UNIT I

FUNDAMENTALS OF RADIATION

Definition of antenna parameters – Gain, Directivity, Effective aperture, Radiation Resistance, Band width, Beam width, Input Impedance. Matching – Baluns, Polarization mismatch, Antenna noise temperature, Radiation from oscillating dipole, Half wave dipole. Folded dipole, Yagi array.

1.1 Definition of antenna parameters:

Antenna is a transition device or a transducer between a guided wave and a free space wave or vice versa. Antenna is also said to be an impedance transforming device

Antenna characteristics

Irrespective of antenna type and applications, all the antennas possess certain basic properties (characteristics) The important properties are:

1) Radiation pattern
    a) Field radiation pattern
    b) Power Radiation pattern
2) Beam solid Angle (Beam Width)
3) Radiation intensity
4) Directive gain and Directivity
5) Power gain
6) Input impedance
7) Polarization
8) Bandwidth
9) Effective Aperture and Effective Length
10) Antenna temperature

1.2 DIRECTIVITY D AND GAIN G

The directivity D and the gain G are probably the most important parameters of an antenna.

The directivity of an antenna is equal to the ratio of the maximum power density $P(\theta,\phi)_{max}\left(\frac{watts}{m^2}\right)$ to its average value over a sphere as observed in the far field of an antenna. Thus

$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}} \qquad \textbf{\textit{Directivity from pattern}}$$

The directivity is a dimensionless ratio ≥ 1.

The average power density over a sphere is given by

$$P(\theta,\phi)_{av} = \frac{1}{4\pi}\int_{\phi=0}^{\phi=2\pi}\int_{\theta=0}^{\theta=\pi} P(\theta,\phi)sin\theta d\theta d\phi$$

$$= \frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)d\Omega \quad (W\ Sr^{-1})$$

There fore , the directivity

$$D = \frac{P(\theta,\phi)_{max}}{\frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)d\Omega} = \frac{1}{\frac{1}{4\pi}\iint_{4\pi}\left(\frac{P(\theta,\phi)}{P(\theta,\phi)_{max}}\right)d\Omega}$$

And

$$D = \frac{4\pi}{\frac{1}{4\pi}\iint_{4\pi} P_n(\theta,\phi)d\Omega} = \frac{4\pi}{\Omega_A} \qquad \textbf{\textit{Directivity from beam area}}\ \boldsymbol{\Omega_A}$$

Where $P_n(\theta,\phi)d\Omega = \left(\frac{P(\theta,\phi)}{P(\theta,\phi)_{max}}\right) = Normalised\ power\ pattern$

Thus , the directivity is the ratio of the area of a sphere ($4\pi\ Sr$) to the beam area Ω_A of the antenna . The smaller the beam area , the larger the directivity D. For an antenna that radiates over only a half a sphere the beam area $\Omega_A = 2\pi$ and the directivity is

$$D = \frac{4\pi}{2\pi} = 2 \qquad (= 3.01\ dBi)$$

Where dBi = decibels over isotropic

Note that the idealized *isotropic antenna* ($\Omega_A = 4\pi\ Sr$) has the lowest possible directivity D=1. All actual antennas have directivities greater than 1 ($D > 1$). The simple short dipole has a beam area $\Omega_A = 2.67\pi\ Sr$ and directivity D = 1.5 (=1.76dBi).

1.3 GAIN

The gain G of an antenna is an actual or realized quantity which is less than the directivity D due to ohmic losses in the antenna. In transmitting , these losses involve power fed to the antenna which is not radiated but heats the antenna structure. A mismatch in feeding the antenna can also reduce the gain. The ratio of the gain to the directivity is the *antenna efficiency factor*. Thus ,

$$G = kD$$

Where k = efficiency factor ($\leq k \leq 1$), dimensionless.

In many well designed antennas, k may be close to unity. In practice , G is always less than D , with D its maximum idealized value.

Gain can be measured by comparing the maximum power density of the Antenna Under Test (AUT) with reference antenna of known gain, such as a short dipole. Thus ,

$$Gain = G = \frac{P_{\max}\ (AUT)}{P_{\max}\ (ref.ant)} \times G(ref.ant)$$

If the half-power beamwidths of an antenna are known, its directivity

$$D = \frac{41253^0}{\theta_{HP}^0 \phi_{HP}^0}$$

Where 41253^0 = number of square degrees in sphere = $4\pi \left(\frac{180}{n}\right)^2$ square degrees

θ_{HP}^0 = half – power beam width in one principal plane

ϕ_{HP}^0 = half – power beam width in other principal plane

If neglects minor lobes , a better approximation is a

$$D = \frac{40000^0}{\theta_{HP}^0 \phi_{HP}^0} \qquad \textbf{approximate directivity}$$

If the antenna has a main half power beam width (HPBW) = 20^0in both principal planes, its directivity

$$D = \frac{40000^0}{400^0} = 100 \; or \; 20dBi$$

Which means that the antenna radiates 100 times the power in the direction of the main beam as a non directional , isotropic antenna.

The directivity –beamwidth product 40000^0 is a rough approximation . for certain types of antennas other values may be more accurate . If an antenna has a main lobe with both half power beam width (HPBW) = 20^0, its directivity is approximately

$$D = \frac{4\pi}{\Omega_A} = \frac{41253^0}{\theta_{HP}^0 \phi_{HP}^0} = \frac{41253^0}{20^0 \times 20^0}$$

$$\cong 103 \cong 20dBi \; (dB \; above \; isotropic)$$

Which means that the antenna radiates a power in the direction of the main lobe maximum which is about 100 times as much as would be radiated by a nondirectional (isotropic) antenna for the same power input.

1.4 EFFECTIVE APERTURE(EFFECTIVE AREA)

The effective aperture of an antenna is the area over which the antenna collects the energy from the incident wave and delivers it to the receiver load.

If the power density in the wave incident from the (θ , ϕ) direction is $'S'$ at the antenna and $P_r(\theta , \phi)$ is the power delivered to the load connected to the antenna , then the effectivee aperture A_e is defined as ,

$$A_e(\theta , \phi) = \frac{P_r(\theta , \phi)}{S} \qquad m^2$$

Refering to the equivalent circuit of the receiving antenna. The power delivered to the load Z_L, connected to the antenna terminals is

$$P_r = \frac{1}{2} |I_R|^2 \, R_L$$

Where , R_L = Real part of the load impedance,

$Z_L = R_L + jX_L$

Let $Z_a = R_a + j\,X_a$ be the antenna impedance.

The real part of the antenna impedance can be divided into two parts. $i.e., R_a = R_{rad} + R_{loss}$

R_{rad} = the radiation resistance

R_{loss} = the loss resistance.

For conjugate match $R_L = R_a$, $or\ Z_L = Z_a^*$, $X_L = -X_a$

For a conjugate match , the current through all three resistances is ,

$$I = \frac{V_a}{R_L + R_{rad} + R_{loss}} = \frac{V_a}{2R_L}$$

and the three powers are computed using the formulae,

$$P_r = \frac{1}{2}|I|^2 R_L = \frac{1}{2}\frac{|V_a|^2}{(2R_L)^2} R_L = \frac{|V_a|^2}{8R_L}$$

$$P_{scat} = \frac{1}{2}|I|^2 R_{rad} = \frac{1}{2}\frac{|V_a|^2}{(2R_L)^2} R_{rad} = \frac{|V_a|^2}{8{R_L}^2} R_{rad}$$

$$P_{loss} = \frac{1}{2}|I|^2 R_{loss} = \frac{1}{2}\frac{|V_a|^2}{(2R_L)^2} R_{loss} = \frac{|V_a|^2}{8{R_L}^2} R_{loss}$$

Where P_r = the power delivered to the receiver load

P_{scat} = the power dissipated in the antenna

P_{loss} = the power scattered

The total power collected by the antenna is sum the three powers.

$$P_C = P_r + P_{scat} + P_{loss}$$

Collective Aperture

If the power density in the incident wave is $'S'$ then the effective collecting aperture A_c $of\ the\ antenna\ is\ the\ equivalent\ area\ from\ which\ the\ power\ is\ collected$

$$A_c(\theta, \phi) = \frac{P_C(\theta, \phi)}{S} \quad m^2$$

This area is split into three parts. The collective aperture is nothing but summation of effective aperture, loss aperture and scattering aperture. It is denoted by A_c.

A_e = the effective aperture corresponding to the power delivered to the receiver load.

A_{loss} = the loss aperture corresponding to the power loss in the antenna

A_s = the scattering aperture corresponding to the power re-radiated by the antenna.

These are given by

$$A_e(\theta , \phi) = \frac{P_r(\theta , \phi)}{S} = \frac{|V_a(\theta , \phi)|^2}{8R_L S} \quad m^2$$

$$A_{loss}(\theta , \phi) = \frac{P_{loss}(\theta , \phi)}{S} = \frac{|V_a(\theta , \phi)|^2}{8{R_L}^2 S} R_{loss} \quad m^2$$

$$A_s(\theta , \phi) = \frac{P_{scat}(\theta , \phi)}{S} = \frac{|V_a(\theta , \phi)|^2}{8{R_L}^2 S} R_{rad} \quad m^2$$

Consider an antenna radiating into free space . Let P_{t1} be the total power input to the antenna. The power density of the antenna is ,

$$S_o = \frac{P_{t1}}{4\pi R^2}$$

If the gain of the antenna is $G_1(\theta , \phi)$, the power density will be larger by that amount in the (θ , ϕ)direction .

$$S = \frac{P_{t1} G_1(\theta , \phi)}{4\pi R^2}$$

Let the transmitting antenna as antenna 1, receiving antenna as antenna 2. Let the effective aperture of this antenna be A_{e2} . Then the power delivered to the matched load connected to the receiving antenna is

$$A_{e2} = \frac{P}{S}$$

$$A_{e2} = \frac{P_{r2} 4\pi R^2}{P_{t1} G_1(\theta , \phi)}$$

$$P_{r2} = \frac{P_{t1} G_1(\theta , \phi) A_{e2}}{4\pi R^2}$$

The power transfer ratio is

$$\frac{P_{r2}}{P_{t1}} = \frac{G_1 A_{e2}}{4\pi R^2}$$

If we interchange the positions of the transmitter and receiver and maintain the conjugate-match at both the antenna ports , the power transfer ratio will be ,

$$\frac{P_{r1}}{P_{t1}} = \frac{G_2 A_{e1}}{4\pi R^2}$$

The ports are assumed to be matched , the power transfer ratio are the same from the reciprocity theorem, we can write

$$\frac{G_1 A_{e2}}{4\pi R^2} = \frac{G_2 A_{e1}}{4\pi R^2}$$

(or) $$\frac{G_1}{A_{e1}} = \frac{G_2}{A_{e2}}$$

The effective aperture area of the Hertzian dipole as,

$$A_e = \frac{|V_a|^2 \times 2\eta}{8R_{rad}\,|E|^2}$$

Substitute V_a & R_{rad} values $V_a = E\ dl$ & $R_{rad} = \frac{\pi\eta}{D}\left(\frac{dl}{\lambda}\right)^2$

Then we get ,

$$\frac{D}{A_e} = \frac{4\pi}{\lambda^2}$$

If the radiation efficiency is unity , we replace D by G and write,

$$\frac{G}{A_e} = \frac{4\pi}{\lambda^2}$$

Thus the ratio of the gain and effective aperture is equal to $\frac{4\pi}{\lambda^2}$ for any antenna.

1.5 RADIATION RESISTANCE

The antenna is a ardiating device in which the power (energy per unit time) is radiated into space in the form of electromagnetic waves. Hence there must be power dissipation which may be expressed as

$$W = I^2 R$$

If the power W can be divided by square of the current I^2, at the point where it is fed to the antenna, a fictitous resistance called as radiation resistance is obtained.

$$R_r = \frac{W}{I^2}$$

The radiation resistance represents the radiation by the antenna. It gives the relation between total energy radiated from a transmitting antenna and current flowing in the antenna.

The radiation resistance R_r is defined as the fictitious resistance which , when substituted in series with the antenna, will consume the same power as is actually radiated by antenna.

