UNIT V
TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

Fundamental relations for Electrostatic and Magneto static fields, Faraday's law for Electromagnetic induction, Transformers, Motional Electromotive forces, Differential form of Maxwell's equations, Integral form of Maxwell's equations, Potential functions, Electromagnetic boundary conditions, Wave equations and their solutions, Poynting's theorem, Time harmonic fields, Electromagnetic Spectrum.

5.1. FUNDAMENTAL RELATIONS FOR ELECTROSTATIC & MAGNETO STATIC FIELDS

TERM	ELECTROSTATICS	MAGNETOSTATICS
Source Element	Stationary Electric Charge	Element carrying steady current
Force Law	Coulomb's Law	Biot and Savart's Law
Relation between Field & force	$E = \frac{F}{q}$	$F = q(VxB)$
Relation: Field & Potential	$E = -\nabla V$ $\varphi_E = -\int_L E.dl$	$B = \nabla xA$
Flux	$\varphi_E = -\int_S E.dS$	$\varphi_m = \iint_S B.dS$
Three vectors	$\bar{E}, \bar{D}, \bar{P}$	$\bar{B}, \bar{M}, \bar{H}$
Relation between the three vectors	$\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$	$\bar{B} = \mu_0(\bar{H} + \bar{M})$
Analogy in vectors	$\bar{E}$ associated with all charges, $\bar{D}$ associated with free charges, $\bar{P}$ associated with induced charges	$\bar{B}$ associated with all currents, $\bar{H}$ associated with true currents, $\bar{M}$ associated with magnetizing currents
Boundary conditions At the interface between 2 media	The tangential component of $\bar{E}$ & the normal component of $\bar{D}$ are continuous across the boundary.	Tangential component of $\bar{H}$ & the normal component of $\bar{B}$ are continuous across the boundary
Theorem/ Law (Integral form)	Gauss Theorem in Vacuum $\varphi_E = \oint_S E.dA = \frac{Q_e}{\varepsilon_0}$.	Ampere's Law $\int_C B.dl = \int_S \nabla xB\ ds = \mu_0 I$
Theorems/ Laws (Differential form)	$\nabla xE = 0$ Field curl-free $\nabla.E = \frac{\rho_V}{\varepsilon}$	$\nabla.B = 0$ Field Divergence=0 $\nabla xB\ ds = \mu_0 I$
Potential	Electric Scalar potential $\varphi or V$	Magnetic Scalar potential φ_m Magnetic Vector potential A
Poisson's Equation	Poisson's Equation	Equation similar to Poisson's $\nabla^2 A = -\mu_0 j$
Gauge ransformation gauge for the scalar potential	transformation $\varphi \rightarrow \varphi + C$, leaves the electric field invariant	transformation $A \rightarrow A - \nabla\psi$, leaves the magnetic field invariant
Dipole	Monopoles & Dipoles both	No Monopoles, Only Dipoles
Dipole Moment	Electric Dipole Moment $p = qd$	Magnetic $\mu = IA$
Potential Energy	$U = -p.E$	$U(\theta) = -\mu.B$

5.2. FARADAY'S LAW FOR ELECTROMAGNETIC INDUCTION

According to Faraday's experiments, a static magnetic field produces no current flow, but a time-varying field produces an induced voltage (called electromotive force or simply emf) in a closed circuit, which causes a flow of current. Faraday discovered that the induced emf. V_{emf} (in volts), in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit. This is called Faraday's law, and it can be expressed as

$$V_{emf} = -N\frac{d\phi}{dt} = -\frac{d\Psi}{dt}$$

where N is the number of turns in the circuit and V is the flux through each turn. The negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it. This is known as Lenz's law,2 and it emphasizes the fact that the direction of current flow in the circuit is such that the induced magnetic field produced by the induced current will oppose the original magnetic field. The electric fields considered so far are caused by electric charges; in such fields, the flux lines begin and end on the charges. However, there are other kinds of electric fields not directly caused by electric charges. These are emf-produced fields. Sources of emf include electric generators, batteries, thermocouples, fuel cells, and photovoltaic cells, which all convert nonelectrical energy into electrical energy. The total electric field at any point is $E = E_f + Ee$

E_f is zero outside the battery, E_f and Ee have opposite directions in the battery, and the direction of Ee inside the battery is opposite to that outside it. If we integrate equation over the closed circuit, $\oint E.dl = \oint E_f.dl + 0 = \int_N^P E_f.dl$ (through battery)

where $\oint E_e.dl = 0$ because Ee is conservative. The emf of the battery is the line integral of the emf-produced field; that is,

$$\int_N^P E_f.dl = -\int_N^P E_e.dl = IR$$

since E_f and Ee are equal but opposite within the battery. It may also be regarded as the potential difference ($V_P - V_N$) between the battery's open-circuit terminals.

It is important to note that:

- An electrostatic field Ee cannot maintain a steady current in a closed circuit

$$-\int_N^P E_e.dl = IR$$

- An emf-produced field E_f is non conservative
- Except in electrostatics, voltage and potential difference are usually not equivalent.

5.3. TRANSFORMERS AND MOTIONAL ELECTROMOTIVE FORCES

Having considered the connection between emf and electric field, Faraday's law links electric and magnetic fields. For a circuit with a single turn (N = 1),

$$V_{emf} = -\frac{d\Psi}{dt}$$

In terms of E and B,

$$V_{emf} = -\oint \bar{E}.dl = -\frac{d}{dt}\int \bar{B}.ds$$

where Ψ has been replaced by $\int \overline{B}.ds$ and S is the surface area of the circuit bounded by the closed path L. It is clear from that in a time-varying situation, both electric and magnetic fields are present and are interrelated. Note that dl and dS are in accordance with the right-hand rule as well as Stokes's theorem. The variation of flux with time may be caused in three ways:

- By having a stationary loop in a time-varying B field
- By having a time-varying loop area in a static B field
- By having a time-varying loop area in a time-varying B field.

