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UNIT V

BLOCKING OSCILLATOR & TIME BASE GENERATORS

5.1 Pulse Transfer

The transformer is a device which transfers electrical energy from one circuit to another without changing frequency. Similarly a pulse transformer is basically a transformer; it couples a source of pulse to load without changing the shape and other properties of pulse. The pulse transformer used in digital circuit to satisfy the following requirements

- i. To provide the isolation between signals.
- ii. Used step up or step down the magnitude of the signal.
- iii. To couple between stages of amplifier.
- iv. To invert the polarity of pulse with the help of centre tapped winding.

v. To act as coupling elements in waveform generator such as blocking oscillators.

The characteristics of pulse transformers are,

- i. Generally iron cores and small in size.
- ii. The leakage inductance is minimum.
- iii. The inter winding capacitance is low.
- iv. The cores have high permeability.
- v. They have high magnetizing inductance.

The ideal pulse transformer model is shown in the below figure.



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Let,	Lp	-	primary	inductance

Le	-	Secondary	inductance
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- M Mutual inductance
 - V_i Source and V_o Output response.
 - R_L Load resistance
 - N_P Primary turns
 - N_s Secondary turns
 - i_P Primary current
 - is Secondary current

The coefficient of coupling between primary and secondary is K. its relation with transformer inductance is given by,

$$K = \frac{M}{\sqrt{L_P L_2}}$$

Note: For ideal transformer K=1

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Transformation ratio is give by,

$$n = \frac{V_0}{V_i} = \frac{i_P}{i_S} = \frac{N_S}{N_P} = \sqrt{\frac{L_S}{L_P}}$$

5.1.1 Practical Equivalent Circuit



5.1.2 Pulse Response Characteristics



Pulse transformer response

Total pulse response can be divided into three sections.

Leading edge response.

i.

At starts there is an overshoot and then the pulses settle down. The response till it settles down after the over shoot is called Leading edge response denoted as X.

ii. Trailing edge response.

The response generally extends below the zero amplitude after the end of pulse width is called back swing. The portion of response from back swing till it settles down is Trailing edge response denoted as Z.

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iii. Flat top response.

The portion of response between the trailing edge and the leading edge is called flat top response.

5.2 Introduction to Blocking Oscillators

A special type of wave generator which is used to produce a single narrow pulse or train of pulse is called **Blocking Oscillator.**

The two important elements of a blocking oscillator are, 1. An active device like transistor 2. A pulse transformer.

Blocking oscillator is not a true multivibrator but similar to some versions of LC sinusoidal oscillators. The energy from the collector circuit is inductively coupled by the base to produce regeneration. This coupling is done by use of iron cored or ferrite cored transformer.

The basic requirement for any switching circuit is that, it should have a positive feedback with loop gain greater than unity. When any amplifier is used in regenerative switching circuit, it must have its loop gain greater than unit.

For satisfying above criterion, we need minimum two transistors is used in the circuit such as multivibrator. Here alternatively one transistor is replaced by a pulse transformer and through it output of the other transistor may be coupled back to input.

There are two types of blocking oscillators

- i. Monostable blocking oscillators
- ii. Astable blocking oscillators

5.3 Monostable or triggered blocking oscillators (Base Timing)

A monostable blocking oscillator circuit which may be triggered by a slowly varying input voltage. It generates a short pulse with large amplitude.

A pulse transformer is connected between collector and base of transistor as shown in figure. Numbers of turns in the base circuit is n times as in the collector circuit. A pulse transformer is a transformer which accepts the pulse at one winding produce similar output at the other winding with 180° out of phase. V_{BB} is connected in the base circuit to avoid triggering and prevent free running operation usually $V_{BB} << V_{CC}$.



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The resistor R connected in series with base controls the pulse duration or timing, hence the circuit is named as **base timing monostable blocking oscillator**.

