**UNIT-II**
**ANGLE MODULATION SYSTEMS**

**2.1 Angle Modulation**

**2.1.1 Definition**

We know that amplitude, frequency or phase of the carrier can be varied by the modulating signal. Amplitude is varied in AM. When frequency or phase of the carrier is varied by the modulating signal, then it is called angle modulation. There are two types of angle modulation.

1. Frequency Modulation: When frequency of the carrier varies as per amplitude variations of modulating signal, then it is called Frequency Modulation (FM). Amplitude of the modulated carrier remains constant.
2. Phase Modulation: When phase of the carrier varies as per amplitude variations of modulating signal, then it is called Phase Modulation (PM). Amplitude of the modulated carrier remains constant.

The angle modulated wave is mathematically expressed as,

$e(t) = E_c \sin[\omega_c t + \theta(t)]$ or $s(t) = A_c\cos(\theta_i(t))$ (2.1)

Here e(t) is angle modulated wave

$E_c$ is peak amplitude of the carrier

$\omega_c$ carrier frequency

$\theta(t)$ instantaneous phase deviation.

The phase deviation takes place in FM as well as PM. Hence phase is direct function of modulating signal. i.e.,

$\theta(t) \propto e_m(t)$

Here $e_m(t)$ is the modulating signal.

**FM equation:**

$e(t) = = E_c \sin\left[\omega_c t + \frac{k_1 E_m}{\omega_m} \sin \omega_m t\right]$ (2.2)

This is an equation for frequency modulated wave. Now let us derive an equation for phase modulated wave. Putting for $\theta$(t) from equation (2.1.5) in equation (2.1.9) we get,

$e(t) = E_c \sin[\omega_c t + k\, e_m(t)]$

PM equation: $e(t) = E_c \sin[\omega_c t + k\, e_m \cos \omega_m t]$ (2.3)

This is an equation for phase modulated wave.

**2.1.2. FM and PM Waveforms:**

Fig.2.1 shows the waveforms of FM and PM.

In this figure following observations can be noted:

i. For FM signal, the maximum frequency deviation takes place when modulating signal is at positive and negative peaks.
ii. For PM signal the maximum frequency deviation takes place near zero crossing of the modulating signal.
iii. Both FM and PM waveforms are identical expect the phase shift.

iv. Form modulated waveform it is difficult to know, whether the modulation is FM or PM.

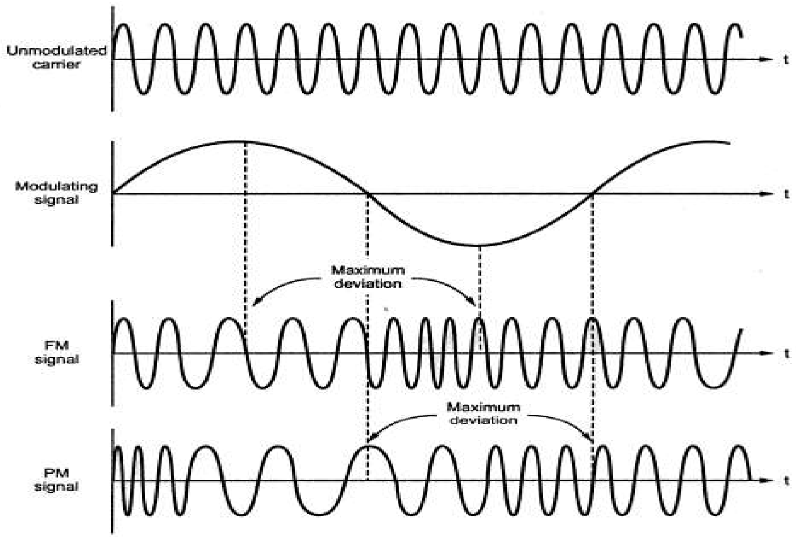



Fig.2.1 waveforms of FM and PM.

**Phase deviation, modulation index and Frequency deviation.**

The FM signal, in general is expressed as,

$e_{FM}$(t) = $E_c$sin[$\omega_{ct}$+m sin$\omega_m t$] (2.4)

And the PM signal, in general is expressed as,

$e_{PM}$(t) = $E_c$sin[$\omega_{ct}$+m cos$\omega_m (t)$] (2.5)

In both the above equations, the term 'm' is called modulation index. Note that the term m sin$\omega_m t$ in equation (2.4) and m cos$\omega_m (t)$ in equation (2.5) indicates instantaneous phase deviation $\theta$(t). Hence 'm' also indicates maximum phase deviation. In other words, modulation index can also be defined as maximum phase deviation.

**Modulation index for PM:**

Comparing equation (2.5) and equation (2.3), we find that

Modulation index in PM : m = k $E_m$ rad (2.6)

Thus modulation index of PM signal is directly proportional to peak modulating voltage. And it's unit is radians.

**Modulation index for FM:**

Comparing equation (2.4) and equation (2.2) we find that,

m = $\frac{K_1 E_m}{\omega_m}$ (2.7)

Thus modulation index of FM is directly proportional to peak modulating voltage, but inversely proportional to modulating signal frequency.

Since $\omega_m$ $=2\pi f_m$ above equation becomes,

Here $\frac{K_1 E_m}{2\pi f_m}$ is called frequency deviation. It is denoted by and its unit is Hz, i.e.,

**Modulation index in FM** :

$$m = \frac{\delta}{f_m} = \frac{\text{Maximum frequency deviation}}{\text{Modulating frequency}} \quad (2.8)$$

Thus modulation index of FM is unit less ratio. From above equation (2.1.14), note that the modulation index is differently defined for FM and PM signals.

**Percentage modulation :**

For angle modulation, the percentage modulation is given as the ratio of actual frequency deviation to maximum allowable frequency deviation. i. e.,

$$\%\ \text{Modulation in} = \frac{\text{Actual frequency deviation}}{\text{Maximum allowable frequency deviation}} \quad (2.9)$$

**Deviation Ratio (DR)** :

The deviation ratio is the ratio of maximum frequency deviation to maximum modulating signal frequency. i.e.,

$$\text{Deviation ratio (DR)} = \frac{\text{Maximum frequency deviation}}{f_{m(max)}} \quad (2.10)$$

Thus the deviation ratio is basically the modulation index corresponding to maximum modulating frequency.

**2.1.3 Frequency spectrum of angle modulated waves**

We know that AM contains only two sidebands per modulating frequency. But angle modulated signal contains large number of sidebands depending upon the modulation index. Since FM and PM have identical modulated waveforms, their frequency content is same. Consider the PM equation for spectrum analysis,

$e_{PM}(t) = E_c \sin[\omega_{ct} + m \cos\omega_m (t)]$

Using Bessel functions, this equation can be expanded as,

$e(t) = E_c \{ J_0 \sin\omega_c t$

$+J_1 [\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)^t]$

$+J_2 [\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)^t]$

$+J_3 [\sin(\omega_c + 3\omega_m)t + \sin(\omega_c - 3\omega_m)^t]$

$+J_4 [\sin(\omega_c + 4\omega_m)t - \sin(\omega_c - 4\omega_m)^t] + \ldots\ldots \}$

Here $J_0, J_1, J_2 \ldots\ldots$ are the Bessel functions.

## 2.2 Frequency Modulation

In FM signals, the carrier amplitude remains constant, while the carrier frequency is changed by the modulating signal. As the amplitude of the information signal or modulating signal increases, the frequency of the carrier signal increases. If the amplitude of the modulating signal decreases, the carrier frequency decreases. The reverse relationship is also implemented. A decreasing modulating signal will increases the carrier frequency above its center value, whereas an increasing modulating signal will decrease the carrier frequency below its center value. The amount of change in carrier-frequency produced by the modulating signal is known as the frequency deviation. Maximum-frequency deviation occurs at the maximum – amplitude of the modulating signal.

The frequency of the modulating signal determines how many times per second the carrier frequency deviates above and below its nominal center frequency. If the modulating signal is a 100Hz sine wave then the carrier frequency will shift above and below the center frequency 100 times per second. This is called the frequency deviation rate.

An FM signal is illustrated in Fig2.1.With no modulating signal, the carrier frequency signal is a constant amplitude sine wave at its constant center frequency. The modulating signal as shown in Fig. 2.1 is a low frequency creases proportionally. The highest frequency occurs at the peak amplitude decreases, the carrier signal frequency decreases. When the modulating signal is at zero amplitude the carrier signal is at center frequency. When the modulating signal goes negative, the carrier frequency will decrease until the peak of the negative half cycle of the modulating signal is reached. When the modulating singles starts increasing towards zero from the peak of the negative cycle, the carrier frequency will again increase The frequency of the modulating signal determines the rate of frequency deviation but has no effect on the amount of deviation, which is strictly a function of the amplitude of the modulating signal.

2.2.1. M**athematical Representation of FM Signal**

Frequency modulation is the form of angle modulation in which instantaneous frequency $f_i(t)$ of the carrier signal is varied linearly with the modulating signal $V_m(t)$ keeping the amplitude of the carrier signal constant.

The instantaneous frequency $f_i(t)$ of the FM signal is given by

$$f_i(t) = f_c + K_f V_m(t) \qquad (2.11)$$

where $f_c$ = Frequency of the unmodulated carrier

$K_f$ = Frequency sensitivity (Hz/Volt)

$V_m(t)$ = Modulating signal or voltage signal

*We know that* $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$ (2.12)

From Eqs.(2.11) and (2.12) we can write

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + K_f V_m(t)$$

Applying integration on both the sides, we get

$\theta_i(t) = \int_0^t 2\pi f_c dt + \int_0^t 2\pi k_f V_m(t)dt$

$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t Vm(t)dt$ (2.13)

The angle of the unmodulated carrier signal is assumed to be zero at time t=0 Using Eq(2.1) the frequency modulated wave can be written as

$S(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t Vm(t)\, dt]$ (2.14)

**Single tone modulation**

Single tone modulation is frequency modulating the carrier signal by single frequency modulating signal.

The modulating signal is given by

$V_m(t) = A_m \cos(2\pi f_m t)$ (2.15)

Where $A_m$ = Amplitude of the modulating signal

$f_m$ = Frequency of the modulating signal

Substituting $V_m(t)$ in Eq (2.11)

We get the instantaneous frequency ($f_i$)

$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$

$f_c = \Delta f \cos(2\pi f_m t)$ (2.16)

Where $\Delta f = k_f A_m$ is called frequency deviation . It is the deviation of instantaneous frequency of the FM signal from the unmodulated carrier signal.