Each of these will be considered separately.

i. Stationary Loop in Time-Varying B Field (transformer emf)

This is the case portrayed in Figure 5.1 where a stationary conducting loop is in a time varying magnetic B field. Equation becomes

$$V_{emf} = - \oint \overline{E}.dl = -\oint \frac{\partial B}{\partial t}.ds$$

This emf induced by the time-varying current (producing the time-varying B field) in a stationary loop is often referred to as transformer emf in power analysis since it is due to transformer action. By applying Stokes's theorem

$$\oint \nabla x\overline{E}.dl = -\oint \frac{\partial B}{\partial t}.ds$$

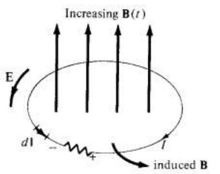

Figure 5.1. Induced emf due to a stationary loop in a time varying B field.

For the two integrals to be equal, their integrands must be equal; that is,

$$\nabla x\overline{E}.dl = -\frac{\partial B}{\partial t}.ds$$

This is one of the Maxwell's equations for time-varying fields. It shows that the time varying E field is not conservative ($\nabla \ X \ E \neq 0$). This does not imply that the principles of energy conservation are violated. The work done in taking a charge about a closed path in a time-varying electric field obeys Lenz's law; the induced current / flows such as to produce a magnetic field that opposes B (f).

ii. Moving Loop in Static B Field (Motional emf)

When a conducting loop is moving in a static B field, an emf is induced in the loop. The force on a charge moving with uniform velocity u in a magnetic field B is Fm = Q(V X B)

The motional electric field Em as

$$E_m = \frac{F_m}{Q} = VxB$$

If consider a conducting loop, moving with uniform velocity V as consisting of a large number of free electrons, the emf induced in the loop is

$$V_{emf} = \oint \nabla x\overline{E}.dl = \oint V\,x\,B\,.dl$$

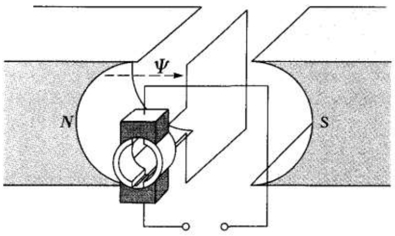

Figure 5.2 A direct-current machine

This type of emf is called motional emf or flux-cutting emf because it is due to motional action. It is the kind of emf found in electrical machines such as motors, generators, and alternators. Figure 5.2 illustrates a two-pole dc machine with one armature coil and a two bar commutator. Although the analysis of the d.c. machine is beyond the scope of this text, Voltage is generated as the coil rotates within the magnetic field. Another example of motional emf is illustrated in Figure 5.3, where a rod is moving between a pair of rails.

$$F_m = I\, l\, x B$$

$$F_m = I\, l\, B$$

$$V_{emf} = uBl$$

$$\int \nabla \, X \, Em. ds = \int \nabla \, X \, (V \, X \, B). ds$$

$$\nabla \, X \, Em = \nabla \, X \, (V \, X \, B)$$

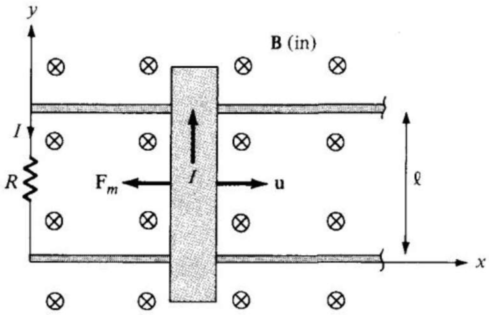

Figure5.3 Induced emf due to a moving loop in a static B field.

The following points should be noted:

1. The integral in equation is zero along the portion of the loop where V = 0. Thus dl is taken along the portion of the loop that is cutting the field , where V has nonzero value.
2. The direction of the induced current is the same as that of Em or V X B. The limits of the integral are selected in the opposite direction to the induced current thereby satisfying Lenz's law. for example, the integration over L is along -ay whereas induced current flows in the rod along ay.

C. Moving Loop in Time-Varying Field

This is the general case in which a moving conducting loop is in a time-varying magnetic field. Both transformer emf and motional emf are present. The total emf as

$$V_{emf} = \oint E. dl = -\oint \frac{\partial B}{\partial t} . ds + \int (V \, X \, B). dl$$

$$\nabla x E = \frac{\partial B}{\partial t} + \nabla x \, (V \, X \, B)$$

5.4. DIFFERENTIAL AND INTEGRAL FORM OF MAXWELL'S EQUATIONS

Maxwell's equation is a set of four expressions derived from Ampere's circuital law , Faraday's law, Gauss law for electric field and Gauss law for magnetic field. These four expressions can be written in the form of

1. Point or differential form
2. Integral form

According to Ampere's circuital law, the line integral of magnetic field intensity around a closed path is equal to the current enclosed by the path.

$$\oint \overline{H}.\,dL = I_{enclosed}$$

Replacing current by surface integral of conductivity of current density J over an area bounded by the path of integration of H

$$\oint \overline{H}.\,dL = \int_s J.\,ds$$

By adding displacement current density to conductor current density,

$$\oint \overline{H}.\,dL = \int_s [J + \frac{\partial D}{\partial t}]ds$$

This is Maxwell equation derived from ampere's circuital law. This equation in integral of H is carried over the closed path bounding the surface s over which the integration is carried out on R.H.S. The equation is also called Mesh relation.

Applying stokes theorem,

$$\oint (\nabla \times H).\,ds = \int [J + \frac{\partial D}{\partial t}].\,ds$$

Assume the surface considered for both integration is same,

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

The equation is point form of differential form of N equation derived from Ampere's circuital law.

Now consider Faraday's law which relates emf induced in a circuit to the time rate of decrease of total magnetic flux linking the circuit

$$\oint E.\,dL = -\int_s \frac{\partial B}{\partial t}.\,ds$$

Assuming that the integration is carried out over the same surface on both sides

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

This is Maxwell's equation derived from faraday's expression in point form or differential form

According to Gauss law the total flux out of the closed surface is equal to the net charge within the surface. this can be

$$\int_s D.\,ds = Q_{enclosecd}$$

If R.H.S of the volume integral of the charge density.

Considered for integration at L.H.S is general form of equation is given by,

$$\int_s \mathrm{D.ds} = \int_v \rho_v \mathrm{d\,v}$$

This equation is called Maxwell's equation for electric field derived from Gauss law, expressed in integral form and applied to finite volume.