Operation and Mathematical Analysis of the Circuit

In the quiescent state, the transistor is Off. Suppose a trigger signal is applied to the collector of transistor such that collector voltage level reduces suddenly. The pulse transformer winding polarities are designed such that the voltage applied at the collector (primary) produces inverted signal at the base (secondary). The phasing dots on the transformer indicate such a phase inversion.

Thus base potential increases as collector potential decreases.

When the voltage is more than the cut in voltage of the transformer, it starts conducting drawing current from the supply. This increases the collector current. Due to this, drop across transformer winding in collector increases. This further lower collector potential and increase base potential. This draws more collector current resulting further decrease in collector potential.

This process is repeated until the ac loop gain is greater than unity, then regeneration takes place and transistor gets driven into saturation from its off state.

To obtain the equation of pulse width, consider the equivalent circuit of pulse transformer neglecting resistance and shunt capacitance. The only importance parameter is the shunt magnetising inductance L.



If 'i' is the current in the ideal transformer collector winding, taking winding polarities into account.

Let V₁- Primary voltage (across collector winding)

V₂- secondaryvoltage (across base winding)

- n Transformation ratio
- L- Shuntmagnetising inductance of transformer
- R- Base timing resistor

We know that, Transformation ratio

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From the equivalent circuit, if $V = V_{CC}$ is the voltage drop across collector winding then nV is the corresponding voltage across base winding.

$$V = V_{CC} \dots \dots \dots \dots (2)$$

From the base circuit,

$$i_B = \frac{nV}{R} = \frac{nV_{CC}}{R} \dots \dots \dots (3)$$

From equation (1)

$$i = ni_B = n\frac{nV_{CC}}{R} = \frac{n^2V_{CC}}{R}\dots\dots\dots(4)$$

If i_m is the magnetising current and Vis constant then,

$$L\frac{di_m}{dt} = V \qquad (or)i_m = \int \frac{V}{L}dt$$
$$i_m = \frac{Vt}{L} = \frac{V_{cc}t}{L} \dots \dots \dots (5)$$

From the equivalent circuit,

For t > 0, the collector current increase even if the base current is constant result in which V_{CE} reduces. Due to this reduce V_{CE}, primary voltage of transistor reduces, it increase the secondary voltage. Thus the transistor enters into saturation region into active region. Because the loop gain exceeds unity in active region the transistor is quickly driven into cut off region thus pulse ends.



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At $t = t_P$ as transistor enters into cutoff, base current reduces to zero.

 $i_C = h_{fe} i_B \dots \dots \dots \dots (7)$

Sub i_B and i_C in equation (7),

$$\frac{n^2 V_{CC}}{R} + \frac{V_{CC} t_p}{L} = \frac{h_{fe} n V_{CC}}{R} (at \ t = t_p)$$
$$t_p = \frac{nL}{R} (h_{fe} - n) \approx \frac{nLh_{fe}}{R}$$

5.4 Monostable or triggered blocking oscillators (Emitter Timing)

This circuit is also called triggered transistor blocking oscillator using emitter timing. This uses a transistor in the emitter circuit which controls the pulse width. The pulse transformer used is a tree winding transformer.

One winding is in collector which is primary winding. The second winding is in the base circuit which has 'n' times as many turns as the collector winding. A load R_L is connected across third winding which has n_1 times as many turns as the collector winding. The resistance R_L acts as load and also helps in improving damping.

The and collector winding must produce polarity inversion thus it provides regenerative feedback.



Monostable blocking oscillator with emitter timing

Mathematical analysis:

Assuming for an ideal transformer leakage inductance, capacitance winding resistance are negligible.

For simplicity of analysis, assume number of turns of primary of transformer which is in collector circuit 1 proportional the number of turns of winding in base circuit is 'n' and that of winding used to connect R_L is n_1 .

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Now 'V' is the voltage across primary collector winding, when transistor is in saturation. The corresponding voltage across secondary winding in base circuit is 'nV' and to phase inversion polarity of 'V' & 'nV' are opposite. The voltage across the resistance R_L is 'n₁V' and the polarity is same as that of 'V'.

