnetic field is quantized and electromagnetic interactions result from the exchange of photons.

In everyday life, magnetic fields are most often encountered as a force created by permanent magnets, which pull on ferromagnetic materials such as iron, cobalt, or nickel, and attract or repel other magnets. Magnetic fields are widely used throughout modern technology, particularly in electrical engineering and electro mechanics. The Earth produces its own magnetic field, which is important in navigation, and it shields the Earth's atmosphere from solar wind. Rotating magnetic fields are used in both electric motors and generators. Magnetic forces give information about the charge carriers in a material through the Hall effect. The interaction of magnetic fields in electric devices such as transformers is studied in the discipline of magnetic circuits.

COMPARE BIOT SAVART LAW AND COULOMBS LAW

COULOMBS LAW	BIOT SAVART LAW
• Basic law of Electric field intensity. • The produced charge in the electric field is a Scalar • It does not depends on Sine angle. • It is inversely proportional to the square of distance.	• Basic law of Magnetic field intensity • The produced charge in the magnetic field is a Vector • It depends on Sine angle. • It is inversely proportional to the square of distance.

3.1. BIOT -SAVART LAW (Ampere's Law)

Biot-Savart's law states that the magnetic field intensity dl produced at a point P, as shown in Figure, by the differential current clement Idl is proportional to the product Idl and the sine of the angle a between the element and the line joining P to the element and is inversely proportional to the square of the distance K between P and the element.

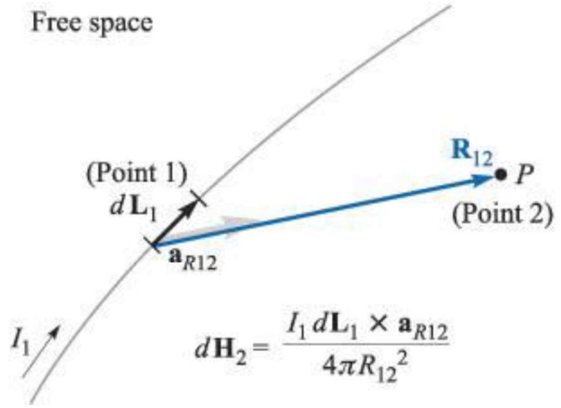


Fig.3.1: magnetic field dH at *P* due to current element I dl.

Magnetic Flux density at any point P due to current element Idl is given by ,

$$dH \alpha \frac{\text{Idl} sin\ \theta}{R^2}$$

where $\mu = \mu_0 \mu_r$ - Permeability of medium

Idl - Current element

r - Distance between point P and current element

$$d\overline{H} = K\frac{\text{Idl}sin\ \theta}{R^2}$$

where K is the constant of proportionality. In SI units, $k = \frac{1}{4\pi}$

$$d\overline{H} = \frac{\text{Idl}sin\ \theta}{4\pi r^2}$$

From the definition of cross product in vector form as,

$$\overline{dH} = \frac{Idl \text{ X } \bar{a}\text{R}}{4\pi r^2} = \frac{Idl \text{ X } \overline{\text{R}}}{4\pi r^3}$$

where $R = |\bar{R}|$ and $\overline{a_R} = \frac{\bar{R}}{|\bar{R}|}$. Thus the direction of $\overline{dH}$ can be determined by the right hand rule with the right-hand thumb pointing in the direction of the current, the right-hand fingers encircling the wire in the direction of $\overline{dH}$ as shown in Figure. Alternatively, use the right-handed screw rule to determine the direction of $\overline{dH}$: with the screw placed along the wire and pointed in the direction of current flow, the direction of advance of the screw is the direction of $\overline{dH}$.

The different current distributions: line current, surface current, and volume current as shown in Figure. If define K as the surface current density (in amperes/meter) and J as the volume current density (in amperes/meter square), the source elements are related as,

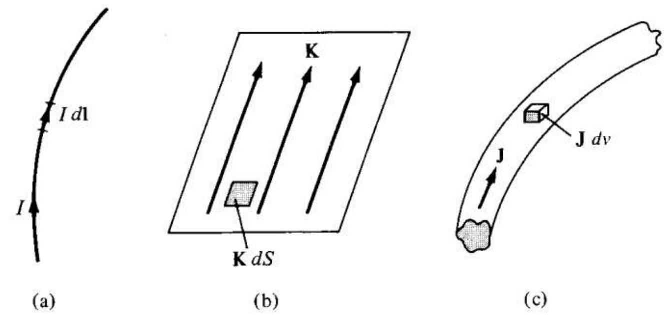


Fig.3.2: Current distributions: (a) line current, (b) surface current, (c) volume current

Idl≡Kds≡JdV

Thus in terms of the distributed current sources, the Biot-Savart law as in equation becomes,

$$\bar{H} = \int \frac{Idl\ X\ \bar{a}R}{4\pi R^2} \quad (Line\ Current)$$

$$\bar{H} = \int \frac{KdsX\ \bar{a}R}{4\pi R^2} \quad (surface\ current)$$

$$\bar{H} = \int \frac{J\ dvX\ \bar{a}R}{4\pi R^2} \quad (volume\ current)$$

Merit and demerit of Biot Savart law:

- It can be used if the path is not closed over which B is constant
- It is more difficult

3.2. MAGNETIC FIELD INTENSITY AND MAGNETIC FLUX

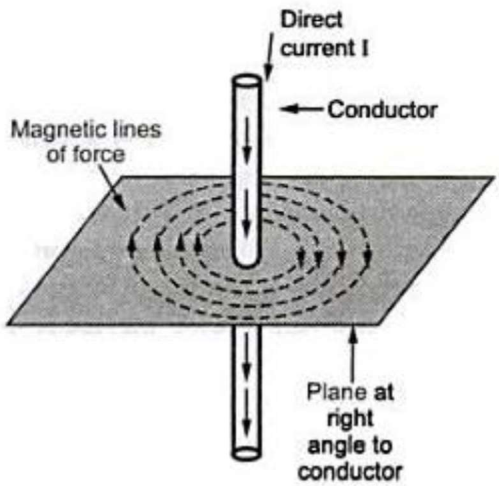


Fig. Magnetic field due to conductor

3.2.1 Magnetic Field Intensity

The quantitative measure of strongness or weakness of the magnetic field is given by magnetic field intensity or magnetic field strength. The magnetic field intensity at any point in the magnetic field is defined as the force experienced by the unit north pole of the Weber strength, when place at that point. The magnetic flux lines are measured in Weber (wb) while magnetic field intensity is measured in Newton/Weber (N/wb) or ampere/meter (A/m). It is denoted as $\bar{H}$. It is the vector quantity. This is similar to the electric fields intensity $\bar{E}$ in electrostatic magnetic field.

3.2.2. Magnetic Flux

The magnetic flux density $\bar{B}$ is defined through the magnetic field intensity $\bar{H}$ at the point

$$\bar{B} = \mu\bar{H} \text{ wb/m}^2$$

The magnetic flux density $\bar{B}$ is also the measure of magnetic flux lines. The magnetic flux density $\bar{B}$ is defined as magnetic flux per unit area.

$$B = \frac{\varphi}{A}\ wb/m^2$$

Where φ is the magnetic flux and $\bar{A}$ is the area. The magnetic flux through an element of area ds is given by the dot product of select the normal components of $\bar{B}$ through the surface ds. For an arbitrary surface ds bounded by a closed contour, the total magnetic flux φ passing through the surface

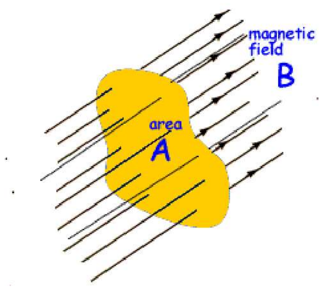


Fig.3.3. Magnetic Flux

$$\varphi = \int \bar{B}.\bar{d}s$$

The unit of φ is Weber while that of $\bar{B}$ is wb/m^2. The electric flux ψ is measure in coulombs and of gauss law, the total flux passing through any closed surface is equal to the charge enclosed.

$$\Psi = \oint_S \bar{D}.\text{ds} = \text{Q}$$

3.3. APPLICATIONS OF BIOT SAVART LAW

3.3.1. Estimation Of Magnetic Field Intensity For Straight Conductors

To determine the field due to a *straight* current carrying filamentary conductor of finite length *AB* as in Figure 3.4.