Using divergence theorem,

$$\int_v (\nabla.\mathrm{D})\mathrm{dv} = \int_v \rho_v \mathrm{d\,v}$$

Assuming same value for integration on both sides,

$$(\nabla.\mathrm{D}) = \rho_v$$

This is Maxwell's magnetic field equation expressed in integral form. This is derived for Gauss law applied to the magnetic fields

Using Divergence theorem, the surface integral can be converted to volume integral as

$$\int_s (\nabla.\mathrm{B})\mathrm{dv} = 0$$

For finite volume $\mathrm{dv} = 0$

$$(\nabla.\mathrm{B}) = 0$$

This is differential form or point form of Maxwell's equation derived from Gauss la applied to magnetic fields

Differential form	Integral form	Significance
$(\nabla \times E) = -\frac{\partial B}{\partial t}$	$\oint E.dL = -\int_s \frac{\partial B}{\partial t}.ds$	Faraday's law
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint H.dL = J + \int_s \frac{\partial D}{\partial t}]ds$	Ampere's circuital law
$(\nabla.D) = \rho$	$\oint_s D.ds = \int_s \rho dv$	Gauss law
$(\nabla.B) = 0$	$\oint_s B.ds = 0$	No isolated magnetic charges

Maxwell equation for free space

For free space a non –conducting medium in which volume charge density ρ_v=0 and conductivityσ is also zero

S.NO	Point form	Integral form
1	$(\nabla \times E) = -\frac{\partial B}{\partial t}$	$\oint E.dL = -\int_s \frac{\partial B}{\partial t}.ds$
2	$\nabla \times H = \frac{\partial D}{\partial t}$	$\oint H.dL = J + \int_s \frac{\partial D}{\partial t}]ds$
3	$(\nabla.D) = 0$	$\oint_s D.ds = 0$
4	$(\nabla.B) = 0$	$\oint_s B.ds = 0$

Maxwell's equation for good conductor

For good conductor conductivity is very high $J \gg \frac{\partial D}{\partial t}$ and $\rho_v=0$

S.NO	Point form	Integral form
1	$(\nabla \times E) = -\frac{\partial B}{\partial t}$	$\oint E.dL = -\int_s \frac{\partial B}{\partial t}.ds$
2	$\nabla \times H = J$	$\oint H.dL = I = \int_s J.ds$
3	$(\nabla.D) = 0$	$\oint_s D.ds = 0$
4	$(\nabla.B) = 0$	$\oint_s B.ds = 0$

Maxwell's equation for harmonically varying field

The electric and magnetic fields are varying harmonically with time. The electric flux density can be $D=D_o e^{j\omega t}$. The magnetic flux density can be written as $B=B_o e^{j\omega t}$

Taking partial derivative with respect to time

$$\frac{\partial D}{\partial t} = j\omega Doe^{j\omega t} = j\omega D \text{ and } \frac{\partial B}{\partial t} = j\omega Boe^{j\omega t} = j\omega B$$

S.NO	Point form	Integral form
1	$\nabla \times E = -j\omega B = -j\omega\mu H$	$\oint E.dL = -\int_s j\omega B.ds = -\int_s j\omega\mu H.ds$ $\oint E.dL = -j\omega\mu H \int_s H.ds$
2	$\nabla \times H = J + j\omega D$ $=\sigma E + j\omega(\varepsilon E)$ $=(\sigma + j\omega\varepsilon)E)$	$\int H.dL = I + \int_s j\omega D.ds$ $=\int_s J.ds + \int_s j\omega\varepsilon E.ds = \int_s \sigma E.ds + \int_s j\omega\varepsilon E.ds$ $= (\sigma + j\omega\varepsilon) \int_s E.ds$
3	$\nabla.D = \rho$	$\int_s D.ds = \oint_v \rho dv$
4	$\nabla.B = 0$	$\oint_s B.ds = 0$

5.5. POTENTIAL FUNCTIONS

For static electric fields the electric scalar potential is given by,

$$V = \int \frac{\rho_V}{4\pi\varepsilon R} dv$$

The static magnetic fields, the magnetic vector potential is given by,

$$\bar{A} = \int_V \frac{\mu \bar{J}}{4\pi R} dV$$

The behavior of these potential when the fields are time varying, For time Varying field,

$$\overline{\nabla} X\bar{A} = \bar{B}$$

$\bar{B}$ is expressed as the curl of a vector magnetic potential $\bar{A}$. The curl of the gradient of a scalar function f is zero.

$$\overline{\nabla} X(\overline{\nabla} f) = 0$$

Any vector with zero curl must be a gradient of some scalar function. Thus if $\overline{\nabla} X\bar{F} = 0$, From the faradays law

$$\overline{\nabla} X\bar{E} = -\frac{\partial B}{\partial t}$$

For time varying field $\overline{\nabla} X\bar{E} \neq 0$ the relation $\bar{E} = -\nabla\bar{V}$ is not sufficient and additional term is required. The additional term may be $\overline{\nabla} X\bar{A} = \bar{B}$. Faradays law becomes,

$$\overline{\nabla} X\bar{E} = -\frac{\partial(\overline{\nabla} X\bar{A})}{\partial t}$$

Which,

$$\overline{\nabla} X\left(\bar{E} + \frac{\partial\bar{A}}{\partial t}\right) = 0$$

The gradient of scalar,

$$\bar{E} + \frac{\partial\bar{A}}{\partial t} = \overline{\nabla} f$$

Where f is a scalar function. If the electric scalar potential V is taken to be this Scalar function, a relation is obtained that satisfies the requirements for both static and time varying situation

$$\bar{E} = -\frac{\partial\bar{A}}{\partial t} - \overline{\nabla} V$$

For the static fields it becomes$\bar{E} = -\overline{\nabla}$V. In general case where the field may vary with time, $\bar{E}$ is given by both a scalar potential V and vector potential $\bar{A}$. If the time variation is harmonic,

$$\bar{E} = -\overline{\nabla} V - j\omega\bar{A}$$

To find some expression for $\bar{A}$ and V

$\nabla.\bar{D} = \rho_V$ is Valid for time varying condition, By taking divergence,

$$\overline{\nabla}.\bar{E} = -\frac{\rho_V}{\varepsilon} = \nabla^2 V - \frac{\partial}{\partial t}(\overline{\nabla}.\bar{A})$$