Polarities of nV are opposite to V



Apply KVL to the outermost loop,

$$-V_{cc} + nV + V = 0$$
$$V = \frac{V_{cc}}{n+1} \dots \dots \dots \dots \dots (1)$$

Apply KVL to the base circuit,

$$nV - (i_B + i_C)R = 0$$

$$nV = (i_B + i_C)R \dots \dots (2)$$

$$(i_B + i_C) = i_E = \frac{nV}{R} = \frac{nV_{CC}}{R(n+1)} \dots \dots \dots (3)$$

The current i_E is the emitter current coming out of the active device.

It is known that sum of the ampere turns in an ideal transformer is always zero.

Amper turns of primary $= i \times 1 = i$

Amper turns of secondary $= i_B \times n = ni_B$

Amper turns of tertiary $= i_1 \times n_1 = n_1 i_1$

The sign of primary and tertiary are same while sign of secondary ampers turns is opposite to primary and tertiary turns.

$$i - ni_B + n_1 i_1 = 0 \dots \dots \dots \dots \dots (4)$$

From the load circuit,

Negative sign is due to assumption of oppositive polarity of $n_1 V$ From (5) equation

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Sub in equation (4)

Apply KCL at the collector node,

Sub in (7)

$$i_{C} - \frac{Vt}{L} - ni_{B} - \frac{n_{1}^{2}V}{R_{L}} = 0$$

$$i_{C} = \frac{Vt}{L} + ni_{B} + \frac{n_{1}^{2}V}{R_{L}} \dots \dots \dots \dots \dots (9)$$

Sub (9) in (3)

Sub (1) in (10)

Sub i_B in (3) we can find i_C

$$\begin{split} i_{B} + i_{C} &= \frac{nV_{CC}}{R(n+1)} \\ i_{C} &= \frac{V_{CC}}{(n+1)} \left(\frac{n}{R}\right) - i_{B} \\ &= \frac{V_{CC}}{(n+1)} \left(\frac{n}{R}\right) - \frac{V_{CC}}{(n+1)^{2}} \left[\frac{n}{R} - \frac{t}{L} - \frac{n_{1}^{2}}{R_{L}}\right] \\ &= \frac{V_{CC}}{(n+1)^{2}} \left(\frac{n^{2} + n - n}{R} + \frac{t}{L} + \frac{n_{1}^{2}}{R_{L}}\right) \\ i_{C} &= \frac{V_{CC}}{(n+1)^{2}} \left(\frac{n^{2}}{R} + \frac{t}{L} + \frac{n_{1}^{2}}{R_{L}}\right) \dots \dots \dots \dots \dots (12) \end{split}$$

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5.4.1 Triggering of Monostable Blocking Oscillator

The triggering method of blocking oscillator is shown below. It is necessary to reduce the collector voltage of T_1 when triggered signal is applied.

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A positive trigger pulse is applied to the base of T_2 . The transistor T_2 provides amplification for the applied trigger signal. The output of T_2 is applied to the collector of T_1 . Due to phase inversion such a voltage gets induced in the secondary which makes the base of T_1 is positive thus the transistor start conducting and normal working of oscillator starts.

When a pulse is switched OFF, then T₂ becomes OFF and it acts as open circuit.

The blocking oscillator is applying a negative pulse through a diode to the collector. When the pulse is applied the diode becomes forward biased and hence reduce the collector voltage of T_1 which turns T_1 out of OFF state when the circuit start working, the pulse is formed. The diode D is reverse biased which avoid the blocking oscillator to react with triggering source.

5.5 AstableBlocking Oscillator (RC Controlled)

In this oscillator RC network is produces a train of pulse thus it is known as RC controlled astable blocking oscillator. This R_1C_1 may be connected in emitter or base terminal according it is named as emitter timing astable blocking oscillator or base timing astable blocking oscillator.

In this circuit another difference is the polarity of V_{BB} is reversed, which is compared with monostable circuit. The function of pulse transformer is same that of emitter timing monostable blocking oscillator.