Similarly the instantaneous angle of the FM signal can be obtained from Eq(2.12).

It is given by

$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^1 A_m \cos(2\pi fmt)$

$= 2\pi f_c t + 2\pi k_f A_m \left[\frac{\sin 2\pi f_m t}{2\pi f_m}\right]$

$= 2\pi f_c t + \frac{2\pi k_f A_m}{2\pi f_m}\sin 2\pi f_m t$

$= 2\pi f_c t + \frac{k_f A_m}{2\pi f_m}\sin 2\pi f_m t$

$= 2\pi f_c t + \frac{\Delta f}{f_m}\sin 2\pi f_m t$ $\quad (\Delta f = k_f A_m)$

$\theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t$ (2.17)

Where $\left(\beta = \frac{\Delta f}{f_m}\right)$ is the modulation index

From Eq (2.17) the maximum and minimum values of angles of FM are

$\theta_I$ (max) $= 2\pi f_c t + \beta$ (2.18)

$\theta_I$ (min) $= 2\pi f_c t - \beta$ (2.19)

The parameter $\beta$ represents the phase deviation of FM which corresponds to the maximum departure of the instantaneous angle $\theta_i(t)$ from the unmodulated carrier angle $2\pi f_c t$

The FM signal can be written using Eq(2.14) as

$s(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$ (2.20)

Depending on $\beta$ , the FM can be distinguished into two types. If the valu of the modulation index $\beta$ is very small compared to one radian, it is called Narrow Band Frequency modulation (NBFM).If the value is very large compared to one radian, it is called WideBand Frequency modulation (WBFM)

*Narrow Band Frequency Modulation (NBFM)*

- Modulation index $\beta << 1$
- Narrow Bandwidth which is equal to twice the message bandwidth.

Wide Bandwidth which is equal to twice the message band width.

*Wide Band Frequency Modulation (WBFM)*

- Modulation index $\beta$ >>1
- Large Bandwidth which is ideally infinite

### 2.2.2 Narrow Band Frequency Modulation (NBFM)

From Eq(2.20),the FM wave s(t) can be written as

$s(t) = A_c cos(2\pi f_c t + \beta sin2 \pi f_m t)$

Expand the above equation using the formula
*Cos (A+B) = cosAcosB -sinAsinB*
The equation s(t) becomes

$s(t) = A_c [cos(2\pi f_c t) cos\ (\beta sin(2 \pi f_m t))]$

$-sin(2\pi f_c t) sin\ (\beta sin(2 \pi f_m t))$ *(2.21)*

Since the value of is very small for NBFM
$cos\ (\beta sin(2 \pi f_m t)) = 1$ (where $cos\theta=1$ if $\theta$ is small)
Substitute the above approximations in Eq(2.21), the resultant equation *s(t)* becomes
$s(t) = A_c [cos(2\pi f_c t) - sin\ (2\pi f_c t)\ (\beta sin(2 \pi f_m t))]$ (2.22)
With the help of Eq(2.22)we can set an arrangement for generating narrow band FM signal as shown in Fig.2.2
The message signal $V_m$ (t) is fed to the integrator and it follows to one input of the product modulator . The other input of the product modulator is obtained from the 90 phase shifter circuit and the input to the 90 phaseshifter circuit is the carrier signal $A_c$ cos$2\pi f_c t$ the output of the adder is nothing but the narrow band FM signal as given in Eq(2.22).

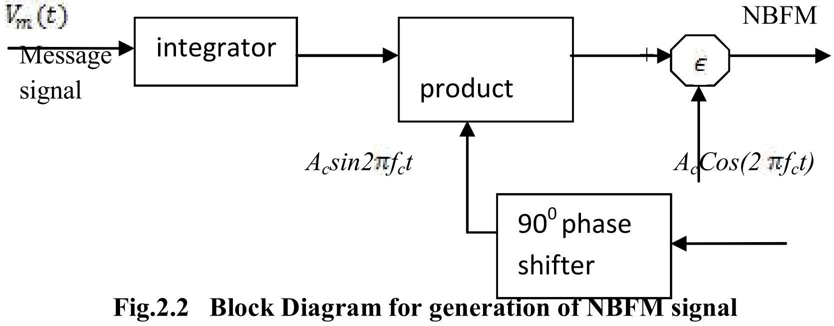



**Fig.2.2 Block Diagram for generation of NBFM signal**

For an ideal condition, the FM signal has a constant envelope and in the case of a sinusoidal modulating signal frequency $f_m$ the angle $\theta_i$ (t) of the FM signal is also sinusoidal with the same frequency modulating signal. But the modulating signal product by NBFM differs from the ideal condition in toe fundamental aspects.

1. The envelope of arrow band FM contains residual amplitude modulation and hence it varies with time.

This disadvantage can verified by writing Eq (2.22) as

$s(t) = v(t)\cos(2\pi f_c t + \emptyset$

Where $v(t) = \sqrt{A_c^2 + \beta A_c^2 sin^2(2\pi f_m t)}$ (2.23)

Eq (2.23) illustrates that the NBFM signal contains residual amplitude modulation and varies with times.

The angle $\emptyset$ is given by

$$\emptyset = \tan^{-1}\left(\frac{\beta A_c \beta \sin(2\pi f_m t)}{A_c}\right)$$

$$\emptyset = \tan^{-1}(\beta A_c \beta \sin(2\pi f_m t)) \qquad (2.24)$$

2. For sinusoidal modulating wave, the angle $\theta_i$ (t) contains harmonic distortion in the form of higher order harmonics of modulating frequency $f_m$ .

This disadvantage can be verified by expanding Eq(2.24)

$$\emptyset = \beta\sin(2\pi f_m t) - \frac{1}{3}\beta^3 sin^3(2\pi f_m t) + \frac{1}{5}\beta^5 sin^5(2\pi f_m t)$$

The above equation is obtained by using power series expansion.

Thus a narrow band FM consists of residual AM and harmonic PM and can be reduced to a negligible value by minimizing modulation index $\beta$ to a small value

Eq (2.22) can be expanded as

$$s(t) = A_c\cos(2\pi f_c t) + \frac{\beta A_c}{2}[\cos(2\pi(f_c+f_m)t) - \cos(2\pi(f_c - f_m)t)] \qquad (2.25)$$

the amplitude modulated signal can be written as

$$s_{AM}(t) = A_c\cos(2\pi f_c t) + \frac{m_a A_c}{2}[\cos(2\pi(f_c+f_m)t) - \cos(2\pi(f_c - f_m)t)] \qquad (2.26)$$

where $m_a$ is the modulation index of AM signal.

Comparing Eq(2.25) and Eq (2.26), we find that in case of sinusoidal modulating wave , the basic difference between an AM signal and a narrow band FM signal is that the sign of the lower band frequency in the NBFM ir reversed. Thus a narrow band FM signal requires twice the bandwidth of the message signal as the AM signal. Therefore the bandwidth of NBFH is $2f_m$.

The narrow band FM signal is represented with phasor diagram as shown in Fig 2.3(a). In the figure it is shown that the carrier phasor used as a reference phasor. The result of the two side band frequency is always at right angles to that of the reference phasor. The resultant phasor is the narrow band FM signal and it is seen that it is having approximately the same amplitude as that of the carrier phasor, but of phase with respect to the carrier phasor.

The AM signal is tepresented with phasor diagram as shown in Fig.2.3(b). In this case the resultant phasor (i.e., AM signal) has an amplitude different to that of the carrier phasor, but always in phase with it.

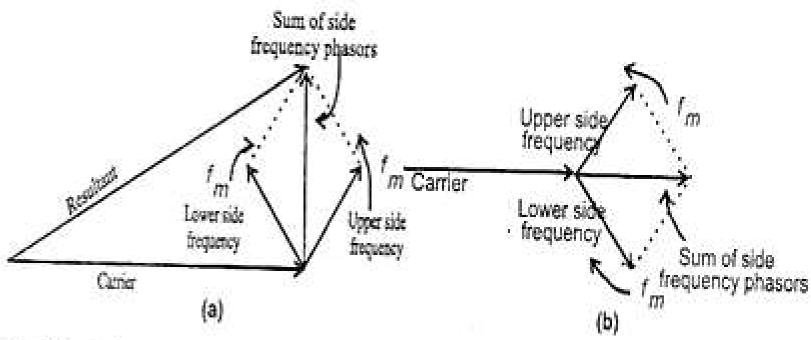



Fig. 2.3 Phasor representation of narrow band FM and AM waves for sinusoidal modulation (a) Narrow band FM wave (b) AM wave

### 2.2.3 Wide Band Frequency Modulation (WBFM)

If the value of the modulation index($\beta$) is very large, then the resulting FM modulation is wide band frequency modulation. The bandwidth of WBFM signal is ideally infinite. In this section we will obtain the spectrum of wideband FM signal

From Eq(2.20) the FM signal is given by

s(t) = $A_c \cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t))$

the above equation can be rewritten as

s(t) = $A_c Re\,[e^{j}(2\pi f_c t) + (\beta \sin(2\pi f_m t))]$

= $A_c Re\; e^{j2\pi f_c t} e^{j\beta \sin 2\pi f_m t}$ (2.27)

The envelope of the FM signal s(t) is given by

$\widetilde{s(t)} = A_c Re e^{j\beta \sin 2\pi f_m t}$ (2.28)

Since the envelop is complex in nature, substitute Eq(2.28) in Eq(2.27). The Eq(2.27) becomes

$s(t) = \text{Re}\left[\widetilde{s(t)} e^{j2\pi f_c t}\right]$ (2.29)

The complex envelops $\widetilde{s(t)}$ is a periodic function of time with a fundamental frequency $f_m$ . Since it is a periodic function it can be represented with the help of complex Fourier series

The complex envelops $\widetilde{s(t)}$ in fourier series representation is given by

$$\widetilde{s(t)} = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_m t} \quad (2.30)$$

Where $C_n$ is the complex Fourier coefficient. The value $C_n$ is given by

$$C_n = f_m \int_{-1/2f_m}^{+1/2f_m} \widetilde{s(t)} e^{-j2\pi n f_m t} dt \quad (2.31)$$