Taking curl of $\overline{\nabla} X\bar{A} = \bar{B}$ and incorporating equations,

$$\overline{\nabla} X\bar{H} = \bar{J} + \frac{\partial D}{\partial t}$$

The potential V and $\bar{A}$ satisfies Poisson equations for time varying conditions.

$$V = \int \frac{\rho_V}{4\pi\varepsilon R} dv$$

$$\bar{A} = \int_V \frac{\mu\bar{J}}{4\pi R} dV$$

5.6. ELECTROMAGNETIC BOUNDARY CONDITIONS

The relationship between the electric flux density $\overline{D}$ electric field intensity $\overline{E}$,Magnetic field intensity $\overline{H}$ and Magnetic flux density $\overline{B}$ can be explained with the help of point form and integral form of Maxwell's equations. The field equations postulated by Maxwell are valid at a point in a continuous medium. The Maxwell equations are useful in determining the conditions at boundary surface of the different media. The concept of linear, isotropic and homogeneous medium can apply. Consider the boundary between medium 1 with parameters ε1,µ1 and σ1 and medium 2 with parameters ε2,µ2 and σ2. In general the boundary condition for time varying fields are same as those for static fields. Thus at the boundary referring boundary conditions for static electric magnetic fields.

i, The tangential component of electric field intensity $\overline{E}$ is continuous at the surface.

$$E_{tan1}=E_{tan2}$$

ii. The tangential component of Magnetic field intensity $\overline{E}$ is continuous across the surface except for a perfect conductor.

$$H_{tan1}=H_{tan2}$$

At the surface of the perfect conductor the tangential component of the magnetic field intensity is discontinuous at the boundary.

$$H_{tan1}-H_{tan2} = K$$

iii. The normal component of the electric flux density is continuous at the boundary is the surface charge density is zero.

$$DN_1=DN_2$$

If ths surface charge density is non zero, then the normal component of the electric flux density is discontinuous at the boundary.

$$DN_1-DN_2 =\rho s$$

iv. The normal component of the magnetic flux density is continuous at the boundary.

$$BN_1=BN_2$$

5.7. WAVE EQUATIONS AND THEIR SOLUTIONS

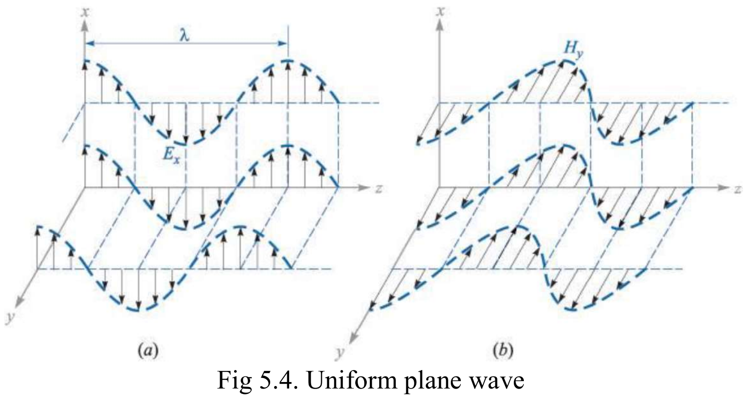

Fig 5.4. Uniform plane wave

In general, waves are means of transporting energy or information. Typical examples of EM waves include radio waves, TV signals, radar beams, and light rays. All forms of EM energy share three fundamental characteristics: they all travel at high velocity; in traveling, they assume the properties of waves; and they radiate outward from a source, without benefit of any discernible physical vehicles. EM wave motion in the following media:

- Free space ($\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$)
- Lossless dielectrics ($\sigma = 0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$ or $\sigma << \omega\varepsilon$)
- Lossy dielectrics ($\sigma \neq 0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$)
- Good conductors ($\sigma = \infty$, $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$ or $\sigma >> \omega\varepsilon$)

5.7.1. DERIVATION OF WAVE EQUATION

The wave equation is obtained by relating the space and time variations of the electric and magnetic fields,

The Maxwell equation,

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$
$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$
$$\nabla . \mathrm{D} = 0$$
$$\nabla . \mathrm{B} = 0$$

In the first equation take curl on both sides

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}(\nabla \times H)$$
$$= -\mu \frac{\partial}{\partial t}[\sigma E + \varepsilon \frac{\partial E}{\partial t}]$$
$$= \mu \sigma \frac{\partial E}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2}$$
$$\nabla \times \nabla \times E = \nabla.(\nabla \times E) - \nabla^2 \mathrm{E}$$

Divergent of curl is zero [$\nabla.(\nabla \times E)$=0]

$$\nabla^2 E = -\mu \sigma \frac{\partial E}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2}$$
$$\nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2}$$

This is the wave equation for electric field

$$\nabla \times \nabla \times H = \nabla \times (\sigma E + \varepsilon \frac{\partial E}{\partial t})$$
$$= \sigma(\nabla \times E) + \varepsilon \frac{\partial}{\partial t}(\nabla \times E)$$
$$= \sigma(-\frac{\partial B}{\partial t}) + \varepsilon \frac{\partial}{\partial t}(\nabla \times E)$$

We have B= μH

$$= -\mu \sigma \frac{\partial H}{\partial t} - \mu \varepsilon \frac{\partial 2H}{\partial t^2}$$
$$\nabla \times \nabla \times H = \nabla.(\nabla \times H) - \nabla^2 \mathrm{H}$$

Divergent of curl is zero [$\nabla.(\nabla \times H)$=0]

$$-\nabla^2 H = \mu \sigma \frac{\partial H}{\partial t} - \mu \varepsilon \frac{\partial^2 H}{\partial t^2}$$
$$\nabla^2 H = \mu \sigma \frac{\partial H}{\partial t} + \mu \varepsilon \frac{\partial^2 H}{\partial t^2}$$

Similarly

$$\nabla^2 B = \mu\,\sigma \frac{\partial B}{\partial t} + \mu\,\varepsilon \frac{\partial^2 B}{\partial t^2}$$

In general,

$$\nabla^2 \begin{bmatrix} \bar{E} \\ \bar{D} \\ \bar{H} \\ \bar{B} \end{bmatrix} = \mu\,\sigma \frac{\partial}{\partial t} \begin{bmatrix} \bar{E} \\ \bar{D} \\ \bar{H} \\ \bar{B} \end{bmatrix} + \mu\,\varepsilon \frac{\partial^2}{\partial t^2} \begin{bmatrix} \bar{E} \\ \bar{D} \\ \bar{H} \\ \bar{B} \end{bmatrix}$$

5.8. POYNTING'S THEOREM

In EM waves an energy can be transported from transmitter to receiver. The energy stored in an electric and magnetic field is transmitted at a certain rate of energy flow which can be calculated with the help of poynting theorem.