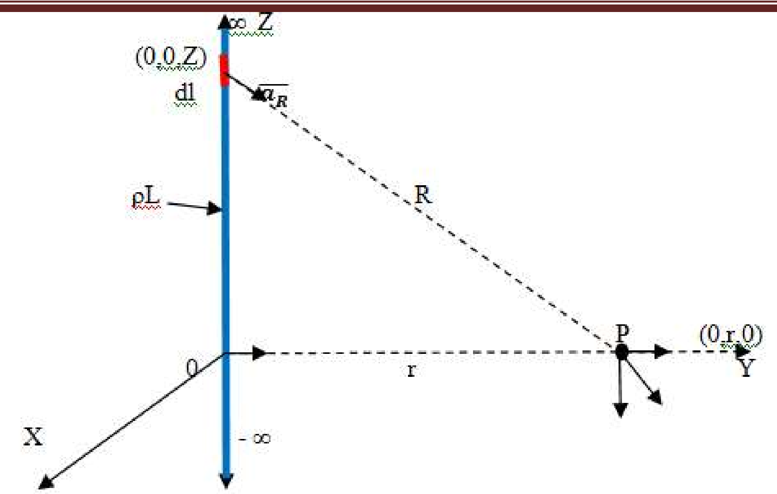


Fig 3.4 Field at a point P due to infinite length current carrying conductor

Consider a infinitely long straight conductor along Z axis. The current passing through the conductor is direct current of I amperes. The field intensity $\overline{H}$ at a point P is also to be calculated which is at a distance r from the Z axis as shown in figure.

Consider small differential element at point 1 along Z axis at a distance Z from origin.

$$\mathbf{I\,\overline{dL} = I\,dZ\,\overline{a_Z}}$$

The distance vector joining point 1 and point 2 is $\overline{R_{12}}$ and

$$\overline{R_{12}} = -Z\overline{a_Z} + r\overline{a_r}$$

$$\overline{a_{R12}} = \frac{\overline{R_{12}}}{|\overline{R_{12}}|} = \frac{-Z\overline{a_Z} + r\overline{a_r}}{\sqrt{r^2 + Z^2}}$$

$$\overline{dL}\,X\overline{a_{R12}} = \begin{vmatrix} \overline{a_r} & \overline{a_\varphi} & \overline{a_z} \\ 0 & 0 & dz \\ r & 0 & -z \end{vmatrix} = rdz\,\overline{a_\varphi}$$

$|\overline{R_{12}}|$ is neglected for convenience

$$I\overline{dL}\,X\overline{a_{R12}} = \frac{Irdz\,\overline{a_\varphi}}{\sqrt{r^2 + Z^2}}$$

According to Biot savart law,

$$\overline{dH} = \frac{I\,\overline{dL}\,X\overline{a_{R12}}}{4\pi {R_{12}}^2} = \frac{Irdz\,\overline{a_\varphi}}{4\pi(r^2 + Z^2)^2\sqrt{r^2 + Z^2}}$$

$$= \frac{Irdz\,\overline{a_\varphi}}{4\pi(r^2 + Z^2)^{3/2}}$$

Thus the total field intensity $\overline{H}$ can be obtained by integrating $\overline{dH}$ over the entire length of the conductor.

$$\overline{H} = \int_{-\infty}^{+\infty} \frac{Irdz\,\overline{a_\varphi}}{4\pi(r^2 + Z^2)^{3/2}}$$

$Put\ z = r\,tan\theta Z^2 = r^2 tan^2\theta$

$$dz = r\,sec^2\theta d\theta, z = -\infty, \theta = -\pi/2\ and\ z = \infty, \theta = \pi/2$$

$$= \int_{-\pi/2}^{+\pi/2} \frac{Irr\ sec^2\theta d\theta\ \overline{a_\varphi}}{4\pi(r^2 + r^2 tan^2\theta)^{3/2}}$$

$$= \int_{-\pi/2}^{+\pi/2} \frac{Ir^2 sec^2\theta d\theta\ \overline{a_\varphi}}{4\pi r^3(1 + tan^2\theta)^{3/2}}\ 1 + tan^2\theta = sec^2\theta$$

$$= \int_{-\pi/2}^{+\pi/2} \frac{Isec^2\theta d\theta\ \overline{a_\varphi}}{4\pi r\ sec^3\theta}\overline{H}$$

$$= \int_{-\pi/2}^{+\pi/2} \frac{Icos\theta d\theta\ \overline{a_\varphi}}{4\pi r}$$

$$= \frac{I\ \overline{a_\varphi}}{4\pi r}\int_{-\pi/2}^{+\pi/2} cos\theta d\theta$$

$$= \frac{I\ \overline{a_\varphi}}{4\pi r}[sin\theta]_{-\pi/2}^{\pi/2}$$

$$= \frac{I\ \overline{a_\varphi}}{4\pi r}[2]$$

$$\overline{H} = \frac{I\ \overline{a_\varphi}}{2\pi r}\ \text{A/m}$$

$$\overline{B} = \mu\overline{H} = \frac{I\mu\ \overline{a_\varphi}}{2\pi r}\ wb/m^2$$

If the conductor is finite

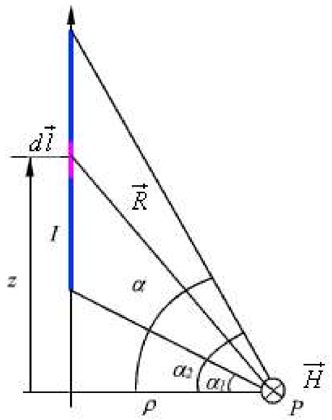


Fig 3.5 Field at a point P due to a finite length current carrying conductor

Consider a 2a length (-a to +a) long straight conductor along Z axis. The current passing through the conductor is direct current of I amperes. The field intensity $\boldsymbol{\overline{H}}$ at a point P is also to be calculated which is at a distance r from the Z axis as shown in figure.

$$\overline{H} = \int_{-a}^{+a} \frac{Irdz\ \overline{a_\varphi}}{4\pi(r^2 + Z^2)^{\frac{3}{2}}}$$

Put z= r tanθZ^2=$r^2 tan^2\theta$

dz= r $\boldsymbol{sec^2\theta}$ dθ, z= -∞, θ= α_1 and z= ∞, θ= α_2

$$= \int_{\alpha1}^{\alpha2} \frac{Irr\ sec^2\theta d\theta\ \overline{a_\varphi}}{4\pi(r^2 + r^2tan^2\theta)^{3/2}}$$

$$= \int_{\alpha1}^{\alpha2} \frac{Ir^2 sec^2\theta d\theta\ \overline{a_\varphi}}{4\pi r^3(1+\ tan^2\theta)^{3/2}} 1 +\ tan^2\theta = sec^2\theta$$

$$= \int_{\alpha1}^{\alpha2} \frac{Isec^2\theta d\theta\ \overline{a_\varphi}}{4\pi r\ sec^3\theta} \overline{H}$$

$$= \int_{\alpha1}^{\alpha2} \frac{Icos\theta d\theta\ \overline{a_\varphi}}{4\pi r}$$

$$= \frac{I\ \overline{a_\varphi}}{4\pi r} \int_{\alpha1}^{\alpha1} cos\theta d\theta$$

$$= \frac{I\ \overline{a_\varphi}}{4\pi r} [sin\theta]_{\alpha1}^{\alpha2}$$

$$\overline{H}\ = \frac{I\ \overline{a_\varphi}}{4\pi r} [\sin\alpha2 - \sin\alpha1]\ \text{A/m}$$

$$\overline{B} = \mu\overline{H} = \frac{I\mu\ \overline{a_\varphi}}{4\pi r} [\sin\alpha2 - \sin\alpha1]\ wb/m^2$$

While using the result if segment carrying current I is not along Z axis then the direction of $\overline{\boldsymbol{H}}$ can not be $\overline{\boldsymbol{a_\varphi}}$. It depends on which plane segment carrying current is placed. The magnitude of $\overline{\boldsymbol{H}}$ is $\frac{I}{4\pi r}[\sin\alpha2 - \sin\alpha1]$ but the direction is always normal to the plane containing the siurce and to be decided by right handed screw rule.

3.3.2. Estimation Of Magnetic Field Intensity For Circular Conductors

Consider a circular loop carrying a direct current I place in XY plane with Z axis. The magnetic field intensity $\overline{\boldsymbol{H}}$ at point P is to be obtained. The point P is at a distance Z from the plane of the circular loop along its axis. The radius of circular loop is r. Consider the differential length $\overline{\boldsymbol{dl}}$ along the circular loop.

