UNIT IV

MAGNETIC FORCES AND MATERIALS

Force on a moving charge, Force on a differential current element, Force between current elements, Force and torque on a closed circuit, The nature of magnetic materials, Magnetization and permeability, Magnetic boundary conditions involving magnetic fields, The magnetic circuit, Potential energy and forces on magnetic materials, Inductance, Basic expressions for self and mutual inductances, Inductance evaluation for solenoid, toroid, coaxial cables and transmission lines, Energy stored in Magnetic fields.

4.1. FORCE ON A MOVING CHARGE

Consider a conductor in which the electron are in motion along with these electrons in motion, the mobile positive ion from a crystalline array structure of a conductor with the magnetic field applied to the conductor. A force is extracted on the electrons and thus their position is slightly shifted. Hence a small displacement is produced between the centers of the clarity of positive and negative charges. But this displacement is opposed by coulombs force between the positive ions and electrons. So when electrons try to move the force of attraction between the electrons and positive ions is opposed. The small voltage across conductor is called Hall voltage and the effect is called Hall effect. The Hall effect is useful in determining whether the given material is P type or n type. Thus the Hall voltage of both the material are different.

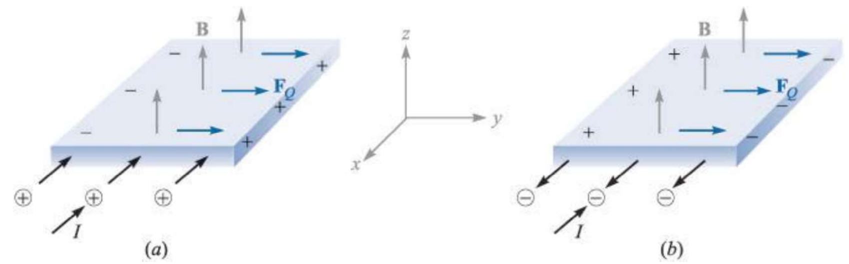

Fig. 4.1 Equal charge directed in the material a. negative charge moving outside b. positive charge moving outside

$$d\bar{F} = dQ\bar{v} \times \bar{B} \quad \mathbf{N}$$

The current density $\bar{J}$ can be expressed in terms of velocity of a volume charge density.

$$\bar{J} = \rho_v \bar{V}$$

But the differential element of charge can be expressed in terms of volume charge. $dQ = \rho_v dV$

$$d\bar{F} = \rho v dV (\bar{v} \times \bar{B})$$

$$d\bar{F} = \bar{J} (\bar{v} \times \bar{B})$$

The relationship between current element has $\bar{J}$ dv = $\bar{K}\, ds = I\, \overline{dl}$

Then the force extracted on a surface current density $\overline{dF} = \bar{K}\, ds$

Similarly force exerted on differential current element is,

$$\overline{dF} = I\, \overline{dL} \times \bar{B}$$

$$\bar{F} = \int_{vol} \bar{J} X \bar{B}\, dv$$

$$\bar{F} = \int_S \bar{K}\, X\, \bar{B}\, dS$$

$$\bar{F} = \int_S I\, \bar{dl}\, X\, \bar{B}$$

If the conductor is straight and the field $\bar{B}$ is uniform along it the integrating equation is

$$\bar{F} = I\bar{L} X \bar{B}$$

The magnitude of the force is given by,

$$F = ILB \sin\theta$$

4.2. FORCE ON A DIFFERENTIAL CURRENT ELEMENT

Consider a conductor in which the electrons are in motion. These motion the immobile positive ions forms crystalline array structure of the conductor, a force is exerted on the electron and thus their position is slightly shifted. Hence a small displacement is produced between the centers of gravity of positive and negative charges.

$$\overline{dF} = dQ\, \bar{V} X \bar{B} \;\; N$$

The current density $\bar{J}$ can be expressed in terms of velocity of a volume charge density is,

$$\bar{J} = \rho_v . \bar{V}$$

But the element of charge is differential

$$dQ = \rho_v dV$$

$$\overline{dF} = \rho_v dV\, \bar{V} X \bar{B}$$

$$\overline{dF} = \bar{J} X \bar{B}\, dV$$

The surface current density force exerted on

$$\overline{dF} = \bar{K} X \bar{B}\, dV$$

For line,

$$\overline{dF} = I\bar{dl} X \bar{B}\, dV$$

$$\bar{F} = \int_V \bar{J} X \bar{B}\, dV \quad \rightarrow \text{Volume}$$

$$\bar{F} = \int_S \bar{K} X \bar{B}\, dV \quad \rightarrow \text{Surface}$$

$$\bar{F} = \int_S I\overline{dl}\, X \bar{B}\, dV \quad \rightarrow \text{line}$$

$$\bar{F} = ILx\bar{B}$$

$$F = ILB \sin\theta$$

4.3. FORCE BETWEEN CURRENT ELEMENTS

If the direction of both currents are same then the conductor experience a force of repulsion. If the direction is opposite a force of attraction. The force exerted on a differential current element is,

$$d(dF_1) = I_1 \overline{dL_1}\, X\, d\overline{B_2}$$

$$d\overline{B_2} = \mu_0 d\overline{H_2} = \mu_0 \left[\frac{I_2 \overline{dL_2} X a\overline{R_{12}}}{4\pi R_{12}^2} \right]$$

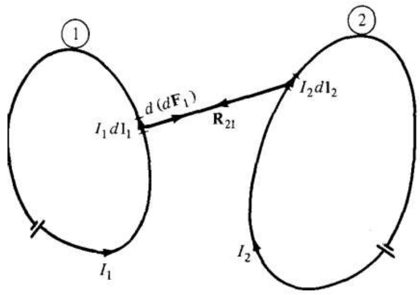

Fig 4.2 Force between to current loops

$$d(dF_1) = \mu_0 \left[\frac{I_1 \overline{dL_1} X I_2 \overline{dL_2} X a\overline{R_{12}}}{4\pi R_{12}^2}\right]$$

$$\overline{F_2} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_2} \oint_{L_1} \frac{\overline{dL_1} X \overline{dL_2} X a\overline{R_{12}}}{R_{12}^2}$$

$$\overline{F_1} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{\overline{dL_1} X \overline{dL_2} X \overline{a_{21}}}{R_{21}^2}$$

For the two current carrying conductor of length l each the force exerted is given by

$$F = \frac{\mu I_1 I_2}{2\pi d}$$

Where $I_1 and\, I_2$ are the currents flowing through conductor 1 and 2. D is the distance of separation between the two conductors.