The energy supplied to an antenna is dissipated

1. In the form of electromgnetic waves and
2. As ohic losses in the antenna wire and near by dielectries i.e., insulators, ground and other surrounding objects.

However all the power fed to the antenna is not radiated into space but a vert small fraction of it is dissipated in the form of hest depending upon the loss resistance (R_l) of the antenna.

$$W_i = W_r + W_l$$

Where W_i = input power to the antenna

W_r = power radiated into space $(I^2 R_r)$

W_l = power dissipated in the form of heat $(I^2 R_l)$

I = r.m.s value of the current flowing through it

$W_i = I^2(R_r + R_l)$

The value of radiation resistance depends on

i) Configuration of antenna.
ii)The point where radiation resistance is considered .
iii) Location of antenna w.r.to ground and other objects.
iv) Ratio of length to diameter of the conductor used.
v)Corona discharge – aluminous discharge round the surface of antenna due to ionization of air.

The presence of ground changes the radiation resistance because the electromagnetic waves radiated from antenna are reflected from ground which (the reflected waves) induced current in the antenna while passing through it.

The magnitude and phase of the induced current depands on the position of antenna with respect to ground i.e., at height above ground. Since the reflected waves are weaker in strength so the fluctuation in radiation resistance decreases as the height is increased.

The knowledge of a resonant antenna is impartant because it acts as load for the transmitter.

Example

i) Radiation resistance of a small current element can be given as

$$R_r = 80\,\pi^2 \left(\frac{dl}{\lambda}\right)^2 \; ohms$$

Where dl = length of the current element

λ = wavelength

ii) Radiation resistance of a half wave dipole antenna is given by

$$R_r = 73.14\; ohms.$$

1.6 BANDWIDTH

- Bandwidth is defined as the width or range of frequency over which the antenna maintains certain required characteristics like gain, pattern, polarization and impedance, etc.
- In general, the antenna bandwidth mainly depends on its impedance and pattern.
- At low frequency of relatively small dimension ($\frac{\lambda}{2}$ *or less*) the bandwidth is usually determined my impedance variation because the pattern characteristic is insensitive to frequency. Under such condition, bandwidth of the antenna is inversely proportional to 'Q' factor of antenna.
- Hence, Bandwidth can be expressed mathematically as

$$\text{Band width (BW)} = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

$$\Delta f = f_2 - f_1 = \frac{f_0}{Q}\ \text{H}_z \qquad (since\ \omega = 2\pi f)$$

$where\ f_0 =$ the centre frequency (or)resonant frequency,

$Q =$ "Q" $factor\ of\ antenna,$

$$Q = 2\pi \frac{Total\ energy\ stored\ by\ antenna}{Energy\ radiated\ per\ cycle}$$

For lower 'Q' Antennas, BW is very high and viceversa.

1.7 ANTENNA BEAM- WIDTH

Antenna Beam width is a measure of directivity of an antenna. Antenna beamwidth is an angular width in degrees, measured on the Major lobe of its radiation pattern between points where the radiated power has fallen to half of its maximum value . This is called as beamwidth between half power points or half power beamwidth (HPBW) because the power at half power points is just half of its maximum value.

Half power beam width is also known as 3dB beam width because at half power points, the power is 3dB down of the maximum power value of the major lobe.

Further at these half power points, the field intensity (i.e., voltage) equals $1/\sqrt{2}$ or 0.707 times its maximum value or 3dB down from maximum value.

Consider the radiation pattern as shown in fig. P_1 and P_2 are the half power points because the power at P_1 and P_2 is half of that at point M.

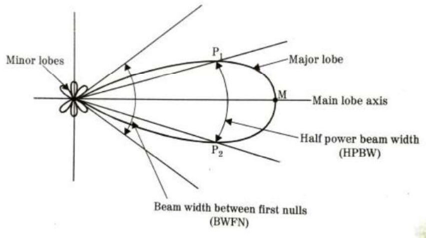


Fig1.1 : Half power beam width

The angular width between first nulls or first side lobes is known as beam wldth between first nulls. It is denoted by BWFN. It is the beam width that is -10dB down from the pattern maximum.

The relationship between Directivity and Beam soild angle or Beam area B is

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B}$$

The beam area is the product of beam widths in horizondal and vertical planes i.e., E planes and H planes.

$$B \approx \theta_E \times \theta_H$$

$$D = \frac{4\pi}{\theta_E \theta_H}$$

Where θ_E and θ_H are in radians

$$D = \frac{41{,}253}{\theta_E \times \theta_H}$$

The factors affecting the beam width of an antenna are

i) The shape of the radiation pattern ii) Dimensions iii) Wavelength

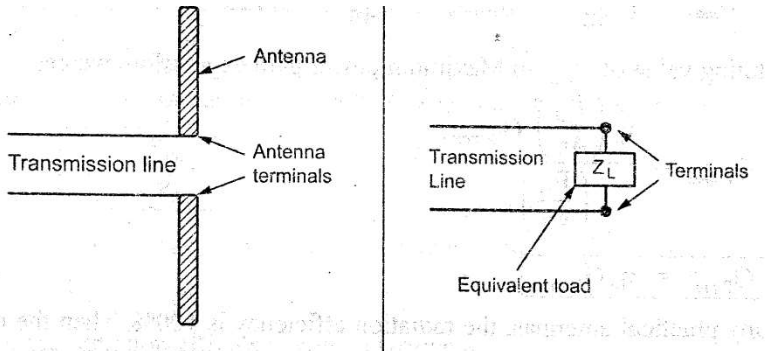
For a direction finding applications, a narrow beam is desirable (less Beam width) and accurancy of direction finding is inversly proportional to beam width .

$$i.e., D \propto \frac{1}{Beam\ width}$$

Hence narrower the beam width , the higher the gain or directivity.

1.8 INPUT IMPEDANCE

- It is the impedance at the point where the transmission line carrying R.F power from the transmitter is connected. It is also called as antenna input impedance, feed point impedance. Driving point impedance (or) Terminal impedance.



(a) Transmission line with Antenna as load *(b) Transmission line with equivalent load Z_L*

Fig . 1.2

- Antenna input impedance is important because it decides the maximum available power from transmitter to the Antenna (or) to extract maximum amount of received energy from the Antenna.
- Actual impedance of an antenna is divided into
  1. Self impedance
  2. Mutual impedance

(i) **Self impedance**

If the Antenna is lossless and isolated (*i,e* away from ground),then the antenna terminal impedance (Z_L) is the same as the self-impedance (Z_{11})of the antenna.

$$Z_{11} = R_{11} + jX_{11}$$

Z_{11} = Self-impedance

R_{11} = Self resistance (or) radiation resistance

X_{11} = Self -reactance

For half-wave centre for antenna, the self impedance is calculated as

$$Z_{11} = R_{11} + jX_{11} = 73+j42.45\Omega$$

- Self impedance of an antenna is defined as its input impedance when all other antenna are completely removed from it .
- Self impedance of an antenna is always positive and its value is same for both transmission and reception.
- In order to make the antenna resonant (i.e, X_{11} = 0), the antenna length can be shortened by small amount.
- By doing this, there is a slight reduction in radiation resistance (or) self-resistance. For example, for half wavelength centre fed antenna, the self resistance. Is calculated to be R_{11} =70 Ω instead of 73 Ω when X_{11}=0.

(ii) Mutual impedance

When two antennas are kept nearby, mutual impedance comes into effect. Let us now consider two coupled antennas and these two antennas are separated by a fraction of wavelength and they are parallel to each other as shown in Fig.

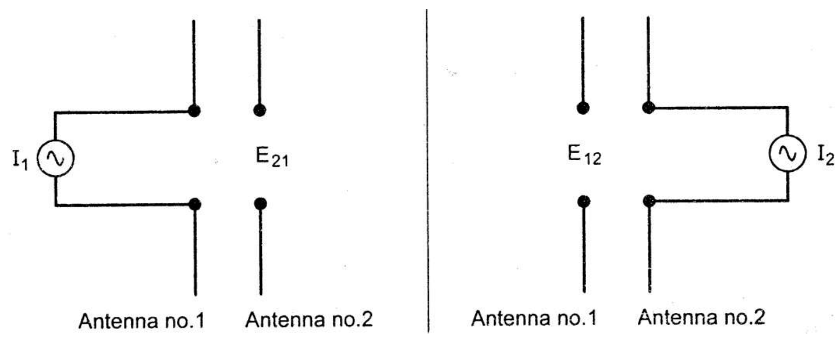


Fig 1.3

- Let a current I_1 in the antenna no.1 induces a voltage E_{21} at the open terminals of antenna no.2, then the ratio of the two is mutual impedance Z_{21}.

  i.e $Z_{21} = -\frac{E_{21}}{I_1}$
- If the generator is shifted to antenna no.2 and a current I_2 of antenna no.2 is inducing a voltage E_{12}at the open terminals of antenna no.1 then

  Mutual impedance$Z_{21} = -\frac{E_{21}}{I_2}$
- By reciprocity theorem, the two mutual impedances are equal,

$$Z_{21} = -\frac{E_{21}}{I_1} = Z_{12} = -\frac{E_{21}}{I2} = Z_m$$

or

$$Z_m = \frac{E_{21}}{I_1} = \frac{E_{21}}{I_2}$$

The mutual impedance, depends on the

(i) Magnitude of induced current,
(ii) Phase relationship between induced and original current and
(iii) Turning conditions of second antenna (or nearby antenna).

- Variation in induced current causes phase of the total current to vary with respect to applied voltage. This means the mutual impedance is a complex quantity having resistive and reactive components.
- Mutual impedance affects the gain of an antenna as it determines the amount of current which will flow for a given amount of power supplied.

1.9 BALUNS

The term balun is an abbreviation of the wors balance and unbalance. It is a device that connects a balanced two-conductor line to an unbalanced coaxial line.

Since baulns add complexity and expensse to a system , let us conside the consequences of not using one. For example , in fig 1.a a horizondal dipole antenna is center fed directly from a coaxial cable. The inner conductor feeds the left half of the dipole while the outer conductor feeds the right half. However , current I_3 will flow down the outside of the outer conductor making it part of the radiating system. The result is a mixture of horizontal polarization as in fig 1.b and vertical polarization as in fig 1.c. Thus , the pattern is not that of a horizontal dipole. For this , a balun is required. However in receiving situations where there is an adequate signal –to –noise (SNR) a balun may not be needed.

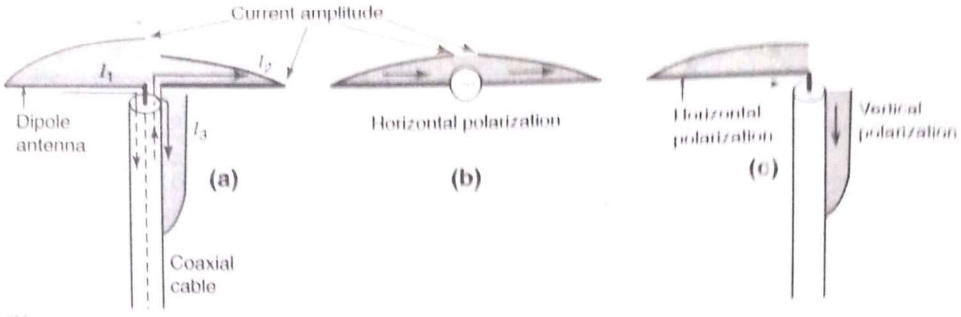


Fig :1.4 A horizontal dipole fed directly from a coaxial line as in (a) produces a mixture of balanced horizontal polarization (b) and of vertical and horizontal polaraization(c)

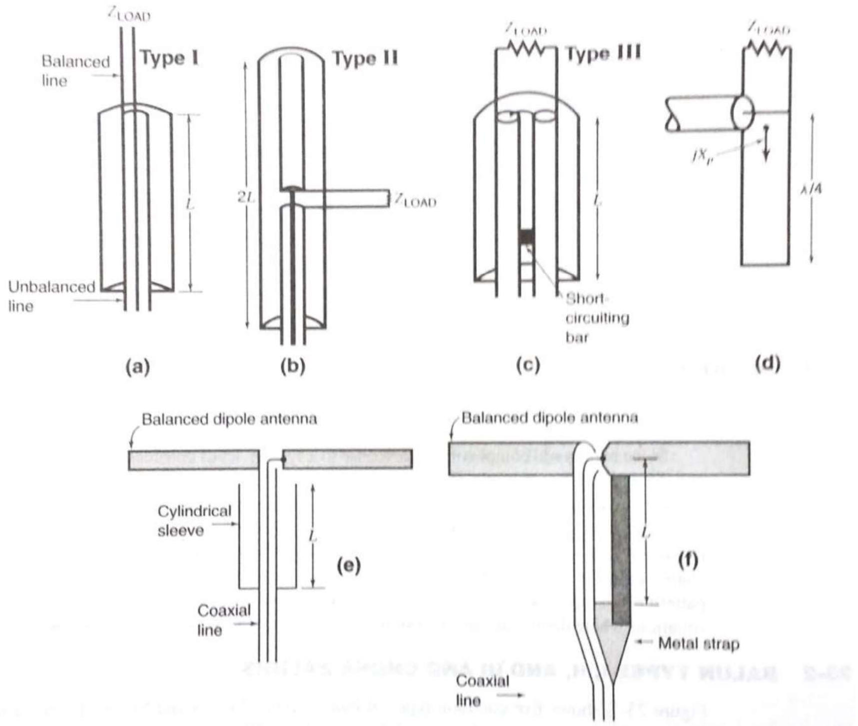


Fig1.5 : (a) Type I balun or "bazooka" (b) Type II balun, (c) Type III balun , (d) Type III balun equivalent circuit, (e) Type I balun with dipole antenna and (f)dipole antenna with Type III balun minus sleeve.