Substitute Eq (2.27) in Eq (2.31) we get

$$C_n = A_c f_m \int_{-1/2f_m}^{+1/2f_m} e^{j(\beta \sin 2\pi f_m t - 2\pi n f_m t)} dt \quad (2.32)$$

The Bessel function of $n^{th}$ order is given by

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx \quad (2.33)$$

From Eq (2.32) the complex Fourier coefficient $C_n$ is given by

$$C_n = A_c \left( f_m \int_{-1/2f_m}^{+1/2f_m} e^{j(\beta \sin 2\pi f_m t - 2\pi n f_m t)} dt \right)$$

Take $u = 2\pi f_m t$

$du = 2\pi f_m dt$

Substitute the new variable u in $C_n$ ,we get

$$C_n = A_c\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin u - nu)}du\right) \quad (2.34)$$

Substitute $J_n(\beta)$ in Eq(2.34)

$$C_n = A_c J_n(\beta) \quad (2.35)$$

Substitute Eq (2.35) in eq (2.30),we get

$$\widetilde{s(t)} = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi n f_m t} \quad (2.36)$$

Substitute Eq(2.36) in Eq(2.29), the resulting FM signal s(t) is given by,

$$s(t) = A_c Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t}\right]$$

$$= A_c Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi(f_c + n f_m)t}\right] \quad (2.37)$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + n f_m)t] \quad (2.38)$$

Eq (2.38 ) gives the fourier series representation of the frequency modulated signal s(t).
Expanding Eq (2.38 ) we get

$$\begin{aligned} s(t) = {} & A_c J_0(\beta)\cos 2\pi f_c t + A_c J_1(\beta)(\cos 2\pi(f_c + f_m)t) \\ & + A_c J_2(\beta)(\cos 2\pi(f_c + 2f_m)t) + \ldots \\ & + A_c J_{-1}(\beta)(\cos 2\pi(f_c - f_m)t) + A_c J_{-2}(\beta)\cos(2\pi(f_c - 2f_m)t) \\ & + A_c J_{-3}(\beta)(\cos 2\pi(f_c - f_m)t) + \ldots \end{aligned}$$

For even values of n

$$J_n(\beta) = J_{-n}(\beta) \quad (2.40)$$

For odd values of n

$$J_n(\beta) = -J_{-n}(\beta)$$

Therefore

$$J_n(\beta) = (-1)^n J_{-n}(\beta) \quad (2.41)$$

By using the properties of Besssel function Eq (2.39) can be simplified as

$$s(t) = A_c J_0(\beta)\cos(2\pi f_c t) - A_c J_1(\beta)[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)]$$
$$+ A_c J_2(\beta)[\cos(2\pi(f_c - 2f_m)t) + \cos(2\pi(f_c + 2f_m)t)]$$
$$- A_c J_3(\beta)[\cos(2\pi(f_c - 3f_m)t) - \cos(2\pi(f_c + 3f_m)t)]$$

(2.42)

The spectrum of s(t) can be obtained by taking Fourier transform on both sides of

$$s(t) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$ 3)

From Eq (2.43) it is observed that the spectrum of FM wave consists of carrier signal with amplitude and a set of sideband frequencies situated on either side of the carrier frequency with a frequency separation of $f_m$, $2f_m$, $3f_m$, etc.
From this aspect we can come to the conclusion that in the case of single tone sinusoidal modulation, AM has only one pair of side frequencies. But FM has infinite number of sideband frequencies.The bandwidth of Wideband FM is infinite.
The variation of the Bessel function $J_n(\beta)$ which determines the amplitude of various sideband frequency components of wideband FM, has been plotted against the modulation index $\beta$ for different values of n in fig.2.4

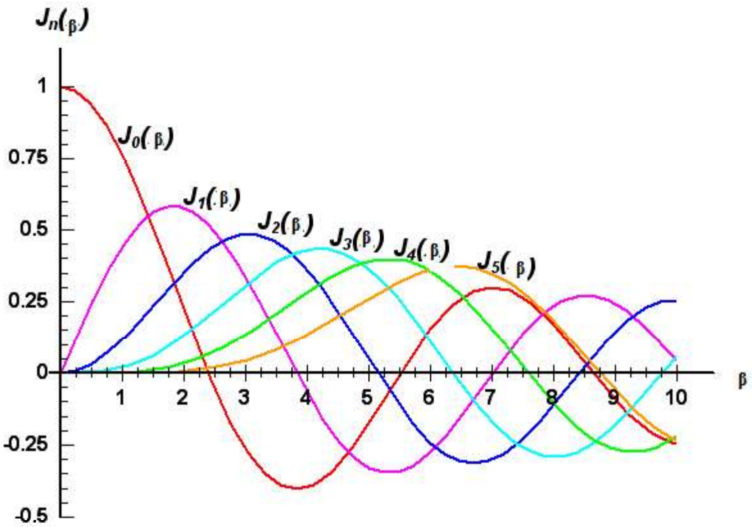



**Fig.2.4 Plots of Bessel function of the first kind with argument (β)**

For small values of modulation index β, the Bessel function $J_n(\beta)$ is
$J_0(\beta) = 1$
$J_1(\beta) = \beta/2$

$J_n$ **(β) = 0 for n** > 2

From the above equation we see that $J_o$**(β)** and $J_1$ (β) have significant values, so that the FM signal is effectively composed of a carrier and a single sideband frequencies at $f_c \pm f_m$ for small values of **β.**

The average power of FM signal can be obtained by the equation

$$\overline{s^2(t)} = \frac{1}{2}A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta) = p \qquad \textbf{(2.44)}$$

By using the property of Bessel function (2.45)

$$\overline{s^2(t)} = p = \frac{1}{2}A_c^2 \qquad \left(\therefore \sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1\right)$$

From Eq (2.45) we see that the average power of FM signal is same as that of the unmodulated carrier power. Unlike AM wave, the amplitude of the carrier component of FM wave is dependent on modulation index β.

**2.3 Transmission Bandwidth of an FM Signal**

From Eq(2.42) it is found that the FM signals consists of infinite number of side bands and the amplitude of each sideband is determined by the Bessel function $j_n(\beta)$. It is concluded that the FM signal has infinite number of sideband components and its bandwidth is infinite for an ideal case. Practically we can ignore the distant sidebands with small amplitude and with significant amplitude are considered in calculating the bandwidth of a FM signal. The number of significant sidebands depends upon the value of frequency deviation ($\Delta f$) and modulating frequency ($f_m$). The modulation index $\beta$ is the ratio of frequency deviation to the modulating frequency

$$\beta = \frac{\Delta f}{f_m} \qquad (2.46)$$

$$= \frac{K_f A_m}{f_m} \qquad /\Delta f = K_f A_m$$

The effective bandwidth is the separation between the two extreme significant sideband frequencies on either side of the carrier. Practically, a sideband frequency is said to be significant if its amplitude is at least one percent of the unmodulated carrier amplitude.

In a spectrum of FM signal there are n significant sideband and the modulating frequency is fm.

Therefore bandwidth (BW) is given by

**BW= 2**$\pi$ $f_m$ (2.47)

Schwarty developed a universal curve for determining the bandwidth of an FM signal if modulation index is known. According to schwarty any frequency component with a signal strength less than 1% of that of the unmodulated carrier is considered to be insignificant. The graph in Fig.2.5 shows the variation of the $\frac{BW}{\Delta f}$ against.

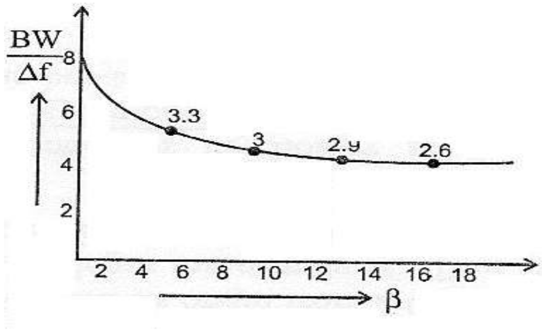



Fig.2.5 **Universal Curve**

**Table 2.1 Number of significant sideband frequencies of a wideband FM signal for varying modulation index.**

| Modulation index ($\beta$) | Number of significant sideband frequencies ($2n_{max}$) |
|---|---|
| 0.1 | 2 |
| 0.3 | 4 |
| 0.5 | 4 |
| 1.0 | 6 |
| 2.0 | 8 |
| 5.0 | 16 |
| 10.0 | 28 |
| 20.0 | 50 |
| 30.0 | 70 |

Table 2.1 gives the number of significant sideband frequencies of a wideband FM signal for various value of modulation index.($\beta$)

According to Carson, the bandwidth of an FM signal is equal to twice the sum of the frequency deviation and the highest modulating frequency. This rule is known as Carsons rule. This rule is just an approximation.

B.W = $2(\Delta f + f_m)$

B.W = $$2\Delta f\left(1+\frac{1}{\beta}\right) \quad (2.48)$$

If $\beta \ll 1$ then Eq (2.48) becomes

$$B.W = 2\frac{\Delta f}{\beta} = 2f_m$$

If $\beta \gg 1$ then Eq(2.48) becomes

$$B.W = 2\Delta f$$

**2.4 Phase Modulation (PM)**

Phase modulation is the type of angle modulation in which the angle is varied linearly with the message signal Vm (t) is given by

$$\theta_i(t) = 2\pi f_c t + k_p V_m(t)$$
$$V_m(t) = A_m \cos\omega_m t$$
$$\text{Therefore} \quad \theta_i(t) = 2\pi f_c t + k_p A_m \cos\omega_m t \qquad (2.49)$$
$$= 2\pi f_c t + \Delta\theta \cos 2\pi f_m t$$
$$\text{Where} \quad \Delta\theta = k_p A_m \qquad \omega_m = 2\pi f_m$$

$\Delta\theta$ is the phase deviation and it is defined as the maximum departure in the phase. The modulated signal s(t)

$$s(t) = A_c \cos(\theta_i(t))$$
$$s(t) = A_c \cos[2\pi f_c t + k_p A_m \cos 2\pi f_m t] \qquad (2.50)$$

The instantaneous frequency is given by

$$f_i = \frac{d\theta_i}{dt} = \frac{d}{dt}[2\pi f_c t + k_p A_m \cos 2\pi f_m t] \qquad (2.51)$$
$$= 2\pi f_c - k_p A_m 2\pi f_m \sin 2\pi f_m t$$

Thus the maximum departure in the frequency is $k_p A_m 2\pi\, f_m$

Which depends upon the modulating frequency $f_m$

The frequency deviation for FM is

$$\Delta f = k_f A_m$$

Therefore for an equal bandwidth in FM and PM, we have.

$$k_f = 2\pi k_p f_m$$

Thus Eq (2.51) gives the relationship between frequency sensitivity $k_f$ and phase sensitivity $k_p$.