4.4. FORCE AND TORQUE ON A CLOSED CIRCUIT

4.4.1. Torque

The moment of a force or torque about a specified point is defined as the vector product of the moment arm $\overline{R}$ and the force $\overline{F}$. It is measured in Newton meter (Nm)

$$\overline{T} = \overline{R}\ X\overline{F}$$

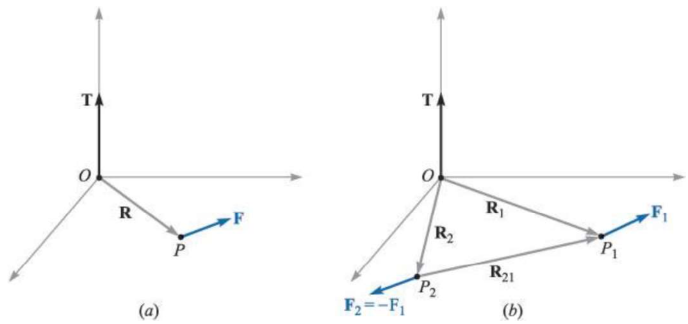

Fig 4.3 a. Torque at a point b. Torque between two pints

Consider a point A at which force $\overline{F}$ is applied as shown if the figure. Let $\overline{R}$ be the arm from origin O to point A. Then the torque $\overline{T}$ about origin is nothing but a vector product of $\overline{R}\ and\overline{F}$.

Now consider that two forces namely $\overline{F_1}$ and $\overline{F_2}$ are applied at point A_1 and A_2 Respectively. Assume that $\overline{F_2} = -\overline{F}_1$. Then torque $\overline{T}$ about the origin due to the two force is given by,

$$\overline{T} = \overline{R}_1 X\overline{F}_1 + \overline{R}_2 X\overline{F}_2$$

$$\overline{T} = (\overline{R}_2 - \overline{R}_1) X\overline{F}$$

$$\overline{T} = \overline{R}_{21} X\overline{F}_1$$

Where $\overline{R}_{21} = \overline{R}_2 - \overline{R}_1$ is a joining $A_2$to A_1 .when total force is zero the torque is independent of the choice of origin.

4.4.2. Force and Torque On A Loop

Consider a differential current loop of rectangular shape with uniform magnetic field everywhere around it as shown in figure4.4. Assume that the loop is placed in XY plane. Let the side AB and CD be parallel to X axis and sides BC and DA be parallel to Y axis. Let dx and dy be the length of the sides of the rectangular loop. As the rectangular loop is differential with the differential length dx and dy the value of magnetic field can be assumed B_0 everywhere. The total force on the loop is zero and the origin for the torque can be selected as the centre of the loop.

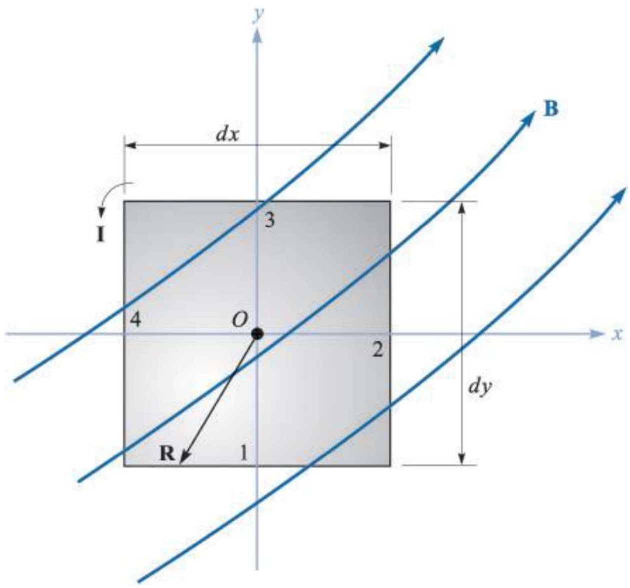

Fig 4.4. Torque On A Loop

The force exerted on the side AB is given by,

$$\overline{dF_1} = \mathrm{Idx}\overline{a_x}\ X\ \overline{B_0}$$

$$= \mathrm{Idx}\ (\overline{a_x}\ X(\overline{B_{0x}}\overline{a_x} + \overline{B_{0y}}\overline{a_y} + \overline{B_{0z}}\overline{a_z})$$

$$\overline{dF_1} = \mathrm{Idx}\ (\overline{B_{0y}}\overline{a_Z} + \overline{B_{0z}}\overline{a_y})$$

For the side AB the level arm exerted from the origin to the midpoint of the side AB.

$$\overline{R_1} = \frac{1}{2}dy(-\overline{a_y}) = -\frac{1}{2}dy\overline{a_y}$$

Hence the torque on side 1 is given by,

$$\overline{dT_1} = \overline{R}_1 Xd\overline{F}_1$$

$$= -\frac{1}{2}dy\overline{a_y} \text{ X Idx } (\overline{B_{0y}}\overline{a_Z} + \overline{B_{0z}}\overline{a_y})$$

$$= -\frac{1}{2} \text{ I dx } dy\overline{B_{0y}}\overline{a_x}$$

The force exerted on the side BC is given by

$$\overline{dF_2} = \text{Idy } \overline{a_y}\ X\ \overline{B_0}$$

$$= \text{Idy } (\overline{a_y}\ X(\overline{B_{0x}}\overline{a_x} + \overline{B_{0y}}\overline{a_y} + \overline{B_{0z}}\overline{a_z})$$

$$\overline{dF_2} = \text{Idy } (\overline{-B_{0x}}\overline{a_Z} + \overline{B_{0z}}\overline{a_x})$$

Thus the lever arm for side BC is given by,

$$\overline{dT_2} = \bar{R}\,_2 X d\bar{F}_2$$

$$= \frac{1}{2}dx\overline{a_x} \text{ X I dy } (\overline{-B_{0x}}\overline{a_Z} + \overline{B_{0z}}\overline{a_x})$$

$$= \frac{1}{2} \text{ I dx } dy\overline{B_{0x}}\overline{a_y}$$

Side3,

$$\overline{dF_3} = \text{-Idx } (\overline{B_{0y}}\overline{a_Z} + \overline{B_{0z}}\overline{a_y})$$

$$\overline{dT_3} = -\frac{1}{2}Idxdy\overline{B_{0y}}\overline{a_x}$$

Side4,

$$\overline{dF_4} = -Idy\ (\overline{-B_{0x}}\overline{a_Z} + \overline{B_{0z}}\overline{a_x})$$

$$\overline{dT_4} = \frac{1}{2}Idxdy\overline{B_{0x}}\overline{a_y}$$

Hence total force is given by,

$$\overline{dF} = \overline{dF_1} + \overline{dF_2} + \overline{dF_3} + \overline{dF_4}$$

$$=0$$

Hence total torque is given by,

$$\overline{dT} = \overline{dT_1} + \overline{dT_2} + \overline{dT_3} + \overline{dT_4}$$

$$= \frac{1}{2}Idxdy\overline{B_{0x}}\overline{a_y} - \frac{1}{2}Idxdy\overline{B_{0x}}\overline{a_y} - \frac{1}{2}Idxdy\overline{B_{0y}}\overline{a_x} - \frac{1}{2}Idxdy\overline{B_{0y}}\overline{a_x}$$