BALUN TYPES I , II, III AND CHOKE BALUNS

Figure shows five common types of baluns. The baluns are Types I ,II and III. Type I has a $\lambda/4$ sleeve which prasents an infinite impedance at the top . Type II has two Type I's in series providing more bandwidth and load balance at all frequencies. Type III is a more compact form. The inner conductors form a two conductor $\lambda/4$ line shorted at the base and presenting an infinite impedence at the top. It also features a sliding short-circuit bar for frequency adjustment. Fig 1.5.e has a Type I balun with dipole. Fig 1.5.f has a dipole fed by a Type III balun minus shielding cavity. The length L of all these baluns is about $\lambda/4$ at the center frequency. In (e) and (f) a reactive impedance $Z = \pm jZ_0 tan\beta L$ appears in parallel with the dipole, where Z_0 = characteristic impedance of the balun line of length L.

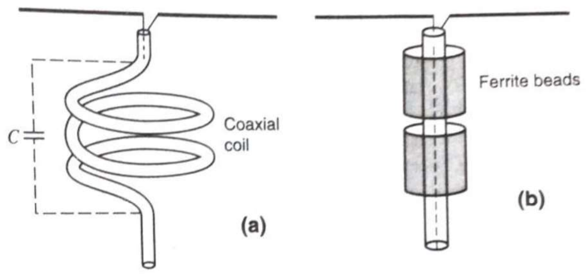


Fig1.6 : Two types of choke balun

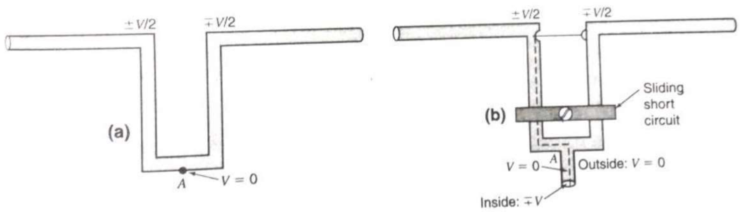


Fig1.7 : Basic structure of Type III balun and (b)the balun with coaxial line connected

Two more baluns are shown in fig. These are choke types. The one in Fig 1.6.a has the coaxial cable would into a coil producing a high impedance on the outside of the coil. The coil and its capacitance C form a parallel LC circuit that should resonate at the operating frequency.

A ferrite – band choke is shown in fig 1.6.b with cylindrical ferrite beads placed on the outside of the coaxial cable. With good – quality ferrite beads large bandwidth may be obtained (an octave or more)

The Type III balun works on the principle that the voltage at point A in Fig 1.7.a is zero. Therefore a cable leaving at this point has zero voltage on its outside. Thus ,with the coaxial line connected as in Fig 1.7.b balance is maintained. To facilitate obtaining an infinite impedance across the terminals at the dipole a sliding short may be used as suggested .

As an example, consider a coaxial cable connected to a half-wave dipole antenna. Here , a coaxial cable is connected to a dipole antenna. For a dipole antenna to operate properly, the currents on both arms of the dipole should be equal in magnitude. When a coaxial cable is connected directly to a dipole antenna the currents will not necessatily be equal.

When coaxial cable is connected to the dipole, the current on the center conductor , (labeled IA) has no where else to go , so must flow along the dipole arm that is connected to it.

However , the current that travels along the inner side of the outer conductor (IB) has two options: it can travel down the dipole antena, or down the reverse (outer) side of the outer conductor of the coaxial cable (labeled IC in fig)

Ideally , the current IC should be zero. In that case, the current along the dipole arm connected to tthr outer conductor of the coaxial cable will be equal to the current on the other dipole arm – a desirable antenna charactereistic. Because the dipole wants equal or balanced currents along its arms, it is the balanced section. As the coaxial cable does not necessarily give this, this is the unbalanced section. i.e., some of the current may travel down the outside of the outer conductor , leadinhg to unbalanced operation.

Some baluns provide impedance transformation in addition to conversion between balanced and unbalanced signal modes; others provide no impedance transformation. For 1:1 baluns (no impedance transformation), the input and output are usually both 50 ohms or 75 ohms. The most common impedance – transformation ratio is 1:4 (alternatively 4:1) . some baluns provide other impedance –transformation ratios, such as 1:9 (and 9:1), 1:10(and 10:1), or 1:16 (and 16:1).

Impedance – transformer baluns having a 1:4 ratio are used between systems with impedances of 50 or 75 ohms (unbalanced) and 200 or 300 ohms (balanced). Most television and FM broadcast receivers are designed for 300 ohm balanced systes, while coaxial cable have characteristic impedances of 50 or 75 ohms. Impedance-transformer baluns with larger ratios are used to match high impedance balanced antennas to low – impedance unbalanced wireless receivers, transmitters, or tranceivers.

In order obtain better efficiency, a balun must be used with loads whose impedances present little or no reactance. Such impedances are callec “purely resistive”. Gererally , well – designed communication antennas present purely resistive loads of 50, 75, or 300 ohms, althrough a few antennas have higher resistive impedances.

1.10 POLARIZATION MISMATCH FOR ANTENNAS

In general, in a given direction, an antenna will radiate an electric field with an E_θ and an E_ϕ component that are not in phase. Thus let the far-zone radiated field be

$$E_\theta = E_0 \frac{e^{-jk_0 r}}{4\pi r} \qquad E_\phi = \tau\, e^{j\beta} E_0 \frac{e^{-jk_0 r}}{4\pi r}$$

in a given direction, where τ and β are real. Thus $E_\phi = \tau e^{j\beta} E_\theta$. In the time domain the fields are

$$E_\theta = \frac{E_0}{4\pi r}\cos(\omega t - k_0 r)$$

$$E_\phi = \frac{\tau E_0}{4\pi r}\cos(\omega t + \beta - k_0 r)$$

if we assume that E_0 is real. Let$k_0 r - \omega t = \alpha$, then

$$E_\theta = \frac{E_0}{4\pi r}\cos\alpha$$

$$E_\phi = \frac{\tau E_0}{4\pi r}(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

To find the resultant total-field magnitude we eliminate the time as follows:

$$\frac{4\pi r E_\theta}{E_0} = \cos\alpha$$

$$1 - \left(\frac{4\pi r E_\theta}{E_0}\right)^2 = \sin^2\alpha$$

From the expression for E_ϕ we can write

$$\left(\frac{4\pi r E_\phi}{\tau E_0} - \cos\beta\,\frac{4\pi r E_\theta}{E_0}\right)^2 = \sin^2\beta\left[1 - \left(\frac{4\pi r E_\theta}{E_0}\right)^2\right]$$

Which can be also expressed as

$$\left(\frac{4\pi r E_\phi}{\tau E_0}\right)^2 + \left(\frac{4\pi r E_\theta}{E_0}\right)^2 - 2\cos\beta\,\frac{(4\pi r)^2 E_\theta E_\phi}{\tau E_0^2} = \sin^2\beta$$

$$\left(\frac{E_\phi}{\tau}\right)^2 + E_\theta^2 - \frac{2\cos\beta}{\tau} E_\theta E_\phi = \frac{E_0^2 \sin^2\beta}{(4\pi r)^2}$$

This is the equation of an ellipse. At a given point in space the resultant field vector traces out an ellipse, once per period in time. If the direction of rotation is clockwise, looking in the direction of propagation, the field is said to be *positive* or *right-elliptical polarized.* If the direction of rotation is counter clockwise the field is *negative* or *left-elliptical polarized.* If $\tau = 1\ and\ \beta = \pm\pi/2$

Then the above Eqn reduces to

$$E_\phi^2 + E_\theta^2 = \frac{E_0^2}{(4\pi r)^2}$$

which is the equation of a circle. For this case the field is circularly polarized, as is illustrated in Fig..

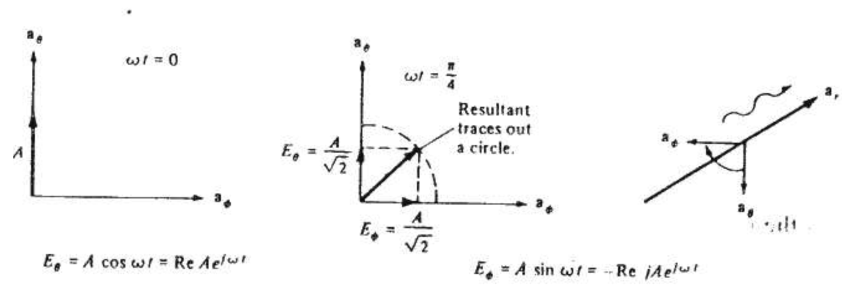


Fig1.8 : Positive or right-circular polarized field. Rotation is from a_θ into a_ϕ. If E $E_\theta = Re\ Ae^{j\omega t}\ and\ E_\phi = Re\ Ae^{j\omega t}$ the field is left- or negative-circular polarized.

It is convenient to express the far-zone field radiated by an antenna relative to that which a unit current element would radiate. Thus let the radiated field be

$$\mathbf{E} = \frac{jZ_0 k_0 I_{in}}{4\pi r}\,\mathbf{h}\,e^{-jk_0 r}$$

where I_{in} is the input current to the antenna and equals VY_{in} while $h = h_\theta\, a_\theta + h_\phi a_\phi$ is a complex vector called the *effective complex length of the antenna.* [Compare the above Eqn with

$$E_\theta = \frac{jZ_0 k_0\, e^{-jk_0 r}}{4\pi r}(I_0\,\Delta l)\sin\theta$$

for the field from a current element.] Note that. h is a function of direction specified by the angles θ and ϕ.

In general, the field incident on an antenna is also elliptically polarized. In order to utilize Eq. (5.2) for the received open-circuit voltage it is convenient to think of the incident field as being produced by two current elements $I_\theta \Delta\, la_\theta$ and $I_\phi \Delta\, la_\phi$, as shown in Fig.1.8. The field that current elements $I_\theta \Delta\, l$ and $I_\phi \Delta\, l$ produce at the receiving antenna is

$$E_\theta = \frac{-jk_0 Z_0 I_\theta\,\Delta l}{4\pi r}e^{-jk_0 r} \qquad E_\phi = \frac{-jk_0 Z_0 I_\phi\,\Delta l}{4\pi r}e^{-jk_0 r}$$

(The negative sign is due to the current orientations.) To reproduce the incident field we must choose

$$I_\theta\,\Delta l = \frac{-4\pi r\, e^{jk_0 r}}{jk_0 Z_0}E_{i\theta}$$

$$I_\phi\,\Delta l = \frac{-4\pi r\, e^{jk_0 r}}{jk_0 Z_0}E_{i\phi}$$

where $E_{i\theta}$ and $E_{i\phi}$ are the actual incident fields

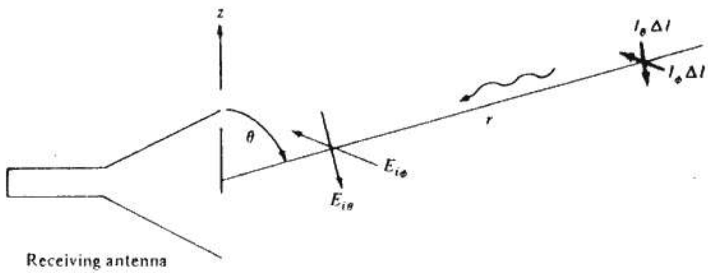


Figure 1.9 : Arbitrary incident field produced by Iwo current elements.