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Working

When V_{BB} is positive, the transistor T is ON and capacitor starts charging a voltage of V_1 volts. Apply KVL to emitter circuit,

$$V_{BB} - V_{BE} - V_1 = 0$$

 $V_{BB} = V_{BE} + V_1(OR)V_1 = V_{BB} - V_{BE}$

If $V_1 > V_{BE} - V_{BE}$, the transistor is enters from ON state to OFF state. Now the capacitor C_1 discharges exponentially to zero through resistance R_1 . When $V_1 = V_{BE} - V_{BE}$ the transistor T becomes forward biased and enters into active region. This starts the regenerative action, same as in monostable circuit and transistor T gets driven immediately into saturation region. Once the transistors enters in saturation the pulse forms, during this capacitor C_1 starts again, at the end of the pulse, the capacitor charged to V_1 which is greater than initial voltage, thus it makes the transistor is OFF. This process is repeated thus it generates continuous train of pulse.



The time required by the capacitor to discharge from V_1 to $V_{BB} - V_{BE}$ is the OFF time of transistor T and it is denoted by V_{f_1} the voltage across the capacitor is,



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At
$$t = t_f$$
, $V_{BB} - V_{BE} = V_1 e^{-\frac{t_f}{RC}}$
 $e^{-\frac{t_f}{RC}} = \frac{V_{BB} - V_{BE}}{V_1}$

Taking log on both sides,

$$t_f = -RC \log_e \left(\frac{V_{BB} - V_{\gamma}}{V_1} \right)$$

where $V_{BE} = V_{\gamma}$

$$t_f = RC \log_e \left(\frac{V_1}{V_{BB} - V_{\gamma}} \right)$$

The total period of RC controlled blocking oscillator is $T = t_P + t_f$ the t_P can be calculated using the equivalent circuit shown in figure

Neglecting $V_{CE} \& V_{BE}$

we get

$$\frac{t_P}{L} - \frac{n}{R} e^{-\frac{t_P}{RC_1}} = -\frac{n_1^2}{R_I}$$

(For monostable blocking oscillator)

We know
$$e^{-x} = 1 - x \dots if x \ll 1$$

 $e^{\frac{t_P}{RC_1}} = 1 - \frac{t_P}{RC_1} + \dots \dots$

Hence,

$$\frac{t_P}{L} - \frac{n}{R} (1 - \frac{t_P}{RC_1}) = -\frac{n_1^2}{R_L}$$
$$\frac{t_P}{L} - \frac{n}{R} + -\frac{nt_P}{R^2C_1} = -\frac{n_1^2}{R_L}$$
$$\frac{t_P}{L} (1 + \frac{nL}{R^2C_1}) = \frac{n}{R} - \frac{n_1^2}{R_L}$$

If C is very large

$$\frac{t_P}{L} = \frac{n}{R} - \frac{n_1^2}{R_L}$$
$$t_P = \frac{nL}{R} - \frac{n_1^2 L}{R_L}$$

Total time $T = t_P + t_f$

$$T = RC \log_e \left(\frac{V_1}{V_{BB} - V_{\gamma}} \right) + \frac{nL}{R} - \frac{n_1^2 L}{R_L}$$

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5.6 UJT Saw Tooth Generators

As the name implies the UJT has only one junction, but three terminal silicon diode. Its difference from BJT and FET is that it has no ability to amplify. Fig shows the sweep circuit using UJT.



Working

Capacitor C gets charged through the resistance R towards supply voltage V_{BB} . As long as the capacitor voltage is less than peak voltage V_{P} the emitter appears as an open circuit.

(1)

$$V_P = \eta V_{BB} + V_D \dots \dots \dots \dots$$

Where

 η - stand off ratio of UJT

 V_D - Cutin voltage of diode.



When the capacitor voltage V_C exceeds the voltage V_P , the UJT fires (Switched ON). The capacitor starts discharging through $R_1 + R_{B1}$ where R_{B1} internal base resistance. As R_{B1} is neglected and hence capacitor discharges through R_1 .