$$\overline{dT} = -\text{I dx } dy\overline{B_{0y}}\overline{a_x} + Idxdy\overline{B_{0x}}\overline{a_y}$$

$$= \text{I dx } dy(-\overline{B_{0y}}\overline{a_x} + \overline{B_{0x}}\overline{a_y})$$

$$= \text{I dx } dy[\overline{a_Z}\ X(\overline{B_{0x}}\overline{a_x} + \overline{B_{0y}}\overline{a_y} + \overline{B_{0z}}\overline{a_z})]$$

$$\overline{dT} = \text{Idx dy } (\overline{a_Z}\ X\ \overline{B_0})$$

When replace the product term dxdy by vector ared of the differential current loop, i.e, $\overline{ds}$

$$\overline{dT} = I\ \overline{ds} \text{ X} \bar{B}$$

4.5. MAGNETIC DIPOLE MOMENT

The magnetic dipole moment is given by

$$\bar{m} = (\text{IS})\overline{a_n} \text{ A.m}^2$$

The torque along the axis of rotation of a planar coil as,

$$\bar{T} = \text{BIS } (-\overline{a_y})$$

Using definition of the magnetic dipole moment the torque can be expressed as

$$\bar{T}=\bar{m} \times \bar{B}$$

When the planar loop or coil is normal to the magnetic field, sum of the force on the planar loop as well as the torque will be zero.

4.6. THE NATURE OF MAGNETIC MATERIALS

4.6.1. Origin of Magnetic Dipole moment in the Material

Basically the magnetic materials are classified on the basis of presence of magnetic dipole moments in the materials. A charged particle with angular momentum always contributes to the permanent magnetic dipole moments. Generally there are three important contributions at the angular moment of an atom.

i. Orbital magnetic dipole moment
ii. Electron spin magnetic moment
iii. Nuclear Spin magnetic moment

In any atom several electrons revolve in the circular orbits around the nucleus. This is very much analogous to a small current loop. The orbital state of motion of an electron in an atom is described with the quantum numbers n, l and ml. The angular momentum of an electron is called spin of the electron. As the electron is a charged particles the spin of the electron produce magnetic dipole moment. The nuclear spin Contribute to the magnetic moment called nuclear spin moment.

4.6.2. Classifications

On the basis of magnetic behavior the magnetic materials are classified as

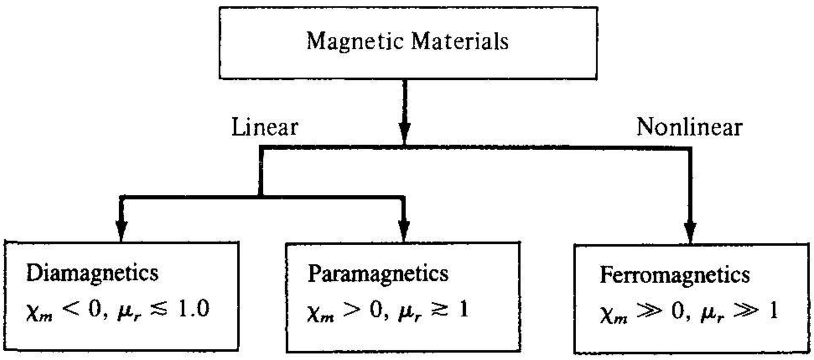

Fig. 4.5. Classification of magnetic materials.

i. Dia magnetic materials

These are materials which do not have dipole moment in the absence of an external applied magnetic field. In a diamagnetic material the atoms have no net magnetic moment when there is no applied field. Under the influence of an applied field (H) the spinning electrons process and this motion, which is a type of electric current, produces a magnetization (M) in the opposite direction to that of the applied field. In these materials magnetization is opposed to the applied field. The relative permeability $\mu_r < 1$.

Example: Silver, Lead, Copper, Water, Gold, Silicon, Graphite, Sodium

ii. Para magnetic materials

These are materials have permanent magnetic dipole moment but the interaction is negligible. So these materials have and light magnetic properties. In these magnetization is the same direction as applied field. It has $\mu_r \geq 1$.

Example: air, aluminium, paradium, potassium, oxygen, tungsten.

iii. Ferro magnetic materials

In these materials, dipole interact strongly and all tend to line up parallel with the applied field, so as to produce a large increase in flux density. Ferromagnetism is only possible when atoms are arranged in a lattice and the atomic magnetic
moments can interact to align parallel to each other. Here $\mu_r >> 1$.

Example: Iron, Nikel, Cobalt, permalloy, Mild steel, Silicon-iron.

iv. Anti ferro magnetic materials

In these materials the adjacent dipole align in anti parallel fashion to the applied field. The net magnetic moment is zero. Anti ferromagnetic materials are very similar to ferromagnetic materials but the exchange interaction between neighboring atoms leads to the anti-parallel alignment of the atomic magnetic moments. Therefore, the magnetic field cancels out and the material appears to behave in the same way as a paramagnetic material. Ant iferro magnetism is only present at relatively low temperature.

Example: Nickel oxide, Ferrous sulphide, Cobalt chloride, Manganese oxide.

v. Ferri magnetic materials

The materials also show an anti parallel alignment of atomic moments, but the moments are not equal. Normally a relatively large increase in flux density in produced as the strong dipole align themselves with the field and weak are aligned anti parallel to the applied field. Ferrimagnetism is only observed in compounds, which have more complex crystal structures than pure elements. Within these materials the exchange interactions lead to parallel alignment of atoms in some of the crystal sites and anti-parallel alignment of others. The material breaks down into magnetic domains, just like a ferromagnetic material and the magnetic behaviour is also very similar, although ferrimagnetic materials usually have lower saturation magnetisations.

Example: Iron oxide magnetite, Nickel Zinc ferrite, Nickel ferrite, etc

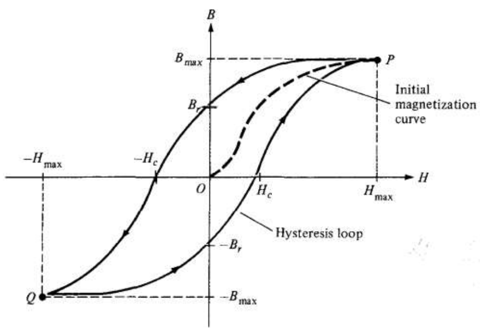

Figure 4.6 Typical magnetization *(B-H)* curve.

vi. Super magnetic materials:

Super magnetic is a form of magnetism, which appears in small ferromagnetic or ferri magnetic nano particles. In sufficiently small nanoparticles, magnetization can randomly flip direction under the influence of temperature. In the absence of an external magnetic field, when the time used to measure the magnetization of the nano particles is much longer than the *relaxation time*, their magnetization appears to be in average zero: they are said to be in the super magnetic state. The applications of Super magnetic materials are Locks for doors, Loudspeakers and headphones, Magnetic bearings and couplings, Benchtop NMR spectrometers