We now apply Eqns

$$V_{oc} V Y_{in} = V_{oc} I_{in} = -\int_V \mathbf{J}_2 \cdot \mathbf{E}_1 \, dV$$

$$= -\frac{jZ_0 k_0 I_{in}}{4\pi r} e^{-jk_0 r} \mathbf{h} \cdot (I_\theta \, \Delta l \mathbf{a}_\theta + I_\phi \, \Delta l \mathbf{a}_\phi)$$

$$= I_{in} \mathbf{h} \cdot \mathbf{E}_i$$

Hence

$$V_{oc} = \mathbf{h} \cdot \mathbf{E}_i$$

This equation further illuminates why h is called the effective length of the antenna, since it shows that $V_\propto$ can be thought of as the voltage induced .-- antenna-polarization is mismatched to that of the incident field. The polarization-mismatch factor p is defined as follows:

$$p = \frac{|\mathbf{h} \cdot \mathbf{E}_i|^2}{|\mathbf{h}|^2 |\mathbf{E}_i|^2}$$

Thus the received power in general is given by

$$P_{rec} = (1 - |\Gamma|^2) p \frac{\lambda_0^2}{4\pi} G(\theta, \phi) P_{inc}$$

and is reduced by the factor *p when the* polarizations are not matched.

1.11 ANTENNA TEMPERATURE(T_A)

- The Antenna temperature is a parameter that depends on the temperature of the region's the antenna is 'looking at'.
- Both the antenna temperature (T_A) and radiation Resistance (R_r) are single valued scalar quantities.
- According Nyquist relation, the noise power available from a resistor 'R' at absolute temperature T^o K is

$$P_a = \text{K T B} \quad (1)$$

Where $P_a \rightarrow$ *Noise power per unit band − width in watts* .

K → Boltzman's constant =1.38×10^{-23} J/k.

T→ Absolute temperature of resistor in ^{0}K.

The power received from the source is given by

$$P = S\ A_e\ B \quad ....(2)$$

Where S→ $Power\ density\ per\ unit\ bandwidth\ in \frac{w}{m} Hz(flux\ density)$.

$A_e \rightarrow Effective\ aperture\ in\ m^2$

B $\rightarrow Bandwidth\ in\ Hz$

Equating both the powers, P= K T B = S A_e B

$$S = \frac{KT_A}{A_e} \quad ....(3)$$

Where T_A →Antenna temperature due to the source in degree K.

$$T_A = \frac{S\,A_e}{K} \text{ degree K}$$

In terms of antenna Beam solid angle Ω_A and source solid angle Ω_s

$$T_A = \frac{\Omega_A}{\Omega_s} T_s \quad ....(5)$$

Where Ω_A → Antenna beam solid angle in steradian

$\Omega s \rightarrow Source\ solid\ angle\ in\ steradian$

$T_A \rightarrow Antenna\ noise\ temperature$

$T_s \rightarrow Source\ temperature\ in\ {}^oK$

- In case , the receiver has a certain noise temperature T_r due to thermal noise in the receiver components ,then the system noise power at the receiver terminals is given by

$$P_s = K(T_A + T_R)B \ldots\ldots\ldots\ldots.(6)$$

Where $T_R \rightarrow$ *Receiver noise temperature at reccvier terminals*

$B \rightarrow$ *Band width*

$T_A \rightarrow$ *Antenna noise temperature at receiver terminals*

$P_s \rightarrow$ *System noise power at receiver terminals.*

- Then the output to noise ratio is given by

$$\frac{S}{N} = \frac{S_A}{(T_A+T_R)KB} \qquad \ldots\ldots\ldots\ldots\ldots..(7)$$

Equivalent noise Temperature of Antenna(T_e)

- It is defined as that fictional temperature at the input of the network which would account for the noise ∇N *at the output*.
  ($\nabla N \rightarrow$ *Additional noise introduced by the network itsself*).
- The noise figure (F) related with effective noise temperature is

$$F = 1 + \frac{T_e}{T_0} \Rightarrow F - 1 = \frac{T_e}{T_0}$$

$$T_e = T_0\ (F-1)$$

Where F = Noise figure (no dimension)

$T_0 = 290^0$ K

- The noise figure 'F' in decibel is given by

$$(F)\ dB = 10 log_{10} F$$

1.12 RADIATION FROM OSCILLATING DIPOLE :

Although a charge moving with uniform velocity along a straight conductor does not radiate , a charge moving back and forth in simple harmonic motion along the conductor is subject to acceleration and radiates.

To illustrate radiation from a dipole antenna, let us consider that the dipole of fig has two equal charges of opposite sign oscillating up and down in harmonic motion with instantaneous separation l (maximum separation l_o) while focusing attention on the electric field . For clarity only a single electric field line is shown.

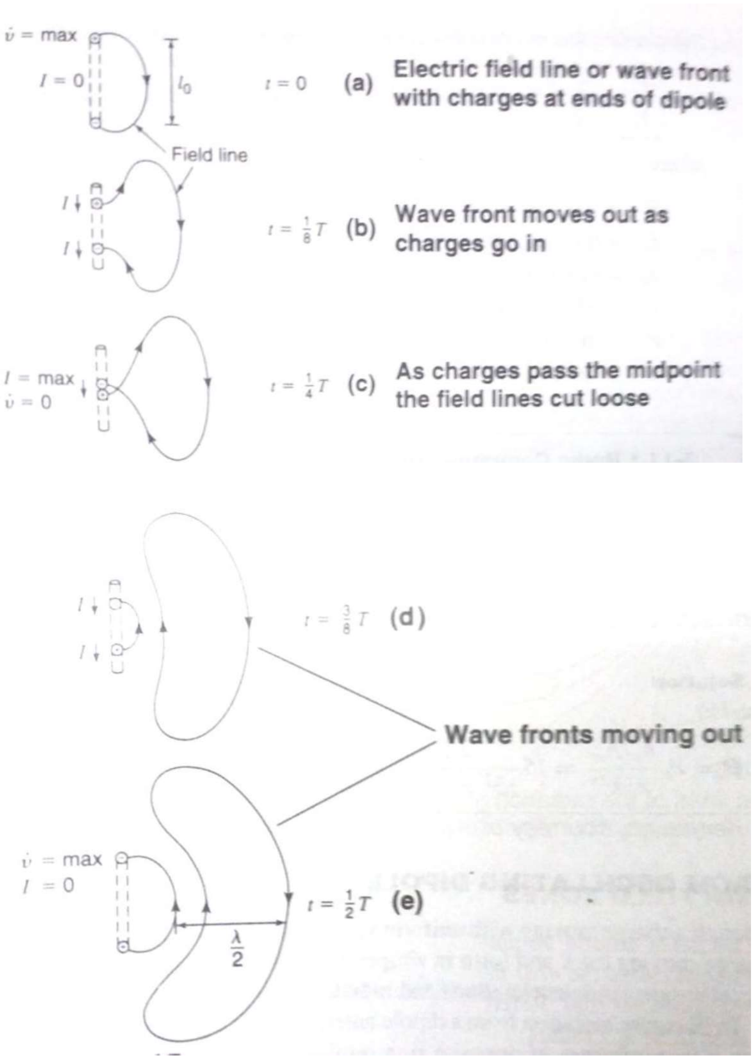


Fig1.10 :Oscillating electric dipole consisting of two electric charges in simple harmonic motion showing propagation of an electric field line and its detachment (radiation) from the dipole. Arrows next to the dipole indicate current (I) direction

At time t = 0 the charges are at maximum separation and undergo maximum acceleration νas they reverse direction (Fig a) . At this instant the current I is zero. At an $\frac{1}{8}$ period later, the charges are moving toward each other (Fig b) and at a $\frac{1}{4}$ period they pass at midpoint (Fig c) . As this happens, the field lines detach and new ones of opposite sign are formed. At this time the equivalent current I is a maximum and the charge acceleration is zero. As time progresses to a $\frac{1}{2}$ period , the fields continue to move out as in fig d and e

An oscillating dipole with more field lines shown in fig at four instants of time.

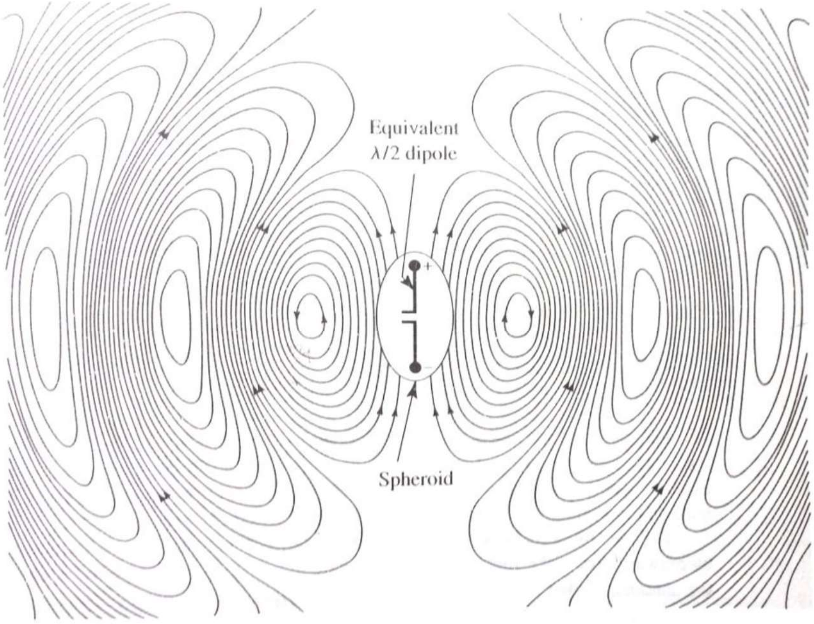


Fig1.11 : Electric field lines of the radiation moving out from $\lambda/2$ dipole antenna.