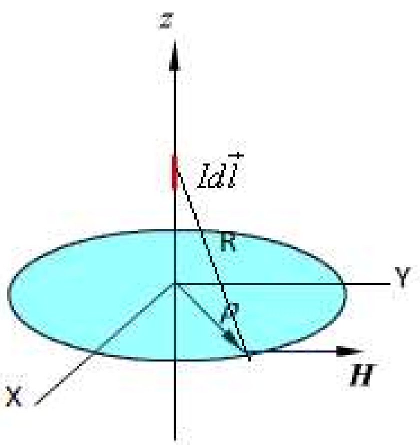


Fig 3.6 Magnetic field due to an circular loop carrying conductor

In cylindrical coordinate system $\overline{dl} = dr\overline{ar} + rd\varphi\overline{a\varphi} + dz\overline{az}$

But $\overline{dl}$ is in the plane for which r is a constant and z=0 constant plane. The I$\overline{dl}$ is tangential point I in $\overline{a_\varphi}$. Direction.

$$I\,\overline{dL} = Ir\,d\varphi\,\overline{a_\varphi}$$

The unit vector $\overline{a_{R12}}$ is in the direction along the line joining different current element to the point P. The distance vector joining point 1 and point 2 is $\overline{R_{12}}$ and

$$\overline{R_{12}} = Z\overline{a_Z} - r\overline{a_r}$$

$$\overline{a_{R12}} = \frac{\overline{R_{12}}}{|\overline{R_{12}}|} = \frac{Z\overline{a_Z} - r\overline{a_r}}{\sqrt{r^2 + Z^2}}$$

$$\overline{dL}\,X\overline{a_{R12}} = \begin{vmatrix} \overline{a_r} & \overline{a_\varphi} & \overline{a_z} \\ 0 & rd\varphi & 0 \\ -r & 0 & z \end{vmatrix} = Zrd\varphi\,\overline{a_r} + r^2 d\varphi\,\overline{a_z}$$

$|\overline{R_{12}}\,|$ is neglected for convenience

$$I\overline{dL}\,X\overline{a_{R12}} = \frac{Irdz\,\overline{a_\varphi}}{\sqrt{r^2 + Z^2}}$$

According to Biot savart law, the differential field strength $\overline{dH}$ at point P is given by

$$\overline{dH} = \frac{I\,\overline{dL}\,X\overline{a_{R12}}}{4\pi {R_{12}}^2} = \frac{I(Zrd\varphi\,\overline{a_r} + r^2 d\varphi\,\overline{a_z})}{4\pi(r^2 + Z^2)^2\sqrt{r^2 + Z^2}}$$

Thus the total field intensity $\overline{H}$ can be obtained by integrating $\overline{dH}$ over φ=0 to 2π.

$$\overline{H} = \int_0^{2\pi} \frac{I(Zrd\varphi\,\overline{a_r} + r^2 d\varphi\,\overline{a_z})}{4\pi(r^2 + Z^2)^{3/2}}$$

$$\overline{H} = \frac{I}{4\pi}\left\{\int_0^{2\pi} \frac{(Zrd\varphi\,)\overline{a_r}}{(r^2 + Z^2)^{3/2}} + \int_0^{2\pi} \frac{(r^2 d\varphi\,)}{(r^2 + Z^2)^{3/2}}\overline{a_z}\right\}$$

Consider the first integral to prove that the value is zero due to radial symmetry,

$$\int_0^{2\pi} \frac{(Zrd\varphi\,)\overline{a_r}}{(r^2 + Z^2)^{3/2}} = \int_0^{2\pi} \frac{(Zrd\varphi\,)}{(r^2 + Z^2)^{\frac{3}{2}}}(\cos\varphi\,\overline{ax} + \sin\varphi\,\overline{ay})$$

The unit vector $\overline{a_r}$ is expressed in rectangular system $\cos\varphi\,\overline{ax} + \sin\varphi\,\overline{ay}$.

Now $\int_0^{2\pi} \cos\varphi\,d\varphi = [\sin\varphi]_0^{2\pi} = \sin 2\pi - \sin 0 = 0$

$$\int_0^{2\pi} \sin\varphi\,d\varphi = [-\cos\varphi]_0^{2\pi} = -\cos 2\pi + \cos 0 = 0$$

$$\therefore \int_0^{2\pi} \frac{(Zrd\varphi\,)\overline{a_r}}{(r^2 + Z^2)^{3/2}} = 0$$

$$\overline{H} = \frac{I}{4\pi}\left\{\int_0^{2\pi} \frac{(r^2 d\varphi\,)}{(r^2 + Z^2)^{3/2}}\overline{a_z}\right\} = \frac{I}{4\pi}\frac{(r^2\,)}{(r^2 + Z^2)^{3/2}}\overline{a_z}\int_0^{2\pi} d\varphi$$

$$= \frac{I}{4\pi}\frac{(r^2\,)}{(r^2 + Z^2)^{3/2}}\overline{a_z}[\varphi]_0^{2\pi}$$

$$= \frac{I}{4\pi} \frac{(r^2\ )}{(r^2 + Z^2)^{3/2}} \overline{a_z}[2\pi]$$

$$\overline{H} = \frac{I(r^2\ )}{2(r^2 + Z^2)^{3/2}} \overline{a_z}\ A/m$$

r- radius of the circular loop

Z- Distance of point P along the axis

At the center of the circular loop Z=0

$$\therefore \overline{H} = \frac{I(r^2\ )}{2(r^2)^{3/2}} \overline{a_z}$$

$$\overline{H} = \frac{I}{2r} \overline{a_z}\ A/m$$

$\overline{B} = \mu\overline{H}$

$$\overline{B} = \frac{\mu I}{2r} \overline{a_z} A/m^2$$

3.4. AMPERE'S CIRCUITAL LAW

Ampere's circuit law states that the line integral of the tangential component of H around a closed path is the same as the net current Ienc enclosed by the path. The line integral of magnetic field intensity $\overline{H}$ around the closed path is exactly equal to the direct current enclosed by the path. The mathematical representation of Amperes circuital law is,

$$\oint \boldsymbol{\overline{H}.\overline{dl}} = \mathrm{I}$$

Proof:

Consider a long straight conductor carrying direct current I placed along Z axis as shown in figure. Consider a closed circular path of radius r which encloses the straight conductor carrying direct current I. The point P is at a perpendicular distance r from the conductor.

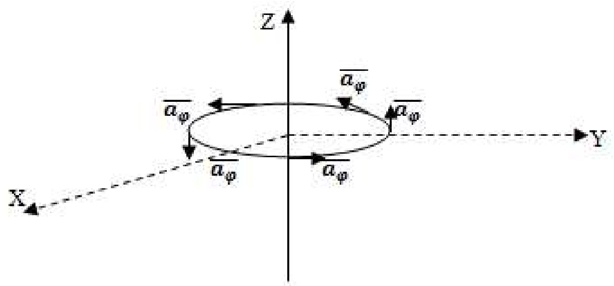


Fig. 3.7: Magnetic field due to an finite line current carrying conductor

Consider $\boldsymbol{\overline{dl}}$ at point P which is in $\boldsymbol{\overline{a_\varphi}}$ direction tangential in circular path at point P.

$$\boldsymbol{\overline{dL} = r\ d\varphi\ \overline{a_\varphi}}$$

While $\boldsymbol{\overline{H}}$ obtained at point P from Biot Savart law due to infinitely long straight conductors,

$$\boldsymbol{\overline{H} = \frac{I\ \overline{a_\varphi}}{2\pi r} A/m}$$

$$\boldsymbol{\overline{H}.\overline{dl} = \frac{I\ \overline{a_\varphi}}{2\pi r} . r\ d\varphi\ \overline{a_\varphi}}$$

$$= \frac{I}{2\pi r} . r\, d\varphi$$

$$= \frac{I}{2\pi} . d\varphi$$

Integrating $\bar{H}.\overline{dl}$ over the entire closed path.

$$\oint \bar{H}.\overline{dl} = \int_0^{2\pi} \frac{I}{2\pi} \, . \, d\varphi$$

$$= \frac{I}{2\pi} [\varphi]_0^{2\pi}$$

$= I$ (current carried by the conductor)

Applications of Amperes circuital Law:

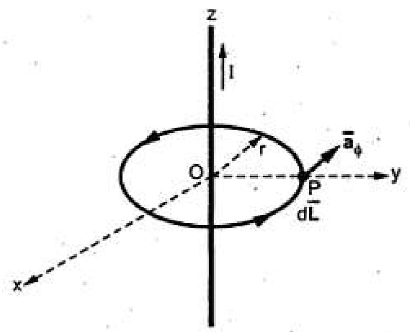
i. $\bar{H}$ due to infinitely long straight conductor
ii. $\bar{H}$ due to co axial cable
iii. $\bar{H}$ due to infinite sheet of current

Merit and Demerit of ampere's circuital law:

- It is simple
- It can't be used if the path is not closed over which B is constant.

APPLICATIONS OF AMPERE CIRCUITAL LAW

$\bar{H}$ Due to Infinitely Long Straight Conductor



Consider an infinitely long straight conductor placed along z-axis, carrying a direct current I as shown in the Fig. . Consider the Amperian closed path, enclosing the conductor as shown in the Fig. . Consider point P on the closed path at which $\bar{H}$ is to be obtained. The radius of the path is r and hence P is at a perpendicular distance r from the conductor.