The product of E and $\bar{H}$ gives a new quantity which is expressed as watt per unit area. This quantity is called power density.

$$\bar{P} = \bar{E} \times \bar{H}$$

Where $\bar{P}$ is called Poynting vector. $\bar{P}$ is the instantaneous power density vector associated with the electromagnetic field at a given point. The direction of $\bar{P}$ indicates instantaneous power flow at the point. To get the net power flowing out of any surface, $\bar{P}$ is integrated over same closed surface.

Poynting theorem states that the net power flowing out of a given volume V is equal to the time rate of decrease in the energy stored within the volume V minus the ohmic power dissipated.

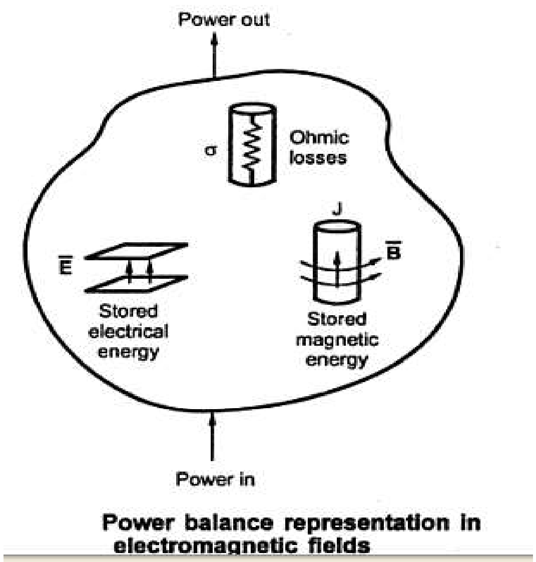

Fig 1.4. Power Balance representation in electromagnetic field

The product of E and H gives a new quantity which is expressed as watt per unit area. This quantity is called power density.

$$\bar{P} = \bar{E} \times \bar{H}$$

Where P is called Poynting vector. Poynting theorem states that the net power flowing out of a given volume is equal to the time rate of decrease in the energy stored within the volume V minus the ohmic power dissipated.

$$E = E_x\overline{ax} \quad and \quad H = H_y\overline{ay} \quad then,$$

$$\overline{P}=\overline{E} \times \overline{H}$$

$$=E_x\overline{ax}\,\mathrm{x}H_y\overline{ay}$$

$$=E_xH_y\overline{az}$$

$$=P_z\overline{az}$$

Consider the electric field propagation in free space given by

$$\mathrm{E}=[\mathrm{E_m}\cos(\omega t - \beta z)]\mathrm{a_x}$$

In the medium, ratio of magnitudes of E and H depends on its intrinsic impedance η for free space,

$$\eta = \eta_0 = \frac{E_m}{H_m} = 120\pi = 377\Omega$$

In free space, electromagnetic waves travel at a speed of light.

$$\mathrm{H}=[\mathrm{H_m}\cos(\omega t - \beta z)]\mathrm{a_y}$$

$$= \frac{Em}{\eta o}\, cos(\omega t - \beta z)]\, a_y$$

According to Poynting theorem,

$$\overline{P}=\overline{E} \times \overline{H}$$

$$= Em\, cos(\omega t - \beta z)\, ax] \times\, [\frac{Em}{\eta_0}\, cos(\omega t - \beta z)\, ay]$$

$$P = \frac{Em^2}{\eta_0} cos^2(\omega t - \beta z)\, a_z$$

The power density measured in watt/m^2.Thus power passing particular area is given by,

Power=power density×Area

Average power density

$$P_{avg} = \frac{1}{T}\int_0^T \frac{Em^2}{\eta_0} cos^2(\omega t - \beta z)\, dt$$

$$P_{avg} = \frac{1}{T}\frac{Em^2}{\eta_0}\int_0^T \frac{1 + \cos(\omega t - \beta z)}{2} dt$$

$$P_{avg} = \frac{1}{T}\frac{Em^2}{\eta_0}\left[\frac{t}{2} + \frac{sin\,2(\omega t - \beta z)}{2}\right]_0^T dt$$

$$= \frac{1}{T}\frac{Em^2}{\eta_0}\left[\frac{T}{2} + \frac{sin2(\omega T - \beta z)}{2} - \frac{\sin(-2\beta z)}{2}\right]$$

$\omega T = 2\pi$

$$P_{avg} = \frac{1}{T}\frac{Em^2}{\eta_0}\left[\frac{T}{2} + \frac{sin2(2\pi - \beta z)}{2} - \frac{\sin(-2\beta z)}{2}\right]$$

$$P_{avg} = \frac{1}{T}\frac{Em^2}{\eta_0}\left[\frac{T}{2} + \frac{sin(4\pi - 2\beta z)}{2} + \frac{\sin(2\beta z)}{2}\right]$$

$$P_{avg} = \frac{1}{T}\frac{Em^2}{\eta_0}\left[\frac{T}{2} - \frac{sin(2\beta z)}{2} + \frac{\sin(2\beta z)}{2}\right]$$

$$P_{avg} = \frac{1}{T}\frac{Em^2}{\eta_0}\left[\frac{T}{2}\right]$$

$$P_{avg} = \frac{1}{2}\frac{Em^2}{\eta_0}\ W/m^2$$

Power flow in a coaxial cable

Consider c coaxial cable in which the power is transferred to the load resistance R along a cable. There are two conductors namely inner conductor and outer conductor concentric to each other. Let the radius of the inner conductor a and outer conductor is b.