4.7. MAGNETIZATION AND PERMEABILITY

The magnetic field strength and flux density are related according to $B = \mu H$. The parameter is called the permeability, which is a property of the specific medium through which the H field passes and in which B is measured. The permeability has dimensions of webers per ampere-meter (Wb/A-m) or henries per meter. (H/m)

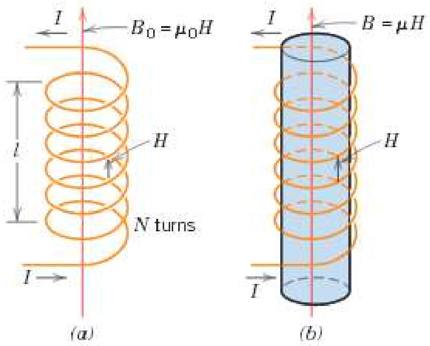

Fig 4.7.Magnetic field strength

(a) The magnetic field H as generated by a cylindrical coil is dependent on the current I, the number of turns N, and the coil length l,
(b) The magnetic flux B as generated by a cylindrical coil is dependent on the current I, the number of turns N, and the coil length l,

In a vacuum, $B = \mu_0 H$. where is the permeability of a vacuum, a universal constant, which has a value of $4\pi \times 10^{-7}$ H/m. The parameter B_0 represents the flux density within a vacuum as demonstrated in Figure 4.7 a .Several parameters may be used to describe the magnetic properties of solids. One of these is the ratio of the permeability in a material to the permeability in a vacuum,

$$\mu_r = \frac{\mu}{\mu_0}$$

where μ_r is called the relative permeability, which is unit less. The permeability or relative permeability of a material is a measure of the degree to which the material can be magnetized, or the ease with which a B field can be induced in the presence of an external H field.

Another field quantity M, called the magnetization of the solid, is defined by

$B = \mu_0 H + \mu_0 M$

In the presence of an H field, the magnetic moments within a material tend to become aligned with the field and to reinforce it by virtue of their magnetic fields; the term $\mu_0 M$. The magnitude of M is proportional to the applied field as

$$M = \chi_m H$$

And χ_m is called the magnetic susceptibility, which is unit less. The magnetic susceptibility and the relative permeability are related as follows

$$\chi_m = \mu_r - 1$$

For ferromagnetic material nonlinear and also depends on the "history" of the material.

$$B = \mu_0 H + \mu_0 \chi_m H$$

$$= \mu_0 (1+\chi_m) H$$

Or

$$B = \mu H$$

Where

$$\mu = \mu_0 (1+\chi_m)$$

is the magnetic permeability of the material and its unit is H/m. The relative permeability is defined as

$$\mu_r = 1+\chi_m$$

The relative permeability and the magnetic behavior of a material can be used as the basis guideline for classifying materials as diamagnetic, paramagnetic, or ferromagnetic.

4.8. MAGNETIC BOUNDARY CONDITIONS INVOLVING MAGNETIC FIELDS

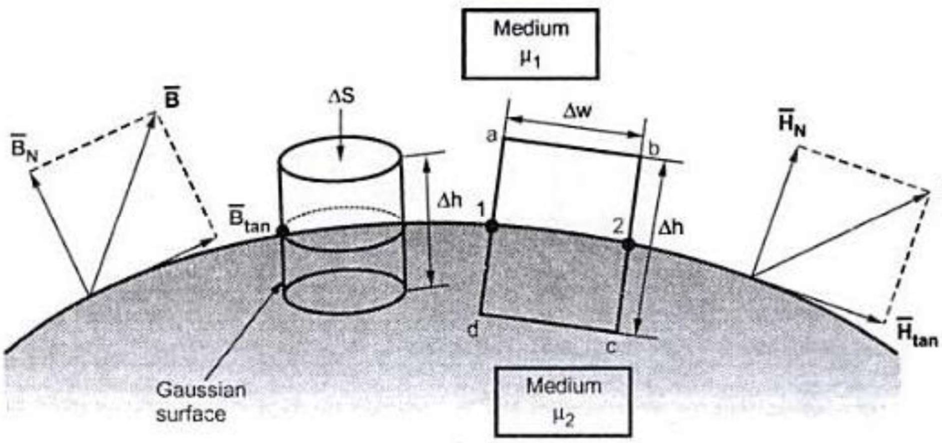

Fig.4.8. Magnetic Boundary Conditions Involving Magnetic Fields

The conditions of the magnetic field existing it the boundary of the two media, when the magnetic field passes one medium to other are called boundary condition for the magnetic boundary condition. The conditions of $\bar{B}$ *and* $\bar{H}$ at the boundary both the vector have two components,

i. Tangential to boundary
ii. Normal to boundary

i. Tangential to boundary

According to Ampere's circuital law,

$$\int \bar{H}.\overline{dL} = I$$

$$\int \overline{H}.\overline{dL} = \oint_a^b \overline{H}.\overline{dL} + \oint_b^1 \overline{H}.\overline{dL} + \oint_1^b \overline{H}.\overline{dL} + \oint_b^c \overline{H}.\overline{dL} + \oint_c^d \overline{H}.\overline{dL} = I$$

The elementary rectangular path with elements height Δh and elementary width Δw.

$$K.dw = H_{\tan 1}(\Delta w) + HN_1\left(\frac{\Delta h}{2}\right) + HN_2\left(\frac{\Delta h}{2}\right) - H_{\tan 2}(\Delta w) - HN_1\left(\frac{\Delta h}{2}\right) - HN_2\left(\frac{\Delta h}{2}\right)$$

At boundry $\Delta h \to 0$ Thus,

$$K.dw = H_{\tan 1}(\Delta w) - H_{\tan 2}(\Delta w)$$
$$H_{\tan 1} - H_{\tan 2} = K$$

In vector form,

$$\overline{H}_{\tan 1} - \overline{H}_{\tan 2} = a\overline{N}_{12} X \overline{K}$$

Where $a\overline{N}_{12}$ is the unit vector in the direction normal at boundry from medium 1 to medium 2.

ii. Normal to boundary

Let the area of the top and bottom are same, equal to ΔS

$$\oint_{top} \overline{B}.d\overline{S} + \oint_{bottom} \overline{B}.d\overline{S} + \oint_{lateral} \overline{B}.d\overline{S} = 0$$

When $\Delta h \to 0$ the magnitude of normal components of $\overline{B}$ be BN_1 and BN_2 in medium 1 and medium 2 respectively.