The magnitude of $\bar{H}$ depends on r and the direction is always tangential to the closed path i.e. $\bar{a}_\phi$. So $\bar{H}$ has only component in $\bar{a}_\phi$ direction say H_ϕ.

Consider elementary length $d\bar{L}$ at point P and in cylindrical co-ordinates it is $r\, d\phi$ in $\bar{a}_\phi$ direction.

$\therefore \quad \bar{H} = H_\phi \bar{a}_\phi \quad$ and $\quad d\bar{L} = r\, d\phi \bar{a}_\phi$

$\therefore \quad \bar{H} \bullet d\bar{L} = H_\phi \bar{a}_\phi \bullet r\, d\phi \bar{a}_\phi = H_\phi r\, d\phi$

According to Ampere's circuital law,

$$\oint \bar{H} \bullet d\bar{L} = I$$

$$\therefore \quad \int_{\phi=0}^{2\pi} H_\phi r\, d\phi = I$$

$$\therefore \quad H_\phi \, r \int_{\phi=0}^{2\pi} d\phi = I$$

$$\therefore \quad H_\phi \, r(2\pi) = I$$

$$\therefore \quad H_\phi = \frac{I}{2\pi r}$$

Hence $\overline{H}$ at point P is given by,

$$\boxed{\overline{H} = H_\phi \, \bar{a}_\phi = \frac{I}{2\pi r} \bar{a}_\phi \quad A/m}$$

$\overline{H}$ Due to a Co-axial Cable

Consider a co-axial cable as shown in the Fig. Its inner conductor is solid with radius a, carrying direct current I. The outer conductor is in the form of concentric cylinder whose inner radius is b and outer radius is c. This cable is placed along z axis. The current I is uniformly distributed in the inner conductor. While – I is uniformly distributed in the outer conductor.

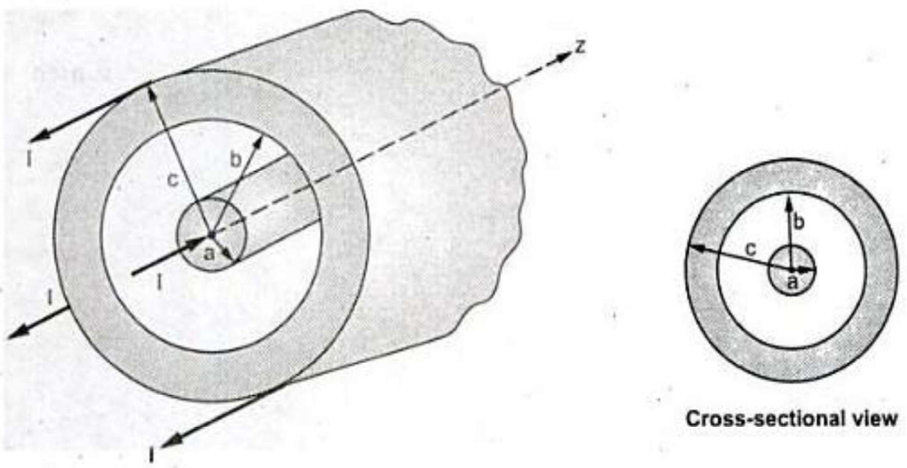
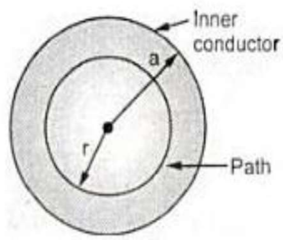


Fig. Co-axial cable

The space between inner and outer conductor is filled with dielectric say air. The calculation of $\overline{H}$ is divided corresponding to various regions of the cable.

Region 1 : Within the inner conductor, r < a. Consider a closed path having radius r < a. Hence it encloses only part of the conductor as shown in the Fig.



The area of cross-section enclosed is πr^2 m^2.

The total current flowing is I through the area πa^2. Hence the current enclosed by the closed path is,

$$I' = \frac{\pi r^2}{\pi a^2} I = \frac{r^2}{a^2} I$$

The $\bar{H}$ is again only in $\bar{a}_\phi$ direction and depends only on r.

$$\therefore \quad \bar{H} = H_\phi \bar{a}_\phi$$

So consider $d\bar{L}$ in the $\bar{a}_\phi$ direction which is $r\, d\phi$.

$$\therefore \quad d\bar{L} = r d\phi\, \bar{a}_\phi$$

$$\therefore \quad \bar{H} \bullet d\bar{L} = H_\phi\, \bar{a}_\phi \bullet r d\phi \bar{a}_\phi = H_\phi\, r\, d\phi$$

According to Ampere's circuital law,

$$\oint \bar{H} \bullet d\bar{L} = I'$$

$$\therefore \quad \oint H_\phi\, r\, d\phi = \frac{r^2}{a^2} I$$

$$\therefore \quad \int_{\phi=0}^{2\pi} H_\phi\, r\, d\phi = \frac{r^2}{a^2} I$$

$$\therefore \quad H_\phi\, r[2\pi] = \frac{r^2}{a^2} I$$

$$\therefore \quad H_\phi = \frac{r^2}{2\pi r a^2} I = \frac{r}{2\pi a^2} I$$

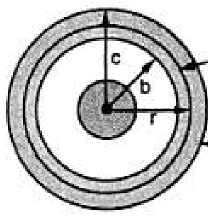
$$\boxed{\bar{H} = \frac{I r}{2\pi a^2} \bar{a}_\phi \quad A/m}$$

$\therefore$ ... r < a within conductor

Region 2 : Within $a < r < b$ consider a circular path which encloses the inner conductor carrying direct current I. This is the case of infinitely long conductor along z-axis. Hence $\overline{H}$ in this region is,

$$\boxed{\overline{H} = \frac{I}{2\pi r}\bar{a}_\phi \text{ A/m}} \qquad \ldots (a < r < b)$$

Region 3 : Within outer conductor, $b < r < c$



Consider the closed path as shown in the Fig. The current enclosed by the closed path is only the part of the current $-$ I, in the outer conductor. The total current $-$ I is flowing through the cross section $\pi(c^2 - b^2)$ while the closed path encloses the cross section $\pi(r^2 - b^2)$.

Hence the current enclosed by the closed path of **outer conductor** is,

$$I' = \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)}(-I) = -\frac{(r^2 - b^2)}{(c^2 - b^2)} I$$

$$\therefore \quad I'' = I = \text{current in inner conductor enclosed}$$

Total current enclosed by the closed path is,

$$I_{enc} = I' + I'' = -\frac{(r^2 - b^2)}{(c^2 - b^2)} I + I$$

$$= I\left[1 - \frac{(r^2 - b^2)}{(c^2 - b^2)}\right] = I\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

According to Ampere's circuital law,

$$\oint \overline{H} \bullet d\overline{L} = I_{enc}$$

Now $\overline{H}$ is again in $\bar{a}_\phi$ direction only and is a function of r only.

$$\therefore \quad \overline{H} = H_\phi \bar{a}_\phi \quad \text{and} \quad d\overline{L} = r\, d\phi\, \bar{a}_\phi$$

$$\therefore \quad \overline{H} \bullet d\overline{L} = H_\phi \bar{a}_\phi \bullet r d\phi\, \bar{a}_\phi = H_\phi\, r\, d\phi$$

$$\therefore \quad \int_{\phi=0}^{2\pi} H_\phi\, r\, d\phi = I_{enc}$$

$$\therefore \quad H_\phi\, r[2\pi] = I\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

$$\therefore \quad H_\phi = \frac{I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

$$\therefore \quad \boxed{\overline{H} = H_\phi \bar{a}_\phi = \frac{I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]\bar{a}_\phi \quad \text{A/m}}$$

Region 4 : Outside the cable, $r > c$.

Consider the closed path with $r > c$ such that it encloses both the conductors i.e. both currents $+I$ and $-I$.

Thus the total current enclosed is,

$$I_{enc} = +I - I = 0 \text{ A}$$

$$\therefore \quad \oint \bar{H} \cdot d\bar{L} = 0 \qquad \text{... Ampere's circuital law}$$

$$\therefore \quad \boxed{\bar{H} = 0 \text{ A/m}} \qquad \text{... } r > c$$

The magnetic field does not exist outside the cable. The variation of $\bar{H}$ against r is shown in the Fig.