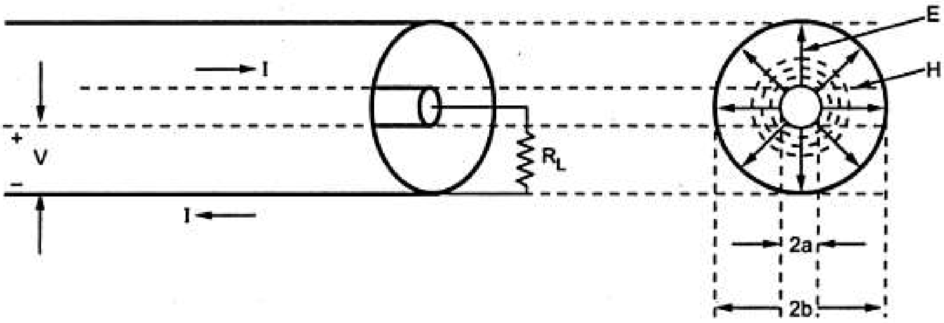

Fig Voltage and Current in Coaxial cable

In the cable there exists a d.c voltage V between the two conductors, while the steady current flow I in the inner and outer conductors as shown in fig 3.5. The magnetic field strength H will be directed in the circular path about the axis. In the region between the two conductors, the current encloses is equal to the magneto motive force around any of the circle of H.

$$\oint \bar{H}.ds = I$$

The magnetic field H is constant along any of the circular path. R be the radius of the circle,

$$\oint \bar{H}.ds = 2\pi rH = I$$

$$H = \frac{I}{2\pi r}$$

The potential difference between inner and outer conductor of the coaxial cable is,

$$V = \frac{q}{2\pi\varepsilon} log\left(\frac{b}{a}\right)$$

q is charge per unit length. The magnitude of electric field intensity for a coaxial cable is,

$$E = \frac{q}{2\pi\varepsilon r}$$

$$E = \frac{V}{\left[log\left(\frac{b}{a}\right)\right] r}$$

According to poynting theorem $\bar{P}=\bar{E}\times\bar{H}$. But the power flow in the direction parallel to the axis of the cable. As $\bar{E}\,and\,\bar{H}$ are mutually perpendicular to each other. The magnitude of poynting vector is P=E.H. The total power flow along the cable is,

$$W = Total\ power\ flow = \int \bar{P}.\overline{da}$$

$$W = \int (\bar{E}\times\bar{H}).\overline{da}$$

$$W = \int_a^b \frac{V}{\left[log\left(\frac{b}{a}\right)\right]r}.\frac{I}{2\pi r}.2\pi r dr$$

$$W = \int_a^b \frac{V.I}{\left[log\left(\frac{b}{a}\right)\right]r}.dr$$

$$W = \frac{V.I}{\left[log\left(\frac{b}{a}\right)\right]}\int_a^b \frac{1}{r}.dr$$

$$= \frac{V.I}{\left[log\left(\frac{b}{a}\right)\right]}[\log r]_a^b$$

$$= \frac{V.I}{\left[log\left(\frac{b}{a}\right)\right]}[logb - \log a]$$

$$= \frac{V.I}{\left[log\left(\frac{b}{a}\right)\right]}log\left(\frac{b}{a}\right)$$

$$W = V.I$$

For the perfect conductor the power flow is entirely external to the conductor.

Integral and Point Forms of Poynting Theorem

Consider Maxwell's equations as given below :

$$\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t} = -\mu\frac{\partial\bar{H}}{\partial t}$$

$$\nabla\times\bar{H} = \bar{J}+\frac{\partial\bar{D}}{\partial t} = \sigma\bar{E}+\varepsilon\frac{\partial\bar{E}}{\partial t}$$

Dotting both the sides of equation with $\bar{E}$, we get,

$$\bar{E}\cdot(\nabla\times\bar{H}) = \bar{E}\cdot(\sigma\bar{E})+\bar{E}\cdot\left(\varepsilon\frac{\partial\bar{E}}{\partial t}\right)$$

Let us make use of a vector identity as given below,

$$\nabla\cdot(\bar{A}\times\bar{B}) = \bar{B}\cdot(\nabla\times\bar{A})-\bar{A}\cdot(\nabla\times\bar{B})$$

Applying above vector identity to equation (4) with $\overline{A} = \overline{E}$ and $\overline{B} = \overline{H}$,

$$\overline{H}\cdot(\nabla\times\overline{E})-\nabla\cdot(\overline{E}\times\overline{H}) = \overline{E}\cdot(\sigma\overline{E})+\dot{\overline{E}}\cdot\left(\varepsilon\frac{\partial\overline{E}}{\partial t}\right)$$

$$\therefore \quad \overline{H}\cdot(\nabla\times\overline{E})-\nabla\cdot(\overline{E}\times\overline{H}) = \sigma E^2+\overline{E}\cdot\left(\varepsilon\frac{\partial\overline{E}}{\partial t}\right)$$

Consider first term on left of equation Putting value of $\nabla\times\overline{E}$ from equatior

$$\overline{H}\cdot(\nabla\times\overline{E}) = \overline{H}\cdot\left(-\mu\frac{\partial\overline{H}}{\partial t}\right)=-\mu\,\overline{H}\cdot\frac{\partial\overline{H}}{\partial t}$$

$$\frac{\partial}{\partial t}(\overline{H}\cdot\overline{H}) = \overline{H}\cdot\frac{\partial\overline{H}}{\partial t}+\overline{H}\cdot\frac{\partial\overline{H}}{\partial t}$$

$$\therefore \quad \frac{\partial}{\partial t}H^2 = 2\overline{H}\cdot\frac{\partial\overline{H}}{\partial t}$$

$$\therefore \quad \frac{1}{2}\frac{\partial}{\partial t}(H^2) = \overline{H}\cdot\frac{\partial\overline{H}}{\partial t}$$

Similarly

$$\frac{1}{2}\frac{\partial}{\partial t}(E^2) = \overline{E}\cdot\frac{\partial\overline{E}}{\partial t}$$