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Due to design of R_1 this discharge is very fast and it produces a pulse across R_1 , when the capacitor voltage falls below V_V the UJT gets turned OFF. The capacitor starts charging again.

The charging equation of the capacitor is given by,

At t = T and $V_C = V_P$ then

$$V_P = V_V + V_{BB} \left[1 - e^{-T/_{RC}} \right] \dots \dots \dots (3)$$

Sub values of V_P in eqn (3)

Neglecting V_{D} and V_{V} to get approximate relation for T

Where f_0 is oscillating frequency

Problems 1 In a UJT sweep circuit, R=100K Ω , C=0.01 μ F and η =0.8. find the frequency of the oscillation

Solution:

$$T = RCln\left[\frac{1}{1-\eta}\right] = [100 \times 10^{3} \times 0.01 \times 10^{-6}]ln\left[\frac{1}{1-0.8}\right]$$
$$= 1.6 \times 10^{-3}sec$$
$$f_{0} = \frac{1}{T} = \frac{1}{1.6 \times 10^{-3}}$$
$$= 621.33Hz$$

Problems 2 For a certain UJT sweep circuit, the resistance is $10k\Omega$ while the capacitance is 0.1μ F. the valley potential is 1.5V when $V_{BB} = 20V$. Assuming diode cut in voltage of 0.7V and stand off ratio as 0.6, calculate the frequency of the oscillations.

Solution:

$$V_{P} = \eta V_{BB} + V_{D} = 0.6 + 20 + 0.7 = 12.7V$$

$$V_{P} = V_{V} + V_{BB} \left[1 - e^{-T} /_{RC} \right]$$
Here $V_{V} = 1.5V$, $R = 10 \times 10^{3} \Omega$, $C = 0.1 \mu F$
 $12.7 = 1.5 + 20 \left[1 - e^{-T} /_{(10 \times 10^{3} \times 0.1 \mu)} \right]$
 $0.56 = 1 - e^{-T} /_{0.0001}$
 $e^{-T} /_{0.0001} = 0.44$
 $-T /_{0.0001} = ln(0.44) = -0.8209$

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$$T = 8.209 \times 10^{-4} Sec$$

$$f_0 = \frac{1}{T} = \frac{1}{8.209 \times 10^{-4}} = 1.218 kHz$$

5.7 Voltage Time Base Generators

Circuits used to generate a linear variation of voltage with time are called voltage time base generators. An important application of such a waveform is in the cathode ray oscilloscope.

Errors of generation of sweep circuit

There are three most commonly used measures of sweep voltage.

Sweep speed error
$$(e_s)$$

$$e_{s} = \frac{Difference in slope at beginning and end of sweep}{initial value of slope}$$

Displacement error (e_d)

It is defined as the maximum difference between the actual sweep voltage and linear sweep which passes through the beginning and end points of the actual sweep.

$$e_d = \frac{|v_S - v_S'|_{max}}{V_S}$$

Transmission error (e_t)

It is defined as the difference between the input and output divided by the input.

Exponential sweep circuit



It consists of resistor, capacitor and switch connected across capacitor. At t=0, the switch 'S' is opened and the sweep voltage V_s is given by,

$$v_s = V\left(1 - e^{-t/_{RC}}\right)$$

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Expression for sweep speed error (e_s)

We know that exponential increasing voltage can be written as,

$$v_s = V\left(1 - e^{-t/_{RC}}\right)$$

Differentiate above equation with respect to t,

Sub (2) ,(3) in (1)

$$e_{s} = \frac{V/_{RC} - V/_{RC} e^{-T_{s}}/_{RC}}{V/_{RC}}$$

$$e^{-T_{s}}/_{RC} = 1 - \frac{T_{s}}/_{RC}$$

$$e_{s} = 1 - (1 - \frac{T_{s}}/_{RC})$$

$$e_{s} = \frac{T_{s}}/_{RC} = \frac{T_{s}}/_{\tau}$$

Where $RC = \tau$

Expression for displacement error (e_d) The displacement error is given by

$$e_d = \frac{|v_S - v_S'|_{max}}{V_S}$$

We know that

$$v_s = V\left(1 - e^{-t/_{RC}}\right)\dots\dots\dots\dots(1)$$

 $1 - e^{-t/_{RC}}$ can be written as,

$$1 - e^{-t/_{RC}} = 1 - \left(1 - \frac{t}{RC} + \frac{\left(\frac{t}{RC}\right)^2}{2} - \frac{\left(\frac{t}{RC}\right)^3}{3} + \cdots\right)$$