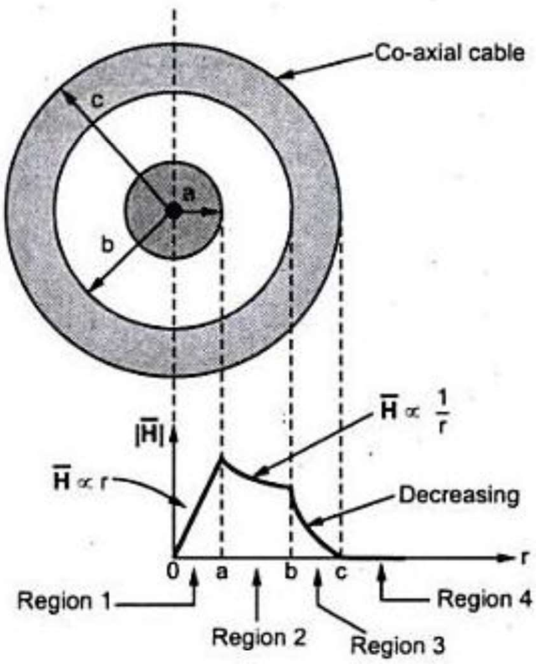
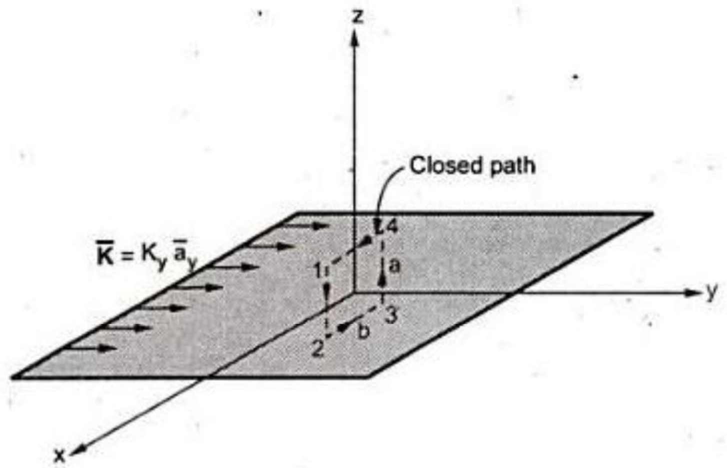


Fig. Variation of $\bar{H}$ against r in co-axial cable

$\bar{H}$ Due to Infinite Sheet of Current

Consider an infinite sheet of current in the $z = 0$ plane. The surface current density is $\bar{K}$. The current is flowing in positive y direction hence $\bar{K} = K_y \bar{a}_y$. This is shown in the Fig.

Consider a closed path 1-2-3-4 as shown in the Fig. The width of the path is b while the height is a. It is perpendicular to the direction of current hence in xz plane.



As current is flowing in y direction, H can not have component in y direction.

So $\overline{H}$ has only component in x direction.

$$\therefore \quad \overline{H} = H_x \bar{a}_x \quad \text{... for } z > 0 \quad \text{...}$$

$$= -H_x \bar{a}_x \quad \text{... for } z < 0 \quad \text{...}$$

Applying Ampere's circuit law,

$$\oint \overline{H} \cdot d\overline{L} = I_{enc}$$

Evaluate the integral along the path 1-2-3-4-1.

For path 1-2, $d\overline{L} = dz\, \bar{a}_z$,

For path 3-4, $d\overline{L} = dz\, \bar{a}_z$

But $\overline{H}$ is in x direction while $\bar{a}_x \cdot \bar{a}_z = 0$.

Hence **along the paths 1-2 and 3-4**, the integral $\oint \overline{H} \cdot dL = 0$.

Consider path 2-3 along which $d\overline{L} = dx\, \bar{a}_x$.

$$\therefore \quad \int_2^3 \overline{H} \cdot d\overline{L} = \int_2^3 (-H_x \bar{a}_x) \cdot (dx \bar{a}_x) = H_x \int_2^3 dx = b H_x$$

The path 2-3 is lying in $z < 0$ region for which $\overline{H}$ is $-H_x \bar{a}_x$. And **limits from 2 to 3, positive x to negative x hence effective sign of the integral is positive.**

Consider path 4-1 along which $d\overline{L} = dx\, \bar{a}_x$ and it is in the region $z > 0$ hence $\overline{H} = H_x \bar{a}_x$.

$$\therefore \quad \int_4^1 \overline{H}\cdot d\overline{L} = \int_4^1 (H_x\,\overline{a}_x)\cdot(dx\,\overline{a}_x) = H_x\int_4^1 dx = b\,H_x$$

$$\therefore \quad \oint \overline{H}\cdot d\overline{L} = b\,H_x + b\,H_x = 2\,b\,H_x$$

Equating this to I_{enc} in equation (6),

$$2\,b\,H_x = K_y\,b$$

$$\therefore \quad H_x = \frac{1}{2}K_y$$

Hence,

$$\overline{H} = \frac{1}{2}K_y\,\overline{a}_x \quad \text{for} \quad z > 0$$

$$= -\frac{1}{2}K_y\,\overline{a}_x \quad \text{for} \quad z < 0$$

In general, for an infinite sheet of current density $\overline{K}$ A/m we can write,

$$\boxed{\overline{H} = \frac{1}{2}\,\overline{K}\times\overline{a}_N}$$

where $\overline{a}_N$ = Unit vector normal from the current sheet to the point at which $\overline{H}$ is to be obtained.

3.5. POINT FORM OF AMPERE'S CIRCUITAL LAW

According to ampere's circuital law, the line integral of magnetic field intensity around a closed path is equal to the current enclosed by the path.

$$\oint H.dL = I_{enclosed}$$

Replacing current by surface integral of conductivity of current density J over an area bounded by the path of integration of H $\quad \oint H.dL = \int_s J.ds$

By adding displacement current density to conductor current density,

$$\oint H.dL = \int_s \frac{\partial D}{\partial t} ds$$

This is Maxwell equation derived from ampere's circuital law.This equation in integral of H is carried over the closed path bounding the surface s over which the integration is carried out on R.H.S.The equation is also called Mesh relation.

Applying stoke's theorem,

$$\oint (\nabla \times H).ds = \int \frac{\partial D}{\partial t}.ds$$

Assume the surface considered for both integration is same,

$$\nabla \times H = \frac{\partial D}{\partial t}$$

The equation is point form of differential form of Ampere's circuital law.

3.6. STOKES THEOREM

Statement

The line integral of a vector around a closed path is equal to surface integral of the normal component of its equal to the integral of the normal component of its curl ever any closed surface.

$$\oint H.dl = \iint_s \nabla \times H\, ds$$

Proof Consider an arbitrary surface. This is broken up into incremental surfaces of areas ∇s as shown in Fig. If H is any field vector, then by definition of the curl to one of these incremental surfaces. The Stokes theorem relates the line integral. It states that, The line integral of H around a closed path L is equal to the integral of curl of H over the open surface S enclosed by the closed path L. Mathematically it is expressed as,

$$\oint_L \boldsymbol{H.dl} = \int_s (\nabla \boldsymbol{xH})\ \boldsymbol{ds}$$

dl- Perimeter of total surface S

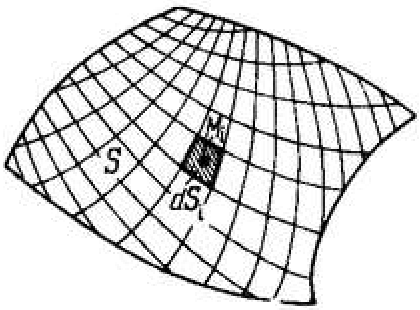


Fig 3.8 Stokes theorem

Stokes theorem is applicable only when H and ∇xH are continuous on the surface S. The path L and open surface S enclosed by the path L.

$$\frac{\oint H.dl \nabla s}{\nabla s} = (\nabla x H) N$$

Where N indicates normal to the surface and dl∇s indicate that the closed path of an incremental area ∇s.

The curl of H normal to the surface can be written as

$$\frac{\oint H.dl \nabla s}{\nabla s} = (\nabla x H).aN,$$

$$\frac{\oint H.dl \nabla s}{\nabla s} = (\nabla x H).aN \nabla s$$

$$= (\nabla xH).\ \nabla s.$$

Where a_N is a unit vector normal to ∇s.

The closed integral for whole surface S is given by the surface s integral of the normal component of curl H.

$$\oint H.dl = \iint_s \nabla\times H.ds$$

∴ Hence proved

3.7. MAGNETIC FLUX DENSITY

The magnetic flux density $\bar{B}$ is analogous to the electric flux density $\bar{D}$. The relation between $\bar{B}$ and $\bar{H}$ is which is through the property of medium called permeability μ. The relation is given by,

$$\bar{B} = \mu\bar{H}$$

For free space μ=μ0= 4π x 10^{-7} H/m hence, $\bar{B} = \mu 0\bar{H}$

The magnetic flux density has unitswb/m^2 and hence it can be defined as the flux in webers passing through unit area in a plane at right angles to the direction flux. If the flux passing through the unit area is not exactly at right angle to the plane consisting the area but making some angle with the plane then the flux φ crossing the area is given by,

$$\varphi = \oint_S \bar{B}.\overline{ds}$$

Where, φ – Magnetic flux in webers

$\bar{B}$ – Magnetic flux density in wb/m^2

$\overline{ds}$ – Open surface through which flux is passing

Consider a closed surface which is defining a certain volume. The magnetic flux lines are always exists in the form of closed loop. The single magnetic pole can notexists like a single isolated electric charges. No magnetic flux can reside in a closed surface is always zero.

$$\oint_S \bar{B}.\overline{ds} = 0$$

This is called law of conservation of magnetic flux (or) Gauss law in integral form for magnetic field.