Using results obtained in equations (i), (ii) and (iii) in equation

$$-\frac{\mu}{2}\frac{\partial}{\partial t}(H^2)-\nabla\cdot(\overline{E}\times\overline{H}) = \sigma E^2+\frac{\varepsilon}{2}\frac{\partial}{\partial t}(E^2)$$

$$\therefore \quad -\nabla\cdot(\overline{E}\times\overline{H}) = \sigma E^2+\frac{1}{2}\frac{\partial}{\partial t}[\mu H^2+\varepsilon E^2]$$

But $\overline{E}\times\overline{H}$ is nothing but Poynting vector; $\overline{P}$, rewriting equation,

$$\therefore \quad \boxed{-\nabla\cdot\overline{P} = \sigma E^2+\frac{1}{2}\frac{\partial}{\partial t}[\mu H^2+\varepsilon E^2]}$$

Equation represents **Poynting theorem in point form.** If integrate this power over a volume, energy distribution as,

$$-\int_v \nabla\cdot\overline{P}\,dv = \int_v \sigma E^2\,dv+\frac{\partial}{\partial t}\int_v \frac{1}{2}[\mu H^2+\varepsilon E^2]\,dv$$

$$-\oint_S \bar{P}\cdot d\bar{S} = \int_v \sigma E^2 dv + \frac{\partial}{\partial t}\int_v \frac{1}{2}[\mu H^2 + \varepsilon E^2] dv$$

Equation represents **Poynting theorem in integral form.** The negative sign on the left of equation indicates that the power is flowing into the surface. The first term on the right gives the total ohmic power loss within the volume, while the second term represents time rate of increase of total energy stored in the electric and magnetic fields. By the law of conservation of energy, the sum of the two terms on right must be equal to the total power flowing into the volume. Thus the **minus sign indicates the power flowing into the volume.** So the total power flowing out of the volume is given by $\oint_S \bar{P}\cdot d\bar{S}$. This can be represented with the help of equation as

$$\oint_S \bar{P}\cdot d\bar{S} = -\int_v \sigma E^2 dv - \frac{\partial}{\partial t}\int_v \frac{1}{2}[\mu H^2 + \varepsilon E^2] dv$$

When we define Poynting vector, both the fields $\bar{E}$ and $\bar{H}$ are assumed to be in the real form. If $\bar{E}$ and $\bar{H}$ are expressed in phasor form, then the average power is given by,

$$\bar{P}_{avg} = \frac{1}{2}Re[\bar{E}\times\bar{H}^*] = \frac{1}{2}Re[\bar{E}^*\times\bar{H}]$$

5.9. TIME HARMONIC FIELDS

A time-harmonic field is one that varies periodically or sinusoidally with time. Not only is sinusoidal analysis of practical value, it can be extended to most waveforms by Fourier transform techniques. Sinusoids are easily expressed in phasors, which are more convenient to work with. Before applying phasors to EM fields, it is worthwhile to have a brief review of the concept of phasors. A phasors z is a complex number that can be written as

$$z = x + jy = r\angle\Phi$$

$$z = re^{j\theta} = r(\cos\Phi + j\sin\Phi)$$

where $j = \sqrt{-1}$, x is the real part of z, y is the imaginary part of z, r is the magnitude of z, given by $r = |z| = \sqrt{x^2 + y^2}$ and Φ is the phase of z, given by $\Phi = \tan^{-1}\frac{y}{x}$

Here x, y, z, r, and Φ should not be mistaken as the coordinate variables although they look similar (different letters could have been used but it is hard to find better ones). The Phasor z can be represented in rectangular form as z = x + jy or in polar form as

$$Z = r\angle\Phi = re^{j\theta}$$

Addition and subtraction of phasors are better performed in rectangular form; multiplication and division are better done in polar form. The electric and magnetic fields are varying harmonically with time.

The electric flux density can be $D = D_o e^{j\omega t}$.

The magnetic flux density can be written as $B = B_o e^{j\omega t}$

Taking partial derivative with respect to time

$$\frac{\partial D}{\partial t} = j\omega D_0 e^{j\omega t} = j\omega D$$

$$\frac{\partial B}{\partial t} = j\omega Boe^{j\omega t} = j\omega B$$

5.10. ELECTROMAGNETIC SPECTRUM

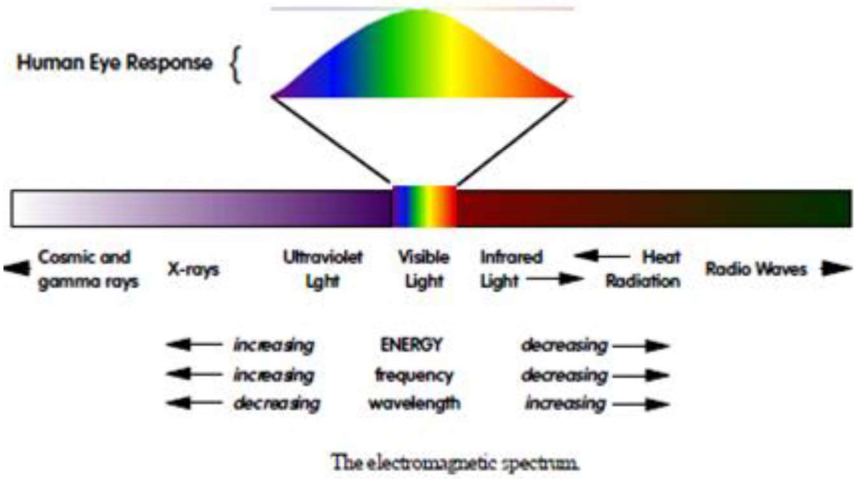

Fig 5.5 Visible spectrum

Electromagnetic radiation is the means for many of interactions with the world: light allows to see; radio waves give TV and radio; microwaves are used in radar communications; X-rays allow glimpses of our internal organs; and gamma rays let eavesdrop on exploding stars thousands of light-years away. Electromagnetic radiation is the messenger, or the signal from sender to receiver. The sender could be a TV station, a star, or the burner on a stove. The receiver could be a TV set, an eye, or an X-ray film. In each case, the sender gives off or reflects some kind of electromagnetic radiation. All these different kinds of electromagnetic radiation actually differ only in a single property their wavelength. When electromagnetic radiation is spread out according to its wavelength, the result is a spectrum, as seen in Fig. 5.5. The visible spectrum, as seen in a rainbow, is only a small part of the whole electromagnetic spectrum. The electromagnetic spectrum is divided into five major types of radiation. As shown in Fig. 5.5, these include radio waves (including microwaves), light (including ultraviolet, visible, and infrared), heat radiation, X-rays, gamma rays, and cosmic rays. Eye can detect only part of the light spectrum. Humans cannot sense any other part of the electromagnetic spectrum without the aid of special equipment. Other animals (such as bees) can see the ultraviolet while some (snakes) can see the infrared. In each case, the eye (or other sense organ) translates radiation (light) into information that (or the bee looking for pollen or the snake looking for prey) can use.