**Conversion of PM to FM**

Frequency modulated signal is obtained by first integrating the message signal of baseband signal and then it is applied to phase modulator. Fig.2.6 shows the Block diagram of the conversion of PM to FM.

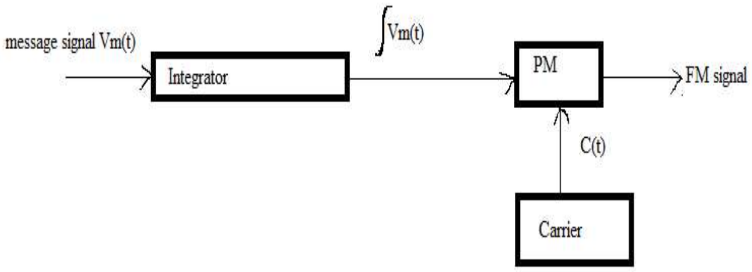



Fig. 2.6 Conversion of PM to FM

**Conversion of FM to PM**

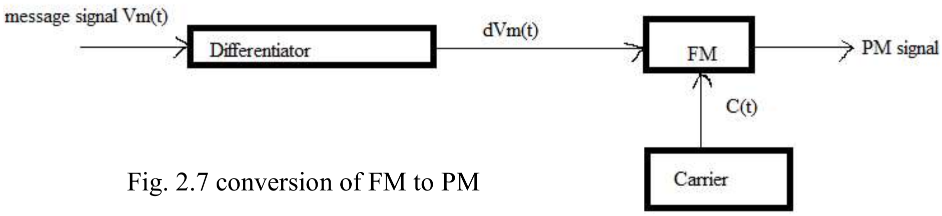



Fig. 2.7 conversion of FM to PM

Phase modulated signal is obtained by first differentiating the message signal Vm(t) and then resultant signal is frequency modulated with the help of the carrier signal. The output of frequency modulator is PM signal.

**2.5 COMPARISOIN OF WIDEBAND AND NARROWBAND FM**

| SI. No. | Parameter Characteristics | Wideband FM | Narrowband FM |
|---|---|---|---|
| 1. | Modulation index | Greater than 1 | Less than (or) slightly greater than 1 |
| 2. | Maximum deviation | 75 KHz | 5 KHz |
| 3. | Range of modulating Frequency | 30 Hz to 15 KHz | 30 Hz to 3 KHz |
| 4. | Bandwidth | Large, about 15 times higher | Small. Approximately same |

| | | | |
|---|---|---|---|
| | | than BW of narrow band FM. BM=2 ($\Delta f + f_m$) | as that of AM. BW=2$f_m$ |
| 5. | Maximum modulation Index | 5 to 2500 | Slightly greater than 1 |
| 6. | Pre-emphasis and De-emphasis | Needed | Needed |
| 7. | Noise | Noise is more suppressed | Less suppressing of noise |
| 8. | Applications | Entertainment broadcasting (can be used for high quality music transmission) | FM mobile communication like Police wireless, ambulances |
| 9. | Side bands | Spectrum contains infinite number of side bands and carrier | Spectrum contains two sidebands and carrier. |

## 2.6 FM and PM Modulators

We know that in FM, the frequency of the carrier is varied according to amplitude changes in the modulating signal. The carrier frequency is generated by LC oscillators. The carrier frequency can be changed by varying either the inductance or capacitance of the tank circuit. The devices like FET, BJT and varactor diodes have the property that their reactance can be varied by varying the voltage across them. Such devices can be used with LC tank circuits to vary the overall reactance. This reactance can be inductive or capacitive. The change in reactance changes the frequency of the oscillator.

There are two types of FM modulators:

**i. Direct FM:**

In this type of angle modulation, the frequency of the carrier is varied directly by the modulating signal. This means, an instantaneous frequency deviation is directly proportional to amplitude of the modulating signal.

**ii. Indirect PM:**

In this type of angle modulation, FM is obtained by phase modulation of the carrier. Instantaneous phase of the carrier is directly proportional to amplitude of the modulating signal.

### 2.6.1 Direct FM

Direct FM can be obtained by using FET and varactor diode. These methods are discussed next.

#### 2.6.1.1. FET Reactance Modulator

Fig 2.8 shows the basic circuit of FET reactance modulator. It behaves as reactance across terminals A-B. The terminals A-B of the circuit may be connected across the tuned circuit of the oscillator to get FM output. The varying voltage (modulating voltage) V, across terminals A-B changes reactance of the FET. This change in reactance can be inductive or capacitive.

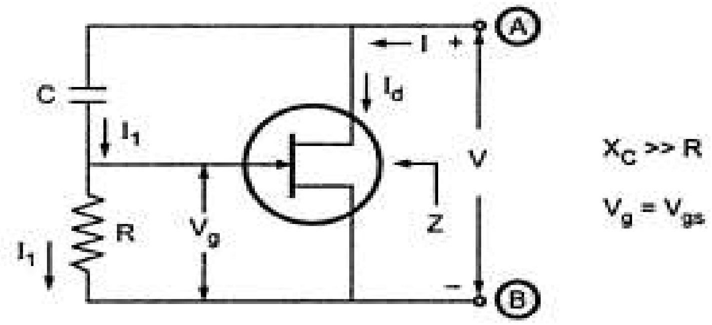



Fig 2.8 FET reactance modulator

Neglecting gate current, let the current through C and R be $I_1$. At the carrier frequency, the reactance of 'C' is much larger than R. We can write equation for $I_1$ as,

$I_1 = \frac{V}{R+\frac{1}{j\omega C}}$

Since $j\omega C$>> R, we can write above education as,

$I_1 = j\omega CV$

From the circuit, $V_g = I_1$R$= j\omega CRV$

For the FET, $I_d = g_m\ V_{gs} = g_m\ V_m$

$= j\omega CR\ g_m$ V (2.52)

From the circuit, impedance of the FET is,

Z $= \frac{V}{I_d} = \frac{V}{j\omega g_m CRV} = \frac{1}{j\omega [g_m\ CR]} = \frac{1}{j\omega C_{eq}}$ (2.53)

Here $C_{eq} = g_m$CR. Thus the impedance of FET is capacitive reactance. By varying the modulating voltage across FET, the operating point $g_m$ can be varied. Hence this varies $C_{eq}$. Thus change in the capacitance will change the frequency of the oscillator. If we connect inductance instead of capacitor, we get inductive reactance in the circuit.

**2.6.1.2 Frequency Modulation using varactor Diode**

All the diodes exhibit small junction capacitance in the reverse biased condition. The varactor diodes are specially designed to optimize this characteristic. The junction capacitance of the varactor diode changes as the reverse bias across it is varied. The variations in capacitance of this diode are wide and linear. The varactor diodes provide the junction capacitance in the range of 1 to 200 pF. Fig 2.9 shows how varactor diode can be used to generate FM. $L_1$ and $C_1$ form the tank circuit of the carrier oscillator. The capacitance of the varactor diode depends upon the fixed bias set by $R_1$ and R2 and the AF modulating signal. Either R1 or Rs is made variable so that the center carrier frequency can be adjusted over a narrow range. The Radio Frequency Choke (RFC) has high reactance at the carrier frequency to prevent the carrier signal from getting into the modulating signal circuits. At positive going modulating signal adds to the reverse bias applied to the varactor diode D, which decreases its capacitance and increases the carrier frequency. A negative going modulating signal subtracts from bias, increasing the capacitance, which decreases the carrier frequency.

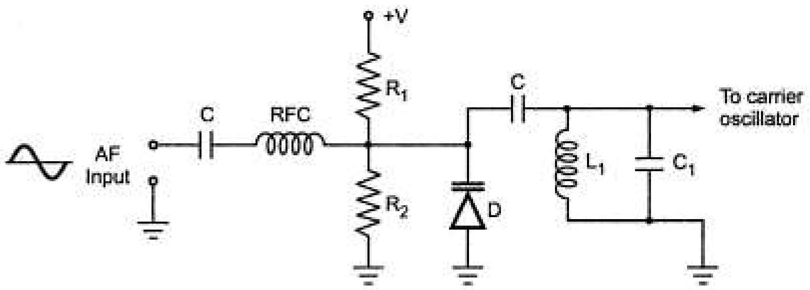



Fig 2.9 Varactor diode for FM generation

The frequency of the LC oscillator changes due to temperature effects. Hence crystals are used in FM generators to provide frequency stability.

**2.6.2Indirect FM**

Fig. 2.10 shows the circuit diagram of indirect FM generation. It consists of the varactor diode $D_1$ in series with tuned $L_1$ $R_1$ network. The complete series and parallel network is in series with crystal oscillator. The modulating signal is applied to varactor diode. The capacitance of varactor diode is changed by modulating signal. This changes phase angle of the complete network. This creates phase shift in the carrier signal from crystal oscillator. The instantaneous phase shift is directly proportional to instantaneous amplitude of modulating signal.

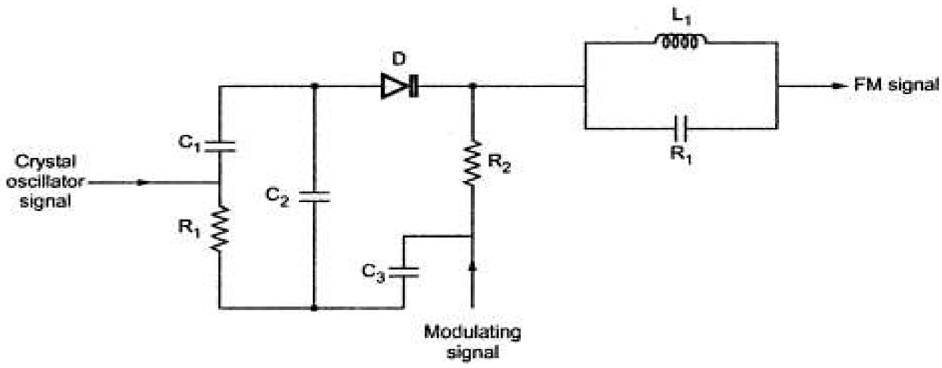



**Fig. 2.10 Indirect FM generation**

**Advantage:**

i) The crystal oscillator is isolated from modulator. Hence frequency stability is more.