OCILLATING DIPOLE : (SHORT DIPOLE)

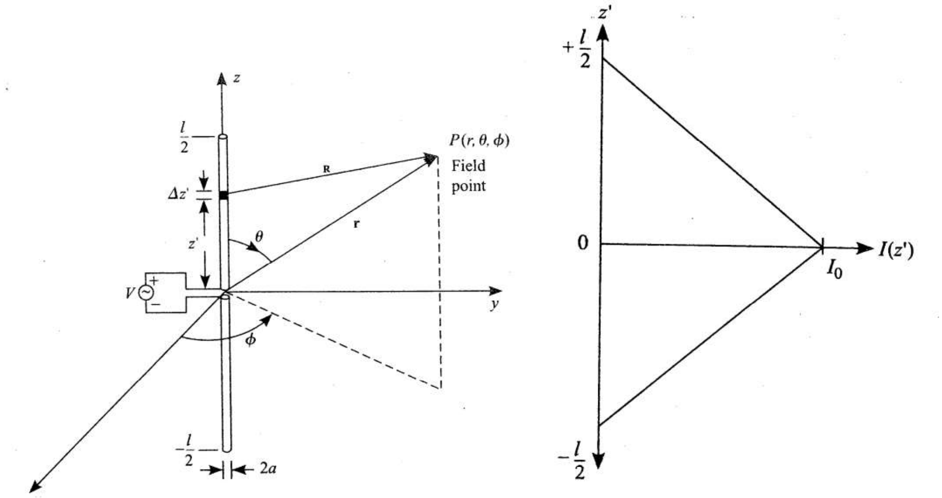


Fig1.12 (a) Geometry of a thin wire dipole. (b) Current distribution on a short dipole excited at the center

Consider a short dipole having length $l\ (l \ll \lambda\ )$ and radius a (a<< λ) , symmetrically placed about the orgin and oriented along the z- axis . The Current on a short wire dipole has a triangular distribution with a maximum at the center

The current on the dipole can be represented by,

$$I_Z(Z') = \begin{cases} \left(1 - \frac{2Z'}{l} I_0\right) ; & 0 \le z' \le l/_2 \\ \left(1 + \frac{2Z'}{l}\ I_0\right); & -l/_2 \le Z' \le 0 \end{cases}$$

;

Since the current is z-directed the magnetic vector potential is expressed as,

$$A(x, y, z) = a_z \frac{\mu}{4\pi} \int_{-l/2}^{l/2} \frac{I_z(z')}{R} e^{-jKR}\ dz' \qquad \ldots(1)$$

Where, R is the distance from source and field point $(x' = 0\ , y' = 0\ , z')$ on the field point (x,y,z) and is given by

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = \sqrt{x^2 + y^2 + (z-z')^2}$$

Expressing the field point (x,y,z) in spherical co-ordinates using the following equation.

$$x^2 + y^2 + z^2 = r^2 \;;$$

$$z = r\cos\theta$$

then

$$R = \sqrt{x^2 + y^2 + z^2 + z'^2 - 2zz'}$$

$$R = \sqrt{r^2 - 2r\cos\theta\, z' + z'^2}$$

$$= \sqrt{r^2[1 - \frac{2z'\cos\theta}{r} + \frac{z'^2}{r^2}]} = r\sqrt{1 + (-\frac{2z'\cos\theta}{r} + \frac{z'^2}{r^2})}$$

Expanded using the Binomial series

$$\therefore \sqrt{1+x} = (1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots..)$$

$$R = r\left(1 + \frac{1}{2}\left(-\frac{2z'\cos\theta}{r} + \frac{z'^2}{r^2}\right) - \frac{1}{8}\left(-\frac{2z'\cos\theta}{r} + \frac{z'^2}{r^2}\right)^2\right)$$

If the field point is far away from the antenna ($r \gg z'$), we can neglect the terms z'/r and higher power of z'/r. Then we get

$$R \approx r - z'\cos\theta$$

For phase we can use the above equation

For amplitude , $R \approx r$

Now equation (1) becomes

$$A(x, y, z) = a_z \frac{\mu}{4\pi} \int_{-l/2}^{l/2} \frac{I(x,y,z)}{r} e^{-jK(r - z'\cos\theta)} \; dz'$$

$$A(x,y,z) = a_z \frac{\mu}{4\pi}\left[\int_{-l/2}^{0}\left(1 + \frac{2}{l}z'\right)\frac{I_0 e^{-jk(r - z'\cos\theta)}}{r} dz' + \int_{0}^{l/2}\left(1 - \frac{2}{l}z'\right)\frac{I_0 e^{-jk(r - z'\cos\theta)}}{r} dz'\right]$$

$$A(x,y,z) = a_z \frac{\mu I_0 e^{-jkr}}{4\pi r}\left[\int_{-l/2}^{0}\left(1 + \frac{2}{l}z'\right) e^{jkz'\cos\theta}\, dz' + \int_{0}^{l/2}\left(1 - \frac{2}{l}z'\right) e^{jkz'\cos\theta}\, dz'\right]$$

……. (2)

Now integrate the terms ,

Let, $I = \left[\int_{-l/2}^{0}\left(1+\frac{2}{l}z'\right)e^{jk z' cos\theta}\,dz' + \int_{0}^{l/2}\left(1-\frac{2}{l}z'\right)e^{jk\, z' \cos\theta}\,dz'\right]$

Put $z' = -z'$ in the first integral and interchange the limits

$I = \left[\int_{-l/2}^{0}\left(1-\frac{2}{l}z'\right)e^{-jk z' cos\theta}\,dz' + \int_{0}^{l/2}\left(1-\frac{2}{l}z'\right)e^{jk\, z' \cos\theta}\,dz'\right]$

$= \left[\int_{0}^{l/2}\left(1-\frac{2}{l}z'\right)[e^{jk\, z' \cos\theta} + e^{-jk z' cos\theta}]dz'\right]$

$= \int_{0}^{l/2}\left(1-\frac{2}{l}z'\right)2\cos(kz'\cos\theta)\,dz'$

$= 2\left\{\left(1-\frac{2}{l}z'\right)\frac{\sin(kz'\cos\theta)}{k\cos\theta} - \left(\frac{-2}{l}\right)\left(\frac{-\cos(kz'\cos\theta)}{cos^2\theta k^2}\right)\right\}$

$= 2\left\{0 - \frac{2}{l}\left[\frac{\cos\left(k\frac{l}{2}\cos\theta\right)}{cos^2\theta k^2}\right] - \left[0 - \frac{2}{lk^2cos^2\theta}\right]\right\}$

$= 4\left\{\frac{-\cos\left(k\frac{l}{2}\cos\theta\right)+1}{lk^2cos^2\theta}\right\}$

$I = 4\,\frac{\left(1-\cos\left(k\frac{l}{2}\cos\theta\right)\right)}{lk^2cos^2\theta}$

$I = \frac{4\times2\left(sin^2\left(k\frac{l}{4}\cos\theta\right)\right)}{l\,k^2\cos^2\theta}$

For $\frac{kl}{4} \ll 1$; then $\sin^2\left(\frac{kl\cos\theta}{4}\right) \approx \frac{(kl\cos\theta)^2}{(4)^2}$

$I = \frac{8\,(kl\cos\theta)^2}{16\,l\,k^2\cos^2\theta}$

$$\boxed{I = \frac{l}{2}}$$

Equation (2) becomes A (x, y, z) = $a_z\,\frac{\mu I_0 e^{-jkr}\,l}{8\pi r}$

The components of the magnetic vector potential in spherical co-ordinates ,

$A_r = A_z\cos\theta = \frac{\mu I_0 e^{-jkr}\,l}{8\pi r}\cos\theta$

$$A_\theta \quad = \quad - \mathrm{A_z} \sin\theta \quad = \quad - \frac{\mu I_0 e^{-jkr}\, l}{8\pi r} \quad \sin\theta$$

$$A_\phi \ = 0$$

To Find the Magnetic field:-

$$\mathrm{H} = \frac{1}{\mu}\,(\nabla \times A)$$

$$\nabla \times A = \frac{1}{\mathrm{r}^2 \sin\,\theta} \begin{vmatrix} a_r & r a_\theta & r\sin\theta a_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & rA_\theta & r\sin\,\theta\, A_\varphi \end{vmatrix}$$

$$= \frac{1}{\mathrm{r}^2 \sin\,\theta} \begin{vmatrix} a_r & r a_\theta & r\sin\theta a_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & r & 0 \end{vmatrix}$$

$$= \frac{1}{\mathrm{r}^2 \sin\,\theta} \left\{ r\sin\theta a_\varphi \left[\frac{\partial}{\partial r}\ (rA_\theta) - \frac{\partial}{\partial \theta}(A_r) \right] \right\}$$

$$= \frac{1}{r} a_\varphi \left[\frac{\partial}{\partial r} \left(\frac{-r\mu\, I_0 e^{-jkr}\ l\sin\,\theta}{8\pi r} \right) - \frac{\partial}{\partial \theta} \left(\frac{\mu\, I_0 e^{-jkr}\ l\cos\,\theta}{8\pi r} \right) \right]$$

$$= \frac{1}{r} a_\varphi \left[\frac{\partial}{\partial r} \left(\frac{-\mu\, I_0 e^{-jkr}\ l\sin\,\theta}{8\pi} \right) - \left(\frac{\mu\, I_0 e^{-jkr}\ l}{8\pi r} \right) \frac{\partial}{\partial \theta}(\cos\theta) \right]$$

$$\nabla \times A = \frac{1}{r} a_\varphi \left[\left(\frac{\mu\, I_0 e^{-jkr}\ (jk)l\sin\,\theta}{8\pi} \right) + \left(\frac{\mu\, I_0 e^{-jkr}\ l\sin\,\theta}{8\pi r} \right) \right]$$

$$\nabla \times A = \frac{\mu\, I_0 e^{-jkr}\ l\sin\,\theta}{8\pi r} a_\varphi [(jkr) + \ 1]$$

$$\nabla \times A = \frac{\mu\, I_0 e^{-jkr}\ l\, jkr\sin\,\theta}{8\pi r} a_\varphi [1 + \ \frac{1}{jkr}]$$

Thus, $H_\varphi \ = \ \frac{1}{\mu} \left[\frac{\mu I_0 e^{-jkr}\ l\, jkr\sin\,\theta}{8\pi r^2} \left(1 + \frac{1}{jkr} \right) \right]$

$$H_\varphi \ = \ \left[\frac{I_0 e^{-jkr}\ l\, jk\sin\,\theta}{8\pi r} \left(1 + \frac{1}{jkr} \right) \right]$$

In far field region,

$$H_{\varphi} = \frac{jkI_0 l e^{-jkr} \sin\theta}{8\pi r}$$

To find electric field:

$$E = \frac{1}{jw\varepsilon} (\nabla \times H)$$

$$E_{\theta} = a_{\theta} j\eta \frac{I_0 e^{-jkr} l k \sin\theta}{8\pi r}$$

Radiation resistance and Directivity:

The directivity is given by

$$D(\theta,\varphi) = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

Radiation intensity , $U(\theta,\varphi) = r^2 s$

Where, S = pointing vector or the average power density is given by

$$S = \frac{1}{2} \mathrm{Re}\left(E_{\theta} H_{\phi}^{*}\right)$$

$$= \frac{1}{2\eta} | E_{\theta} |^2$$

$$= \frac{1}{2\eta} \left| \frac{I_0 e^{-jkr}\, l\, \eta jk \sin\theta}{8\pi r} \right|^2$$

Thus, $$U(\theta,\varphi) = r^2 \times \frac{1}{2\eta} \left| \frac{I_0 e^{-jkr}\, l\, \eta jk \sin\theta}{8\pi r} \right|^2$$

$$= \frac{1}{2\eta} \left| \frac{I_0 e^{-jkr}\, l\, \eta jk \sin\theta}{8\pi} \right|^2$$

$$U(\theta,\varphi) = \frac{\eta}{2} \left| \frac{kI_0 l}{8\pi} \right|^2 sin^2\theta$$

Power, $P_{rad} = \int_0^{\pi} \int_0^{2\pi} S.dA$

Where, $dA = r^2 \sin\theta \, d\theta \, d\varphi$

$$P_{rad} = \int_0^{\pi} \int_0^{2\pi} \frac{\eta}{2} \left| \frac{kI_0 l}{8\pi r^2} \right|^2 sin^2\theta \, . r^2 \sin\theta \, d\theta \, d\varphi$$

$$= \frac{\eta (kI_0 l)^2}{2(8\pi)^2} \int_0^{\pi} \int_0^{2\pi} sin^3\theta \, d\theta \, d\varphi$$

$$= \frac{\eta (kI_0 l)^2}{2(8\pi)^2} \int_0^{\pi} \int_0^{2\pi} \left(\frac{3\sin\theta - \sin 3\theta}{4} \right) d\theta \, d\varphi$$

$$= \frac{\eta (kI_0 l)^2}{2(8\pi)^2} \int_0^{\pi} \left(\frac{3\sin\theta - \sin 3\theta}{4} \right) (2\pi) \, d\theta$$

$$= \frac{\pi\eta (kI_0 l)^2}{4(8\pi)^2} \left[-3\cos\theta + \frac{\cos 3\theta}{3} \right]$$

$$= \frac{\pi\eta (kI_0 l)^2}{4(8\pi)^2} \left[\left(-3(-1) + \frac{(-1)}{3} \right) - \left(3 + \frac{1}{3} \right) \right]$$

$$= \frac{\pi\eta (kI_0 l)^2}{4(8\pi)^2} \left[\frac{16}{3} \right] \qquad (\therefore k = \frac{2\pi}{\lambda})$$

$$P_{rad} = \frac{\pi\eta (2\pi I_0 l)^2}{4\lambda^2 (8\pi)^2} \left[\frac{16}{3} \right] = \frac{\pi\eta I_0^2}{12} \left(\frac{l}{\lambda} \right)^2$$