For the top surface,

$$\oint_{top} \overline{B}.d\overline{S} = BN_1 \oint_{top} d\overline{S} = BN_1 \Delta S$$

For the bottom surface,

$$\oint_{bottom} \overline{B}.d\overline{S} = BN_2 \oint_{bottom} d\overline{S} = BN_2 \Delta S$$

For the lateral surface,

$$\oint_{lateral} \overline{B}.d\overline{S} = 0$$
$$BN_1 \Delta S - BN_2 \Delta S = 0$$
$$BN_1 = BN_2$$

At the magnetic flux density and magnetic field intensity are related by, B=μH

$$\mu_1 HN_1 = \mu_2 HN_2$$
$$\frac{HN_1}{HN_2} = \frac{\mu_2}{\mu_1} = \frac{\mu_{r2}}{\mu_{r1}}$$

From the fig,

$$\text{tam } \alpha_1 = \frac{Btan_1}{BN_1} \qquad \text{tam } \alpha_2 = \frac{Btan_2}{BN_2}$$
$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{Btan_1}{Btan_2} \cdot \frac{BN_1}{BN_2}$$
$$= \frac{Btan_1}{Btan_2} = \frac{\mu_{r2}}{\mu_{r1}}$$

4.9. THE MAGNETIC CIRCUIT

In general magnetic circuits determine the magnetic flux and magnetic field intensities in various parts of the circuits. The magnetic circuits are analogous to the electric circuits. The common examples of the magnetic circuits are transformers, toriods motors, generators, relays and magnetic recording devices. A single magnetic lines of flux may be considered as magnetic circuits. The magneto motive force (m.m.f) is defined as

$$e_m = NI = \oint \bar{H}.d\bar{L}$$

The m.m.f is measured in amperes/turn(A/t)

In the electric circuits resistance is defined as the ratio of voltage to current given by,

$$R = \frac{V}{I}$$

In the case of analogous magnetic circuits, define a new quantity reluctance ($\Re$) as the ratio of the magneto motive force to the total flux

$$\Re = \frac{e_m}{\varphi}$$

$$\text{The reluctance is measured in } \frac{\text{Ampere. turn}}{\text{weber}}$$

The resistance in electric circuit can be expressed in terms of conductivity σ as

$$R = \frac{l}{\sigma S}$$

Where l = length in meter

S = cross section area in m^2

σ = conductivity of the linear isotropic homogeneous material.

In case of magnetic circuit the reluctance

$$\Re = \frac{l}{\mu S}$$

Where μ is the isotropic , linear homogeneous material. For electric circuit, Ohms's law can be expressed in point form as,

$$\bar{J} = \sigma\bar{E}$$

Now consider the magnetic circuit. The magnetic flux density is analogous to the current density,

$$\bar{B} = \mu\bar{H}$$

The basic equation derived from magneto statics are very much helpful in the analysis of the magnetic circuits. These basic equations are

$$\nabla.\bar{B} = 0, \qquad \nabla x\bar{H} = \bar{J}$$

In other form the two equations can be expressed in terms of the total current flowing in the magnetic circuits and total flux density through cross section of the magnetic circuits.

The total current in the magnetic circuits are

$$I = \int_S \bar{J}.\overline{ds}$$

The total magnetic flux density flowing through the cross section of the magnetic circuit is given by,

$$\varphi = \int_S \bar{B}.\overline{ds}$$

In the electric circuit the reciprocal of the resistance is called conductance. In magnetic circuit the reciprocal of the reluctance is called Permeance denoted by *P*. The permeance is measured in Henries (H).

$$P = \frac{\mu S}{l}$$

The analogous electric and magnetic circuits can be represented in simple way is shown in figure 4.9. In electric circuits Kirchhoff's laws can apply such as Kirchhoff's current laws (KCL) and Kirchhoff's Voltage law (KVL). Apply the same laws to the magnetic circuits. There are two laws namely Kirchhoff's flux law and Kirchhoff's m.m.f law. Kirchhoff's flux law states that the total magnetic flux arriving at any junction in a magnetic circuits is equal to the total magnetic field leaving that junction. Using this law parallel magnetic circuits can be easily analyzed.

Mathematically,

$$\sum \varphi = 0$$

Kirchhoff's m.m.f law states that the reluctance m.m.f around a closed magnetic circuit is equal to the algebraic sum of products of flux and reluctance of each part of the closed circuits. For closed magnetic circuits,

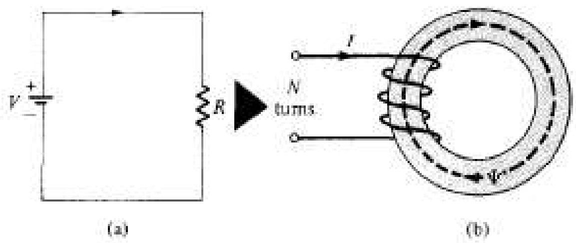

Fig 4.9 Analogous electric and magnetic circuits

$$\sum M.M.F = \sum \varphi \Re$$

Kirchhoff's m.m.f law can be alternatively stated as reluctanct m.m.f around any closed loop of the magnetic field strength and the length of each part of the circuit. Hence,

$$\sum M.M.F = \sum H.l$$

Inspite of having analogous behavior of the magnetic circuits with the electric circuits, it is very difficult to carryout exact analysis of the magnetic circuits. The difficulties in the analysis are due to the following points. In general the magnetic circuits are made up of ferromagnetic materials depends on the magnetic field intensity $\bar{H}$. It is difficult to control and calculate the leakage flux which strays or leaks from the main path of the flux in the magnetic circuit.

If there is an air gap in between the path of magnetic flux it spread and buldge out. This effect is called fringing effect.

Electric Circuits	**Magnetic Circuits**
Field intensity $\bar{E}$	Field intensity $\bar{H}$
Reciprocal of resistance is conductance	Reciprocal of reluctance is permeance
Ohm's law e =IR	Ohm's law em =ΦR
Current Density $J = \frac{I}{s}$	Current Density $B = \frac{\varphi}{s}$
Kirchhoff's law $\sum I = 0$ $\sum M.M.F = 0$	Kirchhoff's law $\sum \varphi = 0$ $\sum M.M.F = \sum H.l$
In the electric circuit the current is actually flow i.e. the movement of electrons	Due to the magneto motive force flux get established and does not flow in the sense in which current flow
The energy must be supplied to the electric field to maintain the flow of current	The energy is required to create the magnetic flux but it not require to maintain it.

4.10. POTENTIAL ENERGY AND FORCES ON MAGNETIC MATERIALS

The Potential energy density function is,

$$W_m = \lim_{\Delta V \to 0} \frac{\Delta W_m}{\Delta V} = \frac{1}{2}\mu H^2 \, J/m^2$$

In differential form,

$$W_m = \frac{1}{2} B\left(\frac{B}{\mu}\right) = \frac{B^2}{2\mu}$$

In linear medium the energy is given by,

$$W_m = \int W_m \, dV$$

$$= \frac{1}{2}\int B.H \, dV = \frac{1}{2}\mu H^2 dV = \frac{1}{2}\int \frac{B^2}{\mu} \, dv$$

In electromagnetically systems many times it required to calculate the magnetic force exerted by a magnetic field on a magnetic material some of the common mechanical systems are relays, rotating machines, electromagnets and magnetic levitation.