**Disadvantage:**

i) Capacitance versus voltage characteristic of varactor diode is nonlinear. This results in distorition in the modulated waveform.

ii) Amplitude of modulating signal should be kept small to avoid distortion.

**Application:**
Indirect FM is used for narrowband low index FM generation.

**2.6.3 Indirect method (Armstrong method)**
In this method of FM generation, the modulating signal is integrated and it is phase modulated by crystal oscillator to get narrow band FM signal which later passed on to a frequency multiplier to get a wideband FM signal. The block diagram of an Armstrong method is shown in Fig 2.11.

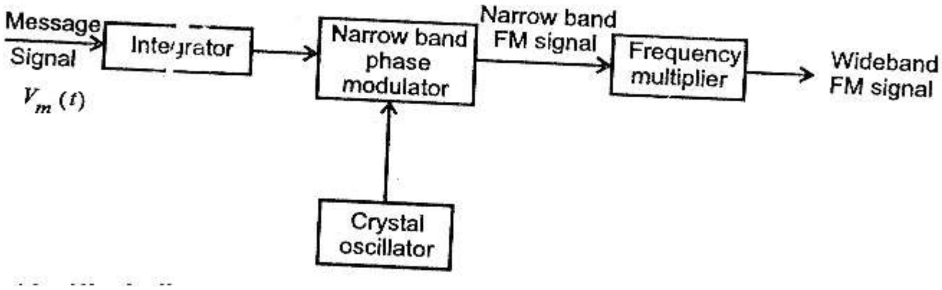



Fig. 2.11 Block diagram of Armstrong method for generating wideband FM signal

The crystal oscillator provides frequency stability. The value $\beta$ is kept small to produce narrow band FM signal. The frequency multiplier consists of a memory less nonlinear device followed by a pandas filter. The block diagram for frequency multiplier is shown in Fig.2.12.

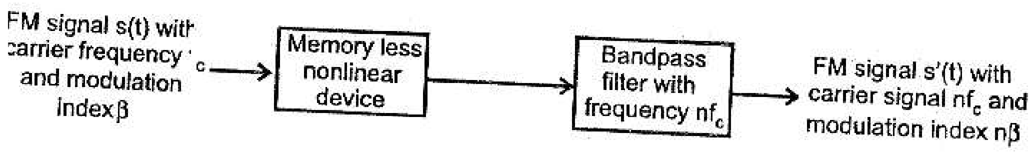



Fig.2.12 Block diagram of Frequency multiplie

If FM input signal is s(t), then outputV0(f) of memory less nonlinear device is given by
$V_0(t) = a_1 s(t) + a_2 s^2(t) + \ldots\ldots + a_n s^n(t)$ (2.54)
Where $a_1$, $a_2$, ------------- $a_n$ are the coefficients determined by the operating point of the device and n is she higher order of non linearity.

The input s(t)
$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t V_m(t)\,dt\right]$

The instantaneous frequency

$f_i(t) = f_c + k_f V_m(t)$

consider the maximum nonlinearity term and then the instantaneous frequency n$f_i(t)$
$n f_i(t) = n f_c + n k_f V_m(t)$
Therefore new wideband FM is given by
$s^1(t) = A_c \cos 2\pi n f_c t + 2\pi n k_f \int_o^t V_m(t)\,dt$
from this it is clear that the frequency of FM signal can be varied by varying the value of *n*.

**Advantages:**
i) FM is generated from PM indirectly.
ii) Modulation takes place at low carrier frequency.

**2.7 FM Demodulators**
The FM receivers also super heterodyne receivers. But they have different types of demodulators or detectors. FM receivers have amplitude limiters which are absent in AM receivers. The AGC system of FM receivers is different than that of AM receivers. RF amplifiers, mixers, local oscillators IF amplifiers, audio amplifiers etc. all are present in FM receivers . the detection of FM is totally different compared to AM. The FM detector should be able to produce the signal whose amplitude is proportional to the deviation in the frequency of signal. Thus the job FM detector is almost similar to frequency to voltage convertor. Here we will discuss these types of FM detectors. Slope detectors, phase discriminator and ratio detector.

**2.7.1 Round – Travis detector or balanced slope detector (frequency Discriminator)**

Fig. 2.13 shows the circuit of balanced slope detector. It consist of two identical circuit connected back to back . The FM signal is applied to the turned LC circuit. Two turned LC circuits are connected in series . The inductance of this secondary turned LC circuit is coupled with inductance of the primary (or input side) LC circuit . Thus it forms a turned transformer. In fig. 2.13, the upper turned circuit is shown as T1 and lower turned circuit is shown as T2. The input side LC circuit is turned to *fc,* carrier frequency. T1 tuned to $fc-\delta f$, which represents the minimum frequency of FM signal. The input FM signal is coupled to *T1* and *T2* $180^\circ$ out of phase. The secondary side tuned circuits (*T1* and *T2)* are connected to diodes *D1* and *D2* with RC loads. The total output $V_{out}$ is equal to difference between $V_{01}$ and$V_{02}$, since they subtract (See Fig. 2.13) Fig. 2.14 shows the characteristic of the balanced slope detector. It shows $V_{out}$ with respect to input frequency.

When the input frequency is equal to $f_c$ both T1 and T2 produce the same voltage. Hence $V_{01}$ and $V_{02}$ are identical and they substract each other. Therefor $V_{out}$ is zero. This is shown in Fig. 2.14. when the input frequency is $f_c + \delta f$,the upper circuit t1 produces maximum voltage since it is tuned to this frequency (i.e. $f_c$). Whereas lower circuit T2 is tuned to $f_c - \delta f$, which is quite away from $f_c + \delta f$. Hence T2 produces minimum voltage. Hence the output $V_{01}$ is maximum where $V_{02}$ is minimum. Therefore $V_{out} = V_{o1} - V_{o2}$ is maximum positive for $f_c + \delta f$.

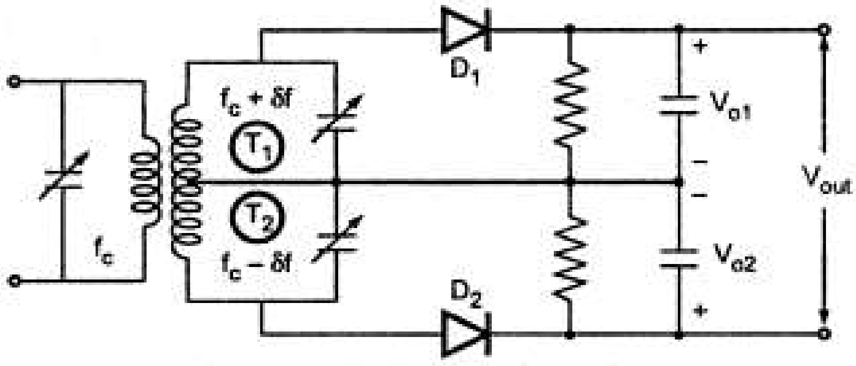



Fig.2.13 Balanced Slope detector

When input frequency is $f_c - \delta f$, the lower circuit T2 produces maximum signal. Hence rectified output $V_{o2}$ is maximum and $V_{o1}$ is minimum. Therefore output $V_{out} = V_{o1} - V_{o2}$ is maximum negative for $f_c - \delta f$. This is shown in Fig. 2.14.

For the other frequencies of input, the output ($V_{out}$) is produced according to the characteristic shown in Fig.2.14. For example if input frequency tries to increase above $f_c$ then $V_{01}$ will be greater than $V_{o2}$ and net output $V_{out}$ will be positive. It is desirable that the characteristic shown in Fig.2.14 should be linear between $f_c - \delta f$ and $f_c + \delta f$, then only proper detection will take place. The linearity of the characteristic depends upon alignment of tuning circuit and coupling characteristics of the tuned coils.

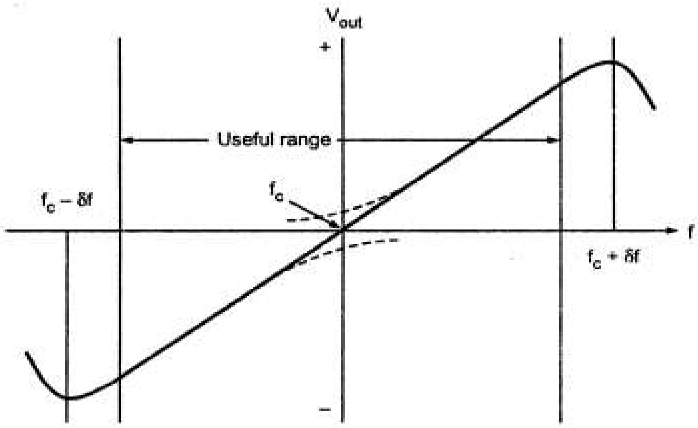



Fig.2.14 Characteristic of balanced slope detector, or 'S' curve

**Frequency Discriminator**

Fig. 2.15 shows the block diagram of a frequency discriminator which is based on the principle of slope detection.

- There are two slope circuits in Fig. 2.15. Their transfer functions are as follows:

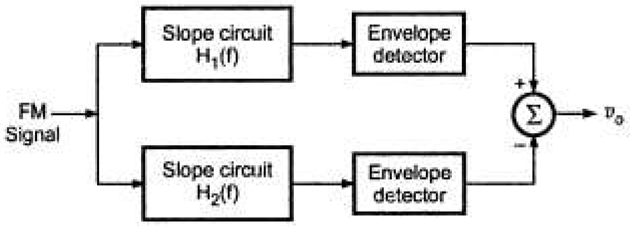



Fig. 2.15 frequency discriminator

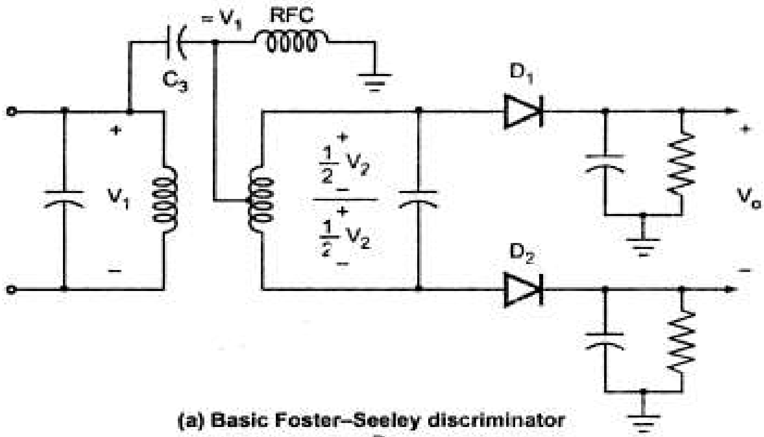
$H_1(f) = H_2(-f) = \begin{cases} j2\pi a\left[f - f_c + \frac{BW}{2}\right] \text{ for } f_c - \frac{BW}{2} \leq f \leq f_c + \frac{BW}{2} \end{cases}$

- When the signal is passed through slope circuits, its amplitude as well as frequency varies as per amplitude of modulating signal $e_m$(t).
- This signal is passed through enveloped detectors. It recovers amplitude variations.
- The outputs of two envelope detectors (one responds to frequency variation below $f_c$) are finally subtracted to give final detected output.