Applying divergence theorem,

$$\oint_S \bar{B}.\overline{ds} = 0 = \int_{vol} \nabla.B\, dv$$

Where dv = volume enclosed by the closed surface but as dv is not zero.

$$\nabla.\bar{B} = 0$$

The divergence theorem of magnetic flux density is always zero. This is called Gauss law in differential form for magnetic field.

3.8. THE SCALAR AND VECTOR MAGNETIC POTENTIALS

3.8.1. Magnetic scalar potential

The concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a scalar magnetic potential and write: $\bar{H} = -\nabla V_m$

From Ampere's law , we know that

$$\nabla X\bar{H} = \bar{J}$$

$$\nabla X(-\nabla V_m) = \bar{J}$$

But using vector identity, $\nabla X(\nabla V) = 0$ we find that $\bar{H} = -\nabla V_m$ is valid only where $\bar{J} = 0$. Thus the scalar magnetic potential is defined only in the region where $\bar{J} = 0$. Moreover, V_m in general is not a single valued function of position.

If V_m is the magnetic potential then, $-\nabla V_m = -\frac{1}{\rho}\frac{\partial V_m}{\partial \varphi}$

$$= \frac{I}{2\pi\rho}$$

$$\therefore V_m = -\frac{I}{2\pi}\varphi + C$$

If we set $V_m = 0$ at $\varphi = 0$ then $c = 0$ and $V_m = -\frac{I}{2\pi}\varphi$

$$\therefore \text{At } \varphi = \varphi_0 \qquad V_m = -\frac{I}{2\pi}\varphi$$

we observe that as we make a complete lap around the current carrying conductor , we reach φ_0 again but V_m this time becomes $\quad V_m = -\frac{I}{2\pi}(\varphi_0 + 2\pi)$

We observe that value of V_m keeps changing as we complete additional laps to pass through the same point. We introduced V_m analogous to electrostatic potential V.

But for static electric fields, $\nabla X\bar{E} = 0$ and $\oint \bar{E}.\bar{dl} = 0$,

whereas for steady magnetic field, $\nabla X\bar{E} = 0$ wherever $\bar{J} = 0$ but $\oint \bar{H}.\bar{dl} = I$ even if $\bar{J} = 0$ along the path of integration.

3.8.2. Magnetic vector potential

The vector magnetic potential which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.

Since , $\nabla X\bar{B} = 0$ and we have the vector identity that for any vector $\bar{A}$, $\nabla.(\nabla X\bar{A}) = 0$, we can write $\bar{B} = \nabla X\bar{A}$.Here, the vector field $\bar{A}$ is called the vector magnetic potential. Its SI unit is Wb/m. Thus if can find $\bar{A}$ of a given current distribution, $\bar{B}$ can be found from $\bar{A}$ through a curl operation.

The vector function $\bar{A}$ and related its curl to $\bar{B}$. A vector function is defined fully in terms of its curl as well as divergence. The choice of $\nabla.\bar{A}$ is made as follows.

$$\nabla X \nabla X\bar{A} = \mu\nabla X\bar{H} = \mu\bar{J}$$

By using vector identity, $\nabla X \nabla X\bar{A} = \nabla(\nabla.\bar{A}) - \nabla^2\bar{A}$

$$\nabla(\nabla.\bar{A}) - \nabla^2\bar{A} = \mu\bar{J}$$

Great deal of simplification can be achieved if we choose $\nabla.\bar{A} = 0$.

Putting $\nabla.\bar{A} = 0$, we get $\nabla^2\bar{A} = -\mu\bar{J}$ which is vector poisson equation. In Cartesian coordinates, the above equation can be written in terms of the components as

$$\nabla^2\bar{A}_x = \mu\bar{J}_x$$

$$\nabla^2\bar{A}_y = \mu\bar{J}_y$$

$$\nabla^2\bar{A}_z = \mu\bar{J}_z$$

The form of all the above equation is same as that of $\nabla^2 V = -\frac{\rho}{\varepsilon}$. for which the solution is

$$V = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho}{R}dv \qquad\qquad R = |\bar{r} - \bar{r}'|$$

In case of time varying fields we shall see that $\nabla^2\bar{A} = \mu\varepsilon\frac{\partial V}{\partial t}$, which is known as Lorentz condition, *V* being the electric potential. Here we are dealing with static magnetic field, so $\nabla.\bar{A} = 0$. By comparison, we can write the solution for Ax as

$$Ax = \frac{\mu}{4\pi}\int_{V'}\frac{J_x}{R}dV'$$

Computing similar solutions for other two components of the vector potential, the vector potential can be written as $\bar{A} = \frac{\mu}{4\pi}\int_{V'}\frac{J}{R}dV'$. This equation enables us to find the vector potential at a given point because of a volume current density $\bar{J}$. Similarly for line or surface current density we can write

$$\bar{A} = \frac{\mu}{4\pi}\int_{L'}\frac{I}{R}dl'.$$

$$\bar{A} = \frac{\mu}{4\pi}\int_{S'}\frac{K}{R}dS'.\text{ respectively}$$

The magnetic flux ψ through a given area S is given by

$$\psi = \int_S \bar{B}.\overline{ds}$$

$$\text{Substituting } \bar{B} = \nabla X\bar{A}.$$

$$\psi = \int_S \nabla X\bar{A}.\overline{ds} = \oint_l \bar{A}.\overline{dl}$$

Vector potential thus have the physical significance that its integral around any closed path is equal to the magnetic flux passing through that path.

3.9. DERIVATION OF STEADY MAGNETIC FIELD LAWS.

In steady magnetic field Biot Savart law and Ampere Circuital law are basic laws and these laws can be derived using the concept of vector magnetic potential.

Biot Savart law

Consider the current element located at point (x_1,y_1,z_1) and another point (x_2,y_2,z_2) vector magnetic potential $\bar{A}$ is given as,

$$A_2 = \int \frac{\mu_0 \bar{J}_1 dV_1}{4\pi R_{12}}$$

The definition of vector magnetic potential

$$\bar{B}_2 = \bar{\nabla}_2 X\,\bar{A}_2 = \mu_0 \bar{H}_2$$

$$\bar{H}_2 = \frac{1}{\mu_0}\bar{\nabla}_2 X\,\bar{A}_2$$

$$\bar{H}_2 = \frac{1}{\mu_0}\bar{\nabla}_2 X \int \frac{\mu_0 \bar{J}_1 dV_1}{4\pi R_{12}}$$

$$\bar{H}_2 = \frac{1}{4\pi}\bar{\nabla}_2 X \int \frac{\bar{J}_1 dV_1}{R_{12}}$$

$$\bar{H}_2 = \frac{1}{4\pi}\int \bar{\nabla}_2 X\,\frac{\bar{J}_1}{R_{12}}dV_1$$

The differential volume dV_1 is a scalar and is function of (x_1,y_1,z_1) only. So it can be tabent out of curl operation. The curl of the product of a scalar and a vector can be solved by using identity.

$$\bar{\nabla}X(S\bar{V}) = (\bar{\nabla}S)X\bar{V} + S(\bar{\nabla}X\bar{V})$$

$$Take\ S = \frac{1}{R_{12}}\ and\ \bar{V} = \bar{J}$$

$$\bar{H}_2 = \frac{1}{4\pi}\int \bar{\nabla}_2 X \frac{S}{J} dV_1$$

$$= \frac{1}{4\pi}\int \left(\bar{\nabla}_2 X \frac{1}{R_{12}}\right) X\bar{J}_1 + \frac{1}{R_{12}} (\bar{\nabla}_2 X \bar{J}_1)\, dV_1$$