Consider an electromagnet made of a materials having constant relative permeability. Also assume that the current I flows through N number of turns of the coil.

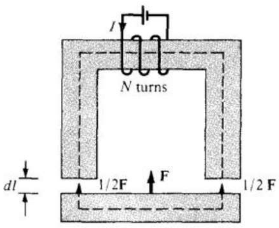

Fig 4.10. An electromagnet

Let the magnetic field in air gap be same as that in iron. The two magnetic circuits are separated by differential element represented by $\overline{dL}$. This differential displacement is equal to the change in the energy in the air gap. The force is exerted on the magnetic materials so as to reduce the air gap. Thus it is a force of attraction represented by negative sign.

$$-F.dL = dW_m = 2\left[\frac{B^2}{2\mu_0} sdL\right]$$

Where S is cross section area of air gap. As there are two air gaps, factor 2 is used to calculate total energy.

$$F = -\frac{B^2S}{\mu_0}$$

Thus force across a single air gap is given by,

$$F = -\frac{B^2S}{2\mu_0}$$

In many applications of electromechanical systems it is required to calculate tractive pressure defined as ratio of force on a magnetic surface per area measured in N/m^2.

$$P = \frac{F}{S} = \frac{B^2}{2\mu_0} = \frac{1}{2}BH = \frac{1}{2}\mu_0 H^2$$

4.11. INDUCTANCE

When a coil with N turns, Carrying current I the flux is produced by it. This flux links with each turn of the coil. Thus total flux linkage of the coil having N turn Nφ wb.turns. The flux linked with the coil is proportional to the current I flowing through it. The ratio of total flux linkage to the current producing that flux is called inductance denoted by L. It is measured in henry (H)

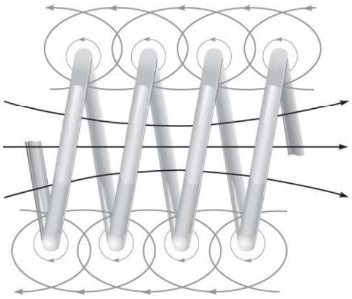

Fig. 4.11. A portion of coil showing flux linkage. The total flux linkage is adding the flux linkage of each turn

$$L = \frac{N\varphi}{I} \quad H$$

In general inductance is also referred as self inductance as the flux produced by the current flowing through the coil links with the coil itself.

4.12. BASIC EXPRESSIONS FOR SELF AND MUTUAL INDUCTANCES

The self induction of a coil is defined as ratio of total magnetic flux linkage to the circuit thro' the coil

$$L = \frac{N\phi}{I}(H)$$

where ϕ - Magnetic flux (Wb)

N - Number of turns

I - Current thro' the coil.

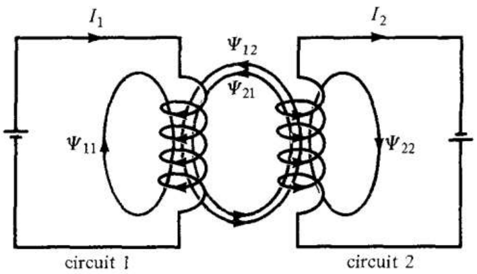

Fig. 4.12. Magnetic interaction between two circuits.

The mutual inductance between two coils is defined as the ratio of induced magnetic flux linkage in one coil to current through in other coil.

$$M = \frac{N_2}{I_1}\varphi_{12}$$

N_2 = Number of turns in coil 2.

ϕ_{12} = Magnetic flux linkage in coil 12

I_1= Current through coil 1.

Relation between Mutual inductance and Self inductance

$$M = K\sqrt{L_1 L_2}$$

where M- Mutual inductance

L_1 - Self inductance of coil 1,

L_2 - Self inductance of coil 2,

K - Coupling coefficient.

4.13. INDUCTANCE EVALUATION FOR SOLENOID

The figure4.13 shows the coil consists of N turns of tine wire carrying a current I. The length of coil is l. The flux density B at the end of along solenoid is less than at the centre. This is caused by flux leakage near the ends of the solenoid. This leakage is mostly confined to a short distance at the ends of solenoid.

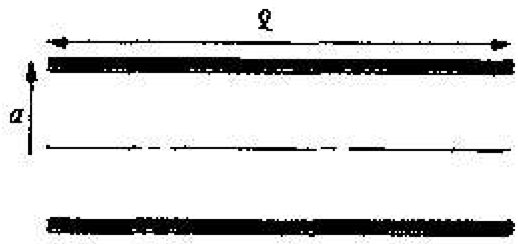

Fig.4.13 Solenoid

So that if solenoid is very long, B is constant over the entire interior of the solenoid and equal to its value at the centre.

The total flux linkage of a long solenoid is

$$N\phi = NBA \qquad (1)$$

Te flux density at the center is

$$B = \frac{\mu NI}{l} \qquad (2)$$

Substitute (2) in (1)

$$\therefore \qquad N\phi = \frac{\mu N^2 IA}{l}$$

But inductance $L = \frac{N\phi}{I}$

$$\therefore \; L = \frac{\mu N^2 A}{l} \text{ Henry.}$$

4.14. INDUCTANCE EVALUATION FOR TOROID

If a long solenoid is bent into a circle and closed on itself, a toroidal coil or Toroid is obtained. When the toroid has a uniform winding of many turns, the magnetic lines of flux are almost entirely continued to the interior of the winding.

Then B is zero outside

The flux linkage $N\phi = NBA \qquad (1)$

The flux density B in toroid is

$$B = \frac{\mu_0 NI}{l}$$

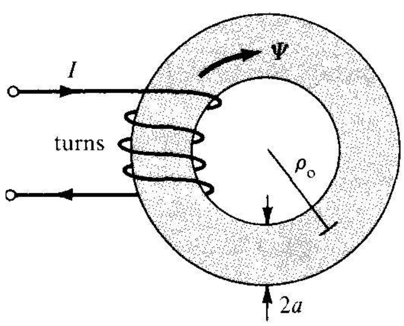

Fig.4.14. Toroid

where l - Mean length of coil,

$$l = 2\pi R$$

$$\therefore\ B = \frac{\mu_0 NI}{2\pi R} \qquad (2)$$

Substitute (2) in (1)

$$= \frac{\mu_0 N^2 AI}{2\pi R}$$

$$= \frac{\mu_0 N^2 \pi r^2 I}{2\pi R} \qquad \because A = \pi r^2$$

$$N\phi = \frac{\mu_0 N^2 r^2 I}{2R}$$

But inductance of a Toroid is

$$L = \frac{N\phi}{I}$$

$$= \frac{\mu_0 N^2 r^2 I}{2IR}$$

$$L = \frac{\mu_0 N^2 r^2}{2R} \text{ Henry.}$$

4.15. INDUCTANCE EVALUATION FOR COAXIAL CABLES

The figure4.14 shows a coaxial transmission line constructed of conducting cylinders of radius ‘a’ and ‘b’. Current on inner conductor is . Current on outer conductor is – I.