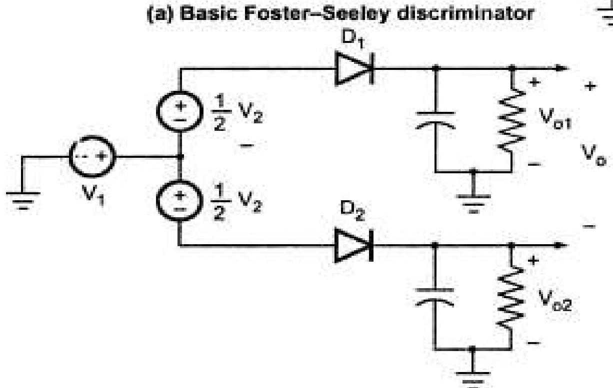
**2.7.2 Foster-Seeley Discriminator (Phase Discriminator)**

The phase shift between the primary and secondary voltages of the tuned transformer is a function of frequency. It can be shown that the secondary voltage lags primary voltage by $90^0$ at the carrier center frequency. The carrier frequency $(f_c)$ is the resonance (or tuned) frequency of the transformer. Foster-Seeley discriminator utilizes this principle for FM detection. Fig 2.16 (a) shows the circuit diagram of basic Foster-Seeley discriminator. In the figure observe that capacitor $C_3$ passes all the frequencies of FM. Thus the voltage $V_1$ is generated across RFC. RFC offers high impedance to frequencies of FM. The voltage $V_1$ thus appears across (RFC) center tap of secondary and ground also. The voltage of secondary is $V_2$ and equally divided across upper half and lower half of the secondary coil.

Fig 2.16 (b) shows the generator equivalent circuit of Foster-Seeley discriminator. In this figure observe that the voltage across diode $D_1$ is $V_{D1} = V_1+0.5V_2$ and that across $D_2$ is $V_{D2} = V_1-0.5V_2$.




(a) Basic Foster–Seeley discriminator




(b) Voltage generator equivalent circuit

Fig.2.16 **Foster-Seeley Discriminator**

The output of upper rectifier is $V_{01}$ and lower rectifier is $V_{02}$. The net output $V_0=V0_1-V_{02}$. Since $V_{01} \approx |V_{D1}|$ and $V_{02} \approx |V_{D2}|$ output V0 $\approx |V_{01}| - |V_{D1}|$.Thus the net output depends upon the difference between magnitudes of $V_{D1}$and $V_{D2}$ .

At the centre frequency both $V_{D1}$and $V_{D2}$ will be equal, since $V_2$ will have $90^0$ phase shift with $V_1$. Fig 2.17(a) shows how $V_{D1}$and $V_{D2}$ are generated from $V_1$ and $V_2$. It shows that $|V_{D1}| = |V_{D2}|$. Hence the net output of the discriminator will be zero. Now consider the situation when input frequency increases above $f_c$. Hence the phase shift between $V_1$ and $V_2$ reduces. Therefore $|V_{D1}|$ is greater than $|V_{D2}|$. This is shown by vector addition in Fig.2.17 (b). Hence the net output $V_0=V0_1-V_{02}$ will be positive. Thus the increase in frequency increases output voltage. Now consider the situation when frequency reduces below $f_c$. This makes $|V_{D1}|$ less than $|V_{D2}|$

This is shown is Fig.2.17. (c). Hence the output $V_0=V0_1-V_{02}$ will be negative. Thus the Foster-Seeley discriminator produces output depending upon the phase shift, The linearity of the output depends upon the linearity between frequency and induced phase shift. The characteristic of the Foster-Seeley discriminator (i.e. S-curve) is similar to that shown in Fig.2.14with more linearity in the operation.

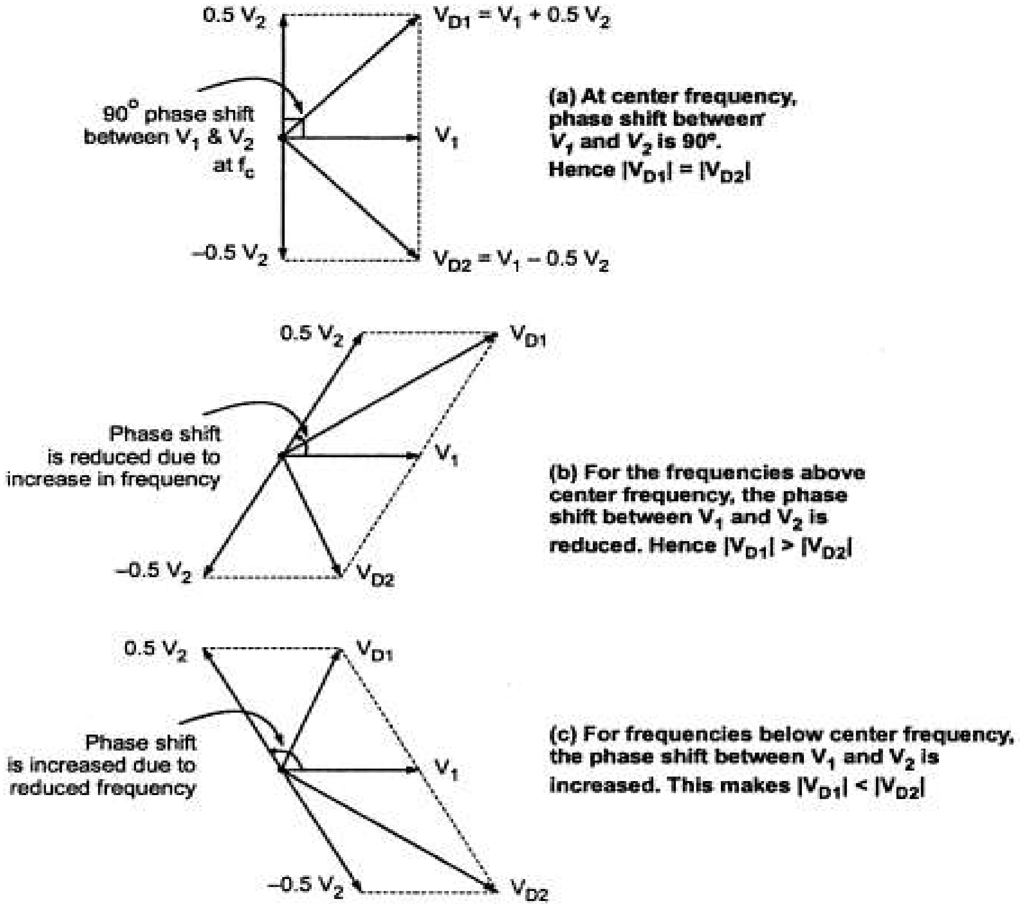



Fig.2.17 Phase shift between $V_1$ and $V_2$.

**2.7.3 Ratio Detector:**

Ratio detector can be obtained by slight modifications in the Fooster-Seeley discriminator. Fig.2.118 shows the circuit diagram of ratio detector .As shown in diagram the diode $D_2$ is reversed, and output is taken from different points.

The polarity of voltage in the lower capacitor is reversed, since connections of diode $D_2$ are reversed. Hence the voltages $V_{01}$ and $V_{02}$ across two capacitors add. We know that when $V_{01}$ increases, $V_{02}$ decreases and vice-versa as we have seen in Foster-Seeley circuit. Since $V'_0$ is sum of $V_{01}$ and $V_{02,}$ it remains constant.

From the circuit of Fig.2.17 we can write two equations for the output voltage $V_0$. The first equation will be

$V_0 = \frac{1}{2} V'_0 - V_{02}$

and $V_0 = -\frac{1}{2} V'_0 + V_{01}$

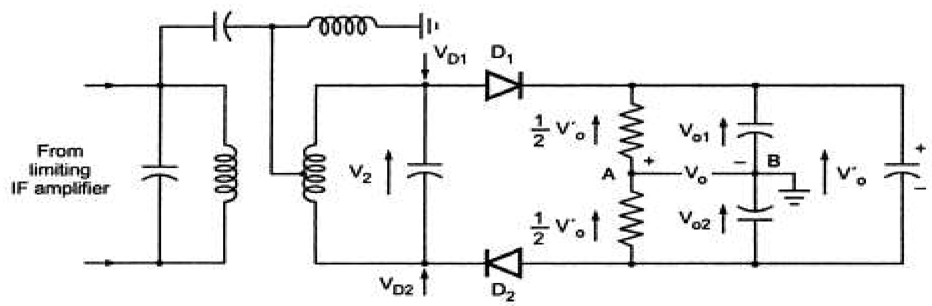



Fig. 2.18 Ratio detector circuit

Adding the above two equations,

$2V_0 = V_{01} - V_{02}$

Therefore $V_0 = \frac{1}{2} (V_{01} - V_{02})$

Since $V_{01} \approx |V_{D1}|$ and $V_{02} \approx |V_{D2}|$, above equation will be,

$V_0 = \frac{1}{2} (|V_{D1}| - |V_{D2}|)$

Here $V_{D1}$and $V_{D2}$ are obtained as discussed earlier in Foster – Seeley circuit. The above equation shows that the output of ratio detector is half compared to that of Foster – Seeley circuit. We have seen earlier that as frequency increases above $f_c$ , $|V_{D1}| > |V_{D2}|$,hence output $V_0$ is positive. Similarly if frequency decreases below $f_c$ , $|V_{D1}| < |V_{D2}|$,hence output $V_0$ is negative.