Figure 5.5 “human eye response” is a magnified portion of the electromagnetic spectrum and represents the sensitivity of the average human eye to electromagnetic radiation. As this graph shows, the human eye is most sensitive to light in the middle part of the visible spectrum: green and yellow. This is why emergency vehicles are often painted garish yellows or green, they stand out in all weather, including fog, and at night better than the “old fashioned” fire-truck red. The eye is much less sensitive toward the red and purple ends of visible light. The infrared and ultraviolet portions of the spectrum are invisible to humans. Since the beginning of the modern age, mankind has expanded its ability to “see” into other parts of the electromagnetic spectrum. X-rays have proved useful for looking inside otherwise opaque objects such as the human body. Radio waves have allowed people to communicate over great distances through both voice and pictures.

SUMMARY

- This is called *Faraday's law,* and it can be expressed as

$$\text{Vemf} = -\frac{d\lambda}{dt} = --\frac{d\Psi}{dt}$$

- An electrostatic field *Ee* cannot maintain a steady current in a closed circuit

$$-\int_N^P E_e . dl = IR$$

- The variation of flux with time may be caused in three ways:
  1. By having a stationary loop in a time-varying B field
  2. By having a time-varying loop area in a static B field
  3. By having a time-varying loop area in a time-varying B field.
- For the two integrals to be equal, their integrands must be equal; that is,

$$\nabla x\bar{E}. \text{dl} = -\frac{\partial \text{B}}{\partial \text{t}} . \text{ds}$$

- When a conducting loop is moving in a static B field, an emf is induced in the loop. The force on a charge moving with uniform velocity u in a magnetic field B is Fm = Qu X B

The *motional electric field* Em as

$$\text{Em} = \frac{F_m}{Q} = \text{u x B}$$

- Boundry condition for EMF

i, The tangential component of electric field intensity $\bar{E}$ is continuous at the surface.

$$E_{tan1}=E_{tan2}$$

ii. The tangential component of Magnetic field intensity $\bar{E}$ is continuous across the surface except for a perfect conductor.

$$H_{tan1}=H_{tan2}$$

At the surface of the perfect conductor the tangential component of the magnetic field intensity is discontinuous at the boundary.

$$H_{tan1}-H_{tan2} = K$$

iii. The normal component of the electric flux density is continuous at the boundary is the surface charge density is zero.

$$DN_1=DN_2$$

- EM wave motion in the following media:
  - Free space ($\sigma= 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$)
  - Lossless dielectrics ($\sigma= 0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$ or $\sigma<<\omega\varepsilon$)
  - Lossy dielectrics ($\sigma\neq0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$)
  - Good conductors ($\sigma= \infty$, $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$ or $\sigma>>\omega\varepsilon$)
- General Wave equation

$$\nabla^2\begin{bmatrix}\bar{E}\\\bar{D}\\\bar{H}\\\bar{B}\end{bmatrix}=\mu\,\sigma\frac{\partial}{\partial t}\begin{bmatrix}\bar{E}\\\bar{D}\\\bar{H}\\\bar{B}\end{bmatrix}+\mu\,\varepsilon\frac{\partial^2}{\partial t^2}\begin{bmatrix}\bar{E}\\\bar{D}\\\bar{H}\\\bar{B}\end{bmatrix}$$

- The product of E and H gives a new quantity which is expressed as watt per unit area. This quantity is called power density. P=E×H
- Time Varying fields

$$\frac{\partial D}{\partial t}=j\omega D_o e^{j\omega t}=j\omega D$$
$$\frac{\partial B}{\partial t}=j\omega Bo e^{j\omega t}=j\omega B$$

- Electromagnetic radiation is the means for many of our interactions with the world: light allows us to see; radio waves give us TV and radio; microwaves are used in radar communications; X-rays allow glimpses of our internal organs; and gamma rays let us eavesdrop on exploding stars thousands of light-years away.

TWO MARKS

1. Write the relations between Electrostatic and Magneto static fields

TERM	ELECTROSTATICS	MAGNETOSTATICS
Source Element	Stationary Electric Charge	Element carrying steady current
Force Law	Coulomb's Law	Biot and Savart's Law
Relation between Field & force	$E=\frac{F}{q}$	$F=q(VxB)$
Relation: Field & Potential	$E=-\nabla V$ $\varphi_E=-\int_L E.dl$	$B=\nabla xA$
Flux	$\varphi_E=-\int_S E.dS$	$\varphi_m=\iint_S B.dS$

2. Define Transformer E.m.f

A stationary conducting loop is in a time varying magnetic B field. Equation becomes

$$V_{emf}=\oint \bar{E}.dl=-\oint \frac{\partial B}{\partial t}.ds$$

This emf induced by the time-varying current (producing the time-varying B field) in a stationary loop is often referred to as *transformer emf* in power analysis since it is due to transformer action. By applying Stokes's theorem

$$\oint \nabla x\bar{E}.dl=-\oint \frac{\partial B}{\partial t}.ds$$

For the two integrals to be equal, their integrands must be equal; that is,

$$\nabla x\bar{E}.dl=-\frac{\partial B}{\partial t}.ds$$

This is one of the Maxwell's equations for time-varying fields. It shows that the time varying E field is not conservative ($\nabla \times E \neq 0$). This does not imply that the principles of energy conservation are violated.

3. Define Motional Electromotive forces

This type of emf is called motional emf or flux-cutting emf because it is due to motional action. It is the kind of emf found in electrical machines such as motors, generators, and alternators.

$$F_m = Il\,xB$$