Neglecting higher order terms in above expression,

$$1 - e^{-t/_{RC}} = 1 - \left(1 - \frac{t}{RC} + \frac{\left(t/_{RC}\right)^2}{2}\right) = 1 - 1 + \frac{t}{RC} - \frac{\left(t/_{RC}\right)^2}{2}$$

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Sub (2) in (1)

$$v_s = \frac{Vt}{RC} \left(1 - \frac{t}{2RC} \right)$$

The slope of linear sweep can be given as

$$v'_{S} = \frac{Vt}{RC} \dots \dots \dots \dots (3)$$

$$v_{S} - v'_{S} = \frac{Vt}{RC} \left(1 - \frac{t}{2RC}\right) - \frac{Vt}{RC}$$

$$|v_{S} - v'_{S}| = \frac{Vt}{RC} \times \frac{t}{2RC}$$

We can realize that derivation is max when $t = T_S/2$

At $t = T_s$ then $v'_s = V_s$ From (3)

Sub (4),(5) in e_d

$$e_{d} = \frac{\frac{V(T_{S}/2)}{RC} \times \frac{(T_{S}/2)}{2RC}}{\frac{VT_{S}}{RC}}$$
$$e_{d} = \frac{T_{S}}{8RC}$$

Expression for transmission error (e_t)

The transmission error is give by

The value of v_s is given by

The slope of linear sweep can be given by

$$v'_{S} = \frac{Vt}{RC}$$

At $t = T_{S}$, $v_{S} = V_{S}$ and $v'_{S} = V'_{S}$
 $V'_{S} = \frac{VT_{S}}{RC} \& V_{S} = \frac{Vt}{RC} \left(1 - \frac{T_{S}}{2RC}\right) \dots \dots \dots \dots (3)$
Sub (3) in (1)

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$$e_{t} = \frac{\frac{VT_{S}}{RC} - \frac{Vt}{RC} \left(1 - \frac{T_{S}}{2RC}\right)}{\frac{VT_{S}}{RC}}$$
$$e_{t} = \frac{T_{S}}{8RC}$$

Relation between e_S , e_d , e_t

We know

$$e_s = \frac{T_s}{RC}, \quad e_d = \frac{T_s}{8RC}, \quad e_t = \frac{T_s}{8RC}$$

 $e_d = \frac{e_s}{8}(or)e_d = \frac{e_t}{4}$

Miller Saw tooth Generator

The basic sweep circuit as shown in fig(a). in which S opens to from the sweep. As shown in fig(b), if we introduce an auxiliary variable generator v and if v is always kept equal to the voltage drop across C, the charging current will be constant at $i = \frac{V}{R}$ and perfect linearity can be achieved.



(a) Exponential charging of capacitor

(b) Constant current charging of capacitor

Let as consider in fig(b) with its Z terminal grounded as in fig(c). with this circuit, linear sweep will appear between terminals Y and ground Z and it will increase negative direction.



(c) Point Z is grounded

Let us now replace the auxiliary variable generator by an amplifier with output terminals YZ and input terminals XZ, as shown in fig(d). Since we have assumed that the

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magnitude of voltage v equals the voltage v_c across the capacitor at every instant of time, then the input v_i to the amplifier is zero. We can say that point Z behaves as a virtual ground. With this situation if we want to obtain finite output, the amplifier gain A should be ideally be infinity. Such a need of amplifier can be satisfied by using operational amplifier and circuit is recognized as the operational integrator amplifier. It is referred to as a Miller integrator or Miller Sweep.