**Advantages:**

1) As compared to Foster – Seeley circuit, this circuit does not respond to amplitude variations.
2) The output is bipolar (i.e., positive as well as negative).

**Disadvantages:**

1) Ratio detector does not tolerate variation in signal strength over performed period.
2) It requires an ACC signal.

**2.8Transmission Bandwidth of FM:**

Theoretically the bandwidth of the FM wave is infinite. But practically it is calculated based on how many sidebands have significant amplitude.

The simplest method to calculate the bandwidth is as follows:

$BW = 2f_m \times n$ radians/sec (2.55)

Where n = number of significant sidebands [ $n= m_f$ ; $m_f >> 1$]

$BW = 2 f_m m_f$ (2.56)

$$m_f = \frac{\Delta f}{fm} \quad (2.57)$$

substitute equation (2.57) into (2.56)

$BW = 2(\Delta f)$ $H_Z$ (2.58)

Thus the approximate bandwidth of a wideband FM system is given as twice the frequency deviation. This approximation holds true for $m_f << 1$.

With increase in modulation index, the number of significant sidebands increase. Thus will increase the bandwidth.

$\Delta f << f_m$ (NBFM), i.e., $m_f << 1$

$BW = 2 f_m$

**Carson's rule (A rule of thumb):**

The second method to find the practical bandwidth is a rule of thumb (Carson's rule)

Carson's rule approximates the bandwidth necessary to transmit and angle modulated wave as twice the sum of the peak frequency deviation and the highest modulating signal frequency.

$BW = 2(\Delta f + f_m)$ $H_Z$ (2.59)

Where,

$\Delta f$ –Peak frequency deviation ($H_Z$)

$f_m$ – Modulating signal frequency ($H_Z$)

Carson's rule defines a bandwidth that includes approximately 98% of the total power in the modulated wave.

But it is an approximation; actually it does give a fairly accurate result if the modulation index is in excess of about 6.

$BW = 2(\Delta f + f_m)$

$= 2\Delta f + 2f_m$

$$f_m = \frac{\Delta f}{mf}$$

$$= 2\Delta f + 2\frac{\Delta f}{mf}$$

$$BW = 2\Delta f\left(1+\frac{1}{mf}\right) \text{ rad/sec}$$

**2.9Advantage, Disadvantage and Application of FM**

**Advantages of FM:**

i. Improved noise immunity
ii. Low power is required to transmit the signal
iii. Covers a large area with the same amount of transmitted power.
iv. Transmitted power remains constant
v. All the transmitted power is useful
vi. Adjacent channel interference is avoided due to guard bands.

**Disadvantages of FM:**

i. Very large bandwidth is required
ii. FM transmission and reception equipments are complex
iii. Compare to AM the area covered by FM is less.

**Application of FM:**

i. Radio broadcasting
ii. Sound broadcasting in TV
iii. Satellite communication
iv. Police wireless
v. Point to point communication
vi. Ambulances
vii. Taxicabs

**2.10Comparison of FM demodulator:**

| S.No. | Parameter | Balanced slope FM detector | Ratio FM detector | Phase FM discriminator |
|---|---|---|---|---|
| 1 | Linearity of output characteristics | Poor | Good | Very good |
| 2 | Output characteristics depends on | Primary and Secondary frequency relation | Primary and Secondary phase relation | Primary and Secondary phase relation |
| 3 | Amplitude limiting | Not provided inherently | Provided inherently | Not provided inherently |
| 4 | Timing procedure | Circuit as three tuned circuit at frequencies | Not critical | Not critical |
| 5 | Applications | Not used in practice | Narrowband FM receiver , TV receiver tuned section | Commercial FM radio receiver , Satellite receiver. |

**2.11Comparison between AM and FM**

| S.No. | Amplitude modulation | Frequency modulation |
|---|---|---|
| 1 | The modulation index is directly proportional to modulating voltage AM and inversely proportional to frequency | The modulation index is proportional to amplitude as well as phase |
| 2 | There are three components in AM. They are Carrier USB, LSB. | There are many frequency components in FM signal. |
| 3 | Power depends on the sideband | Total power remains constant |
| 4 | The bandwidth required is less compared to FM signal and is equal to $2f_{m.}$ B.W = $2f_m$ | Theoretically bandwidth of FM signal is infinite. |
| 5 | AM has poor fidelity due to narrow bandwidth | Since the bandwidth is large, fidelity is better |
| 6 | Adjacent channel interference is present | Adjacent channel interference is absent |
| 7 | Noise interference is more | Noise interference is less |

**2.12PLL-FM Demodulator**
**Basics and block diagram**

A phase-locked loop (PLL) is primarily used in tracking the phase and frequency of the carrier component of an incoming FM signal. PLL is also useful for synchronous demodulator of AM-SC (i.e., Amplitude Modulation with suppressed carrier) signal or signals with few cycle of pilot. Further, PLL is also useful for demodulating FM signals in presence of large noise and low signal power.

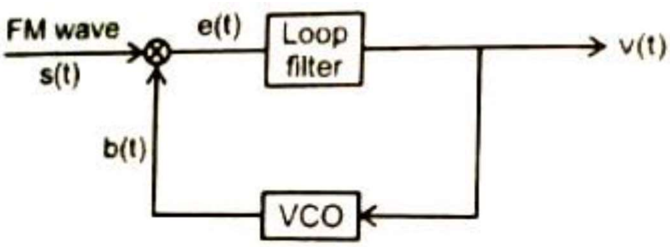



Fig.2.19 Block diagram of PLL

This means that, PLL is most suitable for used in space vehicle – to earth data links or where the loss along the transmission line or path is quite large. Recently, it has found application in commercial FM receivers. A Phase Locked Loop (PLL) is basically a negative feedback system. It consists of three major components. (VCO) corrected together in the form of a feedback loop. A VCO is a sine wave generator whose frequency is determined by the voltage applied to it from an external source.It means that any frequency modulator can work as a VCO.

**Working operation:**

The operation of a PLL is similar to any other feedback system. In any feedback system, the feedback signal tends to follow the input signal. If the signal feedback is not equal to the signal input signal, the error signal will change the value of the feedback signal until it is equal to the input signal. The different signal between *s (t)* and *b (t)* is called an error signal. A PLL operates on a similar principle except for the fact that the quantity feedback is not amplitude, but a generalized phase *φ (t).* The error signal or difference signal *e (t)* is utilised to adjust the VCO frequency in such a way that the instantaneous phase angle comes close to the angle of the incoming signal*s (t).* At this point, the two signals *s (t)* and *b (t)*are in synchronism and the PLL is locked to the signal *s(t).*

**PLL Demodulator**

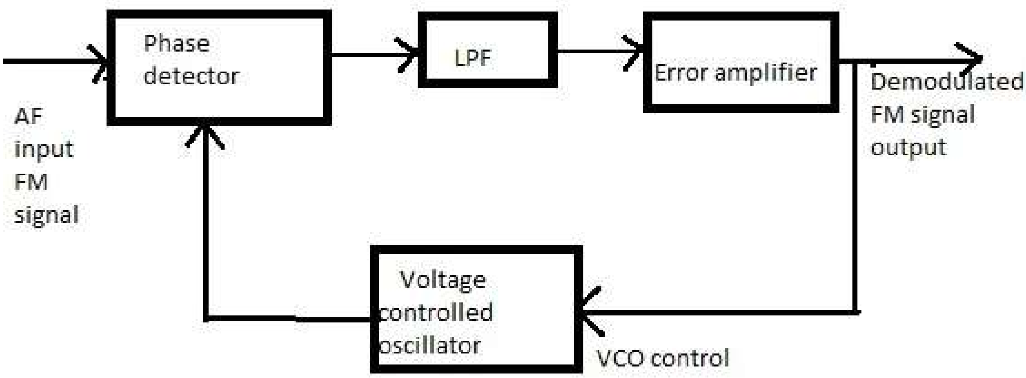



**Fig.2.20 shows the functional block diagram of FM demodulator using PLL**

The operation of FM demodulator using PLL briefly describe in following steps:

- The FM signal is applied to PLL
- As the PLL is locked the FM signal, the VCO starts tracking the instantaneous frequency in the input FM signal

- The error voltage produced at the output of the error amplifier is directly proportional to the frequency deviation
- Due to ac component of the error voltage represents the modulating signal
- Demodulated FM output is obtained at the output of the error amplifier

The following mathematical steps would help us understand how FM demodulator or detection can be performed by using PLL.

Here, we have assumed that the VCO is adjusted initially so that when the control voltage comes to zero, the following two conditions are satisfied:

i. The frequency of the VCO is precisely set at the unmodulated carrier frequency $f_c$ and
ii. The VCO output has a 90° phase-shift w.r.t the unmodulated carrier wave.

Let the input signal applied to the PLL be an FM wave. It is defined as

$s(t) = A \sin[\omega_c t + \varphi_1(t)]$ (2.60)

Where A is the unmodulated carrier amplitude and $\omega_c + 2\pi f_c$= angular carrier frequency

$\varphi_1(t)=2\pi k_f \int_0^t x(t)\,dt$ (2.61)

Where x(t) is the message or baseband signal or modulating signal.

$k_f$=frequency sensitivityof frequency modulator.

Let the VCO output be defined by

$b(t) = A_v \cos[\omega_c t + \varphi_2(t)]$ (2.62)

Where $A_v$= Amplitudeof VCO output when the control voltage applied to the VCO is denoted by V(t),then, we have

$\varphi_2(t)=2\pi k_v \int_0^t V(t)\,dt$ (2.63)

Here,$k_v$ isthe frequency sensitivity of VCO, measured in Hertz/volt.

It may be observed from eq.2.60 & 2.61 that the VCO output and the incoming signals are 90° out of phase, while the VCO frequency is absence of V(t) is precisely equal to the unmodulated frequency of the FM signal. The incoming FM have s(t) and the VCO output b(t) are applied to multiplier.