Since $\bar{\nabla}_2 X \bar{J}_1$ indicates partial derivatives of a function of (x_1,y_1,z_1) taken with respect to variables (x_2,y_2,z_2)

$$\bar{\nabla}_2 X \bar{J}_1 = o$$

$$H_2 = \frac{1}{4\pi}\int \left(\bar{\nabla}_2 X \frac{1}{R_{12}}\right) X\bar{J}_1\, dV_1$$

$$and\ \bar{\nabla}_2 X \frac{1}{R_{12}} = -\frac{\bar{R}_{12}}{R_{12}^3} = \frac{aR_{12}}{R_{12}^2}$$

$$H_2 = \frac{1}{4\pi}\int \left(\frac{aR_{12}}{R_{12}^2} X\bar{J}_1\right) dV_1$$

$$= \frac{1}{4\pi}\int \left(\frac{\bar{J}_1 X aR_{12}}{R_{12}^2}\right) dV_1$$

$$\bar{J}_1 dV_1 = I_1\overline{dL_1} = Kd\bar{s}$$

$$H_2 = \frac{1}{4\pi}\int \left(\frac{I_1\overline{dL_1} X aR_{12}}{R_{12}^2}\right)$$

This is Biot Savart Law, In general,

$$\bar{H} = \int \left(\frac{I\overline{dL} X aR}{4\pi R^2}\right)$$

$$\overline{dH} = \frac{I\overline{dL} X aR}{4\pi R^2}$$

Ampere Circuital law

Ampere circuital law in point form,

$$\bar{\nabla} X\bar{H} = \bar{J}$$

$$\bar{\nabla} X\bar{H} = \bar{\nabla} X \frac{\bar{B}}{\mu_0}$$

By the definition of vector magnetic potential

$$\bar{\nabla} X\bar{A} = \bar{B}$$

$$\bar{\nabla} X\bar{H} = \bar{\nabla} X \frac{\bar{\nabla} X\bar{A}}{\mu_0} = \frac{1}{\mu_0}(\bar{\nabla} X\bar{\nabla} X\bar{A})$$

But $(\bar{\nabla} X\bar{\nabla} X\bar{A}) = \bar{\nabla}(\bar{\nabla}.\bar{A}) - (\bar{\nabla}^2\bar{A})$ where $\bar{\nabla}^2\bar{A}$ is Laplacian of a vector and in Cartesian co ordinates.

$$\bar{\nabla}^2\bar{A} = \bar{\nabla}^2\bar{A}X\bar{a}_x + \bar{\nabla}^2\bar{A}\bar{a}_y + \bar{\nabla}^2\bar{A}\bar{a}_z$$

$$\bar{\nabla} X\bar{H} = \frac{1}{\mu_0}\left(\bar{\nabla}(\bar{\nabla}.\bar{A}) - (\bar{\nabla}^2\bar{A})\right)$$

Divergence of A is given by,

$$\overline{\nabla_2}.\bar{A}_2 = \frac{\mu_0}{4\pi}\int \overline{\nabla_2} X \frac{\bar{J}_1 X dV_1}{R_{12}}$$

Using vector identity,

$$\overline{\nabla}.(S\overline{V}) = \overline{V}(\overline{\nabla}.S) + S(\overline{\nabla}.\overline{V})$$

$$\overline{\nabla_2}.\bar{A}_2 = \frac{\mu_0}{4\pi}\left[\int \bar{J}_1\left(\overline{\nabla_2}\frac{1}{R_{12}}\right) + \frac{1}{R_{12}}(\overline{\nabla}_2.\bar{J}_1)\right] dV_1$$

As $\bar{J}_1$ not a function of (x_2,y_2,z_2)

$$\overline{\nabla}_2 X\,\bar{J}_1 = o$$

$$\overline{\nabla_2}\left(\frac{1}{R_{12}}\right) = -\frac{\overline{R_{12}}}{R_{12}}\; and\; \overline{\nabla_1}\left(\frac{1}{R_{12}}\right) = \frac{\overline{R_{12}}}{R_{12}}$$

$$\overline{\nabla_2}\left(\frac{1}{R_{12}}\right) = -\overline{\nabla_1}\left(\frac{1}{R_{12}}\right)$$

$$\overline{\nabla_2}.\bar{A}_2 = -\frac{\mu_0}{4\pi}\int \bar{J}_1\overline{\nabla_1}\left(\frac{1}{R_{12}}\right) dV_1$$

Again applying vector identity,

$$\overline{\nabla_2}.\bar{A}_2 = -\frac{\mu_0}{4\pi}\int\left[\overline{\nabla_1}\left(\frac{\bar{J}_1}{R_{12}}\right) - \frac{1}{R_{12}}(\overline{\nabla_1}.\bar{J}_1)\right] dV_1$$

$$\overline{\nabla_2}.\bar{A}_2 = \frac{\mu_0}{4\pi}\int\left[\frac{1}{R_{12}}(\overline{\nabla_1}.\bar{J}_1) - \overline{\nabla_1}\left(\frac{\bar{J}_1}{R_{12}}\right)\right] dV_1$$

For steady magnetic field from the continuity equation $\overline{\nabla}.\bar{J} = o$

$$\overline{\nabla_2}.\bar{A}_2 = -\frac{\mu_0}{4\pi}\int\left[\overline{\nabla_1}\left(\frac{\bar{J}_1}{R_{12}}\right)\right] dV_1$$

Using divergence theorem,

$$\overline{\nabla_2}.\bar{A}_2 = -\frac{\mu_0}{4\pi}\oint\frac{\bar{J}_1}{R_{12}}\,dS_1$$

Where the surface S_1 encloses the volume throughout which are integrating. This volume must include all the currents. Since there is no current outside this volume, may integrate over a slightly larger volume or slightly larger enclosing surface without changing vector magnetic potential $\bar{A}$. On this larger surface the current density $\overline{J_1}$ must be zero, and therefore the closed surface integral is zero. Since the integrant is zero. Hence divergence of $\bar{A}$ is zero.

Comparing the X component of $\bar{A}$ with the electrostatic potential

$$\bar{A} = -\frac{\mu_0}{4\pi}\oint\frac{\bar{J}_1 dV_1}{R}\; and\; V = \int\frac{\rho V dV}{4\pi\varepsilon_0 R}$$

$$\mu_0 = \frac{1}{\varepsilon_0}, \bar{J} = \rho\; and\; \bar{A} = V$$

For changing the variable in Poisson equation

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

The Poisson equation becomes,

$$\nabla^2 A_x = -\mu_0 J_x$$
$$\nabla^2 A_y = -\mu_0 J_y$$
$$\nabla^2 A_z = -\mu_0 J_Z$$
$$\text{or}\;\; \nabla^2 A = -\mu_0 J$$

By substituting the divergence and Laplacian of $\bar{A}$ in

$$\bar{\nabla} X\bar{H} = \frac{1}{\mu_0}\left(\bar{\nabla}\,(\bar{\nabla}\,.\bar{A}) - (\bar{\nabla}^2\bar{A})\right)$$

$$\bar{\nabla} X\bar{H} = \bar{J}$$

This is point form of Ampere circuital law.

PROBLEMS

EXAMPLE:3.1

Determine the force between two parallel conductors of length 1m separated by 50 cm in air and carrying currents of 30A. i. in the same dn b). in opposite dn.

Solution:

Given I = 30 A,
l = 1m,
d = 0.5 m.

a). Force of attraction:

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

$$F = \frac{\mu_0 I^2 l}{2\pi d} \qquad I_1 = I_2 = 30A$$

$$= \frac{4\pi\times10^{-7}\times(30)^2\times1}{2\pi\times0.5}$$

$$= 900\times4\times10^{-7}$$

$$= 3600\times10^{-7}$$

$$F = 0.36\times10^{-3}\,N$$

b). Force of Repulsion:

$$F = \frac{\mu_0 I^2 l}{2\pi d} = 0.36\times10^{-3}\,N.$$

EXAMPLE:3.2

Determine the force between two parallel conductors of length 1m separated by 50 cm in air and carrying currents of 30A. i. in the same dn b). in opposite dn.

Solution:

Given

I = 30 A,
l = 1m,
d = 0.5 m.

a). Force of attraction:

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

$$F = \frac{\mu_0 I^2 l}{2\pi d} \qquad I_1 = I_2 = 30A$$

$$= \frac{4\pi \times 10^{-7} \times (30)^2 \times 1}{2\pi \times 0.5}$$

$$= 900 \times 4 \times 10^{-7}$$

$$= 3600 \times 10^{-7}$$

$$F = 0.36 \times 10^{-3}\, N$$

b). Force of Repulsion:

$$F = \frac{\mu_0 I^2 l}{2\pi d} = 0.36 \times 10^{-3}\, N.$$

EXAMPLE:3.3

What is the maximum torque on a square loop of 1000 turns in a field of intensity of 1 Tesla. The loop has 10 cm sides and carries 3A. What is magnetic moment of loop?

Solution:

N = 1000
a = 0.1 m
I = 3A
B = 1 Tesla
Area = $0.01 m^2$
Torque = IAB
= 3 x 0.01 x 1
= 0.03 N-m
Magnetic Moment = IA
= 3 x 0.01
= 0.03 Amp. m^2

EXAMPLE:3.4

A circular coil of radius 10 Cm is made up of 100 turns. It carries a current of 5A. Compute Magnetic field Intensity at centre of coil.