Thus,

$$D(\theta, \phi) = \frac{4\pi \frac{\eta}{2} \left| \frac{kI_0 l}{8\pi} \right|^2 sin^2\theta}{\frac{\pi\eta I_0^2}{12} \left(\frac{l}{\lambda} \right)^2}$$

$$D(\theta, \phi) = \frac{3}{2} \sin^2\theta$$

(or)

$$\boxed{D(\theta, \phi) = 1.5 \sin^2\theta}$$

Radiation Resistance:-

The radiation resistance is given by

$$P_{rad} = \frac{1}{2} I^2 \; R_{rad}$$

Substitute P_{rad} value

$$\frac{\pi\eta \, I_0{}^2}{12} \left(\frac{l}{\lambda}\right)^2 = \frac{1}{2} I^2 \; R_{rad}$$

$$R_{rad} = \frac{\pi\eta}{6} \left(\frac{l}{\lambda}\right)^2 \qquad (\because \eta = 120\pi)$$

$$R_{rad} = 20 \left(\frac{\pi l}{\lambda}\right)^2$$

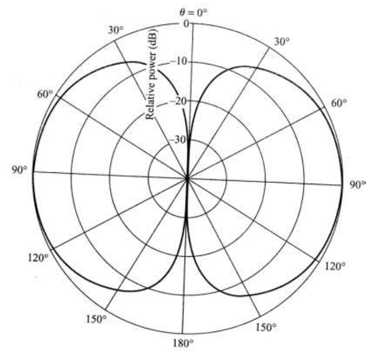


Fig1.13: Radiation pattern for short dipole antenna

1.13 HALF WAVE DIPOLE : $\left(\frac{\lambda}{2}\ \boldsymbol{dipole\ antenna}\right)$

The current distributin on a thin (radius ,$a \ll \lambda$) wire dipole depends on its length. For a very short dipole ($l < 0.1\ \lambda$)it is appropriate to assume that the current distribution is triangular.

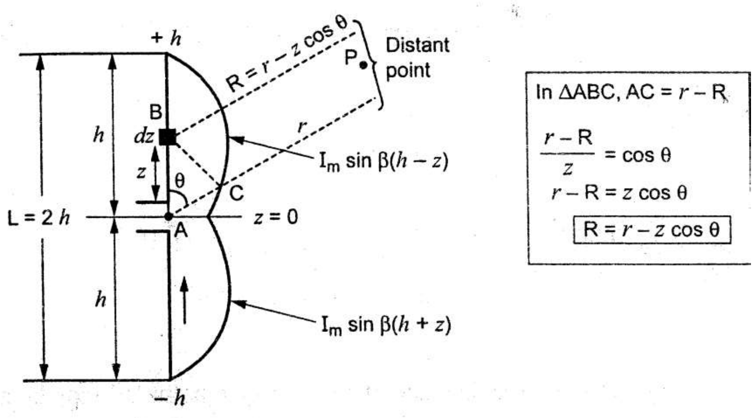


Fig1.12: Half wave dipole

The current on the dipole has only a z- component and is given by,

$$I(z') = \begin{cases} a_z I_0 \sin\left(k\left(\frac{l}{2} - z'\right)\right); & 0 \le z' \le \frac{l}{2} \\ a_z I_0 \sin\left(k\left(\frac{l}{2} + z'\right)\right); & -\frac{l}{2} \le z' \le 0 \end{cases}$$

I_0 is the amplitude of the current distribution

k is the free space propagation constant

First compute the magnetic vector potential in the far field region of the antenna, and find E & H.

The Magnetic vector potential is

$$A(x,y,z) = \frac{\mu}{4\pi}\int_{\frac{-l}{2}}^{\frac{l}{2}} \frac{I(x,y,z)e^{-jkR}}{R}\, dz'$$

Where, For phase angle $R = r - z' \cos\theta$

For amplitude R = r

Thus,

$$A_z = \frac{\mu}{4\pi}\int_{\frac{-l}{2}}^{\frac{l}{2}} \frac{I_z z' e^{-jk\,(r - z'\cos\theta)}}{r}\, dz'$$

$$=\frac{\mu}{4\pi}\left(\int_{-l/2}^{0}\frac{I_0\,\text{sink}\left(\frac{l}{2}+z'\right)}{r}e^{-jk\,(\text{r}-z'\cos\theta)}dz' + \int_{0}^{\frac{l}{2}}\frac{I_0\,\text{sink}\left(\frac{l}{2}-z'\right)}{r}e^{-jk\,(\text{r}-z'\cos\theta)}dz'\right)$$

$$=\frac{\mu I_0 e^{-jkr}}{4\pi r}\left\{\int_{\frac{-l}{2}}^{0}\sin\left[\text{k}\left(\frac{l}{2}+z'\right)\right]\quad e^{jk\;z'\cos\theta}dz' + \int_{0}^{\frac{l}{2}} sin\left[k\left(\frac{l}{2}-z'\right)\right]e^{jk\;z'\cos\theta}dz'\right\}$$

Integrating with respect to $'$, and substituting appropriate limits, the vector potential expression is reduced to,

$$\text{A}_\text{z} = \frac{\mu I_0 e^{-jkr}}{2\pi rk\,sin^2\theta}\left(\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)\right)\ldots\ldots(3)$$

Decomposing A_z into components along the $r\ and\ \theta$ directions , we have,

$$A_r = A_z\ \cos\theta$$

$$\therefore A_r = \frac{\mu I_0 e^{-jkr}}{2\pi rk\ \sin^2\theta}\left(\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)\right)\ \ldots.(4)$$

$$A_\theta = -A_z\ \sin\theta$$

$$\therefore A_\theta = -\frac{\mu I_0 e^{-jkr}}{2\pi rk\ \sin\theta}\left(\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)\right)\ \ldots.(5)$$

$$A_{\varphi=0}$$

To find the electric and magnetic field:-

In the far field region of z directed dipole the component of magnetic vector potential transverse to the direction of propagation is A_θ.

$$A_t = a_\theta A_\theta$$

Now

$$\text{E} = -\text{j}\omega A_t = -\text{j}\omega a_\theta A_\theta$$

$$E = a_\theta j\omega\frac{\mu I_0 e^{-jkr}}{2\pi rk\,sin\,\theta}\left[cos\left(\frac{kl}{2}cos\,\theta\right)-cos\left(\frac{kl}{2}\right)\right]$$

$$E_\theta = \left[\frac{j\eta\,I_0 e^{-jkr}}{2\pi r\ \sin\theta}\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)\right]\quad\ldots\ldots.(a)$$

The Magnetic field is given by ,

$$H = \frac{-J\omega}{\eta} A_t$$

Where,

$$A_t = a_\phi A_\theta$$

$$H_\phi = \frac{-j\omega}{\eta} a_\phi A_\theta$$

$$H_\phi = \frac{-j\omega}{\eta} a_\phi \frac{\mu I_0 e^{-jkr}}{2\pi rk \sin\theta}\left(\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right) \qquad \text{.....(b)}$$

The electric and magnetic field intensities for a half – wave dipole are obtained by substituting $kl/_2 = \pi/_2$ in equations (a) and (b)

$$E_\theta = a_\theta \frac{j\eta I_0 e^{-jkr}}{2\pi r \sin\theta}\left[\cos\left(\frac{kl}{2}\cos\theta\right)\right]$$

$$H_\phi = a_\phi \frac{-j\omega}{\eta} \frac{\mu I_0 e^{-jkr}}{2\pi rk \sin\theta}\left(\cos\left(\frac{kl}{2}\cos\theta\right)\right)$$

Radiation resistance and Directivity:-

Directivity:-

The directivity is given by

$$D(\theta,\varphi) = \frac{4\pi U(\theta,\varphi)}{P_{rad}}$$

The radiation intensity is given by

$$U(\theta,\phi) = r^2\, s$$

The time average power density is given by,

$$S(\theta,\phi) = \frac{1}{2\eta}|E_\theta|^2$$

$$= \frac{1}{2\eta}\left|\frac{j\eta I_0 e^{-jkr}}{2\pi r \sin\theta}\left(\cos\left(\frac{kl}{2}\cos\theta\right)\right)\right|^2$$

$$= \frac{1}{2\eta} \left| \frac{\eta I_0}{2\pi r \sin\theta}\left(\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right)\right|^2$$

Thus

$$\mathrm{U}(\theta,\phi) = r^2 \times \frac{\eta^2}{2\eta r^2}\left|\frac{I_0}{2\pi\sin\theta}\left[\cos\left(\frac{kl}{2}\cos\theta\right)\right]\right|^2$$

$$= \frac{\eta}{2}\left|\frac{I_0}{2\pi\sin\theta}\left[\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right]\right|^2$$

$$\mathrm{U}(\theta,\phi) = \frac{\eta}{2}\left|\frac{I_0}{2\pi\sin\theta}\left[\cos\left(\frac{\pi}{2}\cos\theta\right)\right]\right|^2$$

$$\mathrm{U}(\theta,\phi) = \frac{\eta}{2}\left|\frac{I_0}{2\pi}\right|^2 \frac{\left[cos^2\left(\frac{\pi}{2}\cos\theta\right)\right]}{\sin^2\theta}$$

Radiated power,

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} s.dA$$

$$dA = r^2 \sin\theta \, \mathrm{d}\theta \, d\phi$$

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} s.r^2 \sin\theta \, \mathrm{d}\theta \, d\phi$$

$$= \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} \frac{\eta}{2}\left|\frac{I_0}{2\pi}\right|^2 \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\sin\theta \, \mathrm{d}\theta \, d\phi$$

$$= \frac{\eta}{2}\left|\frac{I_0}{2\pi}\right|^2 \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\sin\theta \, \mathrm{d}\theta \, d\phi$$

$$= \frac{\eta}{2}\left|\frac{I_0}{2\pi}\right|^2 \int_{\theta=0}^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} (2\pi)\mathrm{d}\theta$$

$$= \eta\pi\left|\frac{I_0}{2\pi}\right|^2 \int_{\theta=0}^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \, \mathrm{d}\theta \qquad \ldots\ldots (6)$$

Let $\quad \mathrm{I} = \int_{\theta=0}^{\pi} \frac{cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \, \mathrm{d}\theta$

$$I = \frac{1}{2}\int_{\theta=0}^{\pi} \frac{(1+\cos(\pi\cos\theta))}{\sin\theta} \, \mathrm{d}\theta$$

After integration ,

I= 1.2179

Sub 7 in 6,

$$P_{rad} = \frac{\eta I_0{}^2}{4\pi}(1.2179)$$

$$P_{rad} = \frac{(120\,\pi) I_0{}^2}{4\pi}(1.2179)$$

$$\boxed{P_{rad} = 35.537|I_0|^2}$$

$$Thus\ directivity = D(\theta,\phi) = \frac{4\pi \frac{\eta}{2}\left|\frac{I_0}{2\pi}\right|^2 \frac{\left[cos^2\left(\frac{\pi}{2}\cos\theta\right)\right]}{\sin^2\theta}}{35.537|I_0|^2}$$

$$= 1.642\,\frac{\left[cos^2\left(\frac{\pi}{2}\cos\theta\right)\right]}{\sin^2\theta}$$

The maximum value of directivity occurs along $\theta = \frac{\pi}{2}$

$$D(\theta,\phi) = 1.642\,\frac{[cos^2(0)]}{(1)}$$

$$\boxed{D(\theta,\phi) = 1.642}$$

Directivity in decibel,

$$D_{dB} = 10\,log\,(1.642)$$

$$\boxed{D_{dB} = 2.15\,dB}$$

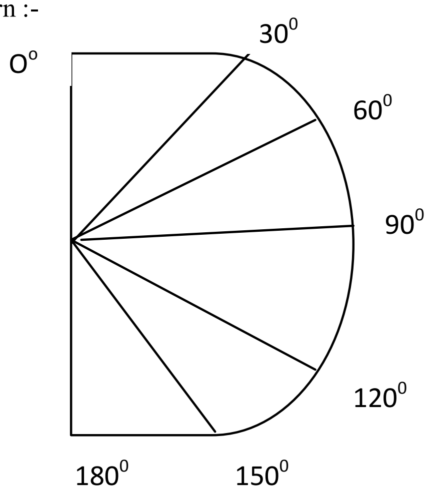
Radiation Resistance,

$$P_{rad} = \frac{1}{2}\,I^2\,R_{rad} \qquad .$$

$$36.537\,I_o^2 = \frac{1}{2}\,I_0{}^2\,R_{rad}$$

$$\boxed{R_{rad} = 73.07 \text{ or } 73\,\Omega}$$

Radiation Pattern :-



1.14 FOLDED DIPOLE:

An improvement over a conventional half wave dipole is the folded dipole shown in fig. In folded dipole antenna , two half wave dipoles (one continuous and the other split at the center) have been folded and joined together in parallel at the ends. The split dipole is fed at the center by balanced transmission line. The two dipoles, therefore have the same voltages at their ends. There are basically two dipoles in parallel as far as radiation fields are concerned. The radiation pattern of a folded dipole and conventional half wave dipole is same but the input impedance of the folded dipole is higher. It has better directivity and bandwidth than simple dipole.