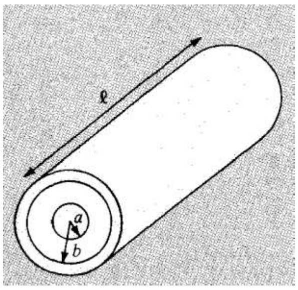

Fig. 4.14. coaxial transmission line

Flux Density B at any r is same at any radius r as this radius form a long straight conductor with the same current.

Flux density B at distance r is $B = \frac{\mu_0 I}{2\pi r}$

The total flux linkage / unit length between a and b is

$$\phi = \int_a^b \frac{\mu_0 I}{2\pi r}.dx$$

$$= \frac{\mu_0 I}{2\pi r}\int_a^b .\frac{dr}{r}$$

$$\phi = \frac{\mu_0 I}{2\pi r} l_n \frac{b}{a}$$

But Inductance $L = \frac{\phi}{I}$

$$L = \frac{\mu_0}{2\pi} l_n \frac{b}{a} \text{ Henry /m.}$$

4.16. INDUCTANCE EVALUATION FOR TRANSMISSION LINES

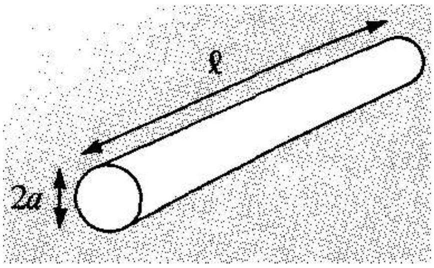

Fig.4.15. Transmission Lines

The figure4.15 shows a two wire transmission line. The radius of wire is a and the spacing between centers of wire is D.

The Flux density B due to the wire at any radius r is.

$$B = \frac{\mu_0 I}{2\pi r}$$

The total flux linkage due to both wires for a length d of line is

$$\phi = 2\int_a^d \frac{\mu_0 I}{2\pi r} dr$$

$$\phi = 2\frac{\mu_0 I}{2\pi} l_n \frac{D}{a}$$

$$L = \frac{\mu_0}{\pi} l_n \frac{D}{a} \text{ Henry /m.}$$

4.17. ENERGY STORED IN MAGNETIC FIELDS.

The energy stored by an inductor is given by,

$$W_m = \frac{1}{2} L I^2$$

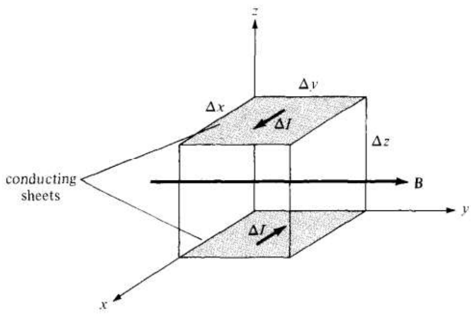

Fig.4.16. A differential volume in a magnetic field.

Consider d differential Volume in a magnetic field $\bar{B}$. The top and bottom surface of a differential volume conducting sheet with current ΔI at present. From the definition of an mutual inductance ΔL of a differential volume.

$$\Delta L = \frac{\Delta\varphi}{\Delta I} = \frac{B\Delta S}{\Delta I}$$

Where ΔS is differential surface area = $\Delta x \Delta y$

$$\Delta L = \frac{B(\Delta x\Delta y)}{\Delta I}$$

$$\Delta L = \frac{\mu H(\Delta x\Delta y)}{\Delta I}$$

Now the differential current ΔIcan be

$$\Delta I = H(\Delta y)$$

The energy stored in the inductance of a differential volume is given by,

$$\Delta W_m = \frac{1}{2}\Delta L.\Delta I^2$$

$$\Delta W_m = \frac{1}{2}\left[\frac{\mu H(\Delta x\Delta y)}{H(\Delta y)}\right].(H\Delta y)^2$$

$$= \frac{1}{2}\mu H^2[\Delta x\Delta y\Delta y]$$

$$\Delta W_m = \frac{1}{2}\mu H^2[\Delta V]$$

The magneto static energy density function is,

$$W_m = \lim_{\Delta V\to 0}\frac{\Delta W_m}{\Delta V} = \frac{1}{2}\mu H^2 \; J/m^2$$

In differential form,

$$W_m = \frac{1}{2}B\left(\frac{B}{\mu}\right) = \frac{B^2}{2\mu}$$

In linear medium the energy is given by,

$$W_m = \int W_m dV$$

$$= \frac{1}{2}\int B.H\, dV = \frac{1}{2}\mu H^2 dV = \frac{1}{2}\int \frac{B^2}{\mu}\, dv$$

PROBLEMS

Example:4.1

Find the Polarization in a dielectric medium with $\varepsilon_r = 2.5$ if D = 5 x 10^{-8} C/m^2

Polarization $p = \chi\varepsilon_0 E$

$$D = \varepsilon_0\varepsilon_r E$$

then $p = \frac{\chi D}{\varepsilon_r}$

But $\chi = \varepsilon_r - 1$

then $p = \left[\frac{\varepsilon_r - 1}{\varepsilon_r}\right] D$

$$p = \left(\frac{2.5-1}{2.5}\right) \times 5 \times 10^{-8}$$

$$p = \frac{1.5-1}{2.5} \times 5 \times 10^{-8}$$

P = 3 x 10^{-8} C/m^2

Example:4.2

Two coils A and B with 800 and 1200 turns respectively are having common magnetic circuit. A current of 0.5 A in coil A reduces a flux of 3mwb and 80% of flux links with coil B. Calculate L_1, L_2 and M.

Coil A

N_1 = 800

I_1 = 0.5 A

$\phi_1 = 3 \times 10^{-3} Wb$

Coil B

N_2 = 1200

$\phi_{21} = 50\% of \phi_1$

= 0.5 x 3 x 10^{-3} Wb

h = 80%

= 0.8

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{800 \times 3 \times 10^{-3}}{0.5} = 4.8H$$

$$M = \frac{1200 \times 0.5 \times 3 \times 10^{-3}}{0.5} = 3.6H$$

$$L_2 = \frac{M^2}{K^2 L_1} = \frac{(3.6)^2}{(0.8)^2 4.8} = 4.2188H.$$

Example:4.3

A and B are two coils with 1000 and 1500 turns respectively and lying in parallel planes, 50% of flux produced by coil A links with coil B. The hysteresis curve indicates that a current of 3A in the coil A produces 0.3 mWb while the same current in coil B produces 0.6 mWb. Calculate Mutual Inductance between them and self inductance of coils.