Solution:

Given a = 10×10^{-2} m
N = 100 turns
I = 5A

$$H = \frac{NI}{2a}$$

EXAMPLE:3.5

Calculate the magnetic flux density due to a circular coil fo 100 amp turns and area of 70 cm^2 on the axis of coil at distance 10 cm from the centre.

Solution:

Given
NI = 100 AT
Area = $70 \pm \times 10^{-4} m^2 = \pi a^2$
d = 0.1m

$$\therefore a^2 = 22.28 \times 10^{-4} m^2$$

Magnetic flux Density B = $\dfrac{\mu_0 N I a^2}{2(a^2+d^2)^{3/2}} Wb/m^2$

$= \dfrac{4\pi\times10^{-7}\times100\times100\times22.28\times10^{-4}}{2(22.28\times10^{-4}+0.01)^{3/2}}$

= 103.7×10^{-6} Tesla.

B = 103.7μT.

EXAMPLE:3.6

Two wires carrying in the same dn of 500A and 800 A are placed with this axes 5cm apart. Calculate Force b/w them.

Solution

Given

I_1 = 500 A

I_2 = 800 A

R = 5×10^{-2}m.

$$F = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi\times10^{-7}\times500\times800}{2\pi\times5\times10^{-2}} = 16\times10^{-5} N.$$

EXAMPLE:3.7

Find the maximum torque on a n 100 turn rectangular coil, 0.2 m by 0.3m, carrying a current of 2A in the field of flux density 5 Wb/m^2?

Solution:

Given

N = 100

A = 0.2 x0.3 = 0.06 m^2

I = 2A

B = 5 Wb/m^2

T_{max} = NIAB

= 100 x 2 x 0.06 x 5

T_{max} = 60 N-m

EXAMPLE:3.8

Determine the force per unit length between two long parallel wires separated by 5 Cm in air and carrying current of 40A in the same direction.

Force/ length = $\dfrac{\mu_0 I_1 I_2}{2\pi d}$

$$= \frac{4\pi\times10^{-7}\times40\times40}{2\pi\times5\times10^{-2}} = 6.4\times10^{-3} N/m.$$

Calculate $\vec{B}$ if the vector potential $\vec{A} = \vec{a}\,5(x^2+y^2+z^2)^{-1}$

Solution:

$\vec{B} = \nabla \times \bar{A}$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 5(x^2+y^2+z^2)^{-1} & 0 & 0 \end{vmatrix}$$

$$= \vec{ax}[0] - \vec{ay}\left[\frac{\partial}{\partial z}(5(x^2+y^2+z^2))^{-1}\right] + \vec{a}_z\left[-\frac{\partial}{\partial z}(5(x^2+y^2+z^2))^{-1}\right]$$

$$= -(1)5\vec{ay}(x^2+y^2+z^2)^{-2}(2z) - 5\vec{a_2}(-1)(x^2+y^2+z^2)^{-1}(2y)$$

$$\vec{B} = -10z(x^2+y^2+z^2)^{-2}\vec{ay} + 10y(x^2+y^2+z^2)^{-2}\vec{a}_z$$

$$= \frac{-10}{(x^2+y^2+x^2)^2}\left[z\vec{ay} - y\vec{a}_z\right]$$

EXAMPLE:3.9

A wire carrying a current of 100A is bent into the form of a circle of diameter 10cm. Calculate a). flux density at the centre of the coil. B). Flux density at appoint on the axis of the coil and 12 cm from it.

Solution:-

a). Flux density at the centre of the coil.

$$B = \frac{\mu_0 NI}{2a}$$

I = 100 A, N = 1 (Let)

Radius $a = \frac{10}{2} = 5cm = -5\times10^{-2}\, m$

$$B = \frac{4\pi\times10^{-7}\times1\times100}{2\times5\times10^{-2}} = 1.256\times10^{-3}\, Wb/m^2$$

$$B = 1.256\times10^{-3}\, Wb/m^2$$

b). Flux Density at appoint on the axis of the coil and 12 cm from it.

$$B = \frac{\mu_0 NIa^2}{2(a^2+a^2)^{3/2}}$$

$$d = 12\times10^{-2}\, m$$

$$B = \frac{4\pi\times10^{-7}\times100\times1\times(5\times10^{-2})^2}{2\left[(5\times10^{-2})^2+(12\times10^{-2})^2\right]^{3/2}}$$

$$B = 0.0714\times10^{-3}\, Tesla$$

$$B = 0.0714m\, Wb/m^2$$

EXAMPLE:3.10

A single phase circuit comprises two parallel conductors A and B, 1 cm in diameter and spaced 1m apart. The conductors carry currents of +100 amps and -100 amps. Determine the field intensity at the surface of each conductor and also in the space exactly midway between A and B.

(i). $\vec{H}$ at conductor surface

$$H = \frac{I}{2\pi a} = \frac{100}{2 \times \pi 0.005} = 3184.7 A/m$$

$$= 3.1847 \times 10^3 A/M$$

ii. Field at any point P between A and B is

H = $H_A + H_B$

$$H = \frac{I}{2\pi r} + \frac{I}{2\pi r_2}$$

At mid point $r_1 = r_2 = 0.5m$

$$H = \frac{I}{2\pi(0.5)} + \frac{I}{2\pi(0.5)} = \frac{I}{\pi} + \frac{I}{\pi} = \frac{2I}{\pi}$$

$$H = \frac{200}{\pi} A/m$$

The variation of magnetic field intensity with respect to r is as follows.

SUMMARY

- Biot –Savart's law states that the magnetic flux density at any point due to current element is proportional to the current element and sine of the angle between the elemental length and inversely proportional to the square of the distance between them

$$dB = \mu_0 Idl \frac{\sin\theta}{4\pi r^2}$$

- Amperes circuital law. The line integral of magnetic field intensity around a closed path is equal to the direct current enclosed by the path .

$$\int H.dL = I$$

- Magnetic scalar potential and magnetic vector potential is defined as dead quantity whose negative gradient gives the magnetic intensity if there is no current source present. H = -∇V m Where V m is the magnetic vector potential
- The quantitative measure of strongness or weakness of the magnetic field is given by magnetic field intensity or magnetic field strength. The magnetic field intensity at any point in the magnetic field is defined as the force experienced by the unit north pole of the Weber strength, when place at that point.
- The total magnetic flux passing through any closed surface is equal to zero.

$$\int B.dS = 0$$

- Integral and point form of Ampere's law.
  - General form: $\int H.dL = I$
  - Integral form: : $\int H.dL = \int J.dS$

- Point form :∇x H = J
- The magnetic flux density $\overline{B}$ is analogous to the electric flux density $\overline{D}$. The relation between $\overline{B}$ and $\overline{H}$ is which is through the property of medium called permeability μ. The relation is given by,

$$\overline{B} = \mu\overline{H}$$

- For free space μ=μ0= 4π x 10^{-7} H/m hence, $\overline{B} = \mu 0\overline{H}$
- The magnetic flux density has unitswb/m^2 and hence it can be defined as the flux in webers passing through unit area in a plane at right angles to the direction flux.
- Magnetic flux density (B) =Magnetic flux area=AΦ webers /m^2 (Tesla)

TWO MARKS

1. Define Magnetic flux density.

   The total magnetic lines of force i.e. magnetic flux crossing a unit area in a plane at right angles to the direction of flux is called magnetic flux density. It is denoted as B .Unit Wb/m2.
2. State Ampere's circuital law.

   The line integral of magnetic field intensity H equal to the direct current enclosed by that path. around a closed path is exactly

   The mathematical representation is $\int H.dL = I$.
3. Define Magnetic field Intensity.

   Magnetic Field intensity at any point in the magnetic field is defined as the force experienced by a unit north pole of one Weber strength, when placed at that point. Unit: N/Wb (or) AT /m.It is denoted as r .
4. What is rotational and irrotational vector field?

   If curl of a vector field exists then the field is called rotational. For irrotational vector field, the curl vanishes i.e. curl is zero.
5. What is the relation between magnetic flux density and magnetic field Intensity**.**

$$\overline{B} = \mu\overline{H}$$

   Where

   B - Magnetic flux density (Tesla)
   H - Magnetic Field Intensity (A/m)
   μ0 - Permeability of free space
   μr - Relative permeability of medium
6. State Biot Savart Law.

   The Biot Savart law states that, The magnetic field intensity dH produced at a point p due to a differential current element IdL is

   - Proportional to the product of the current I and differential length dL
   - The sine of the angle between the element and the line joining point p to the element
   - And inversely proportional to the square of the distance R between point p and the element

   dH

$$dH = \frac{\text{Idl}\sin\theta}{4\pi r^2}$$

7. What is a capacitor?