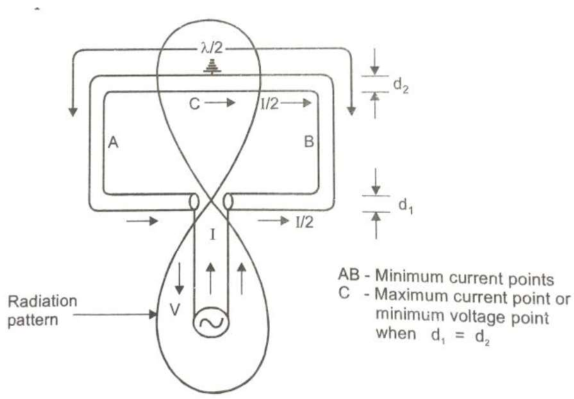


Fig 1.13: Two wire folded dipole with radiation pattern

If the radii of the two conductors are equal , the equal currents flow in both the conductors in the same direction. i.e., Currents are equal in magnitude and phase in the two dipoles. Since the total power developed in folded dipole is equal to that developed in the conventional dipole, therefore the input or terminal impedance of folded dipole is greater than that of the conventional dipole. The input impedance at the terminals of a folded dipole is equal to the square of number of conductors comprising the antenna times the impedance at the terminals of a conventional dipole.

If total current fed at terminal TT' is I then the each dipole have current $I/2$, provided their radii are equal.

Equation of input impedance

The equation for the input impedance or terminal impedance or radiation resistance of a folded dipole antenna can be deduced very simply as follows

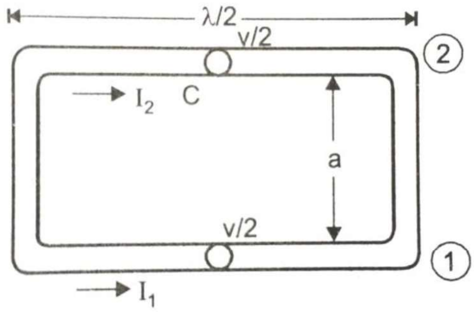


Fig1.14 : Equivalent diagram of two wire folded half wave dipole

Let V be the emf applied at the antenna terminals AA'. This is being divided equally in each dipole. Hence , voltage in each dipole is $V/2$ as shown in fig, and by nodal analysis.

$$\frac{V}{2} = I_1 Z_{11} + I_2 Z_{12} \qquad \ldots\ldots\ldots .(1)$$

Where I_1 = Current at terminals of dipole 1

I_2 = Current at terminals of dipole 2

Z_{11} = Self impedance of dipole 1

Z_{12} = Mutual impedance of dipole 1 and 2

Since $I_1 = I_2$, equation (1) becomes,

$$\frac{V}{2} = I_1(Z_{11} + Z_{12}) \qquad \ldots\ldots\ldots\ldots(2)$$

$$V = 2I_1(Z_{11} + Z_{12}) \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

Since the two dipoles are close , the spacing $'a'$ between two dipoles is the order of $\lambda/100$.

$$Z_{11} \approx Z_{12} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

By applying equation (4) in equation (3), we get

$$V = 2I_1(Z_{11} + Z_{11})$$

$$V = 4I_1 Z_{11}$$

$$Z = \frac{V}{I_1} = 4Z_{11} = 4 \times 73$$

$$\because Z_{11} = 73\Omega \quad for\ a\ dipole$$

$$Z = 292\ \Omega$$

Similarly , for a folded dipole of 3 wires, it can be proved that termination impedance is 657 Ω

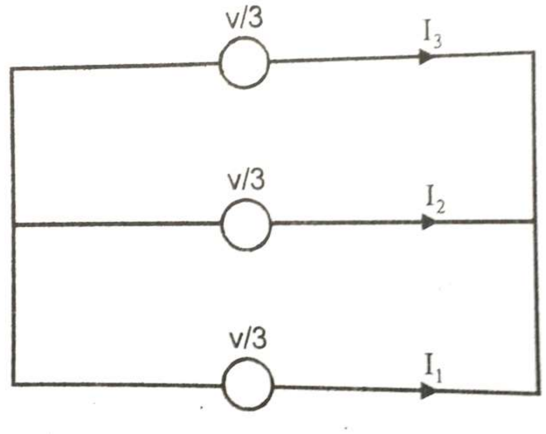


Fig :1.15

$$\frac{V}{3} = I_1(3Z_{11})$$

$$\frac{V}{I_1} = 3^2 Z_{11} = 9 \times 73$$

$$Z = 657\Omega$$

Generalizing , we have,

$$\frac{V}{n} = I_1(nZ_{11})$$

$$Z = \frac{V}{I_1} = n^2 Z_{11}$$

$$Z = n^2 \times 73$$

Where n is the number of half wave dipoles.

Since the impedance transformation is possible by making unequal radii of the two dipoles, the input impedance is given by,

$$Z = Z_{11}\left(1 + \frac{r_1}{r_2}\right)^2 = 73\left(1 + \frac{r_1}{r_2}\right)^2$$

Where $r_1 and\ r_2 = radii\ of\ the\ elements$

If $r_1 = 2r_2$, then $Z = 73\left(1 + \frac{2r_2}{r_2}\right)^2 = 73 \times 9 = 657\Omega$

The input impedance of folded dipole antenna can be increased by increasing the number of dipoles. Alternatively instead of changing the number of dipoles of folded dipole, it is also possible to change the input impedance by keeping the radii of the dipoles unequal. By doing so, larger current flows in thicker dipole, thus it is possible to attain any input impedance that may be desired. With the dipoles of unequal radii, transformation ratio of 1.5 to 25 can be achieved and this ratio can be further increased by increasing the number of dipoles. Folded dipole can also be designed of lengths other than $\lambda/2$. Example ,two element dipole of $3\lambda/4$, each will have input impedance of 450 Ω.

For unequal radii, the input impedance is modified to

$$Z = 73\left(1 + \frac{r_1}{r_2}\right)^2$$

Where r_1 and r_2 are the radii of the dipoles.

Since the impedance transformation also depends on the spacing between the dipoles in addition to radii, the equation will be further modified to

$$Z = 73\left(1 + \frac{(\log a/r_1)}{(\log a/r_2)}\right)$$

Where a is the distance between the dipoles.

r_1 and r_2 are the radii of the dipoles.

Advantages of folded dipole

- High Input Impedance
- Wide band in frequency
- Acts as built –in reactance compensation network

Uses of Folded Dipole:

Folded dipole is used in conjunction with parasitic elements in wide band operation such as television. In the yagi antenna , the driven element is folded dipole and remaining are reflector and director.

1.15 YAGI-UDA ANTENNA

YAGI-UDA or simply yagi antennas are the most high gain antennas. It is named after the professor S.Uda and H.Yagi. This is the most common antenna used for TV reception. The gain of the antenna is around 7 db and its radiation pattern is very iuch directive in one direction (normally receiving direction).

Construction

Yagi-Uda array is an example of a parasitic array. It employs one or more parasitic elements to couple the power electrically from the driven element. As shown in Fig.1.16 , yagi array consists of a driven element, a reflector and one or more directors. The driven element is a resonant half-wave dipole (folded) made up of metallic rod at the frequency of operation.

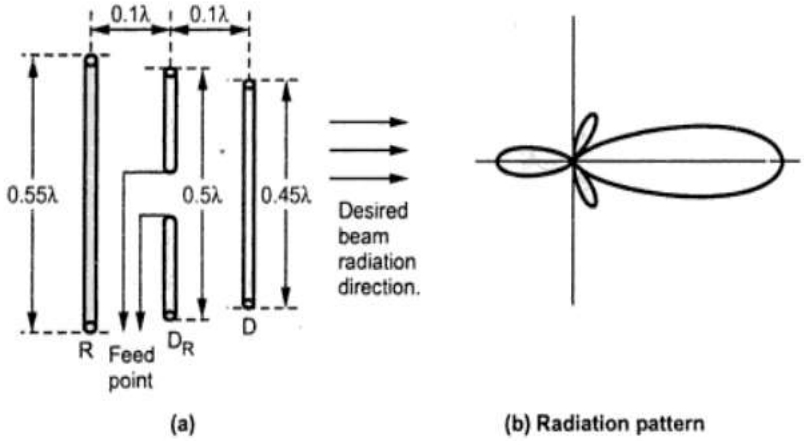


Fig 1.16: Yagi uda antenna and its radiation pattern

It is otherwise called as ***active element*** since power from the transmitter is fed to this driven element. The parasitic element in front of the driven element is known as ***director*** whereas the element in back of it is known as ***reflector***. These parasitic elements derive power by radiation from the nearby driven element.

The length of the reflector is 5% more and the director is 5% less than the driven element which is 1/2 at resonant frequency. In practice, the 3-element yagi array can be designed using the following expressions.

Reflector length $= \frac{500}{f(mHz)}$ **feet (or)** $\frac{152}{f(mHz)}$ **meters**

Driven element length $= \frac{475}{f(mHz)}$ **feet (or)** $\frac{143}{f(mHz)}$ **meters**

Director length $= \frac{455}{f(mHz)}$ **feet (or)** $\frac{137}{f(mHz)}$ **metres**

The spacing between the reflector and driven element is 0.2λ to 0.4λ, and the spacing between the driven element and directors varies from 0.10λ to 0.15λ. The parasitic elements and the driven element could be clamped on a metallic support rod. The clamping over the support rod provides a rigid mechanical structure.

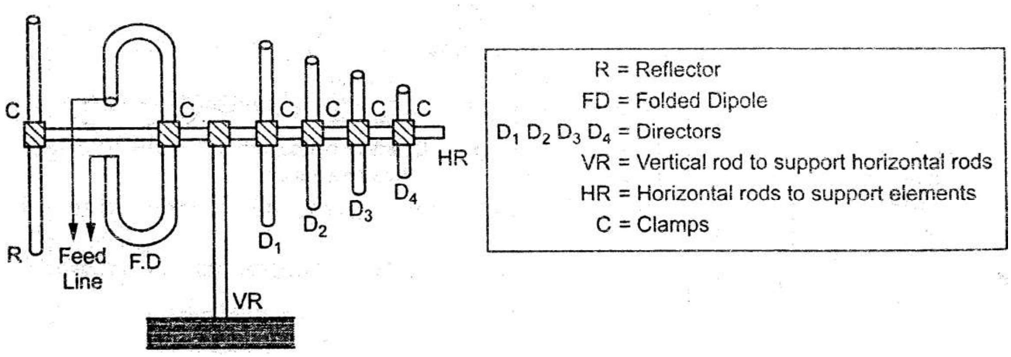


Fig 1.17. 6 element yagi antenna with folded diple

The driven element is fed by a 2 wire balanced transmission line. But the reflector and director are not connected directly with transmission line but they are coupled electrically with driven element.