The output of the multiplier has the following components

I. A high frequency component represented by

$K_mAA_v\sin[2\omega_c t + \varphi_1(t)+ \varphi_2(t)]$ (2.64)

II. A low frequency component represented by

$K_mAA_v\sin[\varphi_1(t)-\varphi_2(t)]$ (2.65)

Where

$K_mA$= Multiplier Gain measured in per volt

The high frequency component can be eliminated by using a filter. Hence discarding the high frequency component, The effective input to the low pass filter(LPF) will be given by

$e(t)=K_mAA_v\sin[\varphi_1(t)-\varphi_2(t)]$

or

$e(t)=K_mAA_v\sin[\varphi_e(t)]$ (2.66)

where $\varphi_2(t)$ is the phase error and is expressed as

$\varphi_e(t)=\varphi_1(t)-\varphi_2(t)$

This means that

$\varphi_e(t)=\varphi_1(t)-2\pi k_v \int_0^t V(t)\,dt$ (2.67)

The loop filter operates on error signal e(t) to produce the output V(t).it is given by

$V(t)=\int_{-\infty}^{\infty} e(\tau)h(t-\tau)d\tau$ (2.68)

Where

H(t)=Impulse response of the low pass filter(LPF)

Using equations (2.66),(2.67)and(2.68),we get

$\varphi_e(t)=\varphi_1(t)-2\pi K_m k_v AA_v\int_0^t \int_{-\infty}^{\infty} \sin[\phi e(t)]h(t-\tau)d\tau dt$

or

$\varphi_e(t)=\varphi_1(t)-2\pi K_0\int_0^t \int_{-\infty}^{\infty} \sin[\phi e(\tau)]\, h(t-\tau)d\tau dt$ (2.69)

where

$k0= K_m\, k_v\, AA_v$ (2.70)

Now differentiatin both the sides of (2.69), we obtain

$$\frac{d\phi e(t)}{dt} = \frac{d\phi 1(t)}{dt} - 2\pi K_0 \int_0^t \int_{-\infty}^{\infty} \sin[\,\phi e(\tau)]\, h(t-\tau)\, d\tau \quad (2.71)$$

Here $K_0$ has the dimension of frequency.on basis of (2.71) equ, we can construct an equivalent model of PLL.it has been shown in fig 2.21.

In this model V(t) and e(t) are also included by utilising their relationship between them as given in equations

$e(t)=K_m A A_v \sin[\,\varphi_e(t)]$and

$V(t)=\int_{-\infty}^{\infty} e(\tau) h(t-\tau) d\tau$

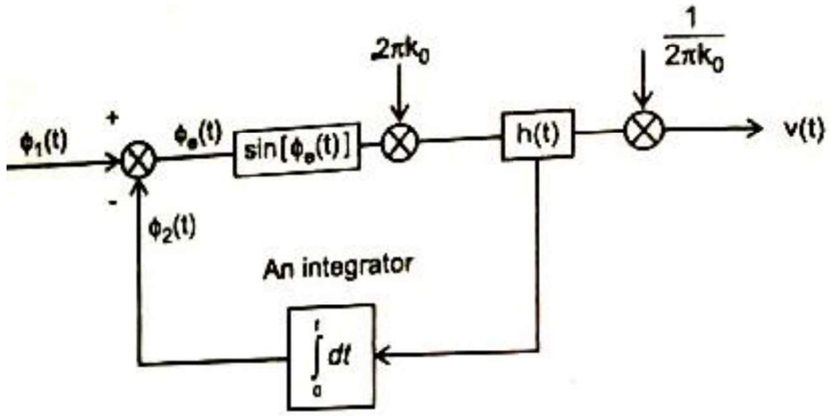



**Fig2.21 non-linear equivalent model of PLL**

When the phase error $\varphi_e(t)$ is zero, then the PLLis said to be phase-locked.hwen the phase error $\varphi_e(t)$ at all items is small compared to 1 radian, then we can approximate $\sin[\varphi_e(t)]$ as $\varphi_2(t)$, ie.,

$$\sin[\varphi_e(t)] \cong \varphi_e(t) \quad (2.72)$$

it is fairly accurate as long as $\varphi_e(t)$ is less than 0.5 radian. In this case,PLL is said to be near-lock condition and the sinusoidal non-linearly can be dis-regraded. The linearised model of PLL is valid under above-mentioned condition as shown in fig.2.22(a). In this model,phase error $\varphi_2(t)$ is related to the input phase $\varphi_1(t)$ by the integro-differential equation. It is expressed as

$$\frac{d\varphi e(t)}{dt} + 2\pi k0 \int_{-\infty}^{\infty} \phi e(\tau)\, h(t-\tau)\, d\tau = \frac{d\phi 1(t)}{dt} \quad (2.73)$$

Taking the fourier transform of both sides of eqn (2.73) we obtain

$\varphi_e(f) = \frac{1}{1 + k0\frac{H(f)}{jf}} \varphi_1(f)$ (2.74)

where $\varphi_e(f)$ and $\varphi_1(f)$ are the fourier transform of $\varphi_e(t)$ and $\varphi_1(t)$ respectively and H(f) is the fourier transform of impulse response h(t) and is known as the transfer function of the loop filter.

The quantity $\frac{k0H(f)}{jf}$ is open loop transfer function of PLL.

$L(f) = \frac{k0H(f)}{jf}$

(2.75)

Substituting (2.75) in(2.74),we obtain

$\varphi_e(f) = \frac{1}{1 + L(f)} \varphi_1(f)$ (2.76)

now, let us consider that for all values of frequency f inside the baseband signal.we make the magnitude of L(f) very large compared to unity.

Thus from Eq(2.74), we obtain

$\Phi_e(f) \rightarrow 0$ as $L(f) \gg 1$
(2.77)

Under above-mentioned condition, the phse of the VCO becomes asymptotically equal to the phase of the incoming wave and the phase back is thereby established.

From fig.2.26(a), we observe that V(f) is related to $\varphi_e(f)$ by

$V(f) = \frac{K0}{Kv} H(f)\varphi_e(f)$ (2.78)

Since, we have

$L(f) = k_0 \frac{H(f)}{jf}$ (2.79)

Eq(2.79),may be obtained by taking fourier transform of both sides of equation given below:

$V(t)=\int_{-\infty}^{\infty} \phi e(\tau) h(t-\tau) d\tau$

And using the approximate of equation,sin[$\varphi_e$(t)]≅$\varphi_e$(t).

Now substituting the value of H(f) from equation(2.79),in equation (2.78),we obtain

$V(f)=\frac{k0}{kv}\left\{\frac{jf}{k0}L(f)\right\}\varphi_e(f)=\frac{\mathbf{jf}}{\mathbf{kv}}\mathbf{L(f)}\varphi_e(f)$ (2.80)

Substituting the value of $\varphi_e$(f) from equation (2.80) in (2.79), we have

$V(f)=\frac{jf}{kv}L(f)\varphi_e(f)=\frac{jf}{kv}L(f)\frac{1}{1+L(f)}\varphi_1(f)$

$V(f)=(\frac{jf}{kv})(\frac{L(f)}{1+L(f)})\varphi_1(f)$ (2.81)

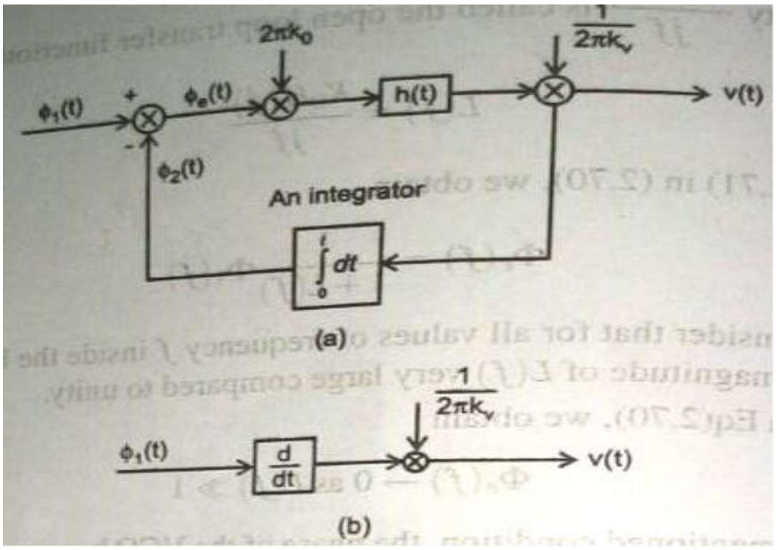



Fig.2.22 illustration of an equivalent PLL

If $|L(f) \gg 1|$ then equation (2.81) may be approximated as

$V(f)=\frac{jf}{kv}[1]\varphi_1(f)=\frac{jf}{kv}\varphi 1(f)$ (2.82)

The corresponding time-domain representation of(2.82) can be obtained by taking inverse fourier transform of both sides of equation (2.82). hence, we have

Inverse FT[V(f)]=Inverse FT$\left[\left(\frac{jf}{kv}\varphi 1(f)\right)\right]$ (2.83)

Or

$V(t)=\frac{1}{2\pi kv}\frac{d\phi 1(t)}{dt}$ (2.84)

**Conclusion**

here, it can be concluded that if the magnitude of L(f) is very large for all frequencies of interest,then PLL may be modelled as a differentiator with its output scaled by a factor $\frac{1}{2\pi kv}$.it has been illustrated in fig.2.22(b).

the simplified model of PLL as shown in fig.2.22(b) may be usd as an FM demodulator. This can be easily verified by substituting the value of $\varphi_1(t)$ from equation.

$\varphi_1(t)=2\pi k_f \int_0^t x(t)dt$ into equation (2.84)

∴ we have

$V(t)=\frac{1}{2\pi kv}\frac{d\phi 1(t)}{dt}=\frac{1}{2\pi kv}\frac{d}{dt}\left\{2\pi kf \int_0^t x(t)\,dt\right\}$

Or

$V(t)=\frac{2\pi kf}{2\pi kv}\frac{d}{dt}\left\{\int_0^t x(t)\,dt\right\}$

$V(t)=\frac{kf}{kv}x(t)$
(2.85)

Hence, we can say the output V(t) of PLL is approximately same except for a scaling factor $\frac{kf}{kv}$, as the original baseband or modulating signal x(t) and the frequency demodulation is performed

**TWO MARKS**

**1. What do you understand by narrowband FM?**

When the modulation index is less than 1, the angle modulated systems are called low index. The bandwidth requirement of low index systems is approximately twice of the modulating.

**2. Define frequency modulation.**

Frequency modulation is defined as the process by which the frequency of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating or message signal.

**3. Define modulation index of frequency modulation.**