(d) Basic Miller circuit

Fig(e) shows the miller circuit with its equivalent circuit. R_i represents input impedence of the amplifier, A represents open circuit voltage gain and R_0 is the output resistance. Here, switch is added, at the closing of switch, the time base waveform will start.



Bootstrap Saw tooth Generator

If Y point in above fig (b) is grounded, the linear sweep will appear between Z and ground and will increase in positive direction. The circuit of fig(b) is referred to as bootstrap



Below fig shows the equivalent circuit for basic bootstrap sweep generator.

5.8 Current Time Base Generator

The circuit which produces a current with linearly increase with time is called current time base generator.

Simple Current Time base generator



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This is also called simple sweep circuit. The basic principle of such circuit is the characteristic behavior of an inductor current.

At t=0, voltage V is applied to a coil of inductance L, with initial current zero then,

$$V = L \frac{di_L}{dt}$$
$$di_L = \frac{v}{L} dt$$
$$i_L = \frac{V}{L} t$$

A transistor Q is used as a switch. An inductor L is connected in series with the transistor Q. a diode D with resistance R_d in series is connected across the inductor L.



A rectangular wave is applied at the base which is called gating waveform. It has two levels. This signal is applied at the base of transistor Q and denoted as V_B .

When the gating waveform is applied base positive, making transistor ON. When gating waveform attains lower, the transistor Q is cutoff.



The equation for inductor current during the sweep is,

$$i_L = \frac{V_{CC}}{L}t$$

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Effect of other Resistance

In the above analysis the effect of yoke internal resistance and collector saturation resistance are neglected.

Let R_L be internal resistance of inductor or yoke.

 R_{CS} be collector saturation resistance

The inductor current equation due to these two resistances becomes,

Expanding exponential term in terms of power series,

$$i_{L} = \frac{V_{CC}}{R_{L} + R_{CS}} \left[1 - \left\{ 1 - \frac{(R_{L} + R_{CS})t}{L} + \frac{(R_{L} + R_{CS})^{2}}{2L^{2}} \dots \right\} \right]$$
$$i_{L} = \frac{V_{CC}}{R_{L} + R_{CS}} \left[\frac{(R_{L} + R_{CS})t}{L} - \frac{(R_{L} + R_{CS})^{2}}{2L^{2}} \dots \right]$$
$$i_{L} = \frac{V_{CC}t}{L} \left[1 - \frac{(R_{L} + R_{CS})t}{2L} \right]$$

From this equation it is clear that the current departs from a linear increase in time. This produces a slope error which is called sweep speed error denoted as e_s

$$e_{S} = \frac{\frac{di_{L}}{dt}\Big|_{t=0} - \frac{di_{L}}{dt}\Big|_{t=T_{S}}}{\frac{di_{L}}{dt}\Big|_{t=0}}$$

Differentiate equation (1) with respect to time,

$$\begin{aligned} \frac{di_{L}}{dt} &= \frac{V_{CC}}{R_{L} + R_{CS}} \left(-e^{-(R_{L} + R_{CS})t} /_{L} \right) \left[\frac{-(R_{L} + R_{CS})t}{L} \right] \\ \frac{di_{L}}{dt} &= \frac{V_{CC}}{L} e^{-(R_{L} + R_{CS})t} /_{L} \\ \frac{di_{L}}{dt} &|_{t=0} = \frac{V_{CC}}{L} \\ \frac{di_{L}}{dt} &|_{t=T_{S}} = \frac{V_{CC}}{L} e^{-(R_{L} + R_{CS})T_{S}} /_{L} \\ e_{S} &= \frac{\frac{V_{CC}}{L} - \frac{V_{CC}}{L} e^{-(R_{L} + R_{CS})T_{S}} /_{L}}{\frac{V_{CC}}{L}} \\ e_{S} &= 1 - \left\{ 1 - \frac{(R_{L} + R_{CS})T_{S}}{L} \right\} \\ e_{S} &= \frac{(R_{L} + R_{CS})T_{S}}{L} \end{aligned}$$

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