Additional directors can be added to the 3 element yagi antenna as shown in Fig.1.17. Increasing the number of directors will increase the power gain but decreases the antenna bandwidth. A 3 element yagi antenna suitable for TV reception of moderate field strength is shown in Fig.1.18.

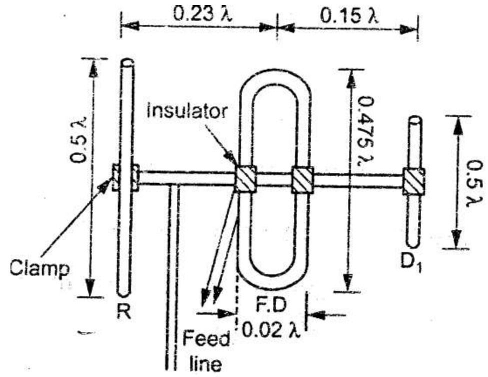


Fig 1.18. A typical TV yagi antenna

Working Principle

- ✓ The length of the parasitic elements and their spacing determine the phases of the current.
- ✓ The length of the reflector is more than the folded dipole (driven element).Therefore it offers inductive reactance (current lag the induced voltage) to the incoming signal.
- ✓ The length of the director is shorter than the dipole. Hence it offers capacitive reactance (current leads the induced voltage).

1. The action of reflector

- ✓ The radiation coming from the front at the reflector is absorbed and it retransmits the radiation towards the dipole in such a way that it adds with the incoming signal.
- ✓ For any radiations coming from the back side, reflector retransmits the radiation in such a way that it is out of phase with the direct radiation from back side at dipole and hence they cancel each other.

2. The action of director

- ✓ For the radiation coming from the front, the director generates its own radiation in such a way that it adds with this radiation at dipole and increases signal strength.
- ✓ For radiation coming from the back, director generates its own radiation such that it cancels the radiation from back at dipole.
- ✓ By suitable dimensioning, the lengths and spacing between two elements, the radiated energy is added up in front and tend to cancel the backward radiation.

3. Compensation for reduction in input impedance

- ✓ If the distance between driven and parasitic element is decreased, then it will load the driven element, irrespective of its length. Therefore the impendence at the input terminals of driven element reduces. That is why a folded dipole is used as driven element so that reduction in input impedance is compensated. (Input impedance of folded dipole = 2^2 x impedance of conventional half wave dipole. i.e., 4 x 7 3 =292Ω)

General Characteristics

- ✓ If three elements array (one reflector, one driven element and one director) is used, then such type of yagi-uda antenna is generally referred to as Beam Antenna.
- ✓ It has unidirectional beam of moderate directivity with light weight, low costand simplicity in feed system design.
- ✓ With spacing of 0.1λ, to 0.15λ,, a frequency bandwidth of the order of 2% is obtained.
- ✓ It provides gain of the order of 8 db or front to back ratio of about 20 dB.
- ✓ It is also known as super directive or super gain antenna due to its high gain and beam-width per unit area of the array.
- ✓ Greater directivity can be achieved by increasing the number of parasitic elements.
- ✓ It is essentially a fixed frequency device (i.e., frequency sensitive) and a bandwidth of about 3% is obtainable. This much bandwidth is sufficient for television reception.

VOLTAGE AND CURRENT RELATIONS IN PARASITIC ANTENNAS

The magnitude and phase of the current in parasitic element will affect the radiation pattern. Therefore it is necessary to study the current and voltage relations in parasitic antennas. Let us consider a yagi-uda antenna with 2 elements, one driven element and one reflector as shown in Fig1.19.

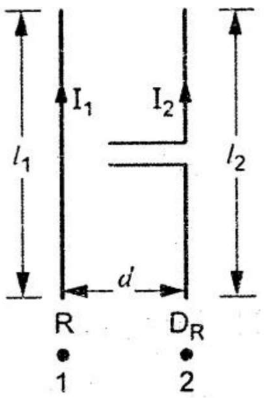


Fig 1.19: Two element yagi-uda antenna

Let V_1= Applied voltage in antenna number 1 i.e . reflector

V_2= Applied voltage in antenna number 2 i.e driven element

I_1 =Current through reflector

I_2= Current through driven element

Z_{11},Z_{21}= Self impedance of reflector and driven element

Z_{12},Z_{22}=Mutual impedance between reflector and driven element.

Now, the applied voltage in the reflector is given by

$$V_1=I_1Z_{11}+I_2Z_{12} \qquad .....1$$

Similarly applied voltage in the driven element is

$$V_2=I_1Z_{21}+I_2Z_{22} \qquad .....2$$

But if the individual antennas are not excised, the applied voltage becomes zero. Here the reflector is not directly excited. Therefore V_1 = 0 and equation (1) becomes.

$$I_1 Z_{11} + I_2 Z_{12} = 0$$

$$I_2 Z_{12} = -I_1 Z_{11}$$

$$I_1 = -\frac{I_2 Z_{12}}{Z_{11}} \quad \ldots\ldots\ldots(3)$$

Substituting equation(3)in (2)

$$V_2 = -I_2 \frac{Z_{12}}{Z_{11}} Z_{21} + I_2 Z_{22}$$

But $Z_{12} = Z_{21}$

$$V_2 = +I_2 \left[Z_{22} - \frac{Z_{12}^2}{Z_{11}} \right]$$

$$I_2 = \frac{V_2}{Z_{22} - \frac{Z_{12}^2}{Z_{11}}} \quad \ldots\ldots\ldots(4)$$

Substituting I_2 in equation (3), we get

$$I_2 = \frac{V_2}{Z_{22} - \frac{Z_{12}^2}{Z_{11}}} \frac{Z_{12}}{Z_{11}}$$

$$= -\frac{\frac{V_2}{Z_{22} Z_{11} Z_{12}^2.}}{Z_{11}} \frac{Z_{12}}{Z_{11}}$$

$$I_1 = \frac{V_2 Z_{12}}{Z_{12}^2 - Z_{22} Z_{11}} \quad \ldots\ldots\ldots\ldots(5)$$

Taking Z_{12} to the denominator

$$I_1 = \frac{V_2}{Z_{12} - \frac{Z_{11} Z_{22}}{Z_{12}}} \quad \ldots\ldots\ldots(6)$$

∴ The input impedance of reflector can be derived from the above equation……(7)

$$Z_1 = \frac{V_2}{I_1} = Z_{12} - \frac{Z_{11} Z_{22}}{Z_{12}}$$

$$\text{i.e} Z_1 = Z_{12} - \frac{Z_{11}Z_{22}}{Z_{12}} \qquad \text{…….(8)}$$

Similarly the input impedance of driven element can be derived from equation (9)

$$Z_2 = \frac{V_2}{I_2} = Z_{22} - \frac{Z_{12}^2}{Z_{11}} \qquad \text{………..(10)}$$

From the equation (9) and (10), it is clear that the input impedance of an element is affected by the presence of parasitic element.

ADVANTAGES OF YAGI-UDA ANTENNA

- ✓ Unidirectional radiation
- ✓ Increased directivity
- ✓ Simple construction.
- ✓ Low cost.
- ✓ Light weight
- ✓ It can transmit over greater distances for a given power level.
- ✓ It can receive weaker signals coming from a particular direction better than an omni-directional antenna.

DISADVANTAGES OF YAGI-UDA ANTENNA

- ✓ It is sensitive to frequency.
- ✓ Bandwidth is reduced if the array is constructed with more number of directors.

APPLICATION OF YAGI UDA ANTENNA

- ✓ Used in television reception.
- ✓ Used as a transmitter in low frequency applications.

SOLVED PROBLEMS

Example 1: Find the radiation resistance of a Hertzian dipole of length $\lambda/60$.

Solution: The radiation resistance of a Hertzian dipole (or) current element (or) infinitesimal dipole is given by

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \Omega$$

$$Given\ dl = \frac{\lambda}{60}$$

$$R_r = 80\pi^2 \left(\frac{\frac{\lambda}{60}}{\lambda}\right)^2 \Omega$$

R_r= 0.219 Ω

Example 2: Find the effective area of a Hertizian dipole operating at 100 MHz.

Solution: $\lambda = \frac{c}{f} = \frac{30\times10^8}{100 MH_z} = \frac{30\times10^8}{100\times10^6}$ = **3m**

Directivity of Hertzian dipole (D) = 1.5

Effective Area $(A_e) = \frac{\lambda^2 D}{4\pi} = \frac{3^2\times1.5}{4\pi}$ = **1.07 m^2**

$A_e = 1.07 m^2$

Example 3: An antenna whose radiation resistance is 300Ω operates at a frequency of 1GHz and with a current of 3 amperes. Find the radiated power.

Solution: Radiated power $P_r = I^2 R_r$

$= 3^2 \times 300$

= 9 × 300

P_r = 2700 watts

Example 4: An antenna is operating at a frequency of 100 MHz. At what distance, radiated field is approximately equal to the induction field.

Given : Operating frequency f = 100MHz

Solution: At distance r $=\frac{\lambda}{2\pi}$, the radiation field is equal to the induction field

Wavelength $\lambda = \frac{c}{f} = \frac{3\times10^8}{100\times10^6}$ = 3 metre

$$\therefore \text{r} = \frac{\lambda}{2\pi} = \frac{3}{2 \times 3.14} = 0.477 \text{ metre}$$

r = 47.7cm

∴ At a distance of 47.7 cm, the induction field is equal to radiation field

Example 5:A dipole antenna has a radiation resistance of 75 ohms and a loss resistance of 20 ohms. Determine its efficiency.

Given : Radiation resistance R_r = 75 ohms

Loss resistance R_L = 20 ohms

Solution: Efficiency $\eta = \frac{R_r}{R_r R_L} \times 100$

= 78.95%

The efficiency of the antenna η = 78.95%

Example 6: A transmitting antenna operating on a wavelength of 1000 metres has an effective height of 100 metres and the antenna current is 100 Amp (rms).Calculate the field strength at a distance of 200 km.

Solution: $\boldsymbol{E_{rms} = \frac{120\pi le I_{rms}}{\lambda r}}$

r = 200 km

le = 100m

I_{rms} = 100 Amp

$$\therefore E_{rms} = \frac{120\pi \times 100 \times 100}{1000 \times 200 \times 10^3} = 18.84 \text{ mV/m}$$

Example 7: Calculate the power radiated by$\lambda/16$ dipole in free space if it carries a uniform current of I = 100 cos ωt amperes. What is its radiation resistance?

Solution: $p_{rad} = \eta_0 \cdot \frac{\pi}{3} \left|\frac{I_m dl}{\lambda}\right|^2$ Watts

Given : dl $= \frac{\lambda}{16}$

I_m = 100

$$\therefore p_{rad} = 120\pi . \frac{\pi}{3} \left|\frac{100}{\lambda} \frac{\lambda}{16}\right|^2$$

$= 40\pi^2 \left(\frac{100\times100}{16\times16}\right) = 1.54056\times 10^4$watts

= **15.4056kW**

$R_r = 80\pi^2 \times \left(\frac{dl}{\lambda}\right)^2$

$= 80\pi^2 \times \left(\frac{\lambda}{16\times\lambda}\right)^2$

$= \frac{80\times(3.14)^2}{256} = \frac{788\times768}{256}$

= **3.0812 ohms**

TWO MARK QUESTIONS

1. Define an antenna .

Antenna is a transition device or a transducer between a guided wave and a free space wave or vice versa. Antenna is also said to be an impedance transforming device.

2 Define Hertzian dipole (oscillating dipole.)

A Hertizan dipole is an elementary sources consisting of a time – harmonic electric current element of a specified direction and infinitesimal length.

3. Define Radiation field.

The radiation field will be produced at a larger distance from the current element, where the distance from the centre of the dipole to the particular point is very large i, e., r >>λ .It is also called as distant field or far field.

4. Define Radiation Resistance.

It is defined as the fictitious resistance which when inserted in series with the antenna will consume the same amount of power as it is actually radiated. The antenna appears to the transmission line as a resistive component and this is known as the radiation resistance.

5. State pointing theorem.

It states that the vector product of electric field intensity vector E and the magnetic field intensity vector H at any point is a measure of the rate energy flow per unit area at the point