Coil A

N_A = 1000

$\phi_A = 0.3 \times 10^{-3}$

I_A = 3A

$$L_A = \frac{N_A \phi_A}{I_A}$$

$$= \frac{1000 \times 0.3 \times 10^{-3}}{3} = 0.1H$$

Coil B

N_B = 1500

$\phi_B = 0.6 \times 10^{-3} Wb$

I_B = 3A

$$L_B = \frac{N_B \phi_B}{I_B}$$

$$= \frac{1500 \times 0.6 \times 10^{-3}}{3} = 0.2H$$

Since the flux linkage is 50% K = 0.5

Mutual Inductance $M = K\sqrt{L_1 L_2}$

$= \sqrt{0.1 \times 0.3}$

M = 0.0866 H.

Example:4.4

Evaluate the loop inductance 1 Km of a single phase transmission circuit compressing two parallel conductor spaced 1m apart and with diameters 0.5 cm and 0.8 cm .

Solution:

d = 1m $a = \frac{0.5}{2} = 0.25 \times 10^{-2} m$ $b = \frac{0.8}{2} = 0.4 \times 10^{-2} m$ $\mu_r = 1$ m.

Inductance of Transmission lines,

$$L = \frac{\mu_0}{4\pi}\left[\mu_r + 2l_n \frac{d^2}{ab}\right]$$

$$= \frac{4\pi \times 10^{-7}}{4\pi}\left[1 + 2l_n\left(\frac{10^7}{0.25 \times 0.4}\right)\right]$$

$= 10^{-7}$ [1+23.026]

$= 24.026 \times 10^{-7}$ H/ m.

$= 24.026 \times 10^{-4}$ H/ Km.

L = 2.4 m H / Km.

Example:4.5

Two coils of self inductances of 0.5 H and 0.8 H with negligible resistance are connected in series. It their mutual inductance is 0.2H. Determine the effective inductance of the combination.

$L_1 = 0.5$ H $L_2 = 0.8$ H M = 0.2 H.

$L = L_1 + L_2 \pm 2M$

L(aiding) $= 0.5 + 0.8 \pm 2 \times 0.2$

= 1.7 H.

L (opposing) = 0.5 + 0.8 - 2 x 0.2

= 0.9 H.

SUMMARY

- The magnitude of the force is given by F= ILB sin θ
- For the two current carrying conductor of length l each the force exerted is given by

$$F = \frac{\mu I_1 I_2}{2\pi d}$$

- The moment of a force or torque about a specified point is defined as the vector product of the moment arm $\bar{R}$ and the force $\bar{F}$. It is measured in Newton meter (Nm)

$$\bar{T} = \bar{R}\ X\bar{F}$$

- When replace the product term dxdy by vector ared of the differential current loop,

$$\overline{dT} = I\ \overline{ds}\ \mathrm{X}\bar{B}$$

- The magnetic dipole moment is given by $\bar{m}$= (IS)$\overline{a_n}$ A.m^2
- The torque along the axis of rotation of a planar coil as, $\bar{T}$ = BIS (-$\overline{a_y}$)
- The magnetic permeability of the material and its unit is H/m. The relative permeability is defined as $\mu_r = 1+ \chi_m$
- The conditions of $\bar{B}\ and\ \bar{H}$ at the boundary both the vector have two components,
    - ❖ Tangential to boundary
    - ❖ Normal to boundary
- The magneto motive force (m.m.f) is defined as $e_m = NI = \oint \bar{H}.d\bar{L}$
- The resistance in electric circuit can be expressed in terms of conductivity σ as $R = \frac{l}{\sigma S}$
- The total magnetic flux density flowing through the cross section of the magnetic circuit is given by, $\varphi = \int_S \bar{B}.\overline{ds}$
- In the electric circuit the reciprocal of the resistance is called conductance. In magnetic circuit the reciprocal of the reluctance is called Permeance denoted by *P.* The permeance is measured in Henries (H). $P = \frac{\mu S}{l}$
- In the electric circuit the current is actually flow i.e. the movement of electrons
- Due to the magneto motive force flux get established and does not flow in the sense in which current fl
- The energy must be supplied to the electric field to maintain the flow of current
- The energy is required to create the magnetic flux but it not require to maintain it.
- The expression for torque experienced by a current carrying loop situated in a magnetic field. T = IAB sinθ
- Torque on a solenoid T = NIAB sinθ
- The expression for magnetic field at the centre of the circular coil.H = I/2a.
- Lorentz force is the force experienced by the test charge .It is maximum if the direction of movement of charge is perpendicular to the orientation of field lines.
- Magnetic moment is defined as the maximum torque on the loop per unit magnetic Induction of flux density. m=IA
- The ratio of total flux linkage to the current producing that flux is called inductance denoted by L. It is measured in henry (H)

$$L = \frac{N\varphi}{I} \quad H$$

- The mutual inductance between two coils is defined as the ratio of induced magnetic flux linkage in one coil to current through in other coil.

$$M = \frac{N_2}{I_1}\varphi_{12}$$

TWO MARKS

1. What is the force exerted in a differential element

   Force exerted on differential current element is, $\overline{\mathbf{dF}} = I\, \overline{\mathbf{dL}} \times \overline{\mathbf{B}}$
2. Define Torque.

   The moment of a force or torque about a specified point is defined as the vector product of the moment arm $\bar{R}$ and the force $\bar{F}$. It is measured in Newton meter (Nm)

   $\bar{T} = \bar{R}\ X\bar{F}$
3. Define Magnetic Dipole Moment

   The magnetic dipole moment is given by

   $\bar{m}$= (IS)$\overline{a_n}$ A.m^2
4. Write down the Classification of magnetic materials.
   - Dia magnetic materials
   - Para magnetic materials
   - Ferro magnetic materials
   - Antiferro magnetic materials
   - Ferri magnetic materials
5. State Kirchhoff's flux law

   Kirchhoff's flux law states that the total magnetic flux arriving at any junction in a magnetic circuits is equal to the total magnetic field leaving that junction. Using this law parallel magnetic circuits can be easily analyzed. Mathematically,

   $$\sum \varphi = 0$$
6. State Kirchhoff's m.m.f law

   Kirchhoff's m.m.f law states that the reluctance m.m.f around a closed magnetic circuit is equal to the algebraic sum of products of flux and reluctance of each part of the closed circuits. For closed magnetic circuits,

   $$\sum M.M.F = \sum \varphi\Re$$

   Kirchhoff's m.m.f law can be alternatively stated as reluctance m.m.f around any closed loop of the magnetic field strength and the length of each part of the circuit. Hence,

   $$\sum M.M.F = \sum H.l$$
7. Define Reluctance

   A new quantity reluctance ($\Re$) as the ratio of the magneto motive force to the total flux

   $$\Re = \frac{e_m}{\varphi}$$